On the use of "piezometric head" in groundwater hydrology

Paulo Herrera R. (paulo.herrera.ricci@gmail.com)

Independent Consultant, Civil Engineer, Ph.D.

Introduction

I was recently put in a difficult situation by a journal editor, who requested to make some changes to an article that had been reviewed twice and been recommended for publication. His request was at first sight confusing and unusual:

"Manuscript refers to 'piezometric head', a term that is generally considered outmoded. You should change it to 'potentiometric head' or 'hydraulic head' or simply 'head'." He later added: *"To be clear, this is not optional or a mere suggestion."*.

The main argument to request such a change was that the term "piezometric head" does not appear in the index of a couple of well-known hydrogeology books, hence it is considered out of common usage. Since I learned early as student to distrust books because they can include errors, I did not find this argument convincing. Moreover, as a Hydraulic Engineer and a former instructor of courses on fluid mechanics, hydraulics and groundwater hydrology; I know that the word "piezometric" in the context of groundwater hydrology has a precise meaning. Therefore, I decided to write this short communication to explain the reasons I have to keep using "piezometric head" and why I recommend so.

Theoretical background

The origin of modern groundwater hydrology as technical discipline can be tracked back to the work of the French Engineer H. Darcy (Darcy, 1856), who postulated a mathematical equation to relate volumetric flow in porous media and energy losses. The modern 3D version of the equation named after Darcy, reads

$$\boldsymbol{q} = -\boldsymbol{K} \cdot \nabla h \qquad (1)$$

where $\boldsymbol{q}$ is the volumetric flow per unit area [L/T], $\boldsymbol{K}$ is the hydraulic conductivity [L/T] and h is the energy or hydraulic head of the flow [L]. Darcy derived this equation, which is similar to other well-known resistance laws in physics, based on experimental results obtained in one-dimensional homogeneous sand columns. However, the Darcy equation can surprisingly well explain groundwater flow observed in natural geological media. Hubert (1940) demonstrated that for isotropic porous media Darcy equation can be expressed in terms of a potential that he defines as: *"A physical quantity capable of measurement at every point in the flow system … flow always occurs from regions in which the quantity has higher values to those in which it has lower values regardless of the direction in space"*.

The difference of the variable h in Darcy equation or the potential according to Hubbert represents the energy available for flow to occur.

In fluid mechanics the total mechanical energy per unit mass of a fluid expressed in units of length is referred to as the total or hydraulic head [L], which corresponds to (e.g. Charberneau, 2006, p. 27):

$$h = \frac{p}{\rho g} + z + \frac{v^2}{2g} \qquad (2)$$

where p is the fluid pressure $[M/T^2/L]$, ρ is the fluid density $[M/L^3]$, g is gravity $[L/T^2]$, z is the elevation measured with respect to some datum [L] and v is the fluid velocity [L/T]. It is standard to call the sum of the two first terms in the right-hand side "piezometric head" (e.g. Bear, 1972, p. 64) for reasons that will become evident later. The third term related to fluid velocity is called velocity head.

In groundwater flow, the velocity head term is negligible in comparison to the first two terms of the right hand side of (2). For example, assuming a relatively high groundwater velocity equal to 100 m/day (1.16 x 10^{-3} m/sec), $\frac{v^2}{2g} = 6.87 \text{ x } 10^{-8}$ m. Therefore, for practical purposes the head in the Darcy equation, h, can be assumed equal to the piezometric head (e.g. Charbeneau, 2006), which is the basis for the extension of Darcy equation to density dependent or multiphase flow, that reads (Marle, 2006; Charbeneau, 2006):

$$\boldsymbol{q} = -\frac{k}{\mu}(\nabla p + \rho g \hat{\boldsymbol{k}}) \qquad (3)$$

where k is the rock permeability $[L^2]$, μ is dynamic viscosity of fluid [M/L/T] and $\hat{\boldsymbol{k}}$ is the unit vector in the vertical direction. Therefore, the common extension of Darcy equation to density dependent or multiphase flow implicitly assumes that h is equal to the piezometric head.

Observations

The standard procedure to measure the hydraulic head of a fluid flow at a given point is to record the elevation of the fluid inside a small-diameter tube called piezometer (Figure 1). Since the fluid inside the piezometer is immobile, the measured hydraulic head corresponds to the sum of the first two terms of the right-hand side of (3). Hence, the sum of those two terms is referred to as "piezometric head". Darcy derived his expression based in experimental results with a sand permeameter that considered h equal to the head measured in piezometers.

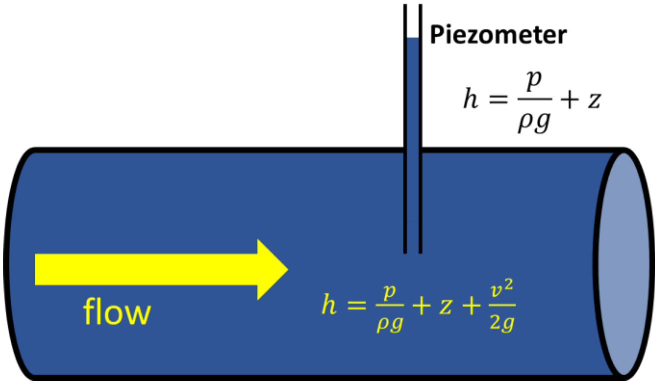

Figure 1: Sketch of a piezometer used to measure hydraulic head.

To apply any version of the Darcy equation to real problems, e.g. estimated groundwater discharge through the section of an alluvial valley, it is necessary to measure the value of *h*. The standard procedure to estimate *h* in groundwater systems consists in measuring the level of the water surface inside wells of different diameter (Figure 2). When wells are of small diameter or are installed for the sole purpose of measuring groundwater levels are also called piezometers. Therefore, levels measured inside wells correspond to piezometric levels, which explains why it has been customary to use "piezometric" in the technical literature related to modern groundwater hydrology since almost its origin (e.g. Theis, 1935). Moreover, even if one of the books I consider the best about hydrogeology (Freeze and Cheery, 1979) does not contain the term "piezometric head" is his index, it does include "piezometer", "piezometer nest", "piezometer test for hydraulic conductivity" and "piezometric surface", recognizing the fact that groundwater levels are measured in piezometers.

It is important to observe that in the case of groundwater piezometers, there is no guarantee of flow velocity being negligible. For example, cross-borehole velocities in piezometers installed in fractured rock can be important (Read et al., 2013). However, measurements in piezometers always relate only to piezometric head.

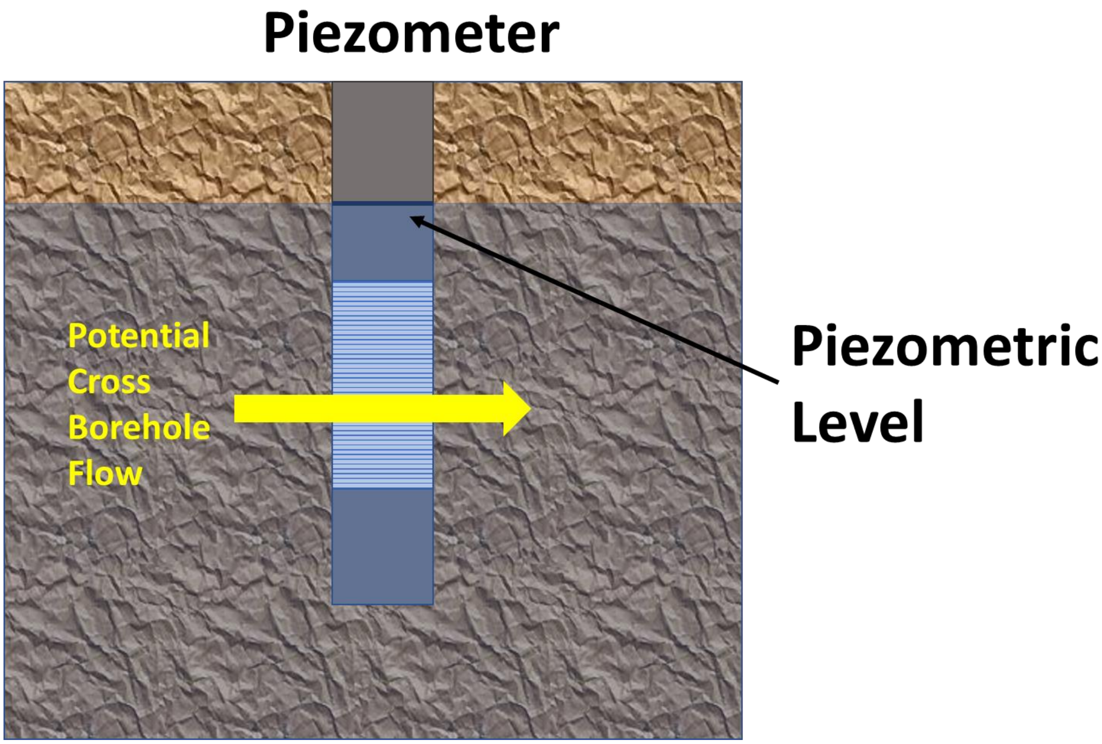

Figure 2: Sketch of a groundwater piezometer used to measure groundwater levels.

Conclusion

The physical quantity that represents the energy that drives groundwater flow in the mathematical equations that represent it corresponds to hydraulic head. In practice, this quantity is measured as the water level inside piezometers. Moreover, in the theoretical development of flow in porous media or groundwater hydrology has been customary to accept that flow velocities are small, so that the energy of the flow is equal to the piezometric head defined as the sum of the pressure height and elevation. Therefore, the quantity that represents the energy of groundwater flow has been traditionally called *"piezometric head"* in groundwater hydrology with good reasons.

The technical and scientific literature is plagued with terms that represent precise concepts. Their use in technical articles not only serve to avoid long explanatory sentences or definitions, but also to express ideas precisely and derive new concepts and formulations. For the case of the use of piezometric head in groundwater hydrology, there are basis in the theoretical treatment of the subject and in the practice that support it now and in the future.

THE END: Considering that the suggestion was not optional, I had to change my preferences and use "hydraulic head", which is more general than the original "piezometric head" I had used.

References

Bear, J. (1972). Dynamics of fluids in porous media. Dover.

Charbeneau, R. (2006). Groundwater hydraulics and pollutant transport. Waveland Press.

Darcy, H. (1856). *Les fontaines publiques de la ville de Dijon exposition et application... par Henry Darcy*.

Freeze, A. and J. Cherry (1979). Groundwater. Prentice Hall.

Hubbert, M. K. (1940). The theory of ground-water motion. *The Journal of Geology*, *48*(8, Part 1), 785-944.

Marle, C. M. (2006). Henry Darcy et les écoulements de fluides en milieu poreux. *Oil & Gas Science and Technology-Revue de l'IFP*, *61*(5), 599-609.

Read, T., Bour, O., Bense, V., Le Borgne, T., Goderniaux, P., Klepikova, M. V., ... & Boschero, V. (2013). Characterizing groundwater flow and heat transport in fractured rock using fiber-optic distributed temperature sensing. *Geophysical Research Letters*, *40*(10), 2055-2059.

Theis, C. V. (1935). The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. *Eos, Transactions American Geophysical Union*, *16*(2), 519-524.