The MIMO data transfer line with complex carrier and time diversity

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I. INTRODUCTION

Until now, the design of communication systems has been based on Shannon's theorem about communication capacity. In this theorem, a real signal with maximum entropy, i.e. "white noise", was considered as a source of information. It followed from this theorem that in order to increase the capacity of a communication channel, it is necessary to use as much of the actual signal plane as possible. In other words, the actual signal should be more similar to the actual noise and have a different amplitude and phase of the carrier frequency, i.e. occupy the entire plane. Accordingly, modulation methods such as M-ary Phase Shift Keying (MPSK), Quadrature Phase Shift Keying (QPSK) and Quadrature Amplitude Modulation (QAM) have appeared [1].

When analyzing these types of modulation, it was convenient to write the signal mathematically in a complex form, since it was displayed on a plane. However, in fact it was real and consisted of two real parts. As is known, in a complex signal the real and imaginary parts are closely related to each other by the Cauchy-Riemann conditions, and the increment of one part of the complex number leads to a corresponding change in the other part. This connection must be taken into account when transmitting and receiving such signals. Therefore, when modulating the signal, a 2×2 scheme must be implemented, as a result of which the two-dimensional (2D) information vector forms a 2D combined vector of the imaginary and real parts of the complex number, which, in its turn, consist of the real and imaginary parts of the information vector. In other words, two information elements must be included in both the real and imaginary parts of the vector obtained by modulation. It is clear that the described complex signal modulation scheme is the Multiple-Input Multiple-Output (MIMO) scheme, i.e. for complex signals it is MIMO 2×2. MPSK and QAM modulation using real signals is a Multiple-Input Single-Output - MISO 2×1 scheme.

Theoretically, it is shown that MIMO increases the capacity of communication channels compared to using signals on the real plane by N times, where N is the number of inputs equal to the number of outputs, i.e. MIMO $N\times N$ [2]. However, the implementation of this scheme still causes great theoretical and technical difficulties. At the present time, only one working Alamouti MIMO 2×2 scheme is known, which provides the implementation of the theoretical gain [3].

In the Alamouti scheme, QPSK signals formed on the real plane are transmitted over the channel [1]. Accordingly, only the real part of the combined vector is used in transmission. The imaginary part is formed due to the properties of the transmission channel, which provides the desired phase shift of the carrier frequency. Mathematically, the communication channel is described by a matrix (2×2) with transmission elements in the form of an exponential mapping of a complex number and rotates a 2D vector on the complex plane and the necessary gain. At the same time, in the Alamouti scheme, in order to form a complex vector, it becomes necessary to transmit the same 2D information signal twice by two antennas with time division. In fact, the operation of carrier modulation with a complex signal with the transmission of the real and imaginary parts of the combined vector is formed in the transmission channel and the associated energy losses due to the retransmission of 2D information vectors are compensated by using two antennas for transmission and reception.

The purpose of the article is to present the modulation method in the 2×2 MIMO scheme as a multiplication of a 2D information vector of two sequentially received information elements by a two-dimensional complex carrier matrix. The complex carrier is a complex number in polar form in a matrix representation with a change in angle over time t with carrier frequency ω_c. In this case, the resulting modulated combined vector consisting of the real and imaginary parts of the complex number is transmitted with time division and coherently demodulated by multiplying on the transposed basis matrices of the complex carrier. Carrier phase determined using known phase locking techniques.

II. MATERIALS AND METHODS FOR SOLVING THE PROBLEM

A complex number consists of a pair of real numbers x, y and is written using the abstract symbol *i* as $(x, y) \mapsto z = x + iy$. The character *i* satisfies the property $i^2 = -1$. The modulus of a complex number is calculated as $|z| = \sqrt{x^2 + y^2}$, and the argument is as $\theta = \mathrm{arctg}(y/x)$.

Physically realizable signals are used in radio engineering, therefore, for the formation and processing of complex signals, it is necessary to represent them in a real vector or matrix extension. Therefore, we write a complex number in the form of a vector and a matrix, as [4]:

$$\mathbf{z} = \begin{bmatrix} x \\ y \end{bmatrix} \text{ и } \mathbf{Z} = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}. \tag{1}$$

The real part of a complex number in matrix representation corresponds to a matrix with units on the main diagonal:

$$\mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{2}$$

The imaginary unit *i* corresponds to a 90° rotation, so in matrix extension it has the form:

$$\mathbf{I} = \begin{bmatrix} \cos\pi/2 & -\sin\pi/2 \\ \sin\pi/2 & \cos\pi/2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{3}$$

The matrix imaginary unit **I** satisfies the properties corresponding to the properties *i*:

$\mathbf{I}^{\mathrm{T}} = -\mathbf{I}$, $\mathbf{I}^2 = -\mathbf{E}$, $\mathbf{I}^3 = -\mathbf{EI} = -\mathbf{I}$, $\mathbf{I}^4 = -\mathbf{II} = \mathbf{E}$, $\mathbf{I}^5 = \mathbf{EI} = \mathbf{I}$ … .

Эти свойства запишем в виде таблицы 1.

Table 1. Result of matrix multiplication (2) and (3).

×	**E**	**I**
E	**E**	**I**
I	**I**	–**E**

The matrices **E** and **I** will be called *the basis matrices of a complex number in algebraic form in the matrix representation*.

Using the basis matrices (2) and (3), we write the complex number $z = x + iy$ in matrix form (1) as

$$\mathbf{Z} = \mathbf{E}x + \mathbf{I}y.$$

Note that the matrix **E** is orthogonal, since $\mathbf{E}^{\mathrm{T}}\mathbf{E} = \mathbf{E}$ and the matrix **I** are orthogonal, since $\mathbf{I}^{\mathrm{T}}\mathbf{I} = \mathbf{I}\mathbf{I}^{\mathrm{T}} = \mathbf{E}$.

All operations with complex numbers in the matrix representation correspond in their results to operations of the same name on complex numbers in algebraic form. The square of the modulus of a complex number in the matrix representation is equal to the determinant of the matrix:

$$|z|^2 = |\mathbf{Z}| = \begin{vmatrix} x & -y \\ y & x \end{vmatrix} = x^2 + y^2.$$

The complex conjugate number $z^* = x - iy$ in matrix form is formed by transposing the original matrix:

$$\mathbf{Z}^* = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \mathbf{E}x - \mathbf{I}y.$$

Consider the product of complex numbers in matrix expansion as a multiplication of a transposed matrix by a non-transposed one:

$$\mathbf{Z}_1^{\mathrm{T}}\mathbf{Z}_2 = \begin{bmatrix} x_1 & y_1 \\ -y_1 & x_1 \end{bmatrix}\begin{bmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{bmatrix} = \begin{bmatrix} x_1x_2 + y_1y_2 & -(x_1y_2 - x_2y_1) \\ x_1y_2 - x_2y_1 & x_1x_2 + y_1y_2 \end{bmatrix}. \tag{4}$$

The product of complex numbers can be written as well as the product of a vector on a matrix:

$$\mathbf{Z}_1^{\mathrm{T}}\mathbf{z}_2 = \begin{bmatrix} x_1 & y_1 \\ -y_1 & x_1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1x_2 + y_1y_2 \\ x_1y_2 - x_2y_1 \end{bmatrix}. \qquad (5)$$

The result corresponds to the first column of the matrix (4) obtained by matrix multiplication.

Writing a complex number in polar form corresponds to an exponential function of a complex number. This function converts the complex number z to a complex number $w = \mathrm{e}^z = \mathrm{e}^{x+iy} = \mathrm{e}^x\mathrm{e}^{iy} = \rho(\cos y + i\sin y) = u + iv$, where $\rho = \mathrm{e}^x$, $u = \cos y$ - real part, $v = \sin y$ - imaginary part of the exponential function of *iy*.

We write the exponential function in matrix form using the basis matrices (2), (3) in the form:

$$w = \rho\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} = \rho\left(\cos\varphi\mathbf{E} + \sin\varphi\mathbf{I}\right). \qquad (6)$$

Since, according to the physical meaning, the imaginary part y of the complex number z in algebraic form is the angle of rotation of the corresponding vector on the complex plane, we will replace its designation with the designation of the angle φ.

In the vector representation (1) of the complex number z, the scalar product is calculated as [4]

$$\mathbf{z}_1\bullet\mathbf{z}_2 = |\mathbf{z}_1||\mathbf{z}_2|\cos\theta = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\bullet\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = x_1x_2 + y_1y_2, \qquad (7)$$

where θ is the angle between the vectors.

The maximum of the scalar product is achieved when the vectors are collinear, i.e. at $\theta = 0°$.

The cross product in the vector representation of a complex number is calculated as

$$\mathbf{z}_1 \times \mathbf{z}_2 = |\mathbf{z}_1||\mathbf{z}_2|\sin\theta = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}\times\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = x_1y_2 - x_2y_1. \qquad (8)$$

The maximum of the vector product is reached at $\theta = 90°$.

Because $z_1^* z_2 = |z_1| e^{-i\alpha} |z_2| e^{i\beta} = |z_1||z_2| e^{i(\beta-\alpha)} = |z_1||z_2| e^{i\theta} = |z_1||z_2|(\cos\theta + i\sin\theta)$, where $\theta = \beta - \alpha$, then as a result of multiplying the conjugate complex number by the non-conjugate one, we obtain the sum of scalar and vector multiplication [4].

As can be seen from the results obtained, the scalar product (7) and vector product (8) are the result of the product of matrices (4) and a vector by matrix (5).

Thus, in the algebraic representation of a complex number, multiplying a conjugate number by a non-conjugate number or in a matrix representation, multiplying a matrix by a transposed one, we get the sum of the scalar and vector product. Real numbers give us only the dot product. This is explained by the fact that complex numbers, unlike real ones, combine the different nature of energy, namely, potential and kinetic. With an exponential display, the real number characterizes the degree of stretching of the complex vector, and the imaginary number characterizes the rotation speed at which potential energy transforms into kinetic energy and vice versa. In other words, the real part of the complex number goes into the imaginary and vice versa, according to the Cauchy-Riemann conditions.

At present, the representation of information transmission lines by a state-space model is becoming more widespread, which has significant advantages over the widely used input-output model [5].

According to this model, the equation of dynamics in continuous time without input action is written as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t), \qquad (9)$$

where the *state transition matrix* (STM) **A** differentiates the state vector $\mathbf{x}(t)$.

The matrix exponent $\mathrm{e}^{\mathbf{A}t}$ is the solution to the differential equation

$$\frac{\partial}{\partial t}\mathrm{e}^{\mathbf{I}\omega t} = \mathbf{I}\omega\mathrm{e}^{\mathbf{I}\omega t} = \mathbf{A}\mathrm{e}^{\mathbf{A}t}.$$

From where, STM $\mathbf{A} = \mathbf{I}\omega$, where ω is the circular frequency, rad/s.
The exponential mapping of the STM **A** when it changes in time is found as the inverse Laplace transform [5]:

$$\mathrm{e}^{\mathbf{A}t} = \mathrm{L}^{-1}\left\{(s\mathbf{E}-\mathbf{A})^{-1}\right\}.$$

As a result of the calculation, we obtain *the fundamental matrix*:

$$\mathbf{\Phi}(\omega,t) = \mathrm{L}^{-1}\left\{\begin{bmatrix} \frac{s}{s^2+\omega^2} & -\frac{\omega}{s^2+\omega^2} \\ \frac{\omega}{s^2+\omega^2} & \frac{s}{s^2+\omega^2} \end{bmatrix}\right\} = \begin{bmatrix} \cos\omega t & -\sin\omega t \\ \sin\omega t & \cos\omega t \end{bmatrix}. \qquad (10)$$

Using the basis matrices (2) and (3), we write the fundamental matrix (10) in the form:

$$\mathbf{\Phi}(\omega_c,t) = e^{\mathbf{I}\omega_c t} = \cos(\omega_c t)\mathbf{E} + \sin(\omega_c t)\mathbf{I}. \qquad (11)$$

where ω_c is the circular frequency of the carrier.

Matrix (11) is called *the complex carrier* in the matrix representation.

Fundamental matrix (11) is orthogonal, because

$$\mathbf{\Phi}^{\mathrm{T}}(\omega_c,t)\mathbf{\Phi}(\omega_c,t) = \mathbf{\Phi}(\omega_c,t)\mathbf{\Phi}^{\mathrm{T}}(\omega_c,t) = e^{-\mathbf{I}\omega_c t}e^{\mathbf{I}\omega_c t} =$$
$$= \left(\cos(\omega_c t)\mathbf{E} - \sin(\omega_c t)\mathbf{I}\right)\left(\cos(\omega_c t)\mathbf{E} + \sin(\omega_c t)\mathbf{I}\right) = \mathbf{E}.$$

Thus, from the above information about complex numbers, we can conclude that in order to obtain a greater energy gain when transmitting information, it is necessary to use complex numbers, consisting of both real and imaginary parts and combining potential and kinetic energy. At the same time, in order to get rid of imaginary units, it is necessary to represent complex numbers in a matrix expansion. This representation leads us to a state-space MIMO 2×2 scheme described in terms of dynamics and output equations.

III. MIMO link model with complex carrier

To form a MIMO channel, we use a complex carrier (11). The sequence of information pulses ± 1, arriving at the input of the transmitter, is divided into pairs of pulses and written as a sequence of information vectors, as the vectors of the initial states $\mathbf{x}(0) = \begin{bmatrix} x_0(0) & x_1(0) \end{bmatrix}^{\mathrm{T}}$ of the dynamic system (9), the solution of which is the complex carrier (11).

For binary transmission, we will have only 4 combinations of initial vectors:

$$\mathbf{x}(0) = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}. \qquad (12)$$

We consider the information vector as a complex number in vector representation (1), so the square of the modulus of the complex number or the square of the norm of each vector will be equal to $|\mathbf{x}(0)|^2 = \|\mathbf{x}(0)\|^2 = \mathbf{x}^{\mathrm{T}}(0)\mathbf{x}(0) = x_0^2(0) + x_1^2(0) = 2$.

Let us perform phase manipulation of the complex carrier with the initial state vector $\mathbf{x}(0)$, for which we multiply (11) by the incoming initial state vector and obtain *the modulated combined output vector*:

$$\mathbf{y}(\omega_c,t) = \mathbf{\Phi}(\omega_c,t)\mathbf{x}(0) = \begin{bmatrix} y_0(\omega_c,t) \\ y_1(\omega_c,t) \end{bmatrix} = \begin{bmatrix} x_0(0)\cos(\omega_c t) - x_1(0)\sin(\omega_c t) \\ x_0(0)\sin(\omega_c t) + x_1(0)\cos(\omega_c t) \end{bmatrix}. \qquad (13)$$

In accordance with the vector notation (1) of a complex number, element $y_0(\omega_c,t)$ represents the real part of the combined vector $\mathbf{y}(\omega_c,t)$, and element $y_1(\omega_c,t)$ represents the imaginary part. Since the fundamental matrix (11) performs an exponential mapping of a complex number, in the process of changing time, the combined output vector rotates counterclockwise with the center at the origin. Due to the orthogonality of $\mathbf{\Phi}(\omega_c t)$, when multiplying the initial vector $\mathbf{x}(0)$ by it, we obtain the output vector (13) with the same norm $\|\mathbf{y}(\omega_c,t)\|^2 = \mathbf{y}^{\mathrm{T}}(\omega_c,t)\mathbf{y}(\omega_c,t) = x_0^2(0)+x_1^2(0)$.

As can be seen from (13), each element of the output vector $\mathbf{y}(\omega_c,t)$ contains information about two elements of the input vector $\mathbf{x}(0)$, i.e. about the transmitted information elements. In other words, we have a MIMO 2×2 scheme. Each element of vector $\mathbf{y}(\omega_c,t)$ is the difference and the sum of cosines and sines with amplitudes corresponding to the values of the information vector. Therefore, they can be drawn on the complex plane and the initial phase can be calculated as the argument of the complex number at t=0. The calculated initial phases of the modulated elements of the combined vectors (13) for the values of the information vectors (12) will be written as a matrix of initial phases:

$$\mathbf{\Theta} = \begin{pmatrix} -45^\circ & -135^\circ & 45^\circ & 135^\circ \\ 45^\circ & -45^\circ & 135^\circ & -135^\circ \end{pmatrix}. \tag{14}$$

As can be seen from (14), the phase difference between the real part of the combined vector and the imaginary part is 90°, which ensures the maximum value of the vector product (8).

Modulated oscillations with initial phases (14) correspond to QPSK. The difference lies in the transmission method. With QPSK, only the real part of the combined vector is transmitted (13) [1]. In our case, we will sequentially transmit first the real part $y_0(\omega_c,t)$, and then the imaginary $y_1(\omega_c,t)$.

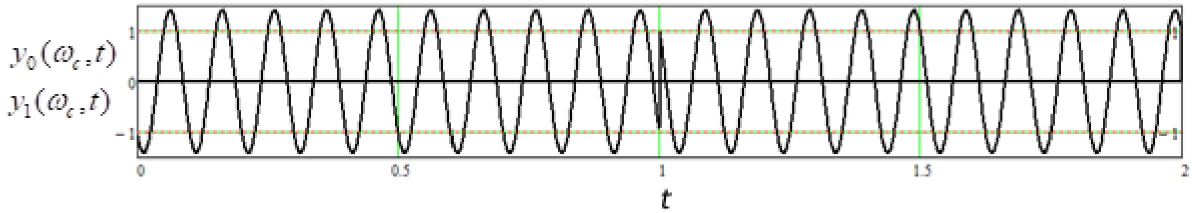


Figure 1. The elements of the modulated combined vector for $\mathbf{x}(0)=\begin{bmatrix}-1 & 1\end{bmatrix}^{\mathrm{T}}$.

Figure 1 shows the elements of the modulated combined vector arranged sequentially in time. The amplitude A of each oscillation (13) for information pulses (12) with their quadrature addition will be equal to $\sqrt{2}$. Therefore, the power of each emitted element (13) will be $P = A^2/2 = 1$. By direct integration over the pulse duration T, we obtain the same result:

$$P = P_0 = P_1 = \frac{1}{T}\int_0^T \left(x_0(0)\cos(\omega_c t) \mp x_1(0)\sin(\omega_c t)\right)^2 dt = \frac{1}{T}\int_0^T \left(\cos^2(\omega_c t)+\sin^2(\omega_c t)\right)dt = 1.$$

The energy of the elements is calculated as $E = E_0 = E_1 = PT = A^2T/2 = T$. Knowing the energy or power of the signal, one can calculate the amplitude of oscillations as $A=\sqrt{2E/T}=\sqrt{2P}$. With E=1 we get $A=\sqrt{2/T}$.

Since we are transmitting two information symbols, when two elements of the output modulated vector (13) are sequentially transmitted, the information transfer rate remains unchanged. However, once again we note that each element of the output modulated vector contains information about two information symbols $x_0(0)$ and $x_1(0)$.

Further, the generated signal is propagated through the communication channel. We will assume that when propagating through the communication channel, interference in the form of white noise is added to the signal. Since the elements of the combined vector are transmitted sequentially in time, the noise for each element will be uncorrelated. We write the model of the received signal in the form:

$$\mathbf{s}(t)=\mathbf{y}(t)+\mathbf{n}(t)=\mathbf{\Phi}(\omega_c t)\mathbf{x}(0)+\mathbf{n}(t), \tag{15}$$

where $\mathbf{s}(t)=\begin{bmatrix}s_0(t) & s_1(t)\end{bmatrix}^{\mathrm{T}}$ - 2D received signal vector, $\mathbf{n}(t)=\begin{bmatrix}n_0(t) & n_1(t)\end{bmatrix}^{\mathrm{T}}$ - 2D complex white noise vector with circular symmetry, i.e. uncorrelated, with variance σ^2 and power spectral density N_0.

Since the noise is complex, i.e. two-dimensional, then the square of the norm of the vector $\|\mathbf{n}(t)\|^2=\sigma^2=\sigma_0^2+\sigma_1^2=\mathrm{N}_{00}/2+\mathrm{N}_{01}/2$, where σ_0^2, σ_1^2 and N_{00}, N_{01} are the dispersions and spectral densities of the elements of the noise vector.

In accordance with the maximum likelihood criterion, the optimal receiver includes a correlator or a matched filter that calculates the scalar product of the received vector (15) and selects the most likely vector [6].

Since the QPSK signal is transmitted as the real part of the combined vector (13), the receiver has only two basic normalized functions: $\phi_1(t)=\sqrt{2/T}\cos\omega_c t$ and $\phi_2(t)=-\sqrt{2/T}\sin\omega_c t$, which form the real plane. The simplest receiver represents two correlators [1]. The signal phase is calculated from the values of the received signal, and then the closest quadrant on the real plane is selected from the phase value.

In our case, we transmit sequentially both the real and imaginary parts of the modulated vector (13). Then, the optimal receiver is a correlator that multiplies the received vector (15) by the basis matrices of the transposed fundamental matrix (11) and compares the result with all possible combinations of information vectors.

As bases, we consider two basis matrices transposed and normalized by response after integration over duration T:

$$\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)=\sqrt{\frac{2}{T}}\cos(\omega_c t)\mathbf{E}=\sqrt{\frac{2}{T}}\begin{bmatrix}\cos(\omega_c t) & 0\\ 0 & \cos(\omega_c t)\end{bmatrix}, \tag{16}$$

$$\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)=-\sqrt{\frac{2}{T}}\sin(\omega_c t)\mathbf{I}=\sqrt{\frac{2}{T}}\begin{bmatrix}0 & \sin(\omega_c t)\\ -\sin(\omega_c t) & 0\end{bmatrix}. \tag{17}$$

Note that in the case of real signals, we have two basis functions that correspond to the coordinate axes, and in the case of complex signals, we operate with two 2D matrices that correspond to two-dimensional planes. The matrices are orthogonal when integrated over duration *T*, since

$$\frac{2}{T}\int_0^T\cos^2(\omega_c t)dt=\frac{2}{T}\frac{T}{2}=1,\qquad \frac{2}{T}\int_0^T\sin^2(\omega_c t)dt=\frac{2}{T}\frac{T}{2}=1.$$

Recall that orthogonal matrices do not change the norm of vectors when they are multiplied by these matrices.

IV. Diversity

From the elements of the modulated signal received in series, we form a vector (15). We first multiply the vector (15) with amplitude $A=\sqrt{2/T}$ by the basis matrix (16) and integrate over the pulse duration *T*. With equal energies of the elements of the signal vector, we obtain the signal component:

$$\mathbf{S}_{\mathrm{E}_{01}}=\frac{2}{T}\begin{bmatrix}x_0(0)\int_0^T\cos^2(\omega_c t)dt\\ x_1(0)\int_0^T\cos^2(\omega_c t)dt\end{bmatrix}=\begin{bmatrix}x_0(0)\\ x_1(0)\end{bmatrix}. \tag{18}$$

Interference component covariance matrix for the same interference power spectral densities:

$$\mathbf{N}_{\mathrm{E}_{01}} = \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix} = \begin{bmatrix} \mathrm{N}_0/2 & 0 \\ 0 & \mathrm{N}_0/2 \end{bmatrix}. \quad (19)$$

As can be seen from the covariance matrix, the interference is not correlated, as it acts on different elements of the combined modulated vector transmitted sequentially. Index E_{01} in the notation of the signal and noise components shows that the real part $\overline{\mathbf{E}}^{\mathrm{T}}(\omega_c, t)$ of the basis matrix $\mathbf{\Phi}(\omega_c, t)$ was used and the cosine component was extracted from the combined vector with complex noise and the sequence of responses corresponds to indices 0 and 1 in the vectors. Note that, due to the orthogonality of the basis matrices (16) and (17), the dispersion of interference elements is equal to the dispersion of interference in the communication channel (15).

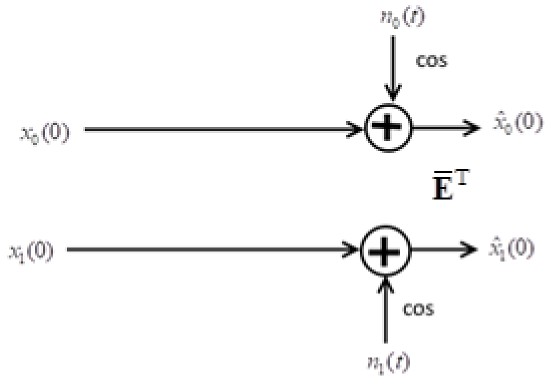


Figure 2. Scheme of the impact of interference during reception using a real matrix (16). The interference is cosine.

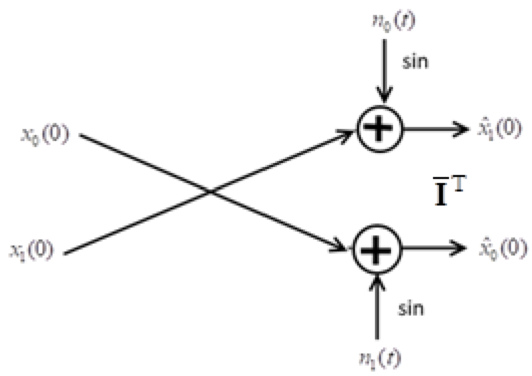


Figure 3. Scheme of the impact of interference during reception using an imaginary matrix (17). The interference is sinus.

Figure 2 schematically shows the order of elements in the signal and interference vector when received using a real basis matrix. It is also symbolically shown that the cosine component of the interference is extracted using the basis matrix $\overline{\mathbf{E}}^{\mathrm{T}}(\omega_c, t)$ and the sequence of responses corresponds to indices 0, 1 in the signal vector and 0, 1 in the noise vector. The diagram also shows that the estimation of information element $\hat{x}(0)$ is fed to the decision device. The estimate of the signal in the form of a sample after integration over time *T* write as:

$$\hat{\mathbf{x}}_{\cos 01}(0) = \left[\hat{x}_{\cos 0}(0) \quad \hat{x}_{\cos 1}(0)\right]^{\mathrm{T}} = \left[\overline{x}_0(0) + \tilde{x}_{\cos 0}(0) \quad \overline{x}_1(0) + \tilde{x}_{\cos 1}(0)\right]^{\mathrm{T}}, \quad (20)$$

where $\overline{x}_0(0)$ and $\overline{x}_1(0)$ - expectation value (18), $\tilde{x}_{\cos 0}(0)$ and $\tilde{x}_{\cos 1}(0)$ - projections onto the real coordinate axis of additive noise with dispersion (19). $\hat{\mathbf{x}}_{\cos}(t)$

Let us now multiply the received signal (15) with amplitude $A = \sqrt{2/T}$ by matrix (17) and integrate over the pulse duration T. As a result, we get the signal component:

$$\mathbf{S}_{\mathrm{I}_{01}} = \frac{2}{T} \begin{bmatrix} x_0(0)\int_0^T \sin^2(\omega_c t)dt \\ x_1(0)\int_0^T \sin^2(\omega_c t)dt \end{bmatrix} = \begin{bmatrix} x_0(0) \\ x_1(0) \end{bmatrix}. \quad (21)$$

Let us calculate the covariance matrix of the interference component:

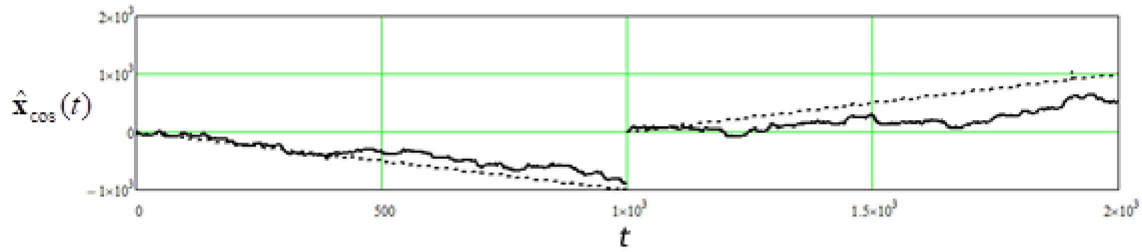
$$\mathbf{N}_{\mathrm{I}_{10}} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_0^2 \end{bmatrix} = \begin{bmatrix} \mathrm{N}_0/2 & 0 \\ 0 & \mathrm{N}_0/2 \end{bmatrix}. \quad (22)$$

By analogy with (20), we write the estimate of the signal in the form of a sample after integration over time T in the form:

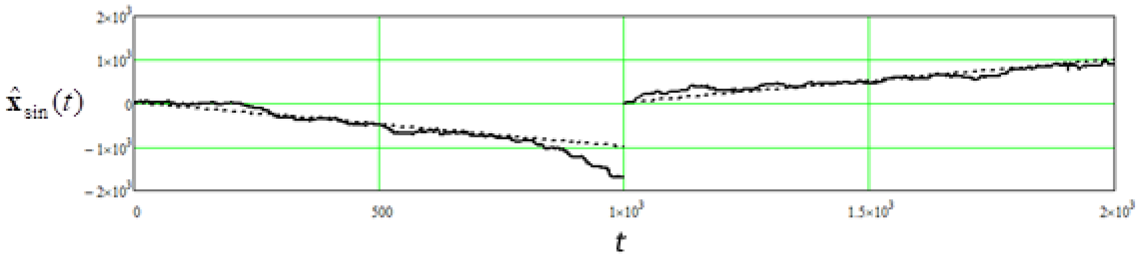
$$\hat{\mathbf{x}}_{\sin 10}(0) = \left[\hat{x}_{\sin 1}(0) \quad \hat{x}_{\sin 0}(0)\right]^{\mathrm{T}} = \left[\overline{x}_1(0) + \tilde{x}_{\sin 0}(0) \quad \overline{x}_0(0) + \tilde{x}_{\sin 1}(0)\right]^{\mathrm{T}}, \quad (23)$$

where $\bar{x}_0(0)$ and $\bar{x}_1(0)$ - expectation value (21), $\tilde{x}_{\sin 0}(0)$ and $\tilde{x}_{\sin 1}(0)$ - projections onto the imaginary coordinate axis of additive noise with dispersion (22).

Figure 3 schematically shows the order of elements in the signal and interference vector when received using an imaginary basis matrix $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)$. The index sin10 in the notation of the signal and noise components shows that the sine component is selected from the combined vector and the sequence of responses corresponds to indices 1, 0 in the signal vector and 0, 1 in the noise vector. As you can see, the order of the elements in the noise vector differs from the order in the signal vector. Figure 3 schematically shows how the order of interference effects on signal elements changes. Combining the circuits shown in Figures 1 and 2, we get a MIMO 2×2 scheme.



a) The result of signal energy accumulation after multiplication by $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)$



b) The result of signal energy accumulation after multiplication by $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)$.

Figure 4. Energy accumulation results for a signal with additive noise - solid lines and a signal without noise - dotted lines.

Figure 4 shows the results of signal reception simulation using the basis matrices (16), (17). As can be seen from the figures, all 4 interferences differ from each other.

Thus, we have 2 different uncorrelated noises that act on elements of the combined modulated signal vector that are different in time. Each of these noises is projected onto the real and imaginary axes of the coordinate system. The responses of these noises after integration are also uncorrelated due to the orthogonality of the coordinate axes. As a result, we obtained two estimates (20) and (23) of the vector of information elements under the influence of various uncorrelated noises.

Consider diversity of the obtained results for different values of information symbols. As is known, when estimating by the maximum likelihood, it is assumed that the parameter is deterministic, but unknown [6]. In our case, the value of information symbols in a 2D information vector acts as a parameter.

We will estimate according to the mathematical expectation (20) and (23). When impacted the "white noise" interference, the mathematical expectation is equal to the values of the information vectors (12). Since the information vector is multiplied by the fundamental matrix (11) during modulation, each information vector forms a complex number in the matrix expansion. Let's represent the relationship of the information vector and the corresponding complex number in the form:

$$\begin{bmatrix}1\\1\end{bmatrix}\Rightarrow\begin{bmatrix}1&-1\\1&1\end{bmatrix},\ \begin{bmatrix}-1\\1\end{bmatrix}\Rightarrow\begin{bmatrix}-1&-1\\1&-1\end{bmatrix},\ \begin{bmatrix}1\\-1\end{bmatrix}\Rightarrow\begin{bmatrix}1&1\\-1&1\end{bmatrix},\ \begin{bmatrix}-1\\-1\end{bmatrix}\Rightarrow\begin{bmatrix}-1&1\\-1&-1\end{bmatrix}. \qquad (24)$$

The diversity between information vectors is calculated as the scalar product (7). The larger the scalar product, the closer the vectors are to each other. In our case, we have a matrix representation of complex numbers on complex planes, therefore, to calculate the diversity of planes, we will use the Frobenius norm for matrices [7]. In other words, we will not calculate the distance between 2D vectors, but the distance between two 2D planes.

When calculating the Frobenius norm for complex numbers in the form (4), we obtain the sum of the scalar and vector product. Квадрат нормы Фробениуса для комплексных чисел в матричном представлении (24) вычисляется, как:

$$\|\mathbf{Z}(0)\|_F^2 = \mathrm{tr}\left\{(\mathbf{E}+\mathbf{I})^{\mathrm{T}}(\mathbf{E}+\mathbf{I})\mathbf{x}(0)\right\} = \mathrm{tr}\{2\mathbf{E}\mathbf{x}(0)\}.$$

At $\mathbf{x}(0) = \pm 1$, $\|\mathbf{Z}(0)\|_F = \mathrm{tr}\{2\mathbf{E}\mathbf{x}(0)\} = 4$. It is known that Binary Phase Shift Keying (BPSK) gives the maximum diversity between real signals equal to 2, since the signals are opposite. In our case, the signals (12) are not opposite and the closest signals differ by ± 1 and their scalar product is 0. However, when calculating the Frobenius norm, we get a diversity equal to 4, which is 2 times greater than the diversity for BPSK.

V. CONCLUSION

Thus, it is shown that the use of complex numbers in the matrix representation makes it possible to increase the diversity of information symbols upon reception by a factor of 2. In this case, unlike the Alamouti scheme, one antenna is used for transmission and one for reception. Moreover, instead of a complex system for estimating complex 2×2 channel gains, it is sufficient to use a conventional phase locking system. The proposed scheme can also be used in wired communication channels.

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