Calculation of structure-borne interior noise in a simplified model of aircraft cabin

Victor V. Krylov

Department of Aeronautical and Automotive Engineering,
Loughborough University,
Loughborough, Leicestershire LE11 3TU, UK

Abstract

The paper describes the application of the earlier proposed simplified analytical approach to the modelling of structural vibration and structural-acoustic coupling in thin-walled elastic structures for calculations of structure-borne interior noise in an aircraft cabin. The structural simplification is based on understanding the processes of generation of influential modes of structural vibration at particular frequencies and of sound radiation by vibrating surfaces into the interior. Analytical results are illustrated by calculations of resonance frequencies of acoustic and structural modes, of their coupling coefficients and of the resulting structure-borne interior noise.

1. Introduction

In the low and medium frequency range, the main sources of interior noise in aircraft and cars are structure-borne. This means that noise is generated mainly by vibrations of structural components excited either by internal sources (engine and other unbalanced rotating components) or by external sources (suspension dynamic forces in cars and aircraft due to road surface irregularities, pressure fluctuations in the surrounding air boundary layer, etc.). A number of different modelling techniques based on finite element calculations or on combined numerical and experimental approaches have been developed, each having its own advantages and disadvantages (see e.g. [1-7]).

The present work describes the application of the earlier proposed analytical approach to the prediction of vehicle and aircraft structure-borne interior noise based on a maximum possible simplification of the model structure and of the acoustic interior [8, 9]. The degree of structural simplification is based on understanding the physics of the problem of both generation of structural vibrations by different sources at particular frequencies and radiation of sound by the excited structural vibrations into the aircraft or vehicle interior. The role of structural-acoustic coupling between some structural and acoustic modes in the formation of frequency spectra of the resulting structure-borne noise is discussed in detail. The results are expressed in terms of relatively simple analytical formulae that give the value of the internal sound pressure as a function of applied forces, resonance frequencies and modal shapes of structural and acoustic modes and of their coupling to each other. Analytical results are illustrated by example calculations of resonance frequencies of acoustic and structural modes, of their coupling coefficients and of the resulting structure-borne interior noise.

2. Outline of the theory

Using a scalar acoustic potential φ related to air particle velocity $\boldsymbol{v}$ and acoustic pressure p' as $\boldsymbol{v} = grad\varphi$ and $p' = -\rho_0\, \partial\varphi/\partial t$ respectively, one can express a time-harmonic acoustic field $\varphi(\boldsymbol{r})$ (the factor $exp(-i\omega t)$ is assumed) inside any closed volume V surrounded by the surface S using the Helmholtz theorem (see, e.g. [10, 11]):

$$\int_S \left(G\frac{\partial\varphi}{\partial\mathbf{n}} - \varphi\frac{\partial G}{\partial\mathbf{n}} \right) dS + \int_V GfdV = \begin{cases} \varphi(r), & \vec{r} \in V \\ 0, & \vec{r} \notin V \end{cases} . \tag{1}$$

Here $G(\boldsymbol{r},\boldsymbol{r}')$ is the acoustic Green's function satisfying certain radiation or boundary conditions, $\boldsymbol{n}$ is a unit vector of inward normal to the surface, and $f(\boldsymbol{r}')$ is a distribution of internal acoustic sources inside a closed volume (Fig. 1).

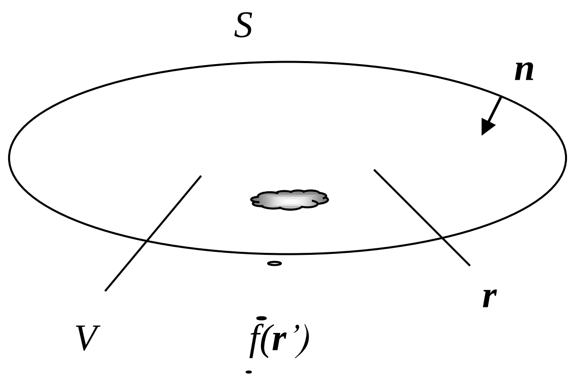

Fig. 1. On the description of the acoustic field in an enclosure.

If to choose the Green's function in such a way that it satisfies Neumann's boundary condition on the surface S, i.e. $\partial G/\partial\boldsymbol{n} = 0$, and to assume that there are no internal acoustic sources within V, i.e. $f(\boldsymbol{r}') = 0$, then it follows from (1) that

$$\varphi(r) = \int_S G(\mathbf{r},\mathbf{r}')v_n(\mathbf{r}')d\mathbf{r}' . \tag{2}$$

Since the acoustic Green's function for the enclosed volume V can be expressed as the sum of the acoustic modes of the volume that are characterised by their resonance frequencies ω_m, attenuation δ_m (so that $k_m = (\omega_m + i\delta_m)/c$) and modal shapes $\Phi_m(\boldsymbol{r})$, the expression (2) can be rewritten in the form:

$$\varphi(\mathbf{r}) = \sum_{m=0}^{\infty} a_m \frac{c^2}{V} \int_S \frac{\Phi_m(\mathbf{r})\Phi_m(\mathbf{r}')}{(\omega_m^{\ 2} - \omega^2 - 2i\delta_m\omega)} v_n(\mathbf{r}')d\mathbf{r}' , \tag{3}$$

where a_m are constants depending on shape of the enclosure and mode type, and the summation over m means summation over the total number of acoustic modes (in practice this means a triple summation if a full 3D case is considered).

To find acoustic resonance frequencies and modal shapes for any particular form of a closed volume and different boundary conditions is a rather difficult and cumbersome task that can be solved analytically only for a limited number of geometrical configurations. We consider here the most simple case, when real enclosures can be approximated by rectangular domains characterised by the length L_x, width L_y, and height L_z, so that the volume $V = L_xL_yL_z$. In this case, the well-known expressions for the acoustic resonance frequencies $\omega_m/2\pi = f_m = f_{ijk}$ and for the acoustic modal shapes $\Phi_m(\mathbf{r}) = \Phi_{ijk}(\mathbf{r})$ can be used (see, e.g. [8-11]).

To calculate analytically the response of an aircraft cabin to external or internal forces one should specify these forces and consider a simplified model structure of certain degree of complexity. The latter means that one has to replace the actual cabin structure by its simple model, so that analytical description would be at all possible.

The problem of introducing suitable model structures requires understanding of main mechanisms responsible for structure-borne interior noise generation under various conditions. There can be different approaches to developing such models, e.g. using combinations of plates, beams, shells, added masses, etc. We will discuss here only the simplest possible models of vehicle or aircraft structures. Namely, considering aircraft cabins, we will assume that they can be modelled by a single thin rectangular plate that is curved along its width and is simply supported along all its edges, including those along its caps (Fig. 2). We will also assume that the smallest radius of the plate curvature is large enough in comparison with the plate flexural wavelengths of interest, so that the shell-type behaviour due to the curvature shall not be accounted for. To simplify the picture even more, the two caps are considered as absolutely rigid vertical plates.

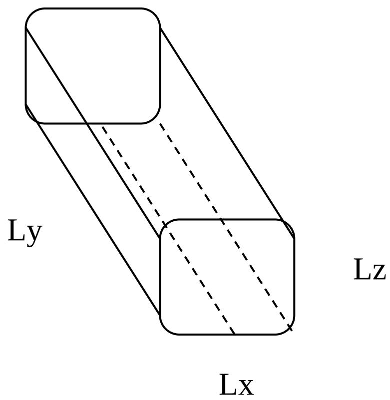

Fig. 2. Simplified structural model of an aircraft cabin

Regarding the effects of external dynamic forces $\boldsymbol{F(t)}$ on aircraft structures, we will consider here in detail only forces that are applied to a cabin from a rough runway during periods of landing and taking off. For example, the effect of a rough runway on a moving aircraft can be described by vertical dynamic forces acting on its bottom and modelling the impacts of wheel suspensions reacting on runway irregularities.

The amplitudes of the wheel suspension forces can be calculated using an aircraft mechanical model taking into account only axle hope resonances, i.e. considering the main body of an aircraft as immobile in a vertical direction. Let us also assume that the runway surface irregularity (roughness) is two-dimensional and characterised by the function $z_1 = g(x)$. For simplicity, we consider this function as a periodic corrugation of height h and space periodicity d: $g(x) = h\cos(2\pi x/d)$. Then, if an aircraft moves along a runway at speed v, the tyre contact displacement z_1 can be described as periodic function of time t: $z_1(t)= h\cos(\omega t)$, where $\omega = 2\pi v/d$. Solving the corresponding mechanical problem in the Fourier domain, one can obtain the expressions for dynamic suspension forces (e.g for the suspension force associated with a front wheel) [8, 12]:

$$F(\omega) = \frac{K_2 \omega_1^{\,2}}{\omega_0^{\,2} - \omega^2 - 2i\alpha\omega} z_1(\omega) = K_2 T(\omega) z_1(\omega)\,, \tag{4}$$

where ω_0 is the wheel hope resonance frequency, ω_1 is the tyre 'jumping' resonance frequency, α is a normalised damping coefficient, and $z_1(\omega)$ is the Fourier component at frequency $\omega = 2\pi v/d$ corresponding to the runway corrugation profile. In the considered case of periodic corrugation $z_1(\omega) = h$.

Let us now consider the response of an aircraft cabin model structure to the applied dynamic forces. Referring, for example, to the above-mentioned simple model of an aircraft structure made up of a single curved plate of width L_y and total length L_l which is simply supported along all its edges, we can use the plate Green's function for flexural displacements $G_s(\boldsymbol{\rho},\boldsymbol{\rho}')$ in which we will neglect the effect of air loading and plate curvature [10]:

$$G_s(\boldsymbol{\rho},\boldsymbol{\rho}') = \frac{4}{M}\sum_{p=1}^{\infty}\frac{\Psi_p(\boldsymbol{\rho})\Psi_p(\boldsymbol{\rho}')}{(\omega_p^{\,2} - \omega^2 - 2i\delta_p\omega)}\;. \tag{5}$$

Here $M = \rho_{0s}h_sL_lL_y$ is the total mass of the plate, ρ_{0s} and h_s are its mass density and thickness respectively, $\omega_p/2\pi = f_p$ and $\Psi(\rho)$ are the corresponding plate resonance frequencies and modal shapes. Note that the assumption of a negligible effect of air loading on plate vibrations usually works well, except for the cases when structural resonance frequencies are close to acoustic ones [11]. To avoid substantial complications associated with taking air loading into account, we follow the above assumption everywhere, keeping in mind that for close acoustic and structural resonance frequencies the obtained results should be considered as rough estimations only. Because of the uncertainties in the modelling and in the definition of vehicle parameters such imprecision in the case of close acoustic and structural resonance frequencies does not seem seriously impairing.

Considering for simplicity only one concentrated force $F(\omega)$ applied to the point of the structural surface characterised by the radius vector $\boldsymbol{\rho}_0$, one can express the distribution of normal particle velocities of the structure as

$$v_n(\boldsymbol{\rho}) = -i\omega F(\omega) G_s(\boldsymbol{\rho},\boldsymbol{\rho}_0)\,. \tag{6}$$

Substituting the expressions (6) and (5) into (3) and using the relationship $p' = -\rho_0\, \partial\varphi/\partial t. = i\omega\rho_0\varphi$, one can obtain the final formula for the acoustic pressure generated within the enclosing structure under consideration [8, 9]:

$$p'(\mathbf{r}) = \frac{4c^2\rho_0 F(\omega)}{\omega^2\rho_{0s}h_sV}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty} a_m F_{mp}(\omega) S_{mp} \Phi_m(\mathbf{r}) \Psi_p(\boldsymbol{\rho}_0)\,, \tag{7}$$

where

$$F_{mp}(\omega)=\frac{\omega^4}{(\omega_m{}^2-\omega^2-2i\delta_m\omega)(\omega_p{}^2-\omega^2-2i\delta_p\omega)} \tag{8}$$

and

$$S_{mp}=\frac{1}{L_l L_y}\int_S \Phi_m(\mathbf{r}')\Psi_p(\boldsymbol{\rho}')d\mathbf{r}' \ . \tag{9}$$

The non-dimensional function $F_{mp}(\omega)$ defined by the expression (8) can be termed as the frequency overlap function of the acoustical and structural modes characterised by the overall indexes m and p.

Similarly, the non-dimensional factor S_{mp} defined by the expression (9) can be considered as the corresponding coefficients of structural-acoustic coupling, which reflect spatial similarity between acoustic and structural modes. Obviously, it is the product $F_{mp}(\omega)S_{mp}$ that determines the amplitudes of the resulting acoustic pressure inside the vehicle compartment.

One can see from (7) that because of the double filtration – over time and over space, described by the products $F_{mp}(\omega)S_{mp}$ – only relatively few of the structural and acoustic modes interact effectively with each other and give noticeable contributions into the resulting structure-borne noise. First of all, it is clear that, because of the time filtration, only those acoustic and structural modes should be taken into account the resonance frequencies of which, ω_m and ω_p respectively, are close enough to each other, so that the frequency functions $(\omega_m^2 - \omega^2 - 2i\delta_m\omega)^{-1}$ and $(\omega_p^2 - \omega^2 - 2i\delta_p\omega)^{-1}$ in (8) overlap effectively, so that their product is far from zero. In addition to this, due to the spatial filtration described by the structural-acoustic coupling coefficients S_{mp} in (9), only those frequency overlapping acoustical and structural modes should be taken into account for which the values of S_{mp} are large enough too.

3. Numerical calculations and discussion

For the purpose of numerical illustration of the above results, we consider a simplified model of a cabin of a medium-sized aircraft (see Fig. 2). For calculation of acoustic modes we approximate a cabin as a parallelepiped characterised by the length along the curved path $L_x = 2.5\ m$, width $L_y = 20\ m$ and height $L_z = 2.2\ m$. For calculation of structural modes, we consider the same cabin as being enveloped by a smoothly curved thin plate with the width equal to L_y and with the total length $L_l = 2L_x + 2L_z$. For calculation purposes we assume that the above-mentioned curved plate is made of steel with $\rho = 7700\ kg/m^3$, $h_s = 0.015\ m$, $E = 1.95\ 10^{11}\ N/m^2$ and $\sigma = 0.31$. Let us also assume that $\delta_m/\omega_m = \delta_p/\omega_p = 5\%$.

The results of calculation of the resonance frequencies of all acoustic modes in the frequency range from 0 to 35 Hz, starting from the lowest order (i,j,k) = (0,1,0), and of some structural modes, starting from (s,t) = (4,5), are shown in Table 1. Note that the lowest acoustic resonance frequencies in the case considered are defined by axial modes (0,j,0) along the cabin length (or along the width of the curved plate modelling its structure).

One can see that there are several pairs of acoustic and structural modes with close resonance frequencies (as a matter of fact, for each acoustic resonance frequency there is a number of very close structural resonance frequencies). For example, the ones characterised by the acoustic frequency of 8.5 Hz (acoustic mode (0,1,0)) and structural frequencies of 8.8, 8.0 and 11.6 Hz (structural modes (4,5), (4,4) and (5.4) respectively), etc.

Acoustic modes		Structural modes		Coefficients of str.-acoust. coupling,
Mode indexes, (i, j, k)	Resonance frequencies, F_{ijk} (Hz)	Mode indexes, (s, t)	Resonance frequencies, f_{st} (Hz)	Smp
(0,1,0)	8.5	(4.5)	8.8	0
(0,1,0)	8.5	(4,4)	8.0	0.022
(0,1,0)	8.5	(5,4)	11.6	0.014
(0,2,0)	17	(5,8)	16.0	0
(0,2,0)	17	(5,9)	17.5	0.006
(0,2,0)	17	(6,5)	16.9	0.0003
(0,2,0)	17	(6,6)	17.9	0
(0,3,0)	25.5	(7,7)	24.4	0
(0,3,0)	25.5	(7,8)	25.7	0.007
(0,3,0)	25.5	(7,9)	27.3	0
(0,3,0)	25.5	(8,1)	26.2	0
(0,4,0)	34	(9,2)	33.4	0
(0,4,0)	34	(9,3)	33.8	-0.0008
(0,4,0)	34	(9,4)	34.4	0.001
(0,4,0)	34	(8,9)	33.4	0.004
(0,4,0)	34	(8,10)	35.1	0.003

Table 1. Calculated resonance frequencies and structural-acoustic coupling coefficients of the acoustic and structural modes for a simplified model of aircraft cabin in the frequency range of 0 - 35 Hz.

To calculate the coefficients of structural-acoustic coupling *Smp* for the pairs of acoustic and structural modes listed in Table 1, we introduce the surface co-ordinate l measured along the curved plate's length and perform the integration along its surface according to (9). As a result, one can obtain the corresponding values of *Smp* that are also shown in Table 1. Substituting pairs of structural and acoustic modes with non-zero values of S_{mp} into (7) and using (4), one can calculate the resulting acoustic pressure in the aircraft cabin as a function of frequency or aircraft ground speed.

Calculations of the acoustic field inside the cabin have been made at a typical passengers position ($l_p = 0.4\ m$, $y_p = 6\ m$ and $z_p = 1.3\ m$) for two co-ordinates of the front wheel suspension along the y-axis ($y_s = 8\ m$ and $y_s = 10\ m$), the l co-ordinate was the same in both cases ($l_s = 0$). The parameters of the supposed sinusoidal irregularity of a runway were: $h = 1$ cm and $d = 15\ cm$. The results of the calculations for the above-mentioned simplified model of aircraft cabin expressed in terms of sound pressure level (SPL, in dB relative to 2 10^{-5} Pa) are shown in Fig. 3 as functions of the aircraft ground speed when it is moving along a runway.

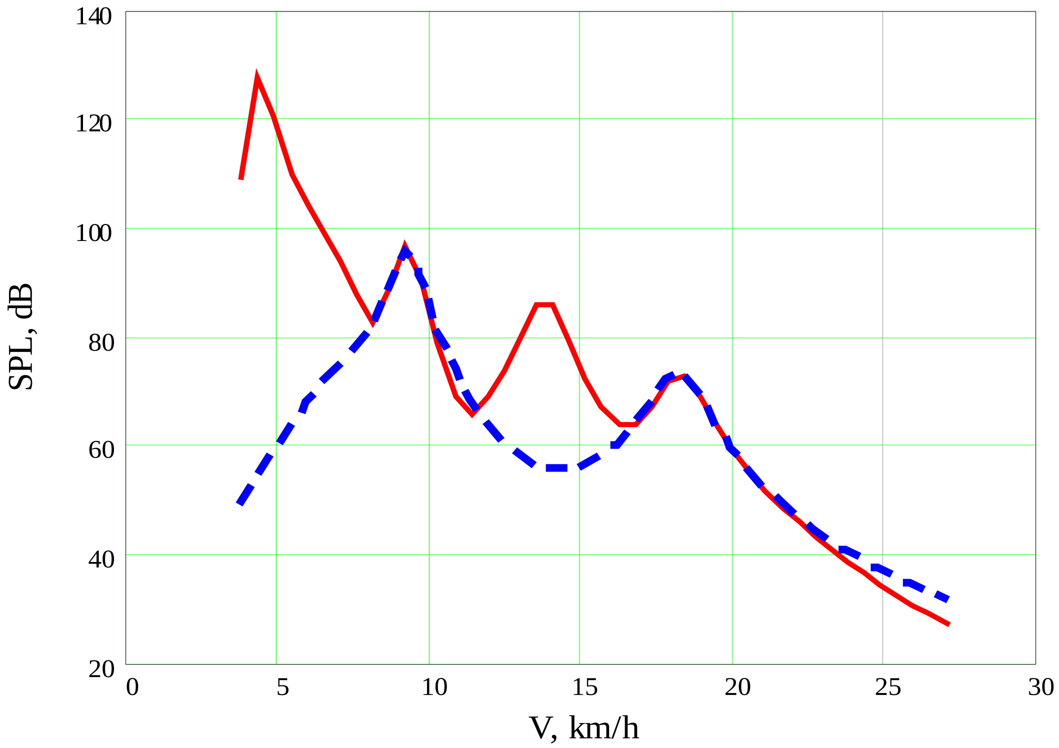

Fig. 3. Calculated sound pressure level (SPL) in the aircraft cabin as a function of aircraft ground speed V on a sinusoidal runway with h = 1 cm and d = 15 cm for the positions of the front wheel suspension: a) y_s = 8 m (solid curve), b) y_s = 10 m (dashed curve).

As one can see from Fig. 3, for the position of the front wheel suspension at $y_s = 8\ m$ (solid curve), the frequency responses of the described simple model associated with the efficiently coupled pairs of acoustic and structural modes have maxima centred at frequencies of 8.5, 17, 25.5 and 34 Hz, that correspond to the aircraft ground speeds on a runway of 4.5, 9, 13.5 and 18 km/h respectively. Note that for the position of the front wheel suspension at $y_s = 10\ m$ (dashed curve) the frequency (ground speed) response has only two maxima, instead of four, and the level of generated structure-borne interior noise is essentially lower at very low speeds (frequencies), i.e. at speeds below 8 km/h, and at speeds ranging from 12 to 18 km/h, as compared to the case of $y_s = 8\ m$. This can be explained by the fact that the corresponding structural modes are excited less efficiently by the roughness-induced dynamic force applied from the wheel suspension located in this position.

4. Conclusions

In the present paper, a simplified analytical approach to the prediction of vehicle and aircraft structure-born interior noise has been described, which includes explicit analytical formulae linking the acoustic pressure in a vehicle or aircraft interior with the external dynamic forces applied to the structures and with the resonance frequencies, modal shapes and coupling coefficients of structural and acoustic modes. The approach is illustrated by the example calculation of structure-borne interior noise in a simplified structural model of aircraft cabin.

References

1. Nefske, D.J., Wolf, J.A., Jr and Howell, L.J. Structural-acoustic finite element analysis of the automobile passenger compartment: a review of current practice, *J. Sound Vib.*, **80**, 247-266 (1982).

2. Kim, S.H., Lee, J.M. and Sung, M.H. Structural-acoustic modal coupling analysis and application to noise reduction in a vehicle passenger compartment, *J. Sound Vib.*; **225**, 989-999 (1999).

3. Lim, T.C. Automotive panel noise contribution modelling based on finite element and measured structural-acoustic spectra, *Applied Acoustics*, **60**, 505-519 (2000).

4. Mixson, J.S. and Wilby, J.F. Interior noise, In: *Aeroacoustics of Flight Vehicles: Theory and Practice*, Ed. H.H. Hubbard, Acoust. Society of America, Woodbury, NY, 1995, Vol. 2, pp. 271-355.

5. Wilby, J.F. Aircraft interior noise, *J. Sound Vib.*, **190**(3), 545-564 (1996).

6. Buehrle, R.D., Fleming, G.A., Pappa, R.S. and Grosveld, F.W. Finite element model development and validation for aircraft fuselage structures, *Proceedings of the 18th International Modal Analysis Conference*, San Antonio, Texas, 7-10 February, 2000, 7 pp.

7. Georgiev, V.B. and Krylov, V.V. Finite element study of the effect of structural modifications on structure-borne vehicle interior noise, *Journal of Vibration and Control*, **15**(4), 483-496 (2009).

8. Krylov, V.V. Simplified analytical models for prediction of vehicle interior noise, *Proc. Int. Conf. on Noise and Vibration Engineering* (ISMA 2002), Leuven, Belgium, 16-18 Sept.2002, Ed. P. Sas, Vol. V, pp. 1973-1980.

9. Georgiev, V.B., Krylov, V.V. and Winward, R.E.T.B. Simplified modelling of vehicle interior noise: Comparison of analytical, numerical and experimental approaches, *Journal of Low Frequency Noise, Vibration and Active Control*, **25**(2), 69-92 (2006).

10. Junger, M.C. and Feit, D. *Sound, Structures and their Interaction*, MIT Press, Cambridge MA, (1972).

11. Fahy, F. *Sound and Structural Vibration*, Academic Press, London, (1985).

12. Krylov, V.V. Generation of ground elastic waves by road vehicles, *J. Computational Acoustics*, **9**, 919-933 (2001).