**The MIMO data transfer line with quaternion carrier and time diversity**

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## I. INTRODUCTION

To increase the capacity of communication channels, multidimensional phase shift keying methods such as M-ary Phase Shift Keying (MPSK), Quadrature Phase Shift Keying (QPSK) and Quadrature Amplitude Modulation (QAM) are widely used. [1]. However, with an increase in the number of initial phase values on the phase plane, the energy distance between information symbols decreases and, therefore, a large signal energy is required to achieve a given error probability.

A natural solution to the problem of increasing the energy distance between information symbols while maintaining the occupied frequency band and, accordingly, the channel capacity, is the transition from two-dimensional signal space (2D) to multidimensional (4D, 8D ...). Currently, methods for transmitting information using the Multiple-Input Multiple-Output scheme, (MIMO) have gained great popularity. Theoretically, it is shown that the MIMO scheme makes it possible to increase the capacity of the communication channel by $N$ times, where $N$ is the number of inputs equal to the number of outputs, i.e. MIMO $N\times N$ [2]. Typically, a MIMO scheme is implemented using multiple transmit and receive antennas. Multidimensional signal space is formed in the communication channel by using the space-time code. However, the realization of such a scheme is very complicated, since it requires knowledge of the state of the communication channel, i.e. complex valued channel transfer matrix. On the receiving side, it is necessary to have a device that generates the corresponding space-time code with the property of orthogonal separation of information symbols. At the same time, to form such a space-time code on the transmitting side, it is necessary to transmit the vector of information symbols several times. The energy losses caused by the repetition of the same information are compensated by the presence of many antennas and the use of the radiated signal power by several antennas at once with diversity reception.

Due to the technical difficulties of implementing a MIMO scheme with many antennas, at present there is only one working Alamouti scheme with two antennas for transmitting and receiving, i.e. MIMO 2×2, which provides the implementation of the theoretical gain [3]. The WiFi MIMO version is defined as the IEEE 802.11n standard [4]. The data transfer rate is 600 Mbps when using 4 antennas for receiving and transmitting with a space-time code according to the Alamouti scheme.

The purpose of the article is to present a 4×4 MIMO scheme in which modulation on the transmitting side is carried out by multiplying the information vector by a quaternion carrier. The quaternion carrier plays the role of the channel matrix of the MIMO channel and is formed from the quaternion in the polar notation in the form of a 4-dimensional matrix when the angle changes in time with the carrier frequency $\omega_c$. At the same time, each obtained elements of the modulated vector contain information about all elements of the 4-dimensional information vector. The output modulated vector elements are transmitted sequentially in time and coherently demodulated by multiplying the received vector by the transposed basis matrices of the quaternion carrier. The frequency phase of the quaternion carrier is determined and maintained using known phase locking techniques.

## II. MATERIALS AND METHODS FOR SOLVING THE PROBLEM

The quaternion refers to hypercomplex numbers and imaginary units $i, j, k$ are used to write the quaternion, which satisfy the following multiplication rules: $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$, $i^2 = j^2 = k^2 = ijk = -1$. The quaternion in algebraic representation has the form [5]:

$$q = s + ix + jy + kz. \qquad (1)$$

The $s$, $x$, $y$, $z$ numbers take real values, and the imaginary units form three orthogonal axes in space. Therefore, given the real part of s, the quaternion is located in 4-dimensional space (4D).

A quaternion with a module equal to one is written in polar representation, as

$$q(t) = e^{\hat{i}\varphi} = \cos\varphi + \hat{i}\sin\varphi, \tag{2}$$

where $\hat{i} = (i + j + k)/\sqrt{3}$, $\hat{i}^2 = -1$, $\varphi$ – angle of rotation of the quaternion in the plane of rotation located in the 3D volume with coordinates *i*, *j*, *k*.

As a rule, in problems of radio engineering, functions change their values according to the time parameter, so we write the quaternion (1) in the parametric representation:

$$q(t) = s(t) + ix(t) + jy(t) + kz(t). \tag{3}$$

We write the function of the parametric quaternion (3) in the form:

$$f\left[q(t)\right] = p\left[q(t)\right] + iu\left[q(t)\right] + jv\left[q(t)\right] + kw\left[q(t)\right]. \tag{4}$$

As a result of applying functions to a quaternion and after grouping by imaginary units, we get various quaternion functions. We write (4) as the sum of time functions on orthogonal axes, leaving the same notation:

$$f(t) = p(t) + iu(t) + jv(t) + kw(t). \tag{5}$$

As functions $p(t)$, $u(t)$, $v(t)$ and $w(t)$ in (5) one can consider any continuous functions with a bounded norm, for example, impulses of finite length. According to (2), the parametric quaternion (3) with a unit modulus $|q| = 1$ and a total circular rotation frequency $\omega$ has the form:

$$q(t) = e^{\hat{i}\omega t} = \cos\omega t + \hat{i}\sin\omega t. \tag{6}$$

Let us represent the quaternion in the algebraic form of writing (1) as a 4 × 4 matrix [5]:

$$\mathbf{Q} = \begin{bmatrix} s & x & y & z \\ -x & s & -z & y \\ -y & z & s & -x \\ -z & -y & x & s \end{bmatrix}. \tag{7}$$

Matrix (7) is decomposed into *basic matrices* **E**, **I**, **J**, **K** and quaternion (7) is written in algebraic form as the sum of matrices:

$$\mathbf{Q} = \mathbf{E}s + \mathbf{I}x + \mathbf{J}y + \mathbf{K}z,$$

where

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{I} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ \mathbf{J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \ \mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

The corresponding basis matrices of the quaternion, as well as the elements *i*, *j* and *k*, are related by the multiplication rules presented in Table 1:

Table 1. Multiplication of basic matrices.

| × | **E** | **I** | **J** | **K** |
|---|---|---|---|---|
| **E** | **E** | **I** | **J** | **K** |
| **I** | **I** | –**E** | **K** | –**J** |
| **J** | **J** | –**K** | –**E** | **I** |
| **K** | **K** | **J** | –**I** | –**E** |

(8)

We write the imaginary part $\hat{i}$ of the quaternion (2) in matrix form using the *imaginary algebraic matrix*:

$$\hat{\mathbf{I}} = \frac{1}{\sqrt{3}}\left(\mathbf{I} + \mathbf{J} + \mathbf{K}\right). \tag{9}$$

The imaginary matrix (9) has the properties of the imaginary unit of a complex number, since $\hat{\mathbf{I}}\hat{\mathbf{I}} = -\mathbf{E}$, $\hat{\mathbf{I}}^{\mathrm{T}}\hat{\mathbf{I}} = \hat{\mathbf{I}}\hat{\mathbf{I}}^{\mathrm{T}} = -\hat{\mathbf{I}}\hat{\mathbf{I}} = \mathbf{E}$. The inverse operator in matrix notation is expressed by the formula: $\mathbf{Q}^{-1} = \mathbf{Q}^{\mathrm{T}} / |\mathbf{Q}|^2$. Since for basis matrices the determinant is equal 1, then $\mathbf{I}^{\mathrm{T}} = \mathbf{I}^{-1} = -\mathbf{I}$, $\mathbf{J}^{\mathrm{T}} = \mathbf{J}^{-1} = -\mathbf{J}$, $\mathbf{K}^{\mathrm{T}} = \mathbf{K}^{-1} = -\mathbf{K}$ and, according to table (8), the basis matrices are orthogonal: $\mathbf{I}\mathbf{I}^{\mathrm{T}} = \mathbf{E}$, $\mathbf{J}\mathbf{J}^{\mathrm{T}} = \mathbf{E}$, $\mathbf{K}\mathbf{K}^{\mathrm{T}} = \mathbf{E}$. Note that the basis matrices of a quaternion are also quaternions.

It is convenient to represent the information transfer model as a model in the state space using the dynamics equation in the form [6]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) . \tag{10}$$

We write the *state transition matrix* using the imaginary matrix (9) in the form

$$\mathbf{A} = \hat{\mathbf{I}}\omega_c . \tag{11}$$

We call the matrix **A** the *quaternion frequency matrix*, where $\omega_c = 2\pi/T = 2\pi f_c$ - angular carrier frequency, radian/s; $T = 1/f_c$ – carrier frequency period $f_c$, s.

The quaternion frequency matrix **A** is the differential operator for the state-space model (11). The solution to equation (10) will be the exponent (6) in matrix representation:

$$e^{\hat{\mathbf{I}}\omega_c t} = \boldsymbol{\Phi}(\omega_c, t) = \mathbf{E}\cos(\omega_c t) + \frac{1}{\sqrt{3}}\mathbf{I}\sin(\omega_c t) + \frac{1}{\sqrt{3}}\mathbf{J}\sin(\omega_c t) + \frac{1}{\sqrt{3}}\mathbf{K}\sin(\omega_c t) =$$

$$= \mathbf{E}\cos(\omega_c t) + \hat{\mathbf{I}}\sin(\omega_c t) . \tag{12}$$

Since the matrix $\boldsymbol{\Phi}(\omega_c, t) = e^{\hat{\mathbf{I}}\omega_c t}$ is a solution to the differential equation (10), then the matrix $\boldsymbol{\Phi}(\omega_c, t)$ is called the *fundamental matrix*.

Fundamental matrix (12) is orthogonal, since $\boldsymbol{\Phi}(\omega_c, t)\boldsymbol{\Phi}^{\mathrm{T}}(\omega, t) = \boldsymbol{\Phi}^{\mathrm{T}}(\omega_c, t)\boldsymbol{\Phi}(\omega_c, t) = \mathbf{E}$. As you know, an orthogonal matrix does not change the modulus of a vector when multiplied. The rank of the fundamental matrix is 4, and the determinant is:

$$\left|\boldsymbol{\Phi}(\omega_c, t)\right| = \det\left[\boldsymbol{\Phi}(\omega_c, t)\right] = \left(\cos^2 \omega_c t + \sin^2 \omega_c t\right)^2 = 1 .$$

Since the fundamental matrix (12) will be used for modulation, we will also call it the *quaternion carrier matrix*.

Denote the basis matrices of the quaternion carrier as

$$\mathbf{E}(\omega_c, t) = \mathbf{E}\cos(\omega_c t) , \ \mathbf{I}(\omega_c, t) = \frac{1}{\sqrt{3}}\mathbf{I}\sin(\omega_c t) , \tag{13}$$

$$\mathbf{J}(\omega_c, t) = \frac{1}{\sqrt{3}}\mathbf{J}\sin(\omega_c t) , \ \mathbf{K}(\omega_c, t) = \frac{1}{\sqrt{3}}\mathbf{K}\sin(\omega_c t) .$$

Then, the quaternion carrier matrix can be written in the form of basis matrices (13), as

$$\boldsymbol{\Phi}(\omega_c, t) = \mathbf{E}(\omega_c, t) + \mathbf{I}(\omega_c, t) + \mathbf{J}(\omega_c, t) + \mathbf{K}(\omega_c, t) . \tag{14}$$

## III. MIMO LINK MODEL WITH QUATERNION CARRIER

Consider the binary manipulation of the quaternion carrier (12) with information pulses that take values of ±1. We will manipulate the quaternion carrier by multiplying the 4D information vector by the matrix of the quaternion carrier:

$$\mathbf{y}(t) = \boldsymbol{\Phi}(\omega_c, t)\mathbf{x}(0) ,$$

where $\mathbf{x}(0) = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 \end{bmatrix}^{\mathrm{T}}$ - 4D vector of information pulses, which is defined for model (10), as a *vector of initial states*, $x_i$ - elements of the information vector, $i = 0,1,2,3$.

When combining pulses into a 4D vector, 16 combinations of binary information pulses at the initial time $t = 0$ are possible:

$$\mathbf{x}(0)=\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1\\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1\\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\end{bmatrix}. \quad (15)$$

The square of the norm of information vectors is calculated as $\|\mathbf{x}(0)\|^2 = x_0^2 + x_1^2 + x_2^2 + x_3^2 = 4$. Since matrix (12) is orthogonal, it does not change the norm of the vector when it is multiplied by this matrix.

As is known, 2D information vectors in the MPSK, QPSK and QAM modulation methods are considered on the complex plane, and a point on this plane corresponds to the information vector with its own phase and amplitude. In our case, the information vector from (15) consists of 4 information impulses and forms a quaternion, so this vector must be considered in a 4D space with orthogonal axes *s*, *i*, *j*, *k*.

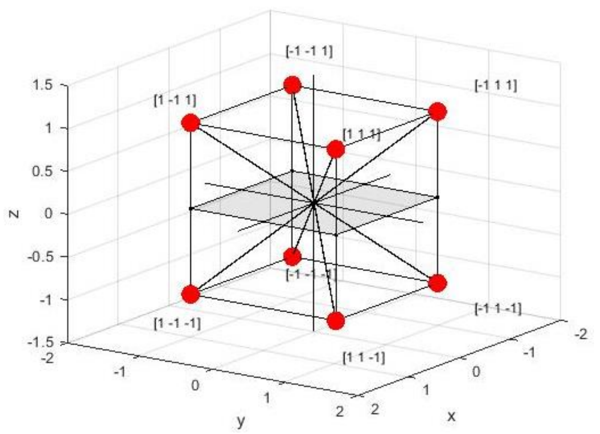

Figure 1. Vectors of information pulses in the form of a quaternion for a positive value of the first element of the information vector.

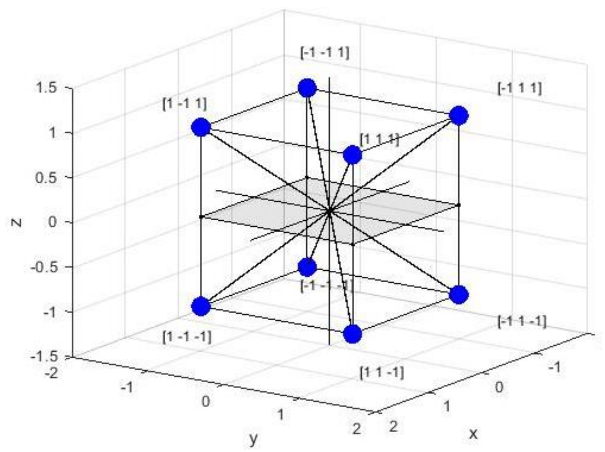

Figure 2. Vectors of information pulses in the form of a quaternion for the negative value of the first element of the information vector.

Since 4D space cannot be depicted on a plane, Figures 1 and 2 show the values of information impulses in 3D space for the last 3 impulses in the vector on single imaginary units *i*, *j*, *k*, which corresponds to the coordinate axes *x*, *y*, *z* of the three-dimensional space. A hypercomplex number consists of one real number and 3 imaginary ones. The imaginary units form orthogonal vectors, which are orthogonal, in turn, to the real part of the hypercomplex number. Figure 1 shows the normalized vectors from (15) with a positive first element (red), and Figure 2 - with a negative one (blue). Since the value of the real part does not depend on the direction, then in 3D space its value is depicted as a point mass, the value of which is proportional to the diameter of the sphere. Thus, we get 16 information vectors located in a 4D volume.

After modulation, i.e. multiplying the information vector by the quaternion carrier matrix (12), we obtain the output vector, which can be represented as:

$$\mathbf{y}(t)=\mathbf{\Phi}(\omega_c,t)\mathbf{x}(0)=\frac{1}{\sqrt{3}}\begin{bmatrix}\sqrt{3}x_0\cos(\omega_c t)+(x_1+x_2+x_3)\sin(\omega_c t)\\ \sqrt{3}x_1\cos(\omega_c t)+(-x_0-x_2+x_3)\sin(\omega_c t)\\ \sqrt{3}x_2\cos(\omega_c t)+(-x_0+x_1-x_3)\sin(\omega_c t)\\ \sqrt{3}x_3\cos(\omega_c t)+(-x_0-x_1+x_2)\sin(\omega_c t)\end{bmatrix}. \quad (16)$$

As can be seen from (16), all 4 information elements $\mathbf{x}(0)$ are part of 4 elements of the output vector $\mathbf{y}(t)$. Therefore, we have a 4×4 MIMO scheme.

Modulation of the quaternion carrier by information pulses corresponds to the exponential mapping $\mathbf{\Phi}(\omega_c,t)=e^{\hat{\mathbf{i}}\omega_c t}$ and the amplitudes of the pulses change according to the harmonic law, i.e. the information vector rotates in 4D space. Since the matrix of the quaternion carrier is orthogonal and, therefore, does not change the norm of the vector when multiplied, the rotation of the point mass occurs in a closed region.

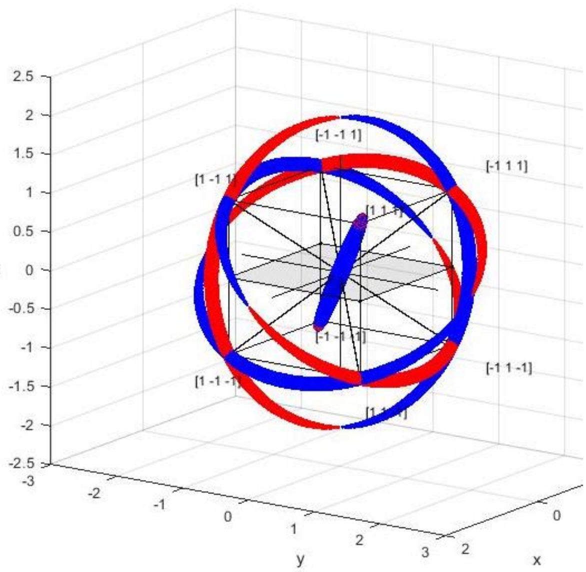


Figure 3. Trajectories of complete rotation of quaternions from 16 initial states [±1 ±1 ±1 ±1].

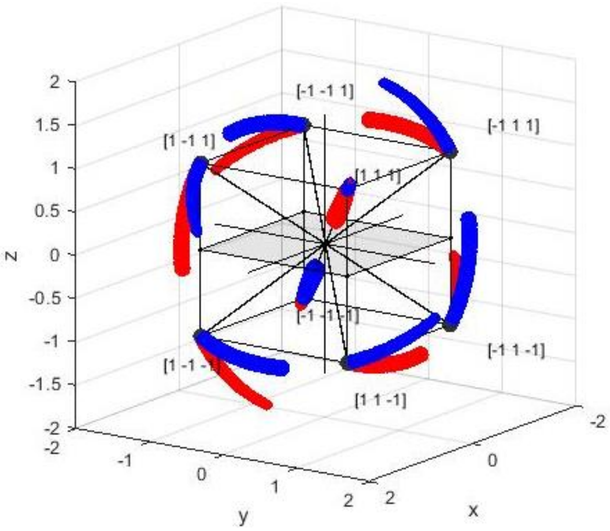


Figure 4. Initial sections of the trajectory of rotation of quaternions from 16 initial states [±1 ±1 ±1 ±1].

Figure 3 shows the rotation trajectories of the normalized modulated output vector (16). Since the considered quaternion has the same frequency of rotation for all coordinates, then in the 3D system of imaginary coordinates, rotation occurs in planes with different orientations. Figure 4 shows the initial trajectories for the positive first element of the information vector (red) and the negative first element of the information vector (blue). As can be seen from Figure 4, when the sign of the first information element changes, the rotation trajectories change. The red trajectory for the positive first element is different from the blue trajectory for the negative first element.

Thus, from the above figures it can be seen that the location of information elements in the 4D region and their rotation in a closed region when modulated by a quaternion carrier provides a greater energy distance between them at a given power than placement on a plane.

Consider a scheme with sequential transmission in time of the combined modulated elements of the output vector (16). Each combined element is transmitted for the duration of the bit, therefore, the information transfer rate will be equal to the information element incoming rate. Recall that in accordance with the MIMO 4×4 scheme, each modulated element contains information about all 4 elements of the information vector $\mathbf{x}(0)$.

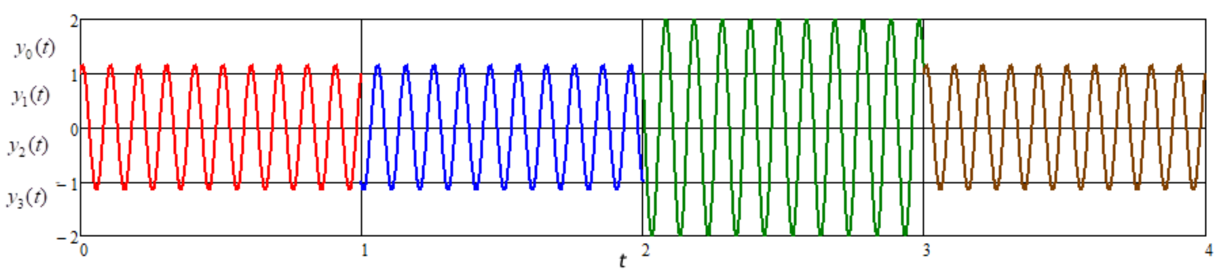


Figure 5. Elements of the output combined vector after quaternion carrier modulation

Figure 5 shows the elements of the output combined vector, located sequentially in time, obtained by modulating the quaternion carrier with the information vector $\mathbf{x}(0) = \begin{bmatrix} 1 & -1 & 1 & 1 \end{bmatrix}$.

In accordance with the values of the information vector, the amplitude of the sine of one of the elements (16) can be equal to ±3, and the other three ±1. Accordingly, with quadrature summation from (16) we obtain that the carrier amplitude can take the values:

$$A=\sqrt{(3+3^2)/3}=\pm 2 \text{ или } A=\sqrt{(3+1^2)/3}=\pm 2/\sqrt{3}.$$

Since the power of the output vector $\mathbf{y}(t)$ is constant and equal to 4, then the power of one of the elements for the duration $T$=1 will be $P$=2, and the power of the other three elements is P=2/3. For example, by direct integration over the pulse duration $T$ for $\mathbf{x}(0)=\begin{bmatrix}1 & -1 & 1 & 1\end{bmatrix}$ we obtain the power of the first element:

$$P_0=\frac{1}{T}\int_0^T\left(x_0\cos\omega_c t+\frac{(x_1+x_2+x_3)}{\sqrt{3}}\sin\omega_c t\right)^2 dt=\frac{1}{T}\left(\frac{1}{2}+\frac{1}{6}\right)=\frac{2}{3T}.$$

Мощность третьего элемента:

$$P_3=\frac{1}{T}\int_0^T\left(x_2\cos\omega_c t+\frac{(-x_0+x_1-x_3)}{\sqrt{3}}\sin\omega_c t\right)^2 dt=\frac{1}{T}\left(\frac{1}{2}+\frac{9}{3}\right)=\frac{2}{T}.$$

Thus, we obtain that at $T$=1 the total power of 4 modulated pulses $P=2+3\times 2/3=4$, and the average pulse power will be $\bar{P}=4/4=1$.

The amplitude of the signal can be calculated with a known energy obtained after integrating the signal for a duration $T$ and, accordingly, it can be calculated as $A=\sqrt{2E/T}=\sqrt{2P}$. With $P=2/3$ we get $A=\sqrt{4/3}=2/\sqrt{3}$, and with $P=2$ we get $A=\sqrt{4}=2$.

Thus, the elements of the output vector are a harmonic signal modulated in phase and amplitude. Based on the values of the amplitudes of the elements in the vector (16), cosine ±1 and sine $\pm 1/\sqrt{3}$ or $\pm\sqrt{3}$, the phases of the quaternion carrier shown in Figure 5 can take values of ±60° for one element with an amplitude of ±2 and value ±30° for three other elements with amplitude: $\theta=\operatorname{arctg}\left(\pm\sqrt{3}\right)=\pm 60°$ and $\theta=\operatorname{arctg}\left(\pm 1/\sqrt{3}\right)=\pm 30°$.

The obtained modulated signal (16) propagates through the communication channel in the form of a sequence of pulses, Figure 5. At the input of the receiver, interference in the form of noise is added to the signal elements. The received signal model will look like:

$$\mathbf{s}(t)=\mathbf{y}(t)+\mathbf{n}(t)=\mathbf{\Phi}(\omega_c t)\mathbf{x}(0)+\mathbf{n}(t), \qquad (17)$$

where $\mathbf{s}(t)=\begin{bmatrix}s_0(t) & s_1(t) & s_2(t) & s_3(t)\end{bmatrix}^{\mathrm{T}}$ - 4D received signal vector, $\mathbf{n}(t)=\begin{bmatrix}n_0(t) & n_1(t) & n_2(t) & n_3(t)\end{bmatrix}^{\mathrm{T}}$ - 4D white noise vector with circular symmetry, i.e. uncorrelated, total variance $\sigma^2$ and power spectral density $\mathrm{N}_0$.

Since the noise is four-dimensional and has the property of circular symmetry, then the square of the norm of the noise vector

$$\|\mathbf{n}(t)\|^2=\sigma^2=\sigma_0^2+\sigma_1^2+\sigma_2^2+\sigma_3^2=\mathrm{N}_{00}/2+\mathrm{N}_{01}/2+\mathrm{N}_{02}/2+\mathrm{N}_{03}/2,$$

where $\sigma_0^2$, $\sigma_1^2$, $\sigma_2^2,\sigma_3^2$ are the dispersions, $\mathrm{N}_{00}$, $\mathrm{N}_{01}$, $\mathrm{N}_{02}$, $\mathrm{N}_{03}$ are the spectral densities of the elements of the noise vector $\mathbf{n}(t)$. With circular symmetry of the noise, the variances and the spectral densities of the noise elements are equal to each other: $\sigma_0^2=\sigma_1^2=\sigma_2^2=\sigma_3^2=\sigma^2/4$ and $\mathrm{N}_{0,0}=\mathrm{N}_{0,1}=\mathrm{N}_{0,2}=\mathrm{N}_{0,3}=\mathrm{N}_0/4$.

Thus, according to (17), the quaternion carrier matrix (12) is a 4×4 MIMO channel matrix. A MIMO signal is formed by multiplying an information vector by a matrix of a quaternion carrier, and not by space-time code when using 4 antennas for transmission and 4 for reception.

## IV. DIVERSITY OF INFORMATION SYMBOLS

### *Signal component*

The separation of information vectors during reception will be carried out using transposed, normalized at the end of integration, basis matrices (13). Since the integral of the cosine and sine on the pulse duration *T*, which is significantly longer than the period of the carrier frequency, is equal to $T/2$, then the normalized transposed basis matrices will take the form:

$$\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)=\sqrt{\frac{2}{T}}\cos(\omega_c t)\mathbf{E}\,,\ \bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)=-\sqrt{\frac{2}{T}}\sin(\omega_c t)\mathbf{I}\,,$$
$$\bar{\mathbf{J}}^{\mathrm{T}}(\omega_c,t)=-\sqrt{\frac{2}{T}}\sin(\omega_c t)\mathbf{J}\,,\ \bar{\mathbf{K}}^{\mathrm{T}}(\omega_c,t)=-\sqrt{\frac{2}{T}}\sin(\omega_c t)\mathbf{K}\,. \tag{18}$$

In order to more clearly present the process of extracting information symbols, we consider the integration of the received signal component $\mathbf{y}(t)$ over each basis matrix (18). We multiply the received signal (17) by the basis matrices (18) and integrate. To do this, we represent the signal component $\mathbf{y}(t)$ using the decomposition into basic matrices of the quaternion carrier (14):

$$\mathbf{y}(t)=\mathbf{\Phi}(\omega_c t)\mathbf{x}(0)=\mathbf{E}(\omega_c,t)\mathbf{x}(0)+\mathbf{I}(\omega_c,t)\mathbf{x}(0)+\mathbf{J}(\omega_c,t)\mathbf{x}(0)+\mathbf{K}(\omega_c,t)\mathbf{x}(0)\,. \tag{19}$$

1) First, consider the estimate of the information vector for the normalized basis matrix $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)$. Multiply (19) by $\sqrt{2/T}\cos(\omega_c t)\mathbf{E}$ and integrate. Since $\int_0^T \cos(\omega_c t)\sin(\omega_c t)dt=0$, the result is:

$$\mathbf{S}_{\mathrm{E}}=\int_0^T \bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)\mathbf{\Phi}(\omega_c t)\mathbf{x}(0)dt=\sqrt{2/T}\mathbf{E}\mathbf{x}(0)\int_0^T\cos^2(\omega_c t)dt=\sqrt{T/2}\mathbf{E}\mathbf{x}(0)\,. \tag{20}$$

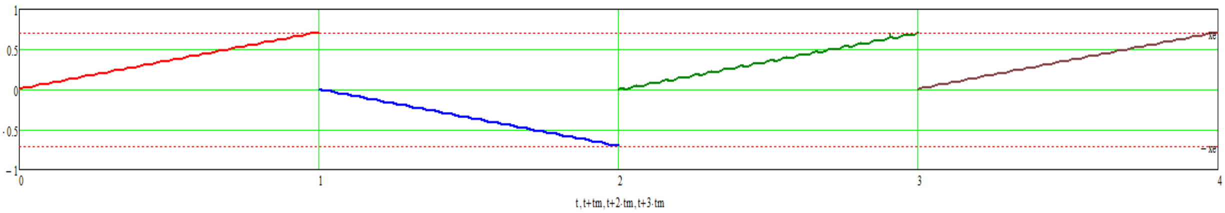


Figure 6 The result of integration when $\mathbf{y}(t)$ multiplied by the basis matrix $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)$.

For $x_0=1$, $x_1=-1$, $x_2=1$, $x_3=1$ and *T*=1 we get $\mathbf{S}_{\mathrm{E}}=\frac{1}{\sqrt{2}}[1\quad -1\quad 1\quad 1]$. As can be seen from Figure 6, the received values of the samples correspond to the values of the transmitted signal, but have an amplitude $1/\sqrt{2}$.

2) Let's find an estimate of the information vector by the basic normalized matrix $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)$. To do this, we multiply (19) by $\sqrt{2/T}\sin(\omega_c t)\mathbf{I}^{\mathrm{T}}$ and get the following expression:

$$\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)\mathbf{y}(t)=\sqrt{2/T}\sin(\omega_c t)\cos(\omega_c t)\mathbf{I}^{\mathrm{T}}\mathbf{E}\mathbf{x}(0)+\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{I}^{\mathrm{T}}\mathbf{I}\mathbf{x}(0)+$$
$$+\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{I}^{\mathrm{T}}\mathbf{J}\mathbf{x}(0)+\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{I}^{\mathrm{T}}\mathbf{K}\mathbf{x}(0)\,.$$

According to Table 1, (8), $\mathbf{I}^{\mathrm{T}}\mathbf{E}=-\mathbf{I}$, $\mathbf{I}^{\mathrm{T}}\mathbf{J}=-\mathbf{K}$, $\mathbf{I}^{\mathrm{T}}\mathbf{K}=\mathbf{J}$. Then the resulting expression will take the form:

$$\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)\mathbf{y}(t)=-\sqrt{2/T}\sin(\omega_c t)\cos(\omega_c t)\mathbf{I}\mathbf{x}(0)+\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{E}\mathbf{x}(0)-$$
$$-\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{K}\mathbf{x}(0)+\sqrt{2/3T}\sin^2(\omega_c t)\mathbf{J}\mathbf{x}(0)\,.$$

After integration, we get

$$\mathbf{S}_{\mathrm{I}} = \bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)\mathbf{y}(t) = \sqrt{T/6}\left(\mathbf{E}-\mathbf{K}+\mathbf{J}\right)\mathbf{x}(0) = \tag{21}$$

$$= \sqrt{T/6}\left[x_0+x_2-x_3 \quad x_1+x_2+x_3 \quad -x_0-x_1+x_2 \quad x_0-x_1+x_3\right]^{\mathrm{T}}.$$

For $x_0=1,\ x_1=-1,\ x_2=1,\ x_3=1$ and $T$=1

$$\mathbf{S}_{\mathrm{I}} = \frac{1}{\sqrt{6}}\left[1+1-1 \quad -1+1+1 \quad -1+1+1 \quad 1+1+1\right]^{\mathrm{T}} = \frac{1}{\sqrt{6}}\left[1 \quad 1 \quad 1 \quad 3\right]^{\mathrm{T}}.$$

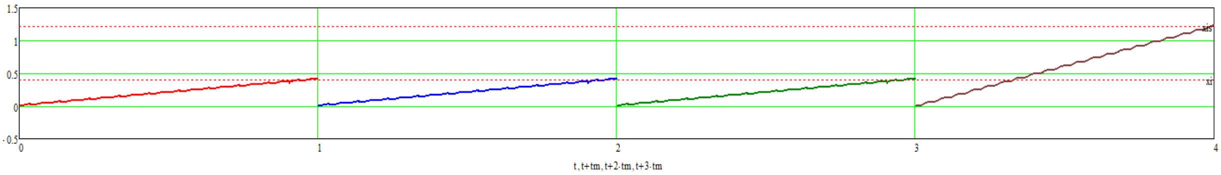

Figure 7 The result of integration when $\mathbf{y}(t)$ multiplied by the basis matrix $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)$.

As can be seen from Figure 7, the value of the received sample of the second element does not correspond to the value of the information element. In this case, the fourth element has an amplitude of $\sqrt{3/2}$, and the remaining 3 elements have an amplitude of $1/\sqrt{6}$.

3) Let's find an estimate of the information vector by the basis matrix $\mathbf{J}^{\mathrm{T}}(\omega_c,t)$. We multiply (19) by $\sqrt{2/T}\sin(\omega_c t)\mathbf{J}^{\mathrm{T}}$ and integrate, as a result we get:

$$\mathbf{S}_{\mathrm{J}} = \sqrt{T/6}\left(\mathbf{E}-\mathbf{I}+\mathbf{K}\right)\mathbf{x}(0) =$$

$$= \sqrt{\frac{T}{6}}\left[x_0-x_1+x_3 \quad x_0+x_1-x_2 \quad x_1+x_2+x_3 \quad -x_0-x_2+x_3\right]^{\mathrm{T}}. \tag{22}$$

For $x_0=1,\ x_1=-1,\ x_2=1,\ x_3=1$ and $T$=1

$$\mathbf{S}_{\mathrm{J}} = \frac{1}{\sqrt{6}}\left[1+1+1 \quad 1-1-1 \quad -1+1+1 \quad -1-1+1\right]^{\mathrm{T}} = \frac{1}{\sqrt{6}}\left[3 \quad -1 \quad 1 \quad -1\right]^{\mathrm{T}}.$$

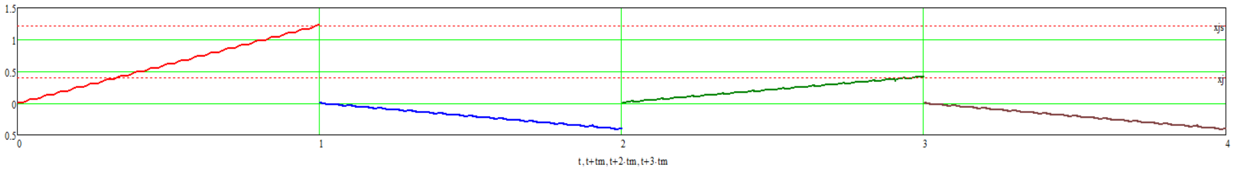

Figure 8 The result of integration when $\mathbf{y}(t)$ multiplied by the basis matrix $\mathbf{J}^{\mathrm{T}}(\omega_c,t)$.

As can be seen from Figure 8, the sample of the first element has an amplitude of $\sqrt{3/2}$, and other samples - $1/\sqrt{6}$. At the same time, the sign of the 4th element differs from the sign of the transmitted information element 4.

4) Let's find an estimate of the information vector by the basis matrix $\bar{\mathbf{K}}^{\mathrm{T}}(\omega_c,t)$. After multiplying (19) by $\sqrt{2/T}\sin(\omega_c t)\mathbf{K}^{\mathrm{T}}$ and integrating, we get the following result:

$$\mathbf{S}_{\mathrm{K}} = \sqrt{T/6}\left(\mathbf{E}+\mathbf{I}-\mathbf{J}\right)\mathbf{x}(0) =$$

$$= \sqrt{\frac{T}{6}}\left[x_0+x_1-x_2 \quad -x_0+x_1-x_3 \quad x_0+x_2-x_3 \quad x_1+x_2+x_3\right]^{\mathrm{T}}. \tag{23}$$

For $x_0=1,\ x_1=-1,\ x_2=1,\ x_3=1$ and $T$=1

$$\mathbf{S}_{\mathrm{K}} = \frac{1}{\sqrt{6}}\left[1-1-1 \quad -1-1-1 \quad 1+1-1 \quad -1+1+1\right]^{\mathrm{T}} = \frac{1}{\sqrt{6}}\left[-1 \quad -3 \quad 1 \quad 1\right]^{\mathrm{T}}.$$

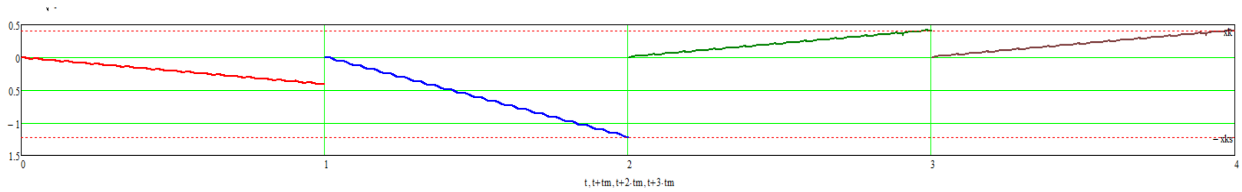


Figure 9 The result of integration when $\mathbf{y}(t)$ multiplied by the basis matrix $\bar{\mathbf{K}}^{\mathrm{T}}(\omega_c,t)$.

As can be seen from Figure 9, the signs of the elements correspond to the signs of the transmitted information vector. In this case, the amplitude of the 2nd reading is $\sqrt{3/2}$, and the remaining $1/\sqrt{6}$.

We use the obtained results of integration over the basis matrices (20) – (23) and write the sum:

$$\mathbf{S}=\mathbf{S}_{\mathrm{E}}+\mathbf{S}_{\mathrm{I}}+\mathbf{S}_{\mathrm{J}}+\mathbf{S}_{\mathrm{K}}= \tag{24}$$

$$=\sqrt{\frac{T}{2}}\mathbf{Ex}(0)+\sqrt{\frac{T}{6}}\left[(\mathbf{E}-\mathbf{K}+\mathbf{J})+(\mathbf{E}-\mathbf{I}+\mathbf{K})+(\mathbf{E}+\mathbf{I}-\mathbf{J})\right]\mathbf{x}(0)=$$

$$=\left(\sqrt{\frac{T}{2}}+3\sqrt{\frac{T}{6}}\right)\mathbf{Ex}(0)=\sqrt{\frac{T}{2}}\left(1+\sqrt{3}\right)\mathbf{Ex}(0).$$

With quadrature addition of the results, taking into account the cosine and sine components, we obtain $\mathbf{S}_{\mathrm{E}}^2+\mathbf{S}_{\mathrm{I}}^2+\mathbf{S}_{\mathrm{J}}^2+\mathbf{S}_{\mathrm{K}}^2=4T/2=2T$.

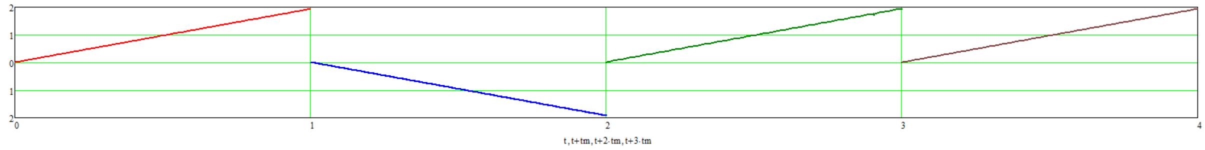


Figure 10. Summary result of integration over normalized basis matrices.

Figure 10 shows the sum of the results of integrating the vector $\mathbf{y}(t)$ of the signal component after multiplying it by the basis matrices for $T=1$. As can be seen from Figure 10, the values of the vector elements after integration correspond to the values of the transmitted information vector. At the same time, the amplitude of values increased by 2 times.

According to formula (24), after multiplying the signal component by the basis matrices with subsequent integration, the terms obtained by multiplying the same-name basis matrices remain, i.e. $\mathbf{E}^{\mathrm{T}}\mathbf{E}+\mathbf{I}^{\mathrm{T}}\mathbf{I}+\mathbf{J}^{\mathrm{T}}\mathbf{J}+\mathbf{K}^{\mathrm{T}}\mathbf{K}=4\mathbf{E}$. Since the normalized basis matrices (18) are orthogonal, they do not change the vector norm during multiplication. Therefore, as a result of adding estimates of information vectors, we will get the power of each information element 4 times more than the power of the original element, and the power of the vector as a whole will be equal to 16, i.e. 4 times more power $\|\mathbf{x}(0)\|^2$.

Let's visualize this result in 4D space. On the transmitting side, we have a set of information vectors (15), which are shown in Figures 1 and 2 in 4D space. When multiplying these vectors by the fundamental matrix (12), we obtain for each vector the corresponding quaternion rotation orbit shown in Figures 3 and 4.

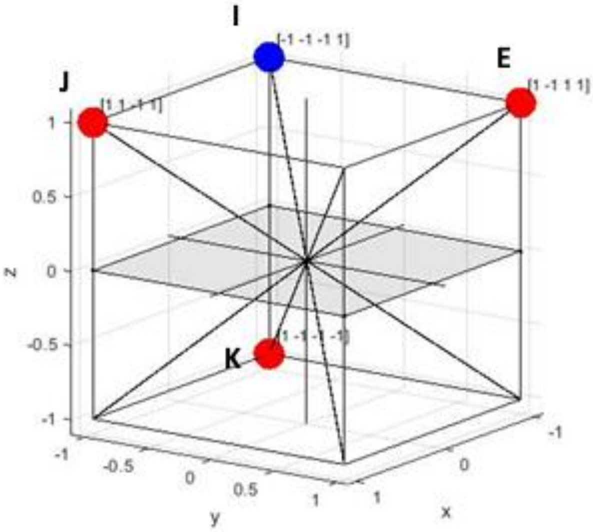


Figure 11. Information vectors obtained from the vector of initial states [1-111] by multiplying by basis matrices.

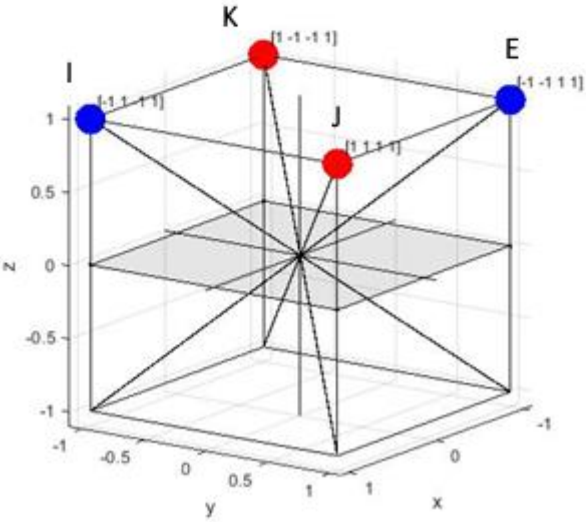


Figure 12 Information vectors obtained from the initial state vector [-1-111] by multiplying by basis matrices.

However, from the point of view of the divisibility of information symbols, it is important to note that the fundamental matrix is decomposed into basic matrices (14). Therefore, when multiplying one of the possible information vectors (15) by the basis matrices **E**, **I**, **J**, **K**, we get 4 information vectors. Figure 11 shows the corresponding 4 vectors obtained after multiplying the information vector $[1 \quad -1 \quad 1 \quad 1]$ by the basis matrices. Figure 12 shows 4 vectors for information vector $[-1 \quad -1 \quad 1 \quad 1]$. Each vector has the same power.

We write these vectors in the form of matrices:

$$\mathbf{F}_5 = \begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}, \quad \mathbf{F}_{13} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

where the matrix indices show the column numbers in (15) for the initial state vector, which is written in the first row.

These matrices are orthogonal because $\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_5 = \mathbf{F}_5\mathbf{F}_5^{\mathrm{T}} = 4\mathbf{E}$ и $\mathbf{F}_{13}^{\mathrm{T}}\mathbf{F}_{13} = \mathbf{F}_{13}\mathbf{F}_{13}^{\mathrm{T}} = 4\mathbf{E}$.

It is shown that all 16 matrices obtained in this way will be orthogonal. The square of the Frobenius norm of these matrices will be $\|\mathbf{F}_n\|_F^2 = 16$, for example, $\mathrm{tr}\left\{\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_5\right\} = \mathrm{tr}\left\{\mathbf{F}_5\mathbf{F}_5^{\mathrm{T}}\right\} = 16$, $\mathrm{tr}\left\{\mathbf{F}_{13}^{\mathrm{T}}\mathbf{F}_{13}\right\} = \mathrm{tr}\left\{\mathbf{F}_{13}\mathbf{F}_{13}^{\mathrm{T}}\right\} = 16$. The squares of the Frobenius norms for matrices formed from opposite vectors will be equal to -16, for example, for matrices $\mathbf{F}_5$ and $\mathbf{F}_{10}$ we get $\mathrm{tr}\left\{\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_{10}\right\} = -16$. Having calculated all possible Frobenius norms for matrix products, we get 4 norms of 8, 4 norms of -8, 6 norms of 0 and 1 norm of -16. Therefore, the minimum distance between the norms will be equal to 16 - 8=8.

Thus, as a result of multiplying the signal component (19) in the received signal (17), we obtain the vector of information pulses (24). As can be seen from (24), the order of the received information elements corresponds to the order of the elements in the information vector, and the amplitude of the elements has increased by 2 times. In this case, the minimum distance between the vectors, obtained as the square of the Frobenius norm, was 8. This is 4 times greater than the minimum distance between opposite elements for BPSK.

***Interference component***

Consider the noise vector $\mathbf{n}(t)$ in (17), in which each element of the noise vector acts on the corresponding elements of the signal vector. The elements of the interference vector in the form of white noise are not correlated. With an equal dispersion of the elements of the noise vector in 4D space, the noise has the property of central symmetry. The interference power in 4D space is divided equally into each coordinate axis of the quaternion space. In other words, the interference in the MIMO channel is projected onto basis matrices, which themselves represent multidimensional volumes.

We multiply by the basic normalized matrix $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)$ the noise vector $\mathbf{n}(t)$, as a result we get:

$$\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)\mathbf{n}(t)=\sqrt{2/T}\cos(\omega_c t)\mathbf{E}\mathbf{n}(t)=\sqrt{2/T}\cos(\omega_c t)\begin{bmatrix}n_0(t) & n_1(t) & n_2(t) & n_3(t)\end{bmatrix}^{\mathrm{T}}.$$

The received interference covariance matrix is calculated as

$$\mathbf{N}_{\mathrm{E}0123}=\mathrm{E}\left\{\frac{2}{T}\cos^2(\omega_c t)\begin{bmatrix}n_0(t)\\ n_1(t)\\ n_2(t)\\ n_3(t)\end{bmatrix}\begin{bmatrix}n_0(t) & n_1(t) & n_2(t) & n_3(t)\end{bmatrix}\right\}=\begin{bmatrix}\sigma_0^2 & 0 & 0 & 0\\ 0 & \sigma_1^2 & 0 & 0\\ 0 & 0 & \sigma_2^2 & 0\\ 0 & 0 & 0 & \sigma_3^2\end{bmatrix},$$

where $\mathrm{E}\{\cdot\}$ – means the calculation of the mathematical expectation.

The index of the covariance matrix shows that the matrix was obtained after multiplying the noise vector by $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c,t)$ and the order of the noise in the resulting vector corresponds to the indices of the original noise 0, 1, 2, 3.

We multiply the noise vector by the basic normalized matrix $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)$, as a result we get the noise vector

$$\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c,t)\mathbf{n}(t)=-\sqrt{2/T}\sin(\omega_c t)\mathbf{I}\mathbf{n}(t)=\sqrt{2/T}\sin(\omega_c t)\begin{bmatrix}-n_1(t) & n_0(t) & n_3(t) & -n_2(t)\end{bmatrix}^{\mathrm{T}}.$$

The covariance matrix of the obtained noise vector will look like:

$$\mathbf{N}_{\mathrm{I}1032}=\mathrm{E}\left\{\frac{2}{T}\sin^2(\omega_c t)\begin{bmatrix}-n_1(t)\\ n_0(t)\\ n_3(t)\\ -n_2(t)\end{bmatrix}\begin{bmatrix}-n_1(t) & n_0(t) & n_3(t) & -n_2(t)\end{bmatrix}\right\}=\begin{bmatrix}\sigma_1^2 & 0 & 0 & 0\\ 0 & \sigma_0^2 & 0 & 0\\ 0 & 0 & \sigma_3^2 & 0\\ 0 & 0 & 0 & \sigma_2^2\end{bmatrix}.$$

As you can see, the order of interference in the vector has changed and corresponds to indices 1, 0, 3, 2.

We now multiply the noise vector by the basic normalized matrix $\bar{\mathbf{J}}^{\mathrm{T}}(\omega_c,t)$:

$$\bar{\mathbf{J}}^{\mathrm{T}}(\omega_c,t)\mathbf{n}(t)=-\sqrt{2/T}\sin(\omega_c t)\mathbf{J}\mathbf{n}(t)=\sqrt{2/T}\sin(\omega_c t)\begin{bmatrix}-n_2(t) & -n_3(t) & n_0(t) & n_1(t)\end{bmatrix}^{\mathrm{T}}.$$

The covariance matrix will be equal to

$$\mathbf{N}_{\mathrm{J}2301}=\mathrm{E}\left\{\frac{2}{T}\sin^2(\omega_c t)\begin{bmatrix}-n_2(t)\\ -n_3(t)\\ n_0(t)\\ n_1(t)\end{bmatrix}\begin{bmatrix}-n_2(t) & -n_3(t) & n_0(t) & n_1(t)\end{bmatrix}\right\}=\begin{bmatrix}\sigma_2^2 & 0 & 0 & 0\\ 0 & \sigma_3^2 & 0 & 0\\ 0 & 0 & \sigma_0^2 & 0\\ 0 & 0 & 0 & \sigma_1^2\end{bmatrix}.$$

The order of interference in the vector corresponds to the indices 2, 3, 0, 1.

When multiplying the noise vector by $\bar{\mathbf{K}}^{\mathrm{T}}(\omega_c,t)$, we get

$$\bar{\mathbf{K}}^{\mathrm{T}}(\omega_c,t)\mathbf{n}(t)=-\sqrt{2/T}\sin(\omega_c t)\mathbf{K}\mathbf{n}(t)=\sqrt{2/T}\sin(\omega_c t)\begin{bmatrix}-n_3(t) & n_2(t) & -n_1(t) & n_0(t)\end{bmatrix}^{\mathrm{T}},$$

with covariance matrix

$$\mathbf{N}_{\mathrm{K}_{3210}} = \mathrm{E}\left\{ \frac{2}{T}\sin^2(\omega_c t) \begin{bmatrix} -n_3(t) \\ n_2(t) \\ -n_1(t) \\ n_0(t) \end{bmatrix} \begin{bmatrix} -n_3(t) & n_2(t) & -n_1(t) & n_0(t) \end{bmatrix} \right\} = \begin{bmatrix} \sigma_3^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_1^2 & 0 \\ 0 & 0 & 0 & \sigma_0^2 \end{bmatrix}$$

and interference order 3, 2, 1, 0.

As can be seen, the noise dispersions do not change, since the basis matrices (18) are orthogonal. However, the order of the elements in the noise vector is changed. Let us depict the order of the effect of interference on signal elements in the form of diagrams.

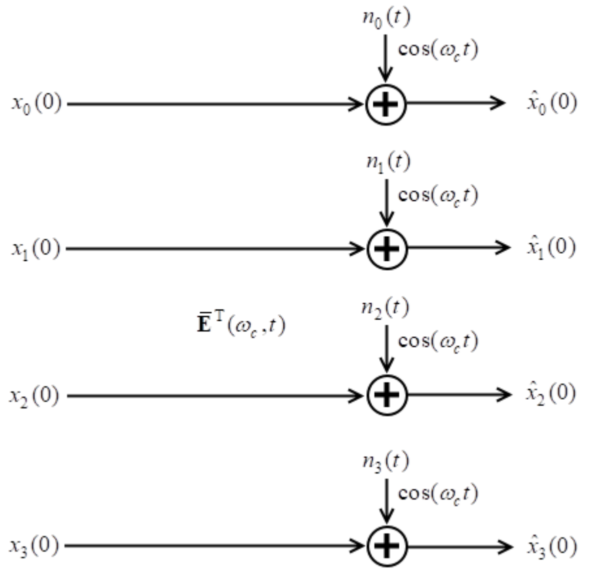


Figure 13 a) Multiplying the noise vector by matrix $\bar{\mathbf{E}}^{\mathrm{T}}(\omega_c, t)$ with extracting the cosine component.

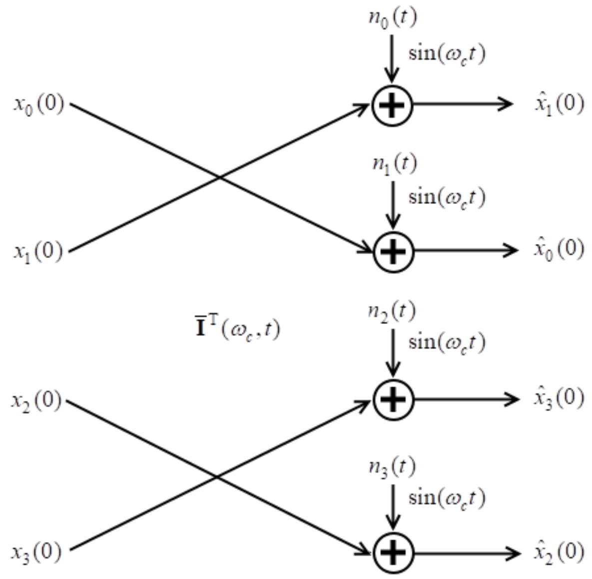


Figure 13 b) Noise vector multiplication by matrix $\bar{\mathbf{I}}^{\mathrm{T}}(\omega_c, t)$ with sine component extracted.

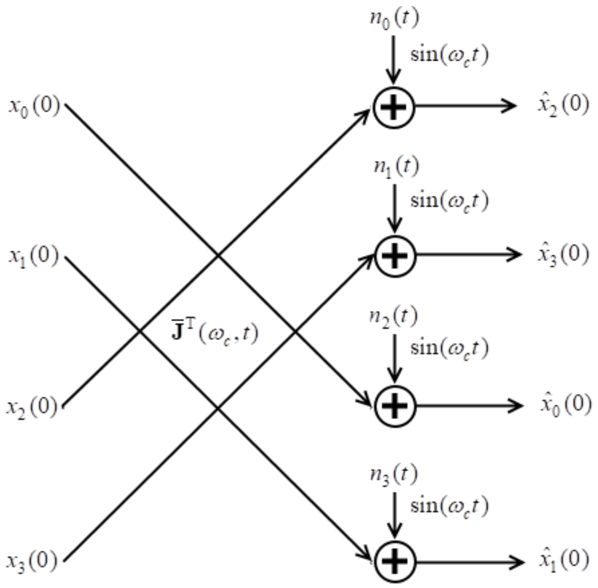


Figure 13 c) Noise vector multiplication by matrix $\bar{\mathbf{J}}^{\mathrm{T}}(\omega_c, t)$ with sine component extracted.

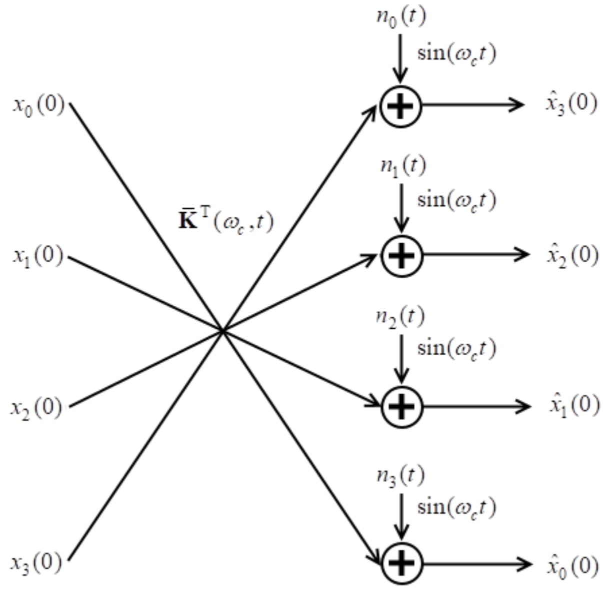


Figure 13 d) Noise vector multiplication by matrix $\bar{\mathbf{K}}^{\mathrm{T}}(\omega_c, t)$ with sine component extracted.

Figure 13 The order of the impact of interference on the elements of the signal vector when multiplied by normalized basis matrices.

Figure 13 schematically shows the order of influence of the elements of the interference vector on the elements of the signal vector after multiplication by the basis matrices (18). The figures also show that the result is fed to the integrator and the decision circuit to obtain a value of $\hat{\mathbf{x}}$. By combining the presented schemes, we obtain a MIMO 4×4 scheme in which the quantification of each information element can be obtained under the influence of various interferences.

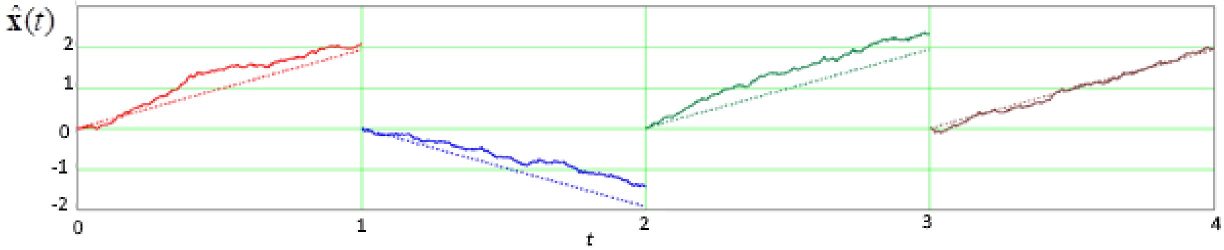


Figure 14 The result of integration when multiplying the received vector by the basic matrices, followed by summing the estimates of the elements. The solid lines show the result with interference, and the dotted lines show the result without interference.

Figure 14 shows the result of integrating the received vector (17) after multiplying by the basis matrices (18) and summing the estimates of the elements of the information vector in accordance with the MIMO 4×4 scheme shown in Figure 13. When summing similar estimates, the noise is also summed up. In accordance with the 4×4 MIMO scheme shown in Figure 13, we write the interference matrix in the form:

$$\mathbf{N} = \sqrt{\frac{2}{T}} \begin{bmatrix} n_0(t)\cos(\omega_c t) & n_1(t)\sin(\omega_c t) & n_2(t)\sin(\omega_c t) & n_3(t)\sin(\omega_c t) \\ -n_1(t)\sin(\omega_c t) & n_0(t)\cos(\omega_c t) & n_3(t)\sin(\omega_c t) & -n_2(t)\sin(\omega_c t) \\ -n_2(t)\sin(\omega_c t) & -n_3(t)\sin(\omega_c t) & n_0(t)\cos(\omega_c t) & n_1(t)\sin(\omega_c t) \\ -n_3(t)\sin(\omega_c t) & n_2(t)\sin(\omega_c t) & -n_1(t)\sin(\omega_c t) & n_0(t)\cos(\omega_c t) \end{bmatrix}. \quad (25)$$

As you can see, the noise matrix (25) is a quaternion, so it is orthogonal. The interference covariance matrix is calculated as

$$\mathrm{E}\left\{\mathbf{N}^{\mathrm{T}}\mathbf{N}\right\} = \mathrm{diag}\left[\sigma^2 \quad \sigma^2 \quad \sigma^2 \quad \sigma^2\right] = \sigma^2\mathbf{E}.$$

Since the noise covariance matrix is diagonal, the noise acting on the elements of the information vector is not correlated.

## V. CONCLUSION

Thus, modulation of the information vector using a quaternion carrier, which acts as a channel matrix for the 4×4 MIMO scheme, made it possible to apply the technology of separate reception of information elements using quaternion basis matrices. At the same time, it became possible to evaluate each element of the information vector under the influence of various interferences. When adding the estimates of the same elements of the information vector, we obtained a signal-to-noise gain of 4 times for each element and 16 times in general for the information vector.

It is known that in the MIMO scheme, the decision on the value of information vectors is possible on the basis of individual elements of the information vector. The MIMO 4×4 scheme, in this case, is 4 transmission lines with BPSK. In this case, we will obtain a 4 times gain in signal-to-noise ratio in each line and an increase in throughput by the corresponding number of times.

It is also possible to use the calculation of the Frobenius norm when making a decision and calculate the maximum likelihood estimate. Since the minimum distance between the Frobenius norms of different information vectors is 8, the coefficient of divisibility of estimates increases by 4 times compared to BPSK. Compared to the MIMO 2×2 scheme based on a complex carrier, the divisibility factor will increase by 2 times [8].

Note that for wireless communication systems, to implement the MIMO 4×4 scheme using a quaternion carrier, it is sufficient to have one antenna for transmitting and one for receiving. In addition, it will be possible to use the MIMO scheme based on a complex or quaternion carrier in wired communication channels and obtain a corresponding increase in throughput.

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