Allocating Reserves in Active Distribution Systems for Tertiary Frequency Regulation

Mukesh Gautam, *Student Member, IEEE*, Mohammad MansourLakouraj, *Student Member, IEEE*, Narayan Bhusal, *Student Member, IEEE*, Mohammed Benidris, *Senior Member, IEEE*, and Hanif Livani, *Senior Member, IEEE*,
Department of Electrical & Biomedical Engineering, University of Nevada, Reno, Reno, NV 89557
{mukesh.gautam, mansour, bhusalnarayan62}@nevada.unr.edu, {mbenidris, hlivani}@unr.edu

***Abstract*—This paper proposes a cooperative game theory-based approach for reserve optimization to enable distributed energy resources (DERs) participate in tertiary frequency regulation. Tertiary frequency regulation schemes ensure that reserve requirements of primary and secondary frequency regulation are fulfilled with a minimum cost. While the available reserve from a single distribution system may not suffice tertiary frequency regulation, stacked reserve from several distribution systems can enable them participate in tertiary frequency regulation at scale. In this paper, a two-stage strategy is proposed to effectively and precisely allocate spinning reserve requirement from each DER in distribution systems. In the first stage, two types of characteristic functions are computed: worthiness index (WI) and power loss reduction (PLR). In the second stage, the equivalent Shapley values are computed based on the characteristic functions, which are used to determine distribution factors for reserve allocation among DERs. The effectiveness of the proposed method for allocating reserves among DERs is demonstrated through several case studies on modified versions of the IEEE 13-node and 33-node distribution systems.**

***Index Terms*—Co-operative game theory, distributed energy resources, Shapley value, tertiary frequency regulation.**

I. Introduction

Integration of distributed energy resources (DERs) has brought several challenges and benefits to power grid operation including frequency control and regulation. Frequency deviation occurs when there is an imbalance between the generation and load, which can happen due to several factors including faults, large load changes, generating unit tripping and islanding parts of the grid [1]. In such scenarios, frequency regulating schemes come into play to compensate frequency deviations. For tertiary frequency regulation, under vertically integrated monopolistic structure of utilities, system operators set operating points of individual generators based on an optimal power flow (OPF) solution, which minimizes the overall operating cost of generation subjected to network and reserve constraints. On the other hand, in deregulated power systems, the main function of tertiary frequency regulation schemes is to maximize the net social welfare through allocating adequate spinning reserve from generators or DERs participating in primary and secondary frequency regulation [2]. Although the contribution of a single DER in frequency control and regulation is not significant, the accumulated contribution from a fleet of DERs can enable them to collectively participate in frequency control and regulation. However, allocating impactful reserves from DERs is a challenging task and requires flexible and efficient solutions.

Various methods have been presented in the literature for tertiary frequency regulation and control in transmission and distribution systems. An approach for optimal tertiary frequency control has been proposed in [3], which also considers regulation based on electricity market. A model predictive control (MPC)-based approach has been proposed in [4] for the activation of tertiary frequency control reserves. A mixed integer linear programming-based optimization tool has been proposed in [5] for the activation of tertiary frequency control reserves at transmission level. In [6], the co-optimization of energy and reserve has been performed in a standalone microgrid considering uncertainties associated with renewable energy sources and loads. In [7], a data-driven approach for the estimation of secondary and tertiary reserve has been presented and tested on a real-life case study. Although several methods and algorithms have been developed and employed for tertiary frequency regulation and control of transmission systems and microgrids, allocating reserves from active distribution systems for the tertiary frequency regulation is still a challenge.

Cooperative game theory-based approaches have been successfully applied in various fields of power systems. A cooperative game theory-based approach has been implemented in [8] for loss reduction allocation of distributed generation using Shapley values. A cooperative game theory-based approach has been proposed in [9] for under frequency load shedding control. A cooperative game theory-based approach for computing participation factors of distributed slack buses has been proposed in [10]. For tertiary frequency regulation, the cooperative game theoretic approaches based on the Shapley value can ensure that the total available reserve is fairly distributed among different DERs taking into account their marginal contributions.

This paper proposes a cooperative game theoretic two-stage approach for fair allocation of reserves among DERs. In the first stage, two types of characteristic functions, viz., worthiness index (WI) and power loss reduction (PLR), are computed for each set of possible coalitions of participating DERs. The equivalent Shapley values and hence distribution factors of DERs are determined in the second stage, which are utilized for the allocation of reserves among DERs. The effectiveness of the proposed approach is demonstrated through case studies

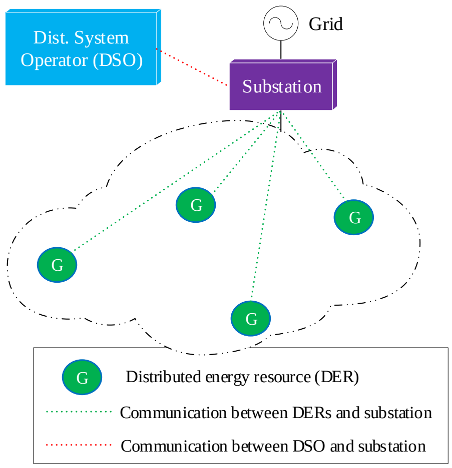

Fig. 1. The Layout of the Proposed Cooperative Game Theoretic Approach

on several test systems such as the IEEE 13-node and the 33-node distribution test systems.

The rest of the paper is organized as follows. Section II describes the cooperative game theory including the Shapley value. Section III explains the proposed approach for tertiary frequency regulation. Section IV presents case studies on the modified IEEE 13-node and 33-node distribution systems. Section V provides concluding remarks.

II. Cooperative Game Theory and Shapley Value

A cooperative game (or coalitional game) is a special class of games in which each player forms alliances with other players to maximize its incentives. In each cooperative game, there are three components as follows: (a) a finite set of players $\mathcal{N}$, and (b) a real-valued set function V, called characteristic function, defined on all sub-sets of $\mathcal{N}$ and satisfies $V(\phi) = 0$.

In game theory terms, $\mathcal{N}$ is defined as the player set, and $V(S) : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is defined as the "worth" or "value" of coalition S, i.e., the total utility that members of S can acquire if a coalition is formed among themselves and the game is played without assistance from players in other coalitions.

A. The Core of a Cooperative Game

The set of feasible allocations that cannot be further improved through any other coalitions is referred to as the core. Generally, outcomes of a cooperative game are expressed as n-tuples of utility: $\alpha = \{\alpha^i : i \in \mathcal{N}\}$, called payoff vectors that are measured in some common unit of money [11].

The core is the set of imputations under which all sets of coalitions have values less than or equal to the sum of its members' payoffs. Thus, α is core if and only if,

$$\alpha.e^S \geq V(S), \forall S \subset \mathcal{N} \tag{1}$$

$$\alpha.e^{\mathcal{N}} = V(\mathcal{N}) \tag{2}$$

where e^S denotes the n-vector having $e_i^S = 1$ if $i \in S$ and $e_i^S = 0$ if $i \in \mathcal{N} - S$. Equation (1) is the stability (or coalitional rationality) criterion and equation (2) is the efficiency criterion.

B. The Shapley Value

The Shapley value fairly allocates the payoff among the players of the cooperative game. The Shapley value of a cooperative game is given as follows [12].

$$\psi_j(V) = \sum_{S \in 2^{\mathcal{N}}, j \in S} \frac{(|S|-1)!(n-|S|)!}{n!} [V(S) - V(S \backslash \{j\})] \tag{3}$$

where $n = |\mathcal{N}|$ is the total number of players.

The Shapley value satisfies the following axioms:

1) *Efficiency:* $\sum_{j \in \mathcal{N}} \psi_j(V) = V(\mathcal{N})$.
2) *Individual Rationality:* $\psi_j(V) \geq V(\{j\}), \forall j \in \mathcal{N}$.
3) *Symmetry:* If j and k are such that $V(S \cup \{j\}) = V(S \cup \{k\})$ for every coalition S not containing j and k, then $\psi_j(V) = \psi_k(V)$.
4) *Dummy Axiom:* If j is such that $V(S) = V(S \cup \{j\})$ for every coalition S not containing j, then $\psi_j(V) = 0$.
5) *Additivity:* If V and W are characteristic functions, then $\psi(V + W) = \psi(V) + \psi(W)$.

III. The Proposed Approach

In this section, an approach for the fair allocation of reserves among participating DERs is developed using a cooperative game theoretic approach. The task of allocation of reserves among DERs is regarded as a cooperative game and the participating DERs are regarded as players of the game. As a motivation for DERs to participate in tertiary frequency regulation, DERs are allowed to send their bid prices for reserves. DERs also send information related to their sellable capacity to a virtual aggregator at the substation, which then sends this information to system operators. Fig. 1 shows the layout of the proposed cooperative game theoretic approach to allocate reserves in an active distribution system for the tertiary frequency regulation.

The proposed cooperative game theoretic approach consists of two stages. In the first stage, the two types of characteristic functions are computed: WI and PLR. The consideration of two types of characteristic functions, here, is equivalent to consideration of two types of objective functions in an optimization problem. In the second stage, Shapley values are computed and then reserve allocations of DERs are determined. WI of each DER indicates the worth or value of each DER for the allocation of reserve so as to maximize the social welfare or benefit. The two factors are taken into consideration for computing WI of each DER, which are available capacity for reserve (ACR) and reserve bid price (RBP).

The ACR of the i^{th} DER available after energy market clearance is calculated based on the total capacity of the i^{th} DER available for selling, P_{ci}, and the accepted capacity of the i^{th} DER after clearing energy market, P_{ei}, as follows.

$$ACR_i = P_{ci} - P_{ei}, \tag{4}$$

The total available capacity for reserve (TACR) from n players (here, DERs) is calculated as follows.

$$TACR = \sum_{i=1}^{n} ACR_i, \tag{5}$$

If the reserve command received from the system operator, P_R, is equal to or higher than TACR, then all reserves are allocated without any optimization. But when P_R is less than TACR, the tertiary frequency controller or regulator allocates P_R optimally among the participating DERs.

The WI of the ith DER is defined as follows.

$$WI_i = \frac{ACR_i}{RBP_i}, \tag{6}$$

where ACR_i is ACR of the i^{th} DER and RBP_i is reserve bid price of the i^{th} DER.

WI of each DER acts as the first characteristic function of the proposed cooperative game and PLR acts as the second characteristic function. PLR is the difference between the active power loss of the system with DERs of a particular coalition and that without any DER.

Shapley values $\psi_{1,i}$ and $\psi_{2,i}$ are calculated using (3) based on each type of characteristic function of participating DERs and their coalitions. The normalized Shapley values corresponding to each characteristic function are then calculated as follows.

$$\psi_{1,i}^{norm} = \psi_{1,i} \Big/ \sum_{k=1}^{n} \psi_{1,k}, \tag{7}$$

$$\psi_{2,i}^{norm} = \psi_{2,i} \Big/ \sum_{k=1}^{n} \psi_{2,k}. \tag{8}$$

Using normalized Shapley values calculated in (7) and (8), the equivalent Shapley value of the i^{th} DER is computed as follows.

$$\psi_i^{eqv} = \frac{\psi_{1,i}^{norm} + \psi_{2,i}^{norm}}{2} \tag{9}$$

The distribution factor (DF) of the i^{th} DER is then calculated using (10). The distribution factors are utilized to distribute P_R among the participating DERs.

$$DF_i = \psi_i^{eqv} \Big/ \sum_{k=1}^{n} \psi_k^{eqv} \tag{10}$$

The allocated reserve of the ith DER is then determined as follows.

$$R_i = P_R \times DF_i \tag{11}$$

The proposed approach or the solution algorithm to determine reserve allocated for DERs for tertiary frequency regulation can be summarized as follows.

1) Read system data related to lines, loads, transformers, and DERs.
2) Read P_c, P_e, and RBP of each DER.
3) Determine available capacity for reserve (ACR) and worthiness index (WI) of each DER.
4) Enumerate all possible coalitions of DERs and compute two types of characteristic functions, viz., WI and PLR, of each coalition.
5) Compute Shapley values using (3) and normalized Shapley values using (7) and (8).
6) Compute the equivalent Shapley values using (9) and the distribution factors using (10).

TABLE I
DIFFERENT PARAMETERS OF DERS IN THE CASE OF THE MODIFIED IEEE 13-NODE SYSTEM

	DER 1	DER 2	DER 3
P_c (kW)	250	350	450
P_e (kW)	220	340	400
RBP ($/kW)	10	20	12

7) Determine the allocated reserves each DER using (11).

The flowchart of the proposed cooperative game theoretic approach is shown in Fig. 2.

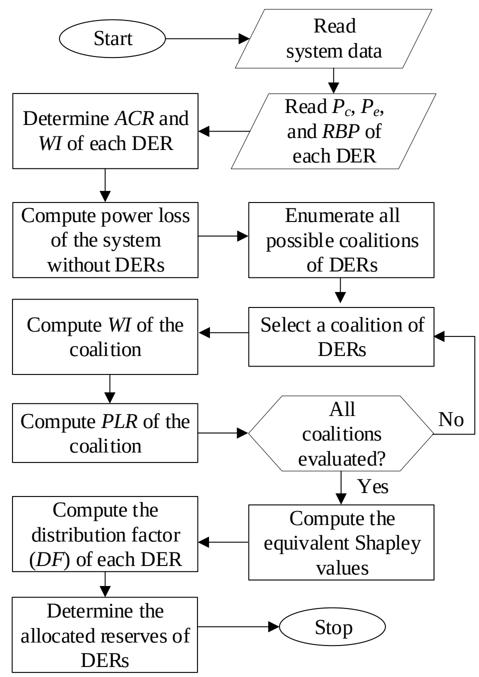

Fig. 2. Flowchart of the proposed approach

IV. Case Studies and Discussions

The proposed approach is implemented on the modified IEEE 13-node and the modified 33-node distribution systems. The IEEE 13-node system is a 4.16 kV distribution test system characterized by having overhead and underground lines, transformers, a voltage regulator, shunt capacitor banks, and unbalanced loading with constant current, power, and impedance models. The total real and reactive loads of this system are, respectively, 3577 kW and 1725 kVAr. For the detailed data of the IEEE 13-node system, the readers are referred to reference [13]. This system is modified by including three DERs at phase 1 of node 652, phase 2 of node 645, and phase 1 of node 675 as shown in Fig. 3.

The 33-node distribution test system is a 12.66 kV radial distribution system with 33 nodes and 32 branches [14]. The total active and reactive power loads are 3715 kW and 2300 kVAr, respectively. In this paper, the 33-node system is modified by placing four DERs of sellable capacity of 300 kW, 200 kW, 400 kW, and 200 kW at nodes 7, 14, 24, and 32, respectively, as shown in Fig. 4.

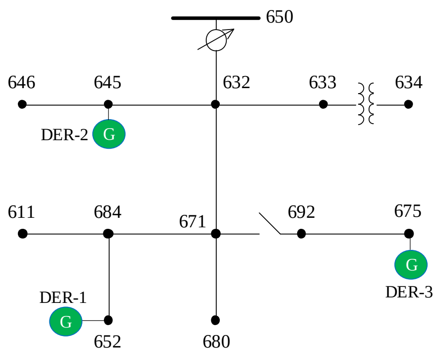

Fig. 3. The modified IEEE 13-node distribution system

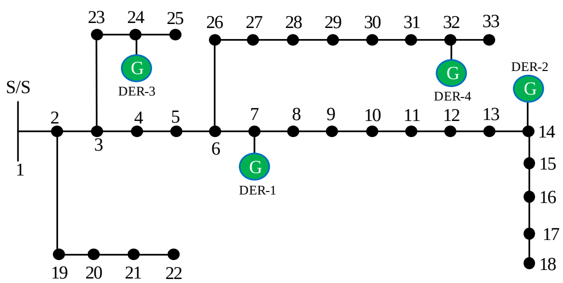

Fig. 4. The modified 33-node distribution system

TABLE II

DIFFERENT PARAMETERS OF DERS IN THE CASE OF THE MODIFIED 33-NODE SYSTEM

	DER 1	DER 2	DER 3	DER 4
P_c (kW)	300	200	400	200
P_e (kW)	280	190	390	170
RBP ($/kW)	10	15	12	10

TABLE III

CHARACTERISTIC FUNCTIONS: WORTHINESS INDEX (WI) AND POWER LOSS REDUCTION FOR THE MODIFIED IEEE 13-NODE SYSTEM

Coalitions of DERs	Worthiness Index (WI)	Power Loss Reduction (kW)
1	3.00	2.60
2	0.50	0.29
3	4.17	4.44
1,2	3.50	2.89
1,3	7.17	6.87
2,3	4.67	4.73
1,2,3	7.67	7.17

The proposed approach starts by by collecting information about sellable capacity (P_c), energy market clearing capacity (P_e), and RBP of each DER. The ACR of each DER is then calculated. For the case of the modified IEEE 13-node system, the sellable capacities of DERs are 250 kW, 350 kW, and 450 kW. The energy market clearing capacities DERs for a particular timestamp under consideration are 220 kW, 340 kW, and 400 kW. Reserve bid prices of DERs are $10/kW, $20/kW, and $12/kW. These values are shown in Table I. Similarly, the different parameters of DERs in the case of the modified 33-node system are shown in Table II.

TABLE IV

CHARACTERISTIC FUNCTIONS: WORTHINESS INDEX (WI) AND POWER LOSS REDUCTION FOR THE MODIFIED 33-NODE SYSTEM

Coalitions of DERs	Worthiness Index (WI)	Power Loss Reduction (kW)
1	2.00	1.66
2	0.67	1.36
3	0.83	0.44
4	3.00	3.73
1, 2	2.67	3.01
1, 3	2.83	2.10
1, 4	5.00	5.37
2, 3	1.50	1.80
2, 4	3.67	5.08
3, 4	3.83	4.17
1, 2, 3	3.50	3.45
1, 2, 4	5.67	6.71
1, 3, 4	5.83	5.80
2, 3, 4	4.50	5.51
1, 2, 3, 4	6.50	7.14

TABLE V

DISTRIBUTION FACTORS AND ALLOCATED RESERVES OF DERS FOR THE MODIFIED IEEE 13-NODE SYSTEM

DERs	Proposed Approach		Capacity-based Approach	
	Dist. Factors	Allocated Reserves (kW)	Dist. Factors	Allocated Reserves (kW)
1	0.3712	18.56	0.2381	11.90
2	0.0530	2.65	0.3333	16.67
3	0.5758	28.79	0.4286	21.43

As explained in Section II, a cooperative game is expressed in terms of a finite set and characteristic functions. In this paper, two types of characteristic functions, viz., WI and PLR, are considered. These characteristic functions are defined for all possible sets of coalitions. PLR for a coalition is the difference between power loss of the system without DERs and that with DERs of that coalition. For the case of the modified IEEE 13-node system, the possible sets of coalitions of DERs and the corresponding values of WI and PLR are shown in Table III. As shown in the table, WI is 3.00 and PLR is 2.60 kW for DER-1. For DER-2, WI is 0.50 and PRL is 0.29 kW. For the coalition of DER 1 and 2, WI is 3.50 and PLR is 2.89 kW. Similarly, for the case of the modified 33-node system, the possible sets of coalitions and the corresponding values of WI and PLR are shown in Table IV.

TABLE VI

DISTRIBUTION FACTORS AND ALLOCATED RESERVES OF DERS FOR THE MODIFIED 33-NODE SYSTEM

DERs	Proposed Approach		Capacity-based Approach	
	Dist. Factors	Allocated Reserves (kW)	Dist. Factors	Allocated Reserves (kW)
1	0.2689	13.45	0.2727	13.64
2	0.1457	7.28	0.1818	9.09
3	0.0948	4.74	0.3636	18.18
4	0.4907	24.53	0.1818	9.09

The characteristic functions shown in Table III and IV are used to calculate Shapley values using (3), and the Shapley values are normalized based on (7) and (8). The equivalent Shapley values are then computed using (9) and finally the distribution factor (DF) of each DER is calculated using

(10). For the case of the modified IEEE 13-node system, the distribution factors of DERs 1, 2, and 3, respectively, are 0.3712, 0.0530, and 0.5758; and the allocated reserves of DERs (for $P_R = 50$ kW) are 18.56 kW, 2.65 kW, and 28.79 kW as shown in Table V. Similarly, for the case of the modified 33-node system, the distribution factors of DERs 1, 2, 3, and 4, respectively, are 0.2689, 0.1457, 0.0948, and 0.4907; and the allocated reserves of DERs (for $P_R = 50$ kW) are 13.45 kW, 7.28 kW, 4.74 kW, and 24.53 kW as shown in Table VI.

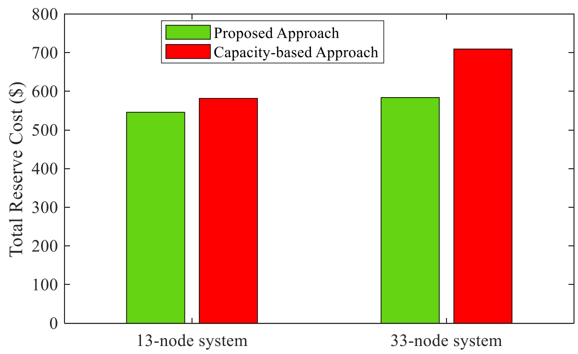

Fig. 5. Bar graph showing the comparison of total reserve costs

For comparison purpose, the allocated reserves computed using the proposed approach are compared with that using a DER-capacity-based approach in terms of total reserve cost. In the DER-capacity-based approach, the distribution factors are computed based on sellable capacities of DERs. The distribution factors and the allocated reserves of DERs obtained using DER-capacity-based approach for the modified IEEE 13-node and the modified 33-node systems are shown in Table V and Table VI respectively. In case of the modified IEEE 13-node system, the total reserve costs obtained using the proposed approach and the DER-capacity-based approach, respectively, are $584.06 and $709.52. Similarly, in case of the modified 33-node system, the total reserve costs obtained using the proposed approach and the DER-capacity-based approach, respectively, are $545.89 and $581.82. Fig. 5 shows the comparison of total reserve costs obtained using the proposed approach and the DER-capacity-based approach for both 13-node and 33-node systems. The result shows that the total reserve cost can be lowered when the reserves are allocated using the proposed approach. This is because of the use of Shapley values, which take marginal contribution of each player into account while allocating the reserves.

When the study is performed on a PC with 64-bit Intel i5 core, 3.15 GHz processor, 8 GB RAM, and Windows OS, the execution time of the proposed approach is around 0.25 seconds for the 13-node and 33-node systems.

V. Conclusion

In this paper, a cooperative game theoretic two-stage approach for tertiary frequency regulation of active distribution systems has been proposed. In the first stage, the two types of characteristic functions, viz., worthiness index (WI) and power loss reduction (PLR), of each set of coalition of DERs were computed. In the second stage, the equivalent Shapley values and distribution factors were computed for fair allocation of reserves among DERs. In developing the proposed approach, the following variables/parameters of DERs have been taken into consideration: sellable capacity, market clearing capacity, and reserve bid price. The case studies were performed on the modified IEEE 13-node and the modified 33-node distribution test systems. The results demonstrate the effectiveness of the proposed approach for tertiary frequency regulation compared to a DER-capacity-based approach.

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