A general model to describe the compression impact behavior of cellular materials

Massimiliano Avalle

Università degli Studi di Genova, Dipartimento di Ingegneria Meccanica, Energetica, Gestionale e dei Trasporti, Via all'Opera Pia 15, 16145 Genova, Italy

massimiliano.avalle@unige.it

Abstract

Cellular materials, commonly known as foams, are largely used for packaging and for the mitigation of the consequences of the impact of vehicles, for their ability to absorb the impact energy while maintaining the loads under critical values. The design of energy absorption systems and devices like car bumpers, road barriers and protections, helmets, packaging, it is useful to have mechanical models of the cellular materials with a higher level of predictability. Such models must describe the stress-strain response in the uniaxial compression first, as well as the tension and multiaxial loading; additionally, influencing factors as strain-rate, temperature, and density must be accounted for. This could help the designer in selecting optimal solutions to minimize weight for a given amount of energy to be dissipated.

In 2005 the authors [1] presented a model of the stress-strain behavior of some polymeric based foam materials, in quasi-static loading. The model was further improved and a complete description of the stress-strain behavior of some metal-based cellular materials including the influence of strain-rate and foam properties was published in [2].

In this paper, that model is applied to some polymeric based cellular materials. Among the considered materials, there are expanded polypropylene (EPP), expanded rigid polyurethane (PUR), expanded polystyrene (EPS). The work will give a review of the identified parameters for each family of materials, drawing the attention on the peculiarity of each of them. The parameters have been identified from previous experimental data from the same authors. Such tests consisted of quasi-static, and impact tests, at various loading/impact speed and energy.

The paper will try to show suitability of the proposed model for most cellular materials in compression, possibly any cellular expanded material, to be considered a valuable tool for design in the safety and packaging applications.

Keywords: Structural foams, Energy absorbing materials, Impact energy management

1. Introduction

Modeling the materials behavior is fundamental for the design of new products in many industrial fields. It is of particular importance for the transportation industry in general, to reduce weight and emissions through detailed design, but also to reduce the time-to-market and the cost of prototypes. Preliminary design towards virtual testing can accelerate the process but it requires accurate and detailed models, especially of the materials.

This is particularly critical in applications related to safety of people and goods where large deformations occur during impact events. The kinetic energy prior of the impact must be transformed deformation energy in the structures, vehicles or packaging depending on the application, possibly in a precisely controlled manner and limiting the amount of force generated.

Cellular materials, or foams, are particularly interesting for these applications due to their capacity to absorb energy through large deformations with a typically low density [3]. Foams are obtained through the expansion of most classes of materials, from polymers to metals, in a cellular structure with a relatively large quantity of voids. The cellular structures can have closed or open cells, bi-dimentional or tri-dimensional organization, more or less regular characteristics, and many other parameters that define, in the end, the foam behavior. The type of base material that is expanded and the relative density, ratio of the foam material to the base material, are usually the most important characteristics that define the foam behavior. In fact, as shown theoretically by Gibson and Ashby [3] and from experiments on polyethylene [5] and polyurethane [6] foams the relative density of the foam summarizes the influence of most of the cell characteristics: from an engineering point of view, this approximation and simplification is largely sufficient. The stress-strain characteristic of a foam of a certain class, that is from a given solid material, can be obtained from experimental tests fitting a few number of curves with a suitable model: this method was implemented in [2] and [4] for aluminum foams, in [7] for polyurethane, and in [8] for polypropylene. Additionally, other influencing factors should are taken into account such as: the strain-rate, like shown in [9] for polyurethane, in [10] and for polypropylene, and in [12] for polyimide; the temperature, in [12] for polyimide, for polypropylene in [8]; the biaxial and triaxial loading [13] and anisotropy [14]

In this paper a general models suitable to fit the behavior of most polymeric expanded materials is presented: the model, which is an advancement of previously proposed foam models proposed in [1]-[2] and [15] will include the effects of the strain-rate, temperature, and – particularly – the influence of the relative density. The model is validated against experimental results from many expanded polymeric materials and can be considered of large usability for most engineering applications.

2. Phenomenological models of the compression behavior of cellular materials

An original, simple, and effective model of the compression stress-strain behavior of foams was introduced by Rusch in 1969 [16]:

$$\sigma(\varepsilon) = a\varepsilon^{p} + b\varepsilon^{n} \qquad (1)$$

Among the disadvantages of this model there is the impossibility to describe a finite value of the slope when the strain tends to zero, and that large plateau ranges cannot be described.

Other models have tried to avoid these problems with different mathematical approximations. In [17] Avalle et al. suggested the following phenomenological model for polymeric foams to better describe the densification phase:

$$\sigma(e) = ae^{p} + b\left(\frac{e}{1-e}\right)^{n} \qquad (2)$$

Later, Avalle et al. in [18] proposed this approximation to improve description of the elastic to plateau transition:

$$\sigma(e) = A\{1-\exp[-(E/A)e(1-e)^{n}]\} + B\left(\frac{e}{1-e}\right)^{n} \qquad (3)$$

This model was further enhanced by including the strain-rate influence of by Jeong et al. [9]:

$$\sigma(e) = \left\{A[1-\exp(-(E/A)e(1-e)^{n})] + B\left(\frac{e}{1-e}\right)^{n}\right\}\left[1+(a+be)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] \qquad (4)$$

The model proposed here, merges the description from the Rusch model [16] for the densification, and from the Avalle et al. model [18] to describe the transition from the elastic range to the plateau. This new model derives from the model by Goga et al. in [15] that, for quasi-static loading, describes the stress-strain behavior as:

$$\sigma(\varepsilon) = \sigma_p[1-\exp(-m\varepsilon)] + \sigma_s\varepsilon + \sigma_D\varepsilon^{n} \qquad (5)$$

Where:

- σ_p plateau stress level
- σ_s linear hardening slope in the intermediate phase
- σ_D Rusch densification parameter
- m linear-plateau transition constant
- n Rusch densification exponent

The strain-rate effect and the influence of the density can be described in the evaluation of the material parameters σ_p, σ_s, and σ_D. For the strain-rate, it can be described by considering the influence in Equation (5) as for the Cowper-Symonds law:

$$\sigma_P = \sigma_{P,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_P(\rho)$$

$$\sigma_P = \sigma_{S,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_S(\rho)$$

$$\sigma_D = \sigma_{D,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_D(\rho) \qquad (6)$$

With:

- $\dot{\varepsilon}$ strain-rate value
- $\dot{\varepsilon}_0$ reference strain-rate value
- p strain-rate exponent
- $\sigma_{p,0}$ plateau stress level in static loading
- $\sigma_{s,0}$ linear hardening slope in the intermediate phase in static loading
- $\sigma_{D,0}$ Rusch densification parameter in static loading
- ρ density of the material
- $f_P(\rho)$ density function for the plateau stress level
- $f_S(\rho)$ density function for the intermediate phase
- $f_D(\rho)$ density function for the densification parameter

The well-known alternative models for the strain-rate effect of Johnson-Cook and Jeong [9] can be used: for the examined materials the Cowper-Symonds better fits the experimental results in the examined ranges of strain-rate.

The influence of the density, or better of the relative density, is included as modifying functions $f_P(\rho)$, $f_s(\rho)$, and $f_D(\rho)$: for the examined materials a power-law sufficiently describe this effect.

Further understanding of the proposed model reveals interesting insights of this description. The first term represents the elastic range and the transition from the elastic phase to the plateau, where the derivative of the stress-strain (5) curve is:

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = m\sigma_p \exp(-m\varepsilon) + \sigma_s\varepsilon + n\sigma_D\varepsilon^{n-1} \qquad (7)$$

The initial slope E of the stress-strain relation is equal to $m\,\sigma_p + \sigma_s$ for values of strain approaching zero, that is the initial foam elastic modulus. It is interesting to observe that this exponential description of the transition of elastic to plastic behavior is equivalent to the so-called universal law of elasto-plastic introduced by Wagoner and his colleague in several papers [19]-[22] for metallic and other types of materials.

The second term describes the progressive compaction of the foam cells. For some foams this term σ_s is near zero and the plateau is horizontal.

The third term describes the densification range as done in the Rusch model, and it appears sufficiently suitable for the expanded materials included in this paper and for other materials [2]. Whereas in that paper metal foams were considered, in the present work polymeric materials are addressed and the experimental result are fitted particularly well in a wide range of materials, strain-rate values, and densities.

3. Experimental tests

The experimental tests which are the basis of the current analysis were performed by the authors in many prior works [1], [17]-[18], [23]. The tests consisted in the uniaxial compression of cylindrical or cubic specimens. Cubic specimens had a side length of 50 mm (for example for expanded polypropylene, EPP; expanded polyphenylene oxide/polystyrene foam known as commercial Noryl-GTX® [1], [23]; rigid expanded polyurethane, PUR [18] and expanded aluminum, FOAMINAL® [24]. Cylindrical specimens with 100 mm diameter and 35 mm height were also used for expanded polystyrene, EPS, or a diameter 50 mm and height 75 mm for advanced pore morphology, APM®, aluminum foam [25]. Results from [24] and [25] were already covered in [2] and not used in this paper.

The tests were made at constant rate of compression in the case of quasi-static tests, normally at 50 mm/s corresponding to 0.02 s^{-1} engineering strain-rate. This rather low value of the speed of loading corresponds to quasi-static loading for all the materials, thus without strain-rate effect. These tests were conducted, typically, up to a compression value of 90% of the initial length or 0.9 engineering strain in compression. The curves of the load-stroke were measured and stored, to obtain engineering stress-engineering strain curves. Closer examination of the compression development proved that, in practice: 1) the transverse area of the specimens remains almost constant; 2) there is no localization of the deformation, that is, especially in quasi-static conditions, the strain is more or less uniform along the axial direction (save at great densification amounts, typically when the compression strain was more than 80%). From this, it can be concluded that: 1) the true stress in the cellular material σ can be considered the same as the engineering stress s, and evaluated from the original transverse area of the specimens; 2) the true strain ε could be evaluated based on the engineering strain e, that is the shortening of the specimen divided by the initial size, with the well known logarithmic expression:

$$\varepsilon = -\ln(1-e) \qquad (8)$$

The minus sign multiplying the value of the engineering stress e and multiplying the logarithm is because, as it is common practice with foam which are mainly loaded in compression, the strain in compression is considered positive. The stress in compression also is usually considered as positive, like a pressure. In equations (1)-(5), these conventions are also implicitly adopted.

The impact tests were done by the author together with co-authors [1], [17]-[18], [23] thanks to a testing facilities of the Vercelli campus of Politecnico di Torino, that is a free fall loading apparatus.

There, a falling mass, variable but usually maintained constant in the tests, is left free to fall and impact the sample. The initial, impact, speed then depends on the height of the fall: the impact energy, of course, is proportional to the impacting mass multiplied by height of the fall. Obviously, the speed could not be constant but decreasing to zero with stroke until the whole kinetic energy was absorbed and converted into strain energy. So, in those experiments, the reported strain-rate value is the initial, the ratio of the impact speed divided by the specimen height. Of course, in those cases the speed was not constant but decreasing to zero with stroke until the kinetic energy of the impacting mass was absorbed by the foam and transformed into strain energy. In those tests, the indicated value of strain-rate is its initial value, the ratio of the impact speed divided by the specimen height. To correctly value the evolution of strain-rate during the development of the compression, it was necessary to compute the instantaneous speed to rate its effect while fitting the experimental curves identifying the coefficients of the strain-rate influence in Equation (6). During the impact tests it was not possible to define a priori the maximum level of compression because the energy necessary to deform the material to a certain level depends on the material response itself: however, this is not a limitation of the method and does not have influence on the results.

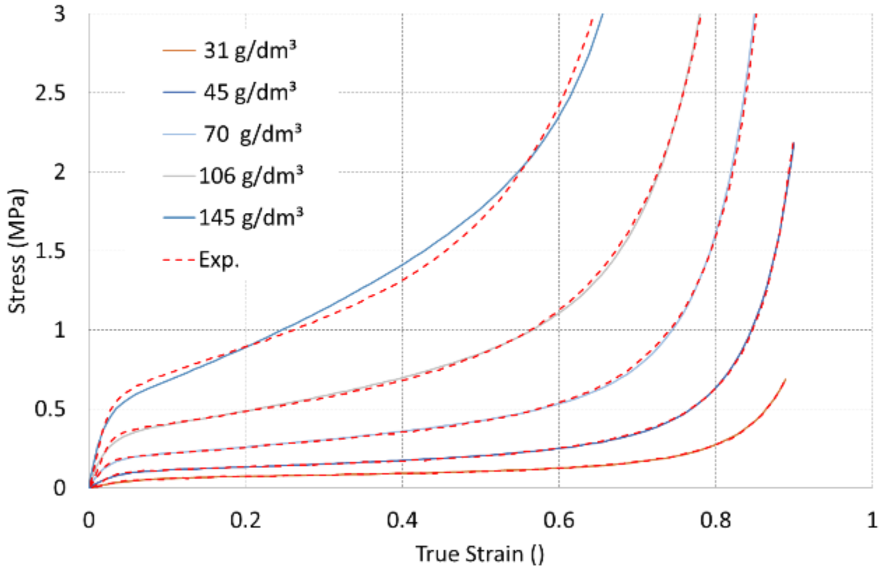
The numerical fitting and the coefficients were obtained, for each sample, by a simple non-linear regression analysis by minimizing the squared sum of the errors with an iterative solver.

3. Identification of the model coefficient for different materials by fitting the experimental results

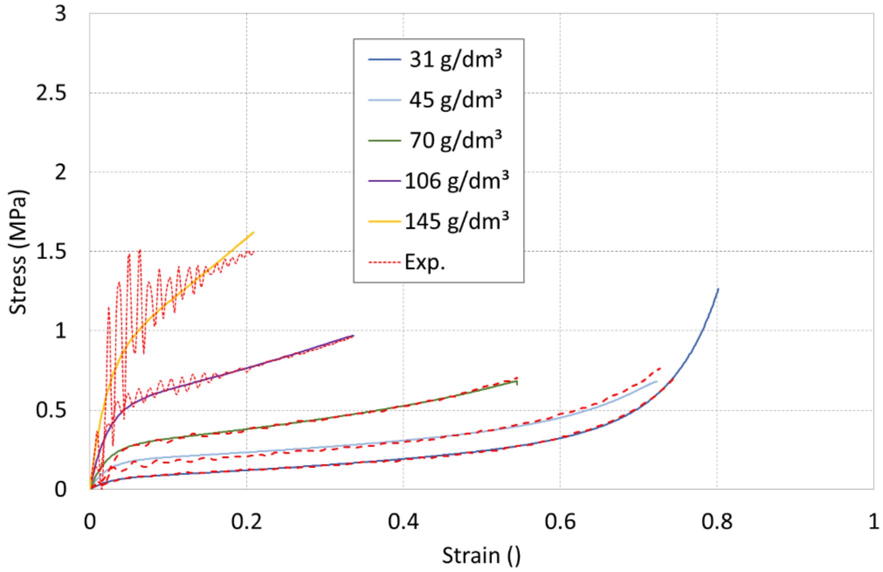
3.1 Expanded polypropylene (EPP)

Results from EPP samples were done by compression of a cubic specimen, 50 mm side, as reported in [1], [2], and [23]. Five values of the foam density, from around 30 to 150 g/dm^3, were tested, in quasi-static (at 0.02 s^{-1}) and impact (at 100 s^{-1} initial strain-rate) conditions. The experimental stress-strain curves were fitted by the model, and the best fit is shown in Fig. 1: for the sake of clarity, the result of one test for each foam density is show. However, for each density at least three samples were tested. For all the values of density, and strain-rate, the repeatability was high, less than some percent, and the fit very effective: experimental and model curves are almost perfectly overlaid in all tests.

In Fig. 2, the influence of the density on the three model parameters σ_{p0}, σ_{s0}, and σ_{D0} is described. The effect of the density modelled by Eq. (6) is well described by a power law. For what concerns the exponents m and n in Eq. (5) they resulted independent from the density and, on average, equal to m = 50 and n = 5.



(a)



(b)

Fig. 1. Experimental results compared with prediction with the proposed model for EPP: (a) static; (b) impact.

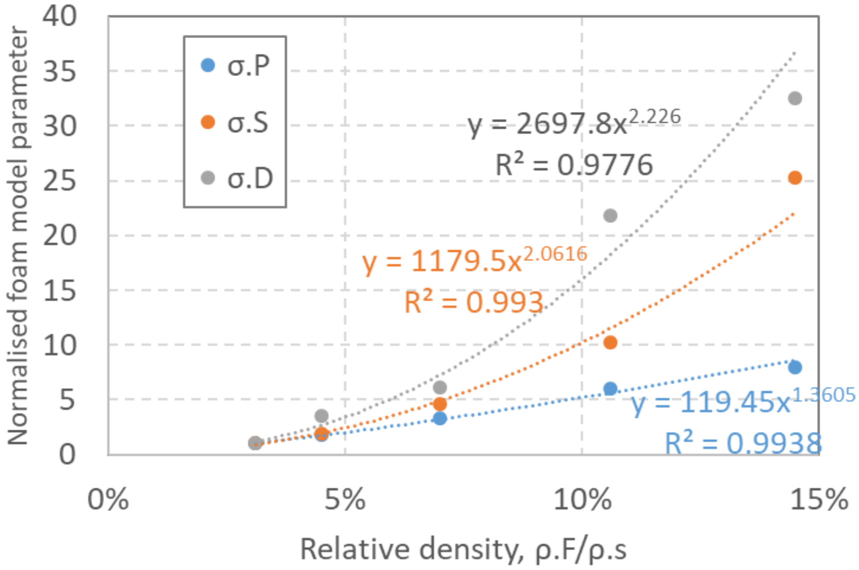
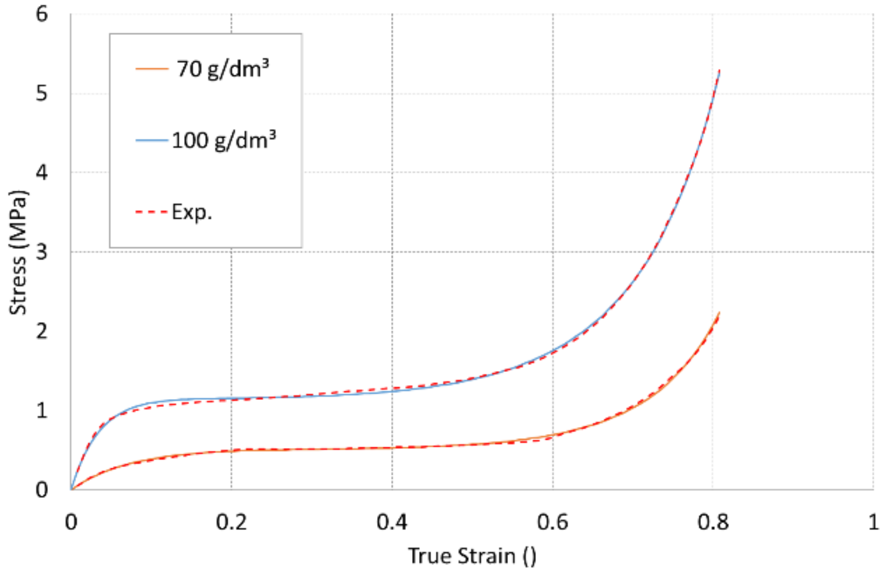


Fig. 2. Influence of the density on the material parameters σ_{p0}, σ_{s0}, and σ_{D0} for EPP

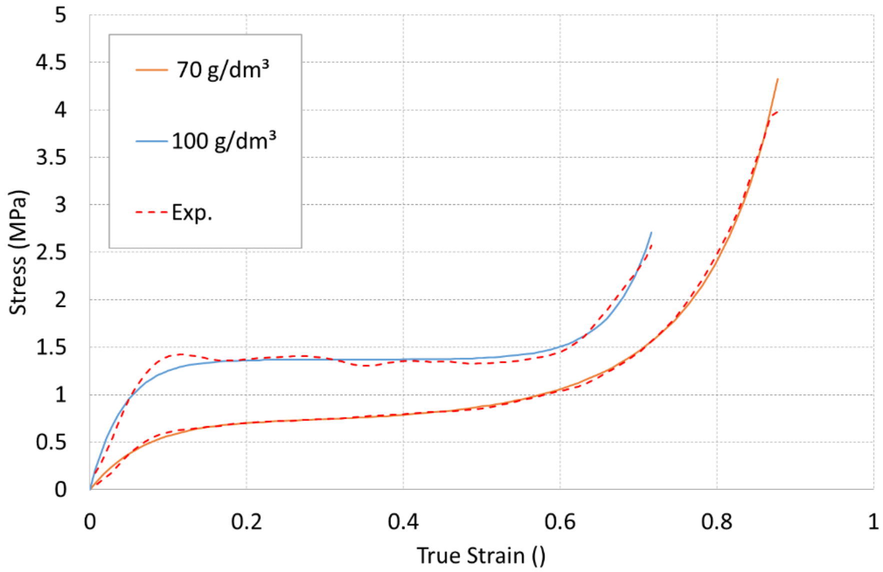
About the strain-rate effect, the coefficients are largely scattered and difficult to identify. For the examined EPP samples, a fair estimation is of P = 0.045 and $\dot{\varepsilon}_0$ = 1500 s^{-1}.

3.2 Expanded polyurethane (PUR)

Experimental results on PUR samples were done on cubic shaped samples too, in [1], [2], [23] or cylindrical samples [18] as discussed in section §2. For this class of material, only two densities were examined. The stress-strain material behavior in compression is shown in Fig. 3 by comparison of the experimental and model curves. In this case also, only the result from a single test is reported, although repeatability was also good for PUR samples. Curves of Fig. 3(a) correspond to quasi-static tests at 4.17×10^{-3} s^{-1} strain-rate, whereas the high-speed tests were carried out at 41.6 s^{-1}. The curves in Fig. 4(a) show the effect of the density on the stress parameters σ_p and σ_D (the slope stress parameter σ_s is zero for PUR foams) while in Fig. 4(b) the effect of the strain-rate is reported with the Cowper-Symonsa like approximation of Eq. (6).

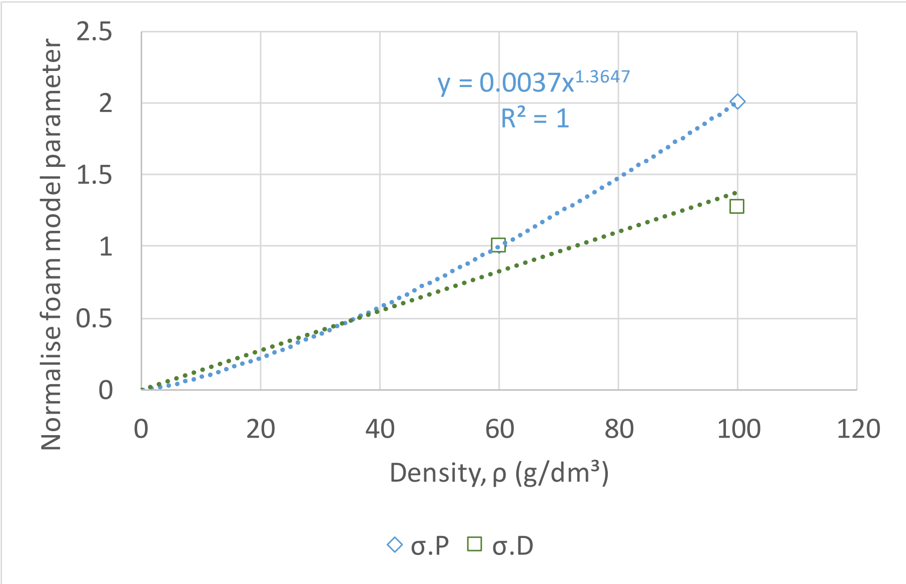


(a)



(b)

Fig. 3. Experimental results and fitted curves of the proposed model for PUR: (a) static; (b) dynamic



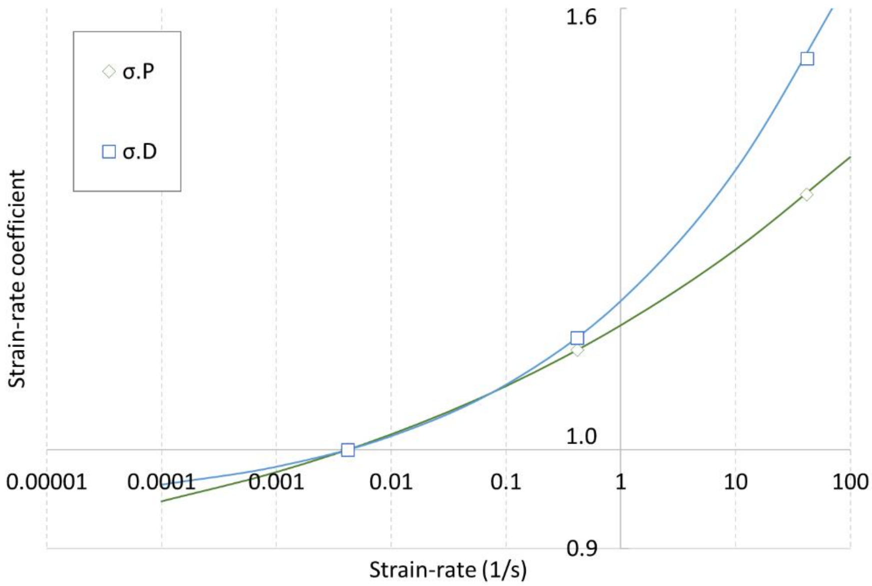


Fig. 4. Modification coefficients for PUR, effects of: (a) density; (b) strain-rate.

3.3 Expanded polystyrene (EPS)

Data from an EPS foam were obtained mostly with a cylindrical specimen [2] as already explained before. Four values of the density were considered (40, 50, 60, 70 kg/m^3). The stress-strain experimental curves are compared with the model in Fig. 5, with again the result from a single repetition only: repeatability was also good for PUR samples. Curves of Fig. 5(a) correspond to quasi-static tests at 0.02×10^{-3} s^{-1} strain-rate. The curves in Fig. 6 show the effect of the density on the stress parameters σ_P, σ_S and σ_D. The influence of the density in Eq. (6) is well approximated by the power law with the coefficients reported in Fig. 6. For the exponents of the stress-strain curves, again the scatter in the values of m and n is relatively high, but the values of m = 75 and of n = 5.2 can be used independently from the density. About the strain-rate effect, the coefficients are largely scattered and difficult to identify. For the examined EPS samples, a fair estimation of P is 0.030 and $\dot{\varepsilon_0}$ = 2000 s^{-1}.

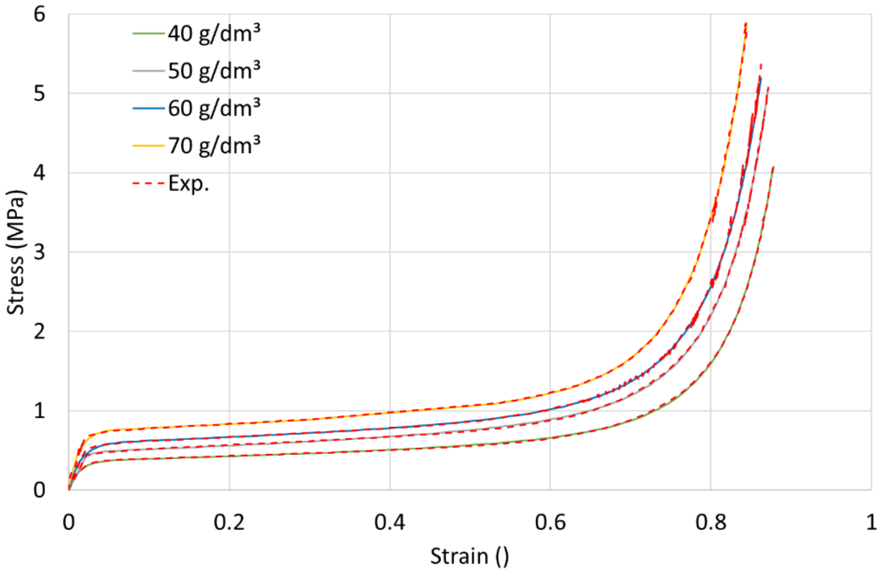


Fig. 5. Comparison of experimental results and the proposed model for EPS

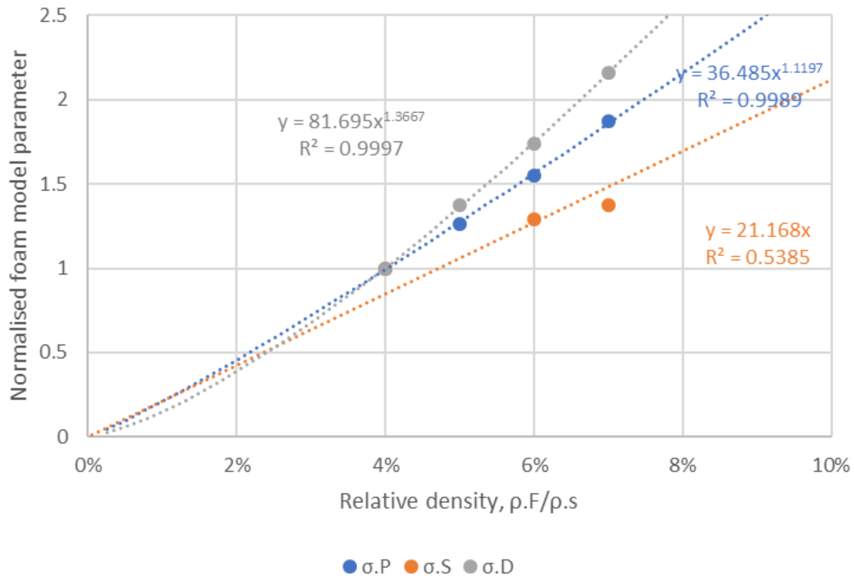


Fig. 6. Modification coefficients for EPS, effects of density

3.4 Modified polyphenylene oxide/polystyrene (PPO/PS, Noryl-GTX®)

Data from a Noryl-GTX®) foam were obtained on the same cubic sample 50 mm side [1], [23] as shown previously. Two values of the density were considered (70 and 100 kg/m^3). In Fig. 9 the comparison of the experimental stress-strain curves with the model approximation is shown (again the curve of one single test is shown: repeatability was also good for Noryl-GTX®). Curves of Fig. 7 correspond to quasi-static tests at 4×10^{-3} s^{-1} strain-rate. The curves in Fig. 8 show the effect of the density on the parameters σ_p, σ_S, and σ_D. The effect of strain-rate was moderate for this foam and more or less only changing the value of the σ_p level: the other parameters are almost not influenced by the strain-rate.

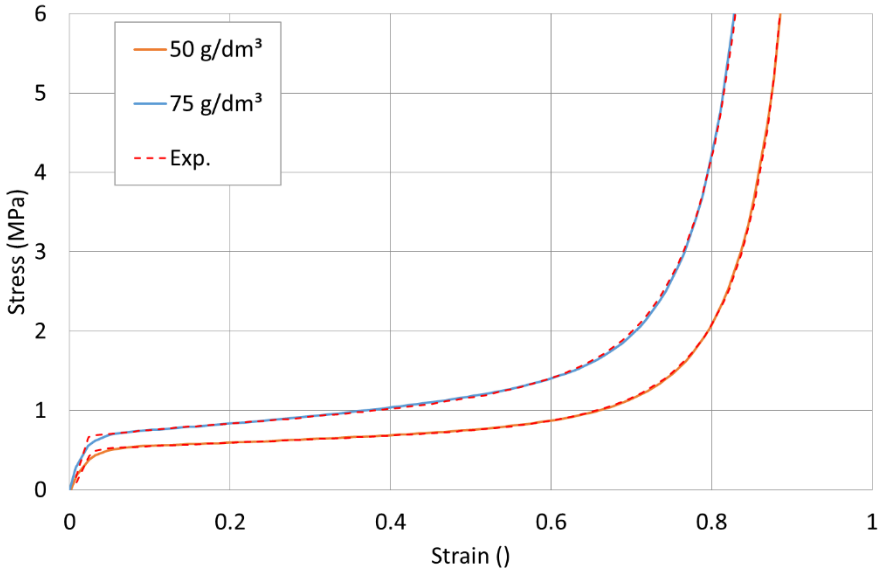


Fig. 7. Comparison of experimental results and the proposed model for Noryl-GTX®

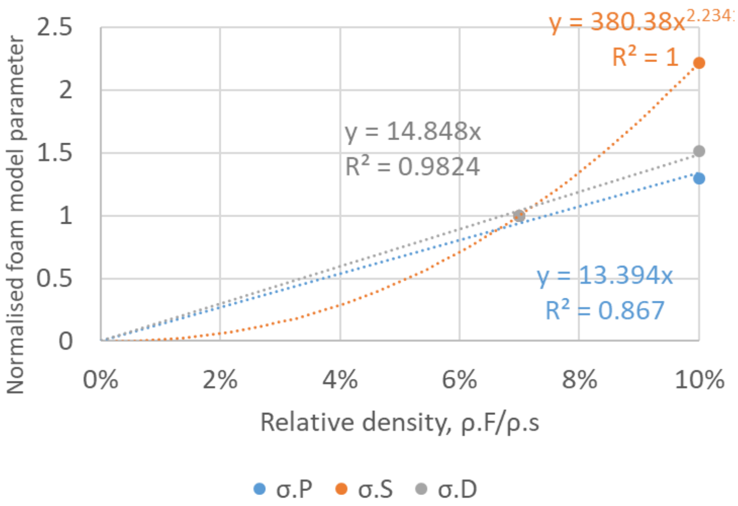


Fig. 8. Modification coefficients for Noryl-GTX®

4. Conclusions

A new model to predict the stress-strain characteristics of many types of foams has been presented, based on experimental data with different densities and in different loading conditions (strain-rate). The model fits very well the stress-strain curves of the tests and allows for a sound description of the effects of the foam density and of the strain-rate. The representation of the compressive stress-strain characteristic of these foams is based on the sum a three terms approximation: the first describe the elastic-plastic phase with a smooth transition; the second term adds the almost linear slope encountered in many materials after the onset of the plateau phase and before densification; the third term models the densification with a power law approximation similar to the approach first proposed by Rusch. Three parameters relative to the three terms and two coefficients are able to describe each foam material in given loading conditions. Strain-rate effects can then be considered in a multiplicative form by modifying the three parameters of the general model: from the analysis of several polymeric and aluminum foams, it appears that the effect of the strain-rate can be modeled as in the Cowper-Symonds law well known to describe this effect on the behavior of metals and other materials. The model parameters can also be expressed depending on the foam density: for the most part of cases a power-law is adequate. Values of the parameters and coefficients for some common foam materials were reported in the work.

The advantage of this almost universal model is that given a limited set of experimental tests, after proper identification of the parameters and coefficients, the material behavior in different conditions can be predicted. The designer can use these predictions during the development of new applications when the optimal value of the density is not known a priori. The model can be easily implemented and used for simulations of components made of virtually any foams of a given class of materials,

References

[1] Avalle M, Belingardi G, Ibba A. Mechanical models of cellular solids: parameters identification from experimental tests. In Impact Loading of Lightweight Structures, M. Alves & N. Jones (Editors). WIT Transactions on Engineering Sciences, WIT Press, ISSN 1743-3533. 2005, 49: 75-87.

[2] Avalle M, Belingardi G. A Mechanical Model of Cellular Solids for Energy Absorption, Advanced Engineering Materials. 2018, 1800457.

[3] Gibson, L.J., Ashby, M.F. Cellular Solids: Structure and Properties, Cambridge University Press, 1999.

[4] Butt, H.S.U., Xue, P., Hou, B. Evaluation of phenomenological model parameters using density dependent laws for prediction of mechanical response of cellular materials. Bulletin of the Polish Academy of Sciences, Technical Sciences 63(1) DOI:10.1515/bpasts-2015-0021

[5] Throne, J.L. Effect of Density on Compressive Stress-Strain Behavior of Uniform Density Closed Cell Foams. Journal of Cellular Plastics. 1985, 178-182

[6] Alzoubi, M.F., Al-Hallaj, S., Abu-Ayyad, M. Modeling of Compression Curves of Flexible Polyurethane Foam with Variable Density, Chemical Formulations and Strain Rates. Journal of Solid Mechanics. 2014, 16(1) 82-97

[7] Kirpluks, M., Cabulis, U., Andersons, J., Japins G. and Kalnins, K. Modeling the Effect of Foam Density and Strain Rate on the Compressive Response of Polyurethane Foams. SAE International Journal of Materials and Manufacturing. 2018. 11(2): 131-138.

[8] Morton, D.T., Reyes, A., Clausen, A.H., Hopperstad, O.S. Mechanical response of low density expanded polypropylene foams in compression and tension at different loading rates and temperatures, Materials Today Comm. 2020. 21, 100917 DOI 10.1016/j.mtcomm.2020.100917

[9] Jeong, K.Y., Cheon, S.S. & Munshi, M.B. A constitutive model for polyurethane foam with strain rate sensitivity. J Mech Sci Technol 26, 2033–2038 (2012). https://doi.org/10.1007/s12206-012-0509-1

[10] Kim, K.K, Cho, I.U., Jeong, K.Y, Chun, Y., Carman, G.P., Cheon. S.S. A coupled constitutive relation with impulse-momentum for compressive impact behavior of the expanded polypropylene foam. Polymer Engineering and Science. 2019, 59(1): 49-57 https://doi.org/10.1002/pen.24865

[11] Viot P. and Beani F. Polypropylene foam behavior under compressive loading at high strain rate. Proceedings of the 8th Conference on Structures Under Shock and Impact. 2004, WIT Press, 507-516.

[12] Yoshimoto, A., Kobayashi, H., Horikawa, K., Tanigak, K. The compressive behaviour and constitutive equation of polyimide foam in wide strain rate and temperature. EPJ Web of Conferences. 2015, 94: DOI: 10.1051/epjconf/20159401047

[13] Shim, V.P.W., Tu, Z.H., Lim, C.T. Two-dimensional response of crushable polyurethane foam to low velocity impact. International Journal of Impact Engineering. 2000, 24:6-7, 703-731.

[14] Linul, E., Mohavedi, N., Marsavina, L. The temperature and anisotropy effect on compressive behavior of cylindrical closed-cell aluminum-alloy foams. Journal of Alloys and Compounds. 2018 , 740:1172-1179 DOI:10.1016/j.jallcom.2018.01.102

[15] Goga, V., Hučko, Phenomenological Material Models of Foam Solids. B. J. Mech. Eng. Sci. 2015, 65, 5.

[16] Rusch K C. Energy-absorbing characteristics of foamed polymers, Journal of Applied Polymer Science. 1970, 14: 1433-1447.

[17] Avalle M, Belingardi G, Ibba A. Mechanical models of cellular solids: Parameters identification from experimental tests, International Journal of Impact Engineering. 2007, 34: 3-27.

[18] Peroni L., Avalle M, Peroni M. The mechanical behaviour of polyurethane foam: multiaxial and dynamic behaviour. Int. J. Materials Engineering Innovation, 2009, 1(2): 154-174.

[19] Chen Z, Bong H J, Li D, Wagoner R H. Elastic-plastic transition: a universal law, NUMIFORM 2016, MATEC Web of conferences. 2016, 80: 1-8.

[20] Chen Z, Bong H J, Li D, Wagoner R H. The elastic-plastic transition of metals, International Journal of Plasticity. 2016, 83: 178-201.

[21] Lee J, Lee Y-Y, Barlat F, Wagoner R H, Chung K, Lee M-G. Extension of quasi-plastic–elastic approach to incorporate complex plastic flow behavior – application to springback of advanced high-strength steels. International Journal of Plasticity 45 (2013) 140–159

[22] Sun L, Wagoner R H. Complex unloading behavior: Nature of the deformation and its consistent constitutive representation. International Journal of Plasticity 27 (2011) 1126–1144

[23] Avalle M, Belingardi G, Montanini R. Characterization of polymeric structural foams under compressive impact loading by means of energy-absorption diagram, International Journal of Impact Engineering. 2001, 25: 455-472

[24] Peroni L., Avalle M, Peroni M. The mechanical behaviour of aluminium foam structures in different loading conditions, International journal of impact engineering. 2008, 35: 644-658

[25] Stöbener K, Lehmhus D, Avalle M, Peroni L, Busse M. Aluminum foam-polymer hybrid structures (APM aluminum foam) in compression testing. International Journal of Solids and Structures 45 (2008) 5627–5641