Space-Time Fresnel Prism for Practical Dynamic Systems

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Abstract – **This paper presents the concept of a space-time Fresnel prism, a space-time version of the conventional Fresnel prism. This prism emulates the operation of a space-time interface, the building brick of any modulation-based space-time metamaterial, while exhibiting a finite size and being capable to operate in the continuous-wave regime, unlike previously reported space-time and time systems. The main characteristics of the prism are established and its validity is demonstrated by full-wave simulation. This prism may play an instrumental role in the practical development of space-time metamaterial systems, such nonreciprocal, matching beyond the Bode-Fan bound, temporal cloaking, etc.**

I. Introduction

Dynamic metamaterials and metasurfaces represent a rapidly growing research field that extends static media to time-varying and, more generally, spacetime media [1]. Time-varying metamaterials are formed by a *standing-wave* modulation of some host-medium parameter. Introducing time as an additional dimension, they break time-reversal symmetry, which leads to new effects and applications, such as the inverse prism [2], temporal aiming [3], temporal Faraday rotation [4], enhanced matching efficiency [5] and analog computing [6]. In contrast, space-time metamaterials are based on *traveling-wave* modulations. They include thus spacetime discontinuities in the medium parameters, without involving any net motion of matter. They may be considered as generalizations of purely time-varying metamaterials [7,8], and offer therefore more degrees of freedom for novel physics effects and applications [8–10].

The majority of dynamic metamaterials reported to date are of infinite extent, while practical structures, of course, can only be of finite in size. Some of the reported dynamic metamaterials have a finite size [7–10], but they can only manipulate short pulses, using discontinuities synchronized with the arrival time of the pulses on the structure [3–6].

This paper presents the concept of *space-time Fresnel prism* to resolve these issues, namely to make dynamic metamaterials finite in size and capable to handle continuous waves. It further provides related design rules and full-wave numerical demonstrations.

II. Proposed Fresnel Prism Concept: from Space-Space to Space-Time

The proposed space-time Fresnel prism is obtained by a space-to-time transformation of the conventional Fresnel prism [11], as illustrated in Fig. 1. The conventional Fresnel prism is a particular case of the Fresnel lens with the curvature being reduced to zero. It allows to dramatically reduce the volume, and hence the weight and cost of the regular prism, shown in Fig. 1(a), by reducing it to a much thinner structure, formed by a periodic repetition of geometrically similar sub-prisms, as shown in Fig. 1(b).

Figure 1(c) shows a space-time interface between two media, of refractive indices n_1 and n_2, and which might be seen as a space-time prism. Such a prism may be considered as the fundamental building brick of any space-time metamaterial [1]. Unfortunately, it suffers from a similar problem as the conventional prism [Fig. 1(a)]: it tends to be excessively thick (z direction in the figure); worse, it would need to have an infinite spatial extent to handle a continuous wave (CW), since such a wave is time-unlimited (ct direction in the figure). Applying the Fresnel concept that reduced the conventional prism to the Fresnel prism with the structure length being replaced by the interface duration leads to the proposed space-time Fresnel prism concept that is depicted in Fig. 1(d). This device has a *finite* thickness ($\infty \rightarrow z_\mathrm{F}$ in the figure) and hence resolves the problem of unlimited size issue plaguing the space-time interface in Fig. 1(c).

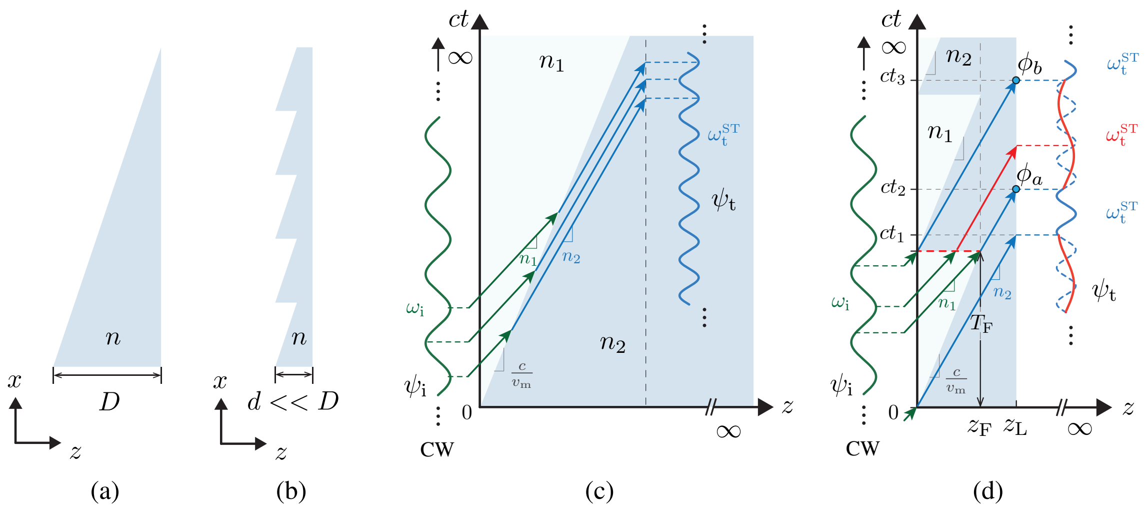


Fig. 1: Concept of the space-time Fresnel prism, obtained by transforming one of the two dimensions of its conventional counterpart from space to time. (a) Conventional prism and (b) and corresponding Fresnel prism. (c) Space-time prism (interface between media n_1 and n_2) and (d) corresponding space-time Fresnel prism. The pure-time duration has been made intentionally excessive for the sake of visualization.

The space-time Fresnel prism in Fig. 1(d) is imperfect because the space-time (ST) interface sections are interconnected by pure-time (PT) interface sections, which imply different (vertical) frequency transitions than the space-time (oblique) frequency transitions [8]. This spurious effect might be considered as the counterpart of the spurious diffraction effect at the corners of the conventional Fresnel prism [Fig. 1(b)].

III. Characterization of the Prism

We wish here to determine the fundamental constraints of the space-time Fresnel prism in Fig. 1(d), assuming an incident harmonic wave with angular frequency ω_i. For this purpose, we first calculate the ratio of the duration of the desired transmitted wave over that of the total transmitted wave; this ratio may be found geometrically in the figure as $\eta_t = (t_2 - t_1)/(t_3 - t_1) = 1 - \beta n_2$, where $\beta = v_\text{m}/c$, and is seen to decrease with increasing modulation velocity (v_m). We then calculate the ratio of the frequency of the desired transmitted wave over that of the total incident wave, which may be found from the dispersion diagram of the space-time interface as $\eta_\omega = \omega_\text{t}^\text{ST}/\omega_\text{i} = 1 + \beta(n_2 - n_1)/(1 - \beta n_2)$ [8], which is seen to increase as v_m increases[1]. Thus, the space-time Fresnel prism features a fundamental trade-off between the transmission time and the frequency change of the desired wave.

The load, placed at the position $z_\text{L} > z_\text{F}$, receives both the (desired) space-time part of the transmitted wave [blue curves in Fig.1(d)] and the (spurious) pure-time part of the transmitted waves [red curves in Fig.1(d)], which have different frequencies and amplitudes. The prism can be designed to ensure phase coherence with target wave of the uniform prism [Fig.1(c)], i.e., to produce the same phase at time t_3 as at time t_2, or $\phi_b(z_\text{L}, t_3) - \phi_a(z_\text{L}, t_2) = \omega_\text{t}^\text{ST}(t_3 - t_2) = 2\pi q$, where $q = 1, 2, 3, \ldots$.

Substituting the above result $\omega_\text{t}^\text{ST} = \omega_\text{i}(1 - \beta n_1)/(1 - \beta n_2)$ into the last relation for the phase difference and writing, from the geometry in Fig. 1(d), $t_3 - t_2 = z_\text{F} n_2/c$, yields then the following results for the thickness and the duration of each sub-prism:

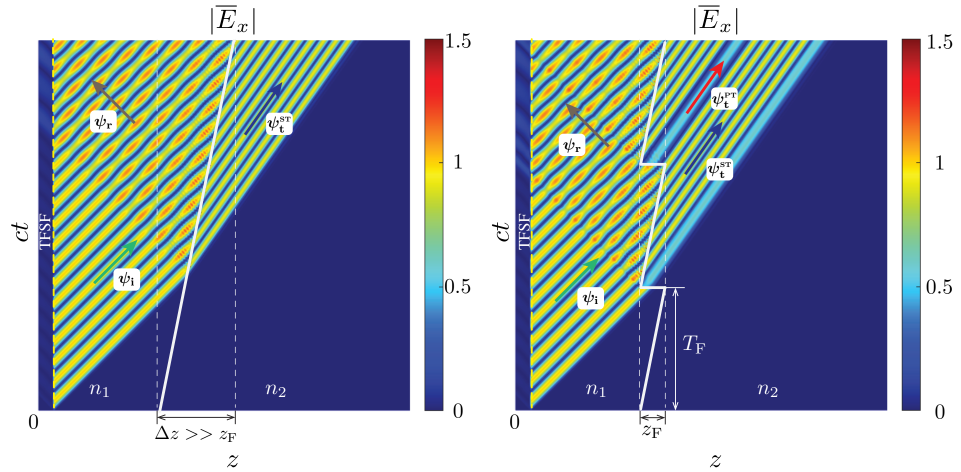
$$z_\text{F} = q\frac{1 - \beta n_2}{n_2(1 - \beta n_1)}\lambda_0 \quad \text{and} \quad T_\text{F} = \frac{z_\text{F}}{v_\text{m}} = q\frac{1 - \beta n_2}{\beta n_2(1 - \beta n_1)}T_0, \tag{1}$$

where $\lambda_0 = 2\pi c/\omega_\text{i}$ and $T_0 = 2\pi/\omega_\text{i}$ are the incident wavelength and period in free space, respectively. Note these spatial and temporal periods are dependent on ω_i, which limits the flexibility and applicability of the proposed space-time Fresnel prism.

[1]We assume here $n_1 < n_2$, corresponding to a frequency increase across the prism interface.

IV. Results and Discussion

Figure 2 plots the space-time diagrams of the normalized field $|\overline{E}_x| = |E_x/E_{x\mathrm{i}}|$ scattered on a uniform space-time prism [Fig. 2(a)] and its Fresnel counterpart [Fig. 2(b)] obtained by full-wave Finite-Difference Time-Domain (FDTD) simulation [12]. In both cases, the white solid lines are the trajectories of the modulation interfaces separating the two media. In Fig. 2(b), the Fresnel prism is designed according to the phase coherence formulas in Eqs. (1). The simulation results validate the theory.



(a) (b)

Fig. 2: FDTD computation of the space-time diagram showing the scattering on (a) a uniform space-time prism [Fig. 1(c)] and (b) a space-time Fresnel prism [Fig. 1(d)], for the parameters $n_1 = 1$, $n_2 = \sqrt{2}$ and $v_\mathrm{m} = 0.2c$.

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