A Differential Equation and the Lipschitz Condition for Pile Ultimate Bearing Capacity Based on Static Load Test

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Abstract

This paper provides an alternative method for determining the ultimate bearing capacity of pile foundations based on Static Load Test (SLT) results. A differential equation modelling the relation between loads and settlements of SLT is presented as well as its solution in a form of continuous function. The notion of Lipschitz continuity in mathematical analysis is adopted and is applied to the function for finding the most critical load, which is defined as the ultimate bearing capacity. For a validation, interpretations of an SLT data using Davisson's method and the method proposed in this paper are compared. The result shows that the interpretation using the proposed method is slightly greater than that of Davisson's and gives a valid result.

Keywords: Static Load Test, Lipschitz Function, Differential Equation, Bearing Capacity

1 Introduction

Static Load Test (SLT) is an in-situ loading test for pile foundations (Das, 2011). The primary reason for the test to be conducted is the unreliability of available prediction methods according to the pre-existing data (Das, 2011). The standard procedure of the SLT may be different for each country in accordance with its respective applicable codes.

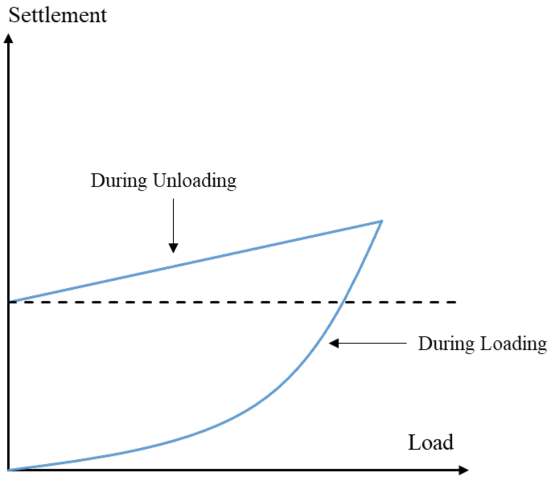

Figure 1: Load vs Settlement Graphs of SLT

The resulting data of SLT can be plotted for the values of loads vs settlements, and will have a typical form as illustrated by figure 1 (Das, 2011). During the loading process, the settlement tends to increase further up to the point of the maximum test load. During the unloading process, it tends to come back to its initial state before settlements even though it never really reaches its initial state and stops at some settlement value. Through out this paper, it will be assumed that the settlement values are expressed in milimeters (mm), while the load values are expressed in kilonewtons (kN).

The ultimate bearing capacity of piles can naturally be determined from the curve of the loading process. A loading point q_u in the load interval is the ultimate bearing capacity of pile if the differential of the settlement at q_u is significantly increase than that of points before q_u. This notion is made an axiom of the ultimate bearing capacity formulation in this paper.

As a preliminary, the readers are encouraged to explore the foundational concepts in mathematics that are incorporated in this paper, namely formal logic (Bergmann et al., 2014), set theory (Stoll, 1963), mathematical analysis (Rudin, 1964), metric spaces (Kreyszig, 1978; André, 2020) as well as Lipschitz functions (Searcóid, 2007) and Ordinary Differential Equation (ODE) (Kreyszig, 2011).

2 Lipschitz Continuity of Functions

Lipschitz continuity is a stronger continuity form of uniform continuity concept of functions (Searcóid, 2007), named after German mathematician Rudolf Lipschitz. This concept is applicable to functions between metric spaces (André, 2020; Kreyszig, 1978). The formal definition of Lipschitz continuity is presented as follows.

Definition 1 (Lispchitz Continuity)**.** Let $(X,\, d_X)$ and $(Y,\, d_Y)$ be metric spaces (Searcóid, 2007; André, 2020). A function (Stoll, 1963; Judson, 2012) $f : X \to Y$ is a Lipschitz function (Searcóid, 2007) if there exists some $K \geq 0$ such that

$$\forall s, t \in X\,,\ d_Y(f(s),\, f(t)) \leq K d_X(s,\, t)$$

holds. The function f is also referred to as a K-Lipschitz function (Searcóid, 2007). And the constant K is also referred to as a Lipschitz constant.

An alternative way in defining Lipschitz continuity involving difference expressions is presented on the following remark.

Remark 1. Let $(X,\, d_X)$ and $(Y,\, d_Y)$ be metric spaces (Searcóid, 2007; André, 2020). Suppose a function $f : X \to Y$. For a pair of points $u, v \in X$, the inequality

$$d_Y(f(u),\, f(v)) \leq K d_X(u,\, v)$$

is trivially satisfied by an equality in case $u = v$, as shown by

$$d_Y(f(u),\, f(v)) = d_Y(f(u),\, f(u)) = 0 = K \cdot 0 = K d_X(u,\, u) = K d_X(u,\, v)\,.$$

The expression above always hold as long as $(X,\, d_X)$ and $(Y,\, d_Y)$ are metric spaces. And it can be shown that f is K-Lipschitz if

$$\frac{d_Y(f(u),\, f(v))}{d_X(u,\, v)} \leq K$$

is satisfied with $u \neq v$.

A derivative notion from Lipschitz continuity is presented on the following definition. Note that this notion will be important to the formulation of the load vs settlement graph as a function on the following section.

Definition 2 (Locally Lipschitz)**.** Let $(X,\, d_X)$ and $(Y,\, d_Y)$ be metric spaces (Searcóid, 2007; André, 2020). A function $f : X \to Y$ is called locally K-Lispchitz if for every point $x \in X$, there exists a neighbourhood $U \subseteq X$ of x such that the restriction map (Stoll, 1963; Judson, 2012; Rudin, 1964) $f_U : U \to Y$ is K-Lipschitz.

3 Settlement Function

Suppose a data set of a SLT result. Suppose there are α incremental applications of loading, for some $\alpha \in \mathbb{N}$. Let $Q := \{q_0, q_1, \ldots, q_\alpha\}$ be the set of the load increments. Since Q is the set of load increments, then

$$\forall k \in \{1, \ldots, n\}\big[q_k \in [0, \infty)\big] \;\wedge\; \forall k \in \{1, \ldots, n\}\big[q_k - q_{k-1} > 0\big] \tag{1}$$

is necessarily satisfied. Also, suppose $q_0 := 0$, since at the initial state before the application of loading, settlements are not assumed to happen. Consequently, q_α is the maximum load of the SLT. Suppose a map $s : Q \to \mathbb{R}^+$ is the settlement map of the loading process, where

$$\mathbb{R}^+ := \{x \in \mathbb{R} \mid x \geq 0\}\,.$$

We may denote $s_k := s(q_k)$, for the settlement due to the load q_k, for every $k \in \{1, \ldots, \alpha\}$.

3.1 Differential Equation and Solution of Settlement

The map $s : Q \to \mathbb{R}^+$ is necessarily approximated with a continuous function $S : \mathbb{R}^+ \to \mathbb{R}^+$. The function S can be determined from a differential equation describing the load and settlement relationship from the SLT. Empirical results as represented by figure 1 show that the settlement keeps increasing as the load increases, and the rate of change of the settlement is proportional to the settlement itself. It can be inferred that

$$\frac{dS}{dq} \propto S\,.$$

Thus, a differential equation describing S is given by

$$\forall q \in \mathbb{R}^+\,,\; \frac{d}{dq}S(q) = \tau S(q) + \gamma\,, \tag{2}$$

where $\tau, \gamma \in \mathbb{R}$ are some constants. Note that equation 2 above is separable (Kreyszig, 2011), hence we obtain

$$\forall q \in \mathbb{R}^+\,,\; \frac{dS(q)}{\tau S(q) + \gamma} = dq\,.$$

By integrating the expression above, we have

$$\int \frac{dS}{\tau S + \gamma} = \int dq\,.$$

Suppose $W := \tau S + \gamma$. Then, $dW/dS = \tau$ and consequently, $dS = dW/\tau W$. And the expression above becomes

$$\frac{1}{\tau}\int \frac{dW}{W} = q + c_1 \qquad \therefore \ln|W| = \tau q + c_2\,,$$

which again implies

$$\forall q \in \mathbb{R}^+\,,\; S(q) = ce^{\tau q} - \frac{\gamma}{\tau}\,.$$

For convenience, let $r := \gamma/\tau$, and we have

$$\forall q \in \mathbb{R}^+\,,\; S(q) := ce^{\tau q} - r\,, \tag{3}$$

which is the general solution of S (Kreyszig, 2011).

A set of initial conditions shall be presented (Kreyszig, 2011) in order to determine r and τ. At the load of 0, the settlement has not occurred, thus $S(0) = 0$ holds. And it is assumed that there exists a point $\tilde{q} \in \mathbb{R}^+$ such that there exist $\nu, \xi \in \{1, \ldots, \alpha\}$ with $q_\nu = \tilde{q}$ and $q_\xi = 2\tilde{q}$. In other words, $\tilde{q}, 2\tilde{q} \in Q$ and $S(\tilde{q}) = s_\nu$ as well as $S(2\tilde{q}) = s_\xi$. Empirical results show that $2s_\nu < s_\xi$.

Follows from $S(0) = 0$, we have

$$0 = S(0) = ce^{\tau \cdot 0} - r = c - r$$

which implies that $c = r$. Thus,

$$\forall q \in \mathbb{R}^+ , \; S(q) = re^{\tau q} - r \, . \tag{4}$$

Follows from $S(\tilde{q}) = s_\nu$ and $S(2\tilde{q}) = s_\xi$, we have

$$re^{\tau \tilde{q}} - r = s_\nu$$

and

$$re^{\tau 2\tilde{q}} - r = s_\xi \, ,$$

which imply

$$\tau = \ln \sqrt[\tilde{q}]{\frac{s_\nu + r}{r}} = \ln \sqrt[2\tilde{q}]{\frac{s_\xi + r}{r}} \, . \tag{5}$$

Follows from equation 5 above, r can be solved as follows. By the equality on equation 5, we obtain

$$0 = \ln \sqrt[\tilde{q}]{\frac{s_\nu + r}{r}} - \ln \sqrt[2\tilde{q}]{\frac{s_\xi + r}{r}} = \ln \frac{\sqrt[\tilde{q}]{\frac{s_\nu + r}{r}}}{\sqrt[2\tilde{q}]{\frac{s_\xi + r}{r}}} \, ,$$

which implies

$$\frac{\sqrt[\tilde{q}]{\frac{s_\nu + r}{r}}}{\sqrt[2\tilde{q}]{\frac{s_\xi + r}{r}}} = e^0 = 1 \, .$$

By multiplying both sides with the denominator of the far left side, we obtain

$$\sqrt[\tilde{q}]{\frac{s_\nu + r}{r}} = \sqrt[2\tilde{q}]{\frac{s_\xi + r}{r}} \, .$$

And by raising both sides to the power of $2\tilde{q}$, we obtain

$$\begin{aligned} (s_\nu + r)^2 &= r(s_\xi + r) \\ s_\nu^2 + 2rs_\nu + r^2 &= rs_\xi + r^2 \, , \end{aligned}$$

which implies

$$s_\nu^2 + 2rs_\nu = rs_\xi \, .$$

And consequently, we obtain

$$r = \frac{s_\nu^2}{s_\xi - 2s_\nu} \, . \tag{6}$$

Since s_ν and s_ξ can be obtained from the SLT, thus, the exact value of r can be determined by applying equation 6. As r has already been determined, τ can be determined by applying

$$\tau = \frac{\ln\left(\frac{s_\nu}{r} + 1\right)}{\tilde{q}} \, , \tag{7}$$

which is implied from equation 5. Thus, S can be determined by applying equation 4. A further analysis on the settlement function S will be presented on the following subsection.

3.2 Analysis on Settlement Function

Follows from the empirical result that $2s_\nu < s_\xi$, then there exists some $\varepsilon > 0$ such that

$$\frac{s_\xi}{s_\nu} = 2 + \varepsilon \, .$$

Consequently,

$$r = \frac{s_\nu^2}{s_\xi - 2s_\nu} = \frac{s_\nu}{\frac{s_\xi}{s_\nu} - 2} = \frac{s_\nu}{\varepsilon} > 0 \, .$$

This result gives an implication

$$\sqrt[\tilde{q}]{\frac{s_\nu + r}{r}} = \sqrt[\tilde{q}]{\frac{s_\nu}{r} + 1} = \sqrt[\tilde{q}]{\varepsilon + 1} > 1\,,$$

which shows that

$$\tau = \ln \sqrt[\tilde{q}]{\frac{s_\nu + r}{r}} = \ln \sqrt[\tilde{q}]{\varepsilon + 1} > \ln 1 = 0\,.$$

Hence we conclude that $r, \tau > 0$. A direct consequence to these results is presented on the following lemma.

Lemma 1. *The settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$ as defined in accordance with equation 3 is a strictly increasing function (Bartle, 1991).*

Proof. The settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$ is said to be strictly increasing if and only if

$$\forall u, v \in \mathbb{R}^+ \left[u < v \iff S(u) < S(v)\right]$$

holds. Let $p, q \in \mathbb{R}^+$. (*Forward part of the proof*). Assume that $p < q$. Then

$$\frac{S(p) + r}{S(q) + r} = \frac{re^{\tau p} - r + r}{re^{\tau q} - r + r} = e^{\tau(q-p)}\,.$$

Since r, τ have been shown to be positive on section 3.2, then $e^{\tau(q-p)} > 1$, which implies

$$\frac{S(q) + r}{S(p) + r} > 1\,.$$

It shows that

$$S(p) + r < S(q) + r\,,$$

which again implies $S(p) < S(q)$. (*Backward part of the proof*). For the backward part, let us assume that $S(p) < S(q)$. Now we need to show that $p < q$, which can be done by reversing the forward part. We have

$$1 < \frac{S(q) + r}{S(p) + r} = \frac{e^{\tau q}}{e^{\tau p}} = e^{\tau(q-p)}$$

which implies

$$q - p = \tau \ln e^{\tau(q-p)} > \tau \ln 1 = \tau \cdot 0 = 0\,,$$

which shows that $p < q$. Hence, we conclude that S is strictly increasing. ☐

Another property of S is the differentiabiliy of S . It is in fact a natural consequence that S is differentiable since it is a solution of a differential equation (Kreyszig, 2011). Thus, we present this property on the following remark.

Remark 2. The settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$ is differentiable. Moreover, given any pair of points $p, q \in \mathbb{R}^+$ such that $p < q$, then we have

$$S'(p) < S'(q)\,,$$

as shown by

$$\frac{S'(p)}{S'(q)} = \frac{\tau r e^{\tau p}}{\tau r e^{\tau q}} = \frac{e^{\tau p}}{e^{\tau q}} = e^{\tau(q-p)} > e^{\tau(p-p)} = e^0 = 1\,.$$

An important property of the settlement function in this context is that it is not Lipschitz. This property is presented on the following theorem.

Theorem 1. *The settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$ defined in accordance with equation 3 is not Lipschitz.*

Proof. Note that the underlying metric spaces in this context are the same for both the domain and the codomain of S, given by $(\mathbb{R}^+, d_s)$, which is a metric subspace (André, 2020) of the usual metric space (André, 2020) on $\mathbb{R}$. Thus, the metric d_s is defined by

$$\forall u, v \in \mathbb{R}^+ ,\ d_s(u, v) := |u - v| .$$

We will prove this theorem by contradiction. Let $K > 0$. And assume that S is K-Lipschitz. Then we have

$$\forall u, v \in \mathbb{R}^+ ,\ u < v \implies \frac{|S(u) - S(v)|}{|u - v|} \leq K .$$

Let $a, b \in \mathbb{R}^+$ such that $a < b$. By lemma 1, we have that S is strictly increasing. Thus,

$$\frac{S(b) - S(a)}{b - a} = \frac{|S(a) - S(b)|}{|a - b|}$$

holds. And the Lipschitz condition for S–as assumed–can simply be expressed by

$$\frac{S(b) - S(a)}{b - a} \leq K .$$

Follows from remark 2, S is differentiable with a strictly increasing derivative. Let $c \in \mathbb{R}^+$ such that $c > b$. Then by mean value theorem (Rudin, 1964) there exists a point $\tilde{b} \in (a, b)$ and a point $\tilde{c} \in (b, c)$ such that

$$\frac{S(b) - S(a)}{b - a} = S'(\tilde{b}) < S'(\tilde{c}) ,$$

since $\tilde{b} < \tilde{c}$. Let $K := S'(\tilde{c})$. Given another point $d \in \mathbb{R}^+$ with $d > c$, follows from mean value theorem there exists a point $\tilde{d} \in (c, d)$ and we have

$$\frac{S(d) - S(c)}{d - c} = S'(\tilde{d}) > S'(c) = K ,$$

which shows that S is not K-Lispchitz. Given any larger $\tilde{K} > K$, with the similar process we will obtain that S is not $\tilde{K}$-Lipschitz. This is a contradiction to the initial assumption. Thus, S is not a Lipschitz function. □

4 Lipschitz Condition for Ultimate Bearing Capacity

In order to find the ultimate bearing capacity of piles, the function S will be restricted (Stoll, 1963; Judson, 2012) to a connected bounded subset of $\mathbb{R}^+$ (André, 2020). Suppose $Q_u \subset \mathbb{R}^+$ is the subset. It is necessarily assumed that

$$\inf Q_u = 0 .$$

And since Q_u is bounded, then

$$\sup Q_u < \infty$$

holds.

Follows from theorem 1, we find that $S : \mathbb{R}^+ \to \mathbb{R}^+$ is not Lipschitz. However, it will be shown that the restriction $S\big|_{Q_u} : Q_u \to \mathbb{R}^+$ is Lipschitz, and consequently, S is locally Lipschitz.

Theorem 2. *The restriction $\tilde{S} := S\big|_{Q_u} : Q_u \to \mathbb{R}^+$ on the settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$ as defined in accordance with equation 3 is a R-Lipschitz function.*

Proof. By the boundedness of Q_u, then let

$$q_u := \sup Q_u .$$

Let $R := S'(q_u)$. By remark 2, we have

$$\forall u \in Q_u ,\ \tilde{S}'(u) = S'(u) \leq S'(q_u) = R .$$

Given a pair of point $p, q \in Q_u$ with $p < q$, then by mean value theorem (Rudin, 1964) there exists some point $s \in (p,\, q)$ such that

$$\frac{\tilde{S}(q) - \tilde{S}(p)}{q - p} = \tilde{S}'(s)\,.$$

Note that $s \leq q_u$, since every point in Q_u is less than or equal to q_u. Follows from remark 2, we obtain

$$\frac{|\tilde{S}(p) - \tilde{S}(q)|}{|p - q|} = \frac{\tilde{S}(q) - \tilde{S}(p)}{q - p} = \tilde{S}'(s) \leq S'(q_u) = R\,,$$

which shows that $\tilde{S}$ is R-Lipschitz. □

Remark 3. By theorem 1, the restriction $\tilde{S} := S\big|_{Q_u} : Q_u \to \mathbb{R}^+$ of $S : \mathbb{R}^+ \to \mathbb{R}^+$ is R-Lipschitz. Thus, by definition 2, S is locally Lipschitz.

The set Q_u is interpreted in terms of SLT as the points of loading in which the pile is still safe. The pile is assmued to collapse at the least upper bound of Q_u, that is, the supremum (Rudin, 1964) of Q_s. Thus, the ultimate bearing capacity of piles based on SLT is defined by

$$q_u := \sup Q_u\,. \tag{8}$$

In order to find q_u, we need to determine the best constant R. We propose two methods for this case.

The first method is by considering the limit of settlement as determined by the standard (or code) in charge or by an empirical justification. In this scenario, the notion of the Lipschitz continuity of S is redundant in terms of finding q_u, but is still necessary for describing the nature of S. The formal description of this method is presented as follows.

Formulation 1 (Limit Settlement Method)**.** Suppose the settlement of piles has been limited to some number according to the standard in charge. Let $L > 0$ be the limit of the settlement. Follows from equation 3, the ultimate bearing capacity of piles q_u is assumed to satisfy

$$L = S(q_u) = re^{\tau q_u} - r\,. \tag{9}$$

And q_u is given by

$$q_u = \frac{\ln\left(\frac{L}{r} + 1\right)}{\tau}\,. \tag{10}$$

The second method, we limit the speed of settlement relative to the initial settlement. We postulate that the speed of the settlement at every point in Q_u cannot exceed twice the speed of the initial settlement. And the formal description of this postulate is presented as follows.

Postulate 1. The speed of the settlement is the first derivative of the settlement function $S : \mathbb{R}^+ \to \mathbb{R}^+$. The derivative of S exists as shown by remark 2, and we obtain

$$\forall q \in \mathbb{R}^+\,,\; S'(q) = \tau rc^{\tau q} < S'(q_u)\,.$$

Then it is postulated that

$$\frac{S'(q_u)}{S'(0)} = 2\,,$$

which implies

$$S'(q_u) = 2S'(0) = 2\tau r\,. \tag{11}$$

Consequently, the restriction $\tilde{S} := S\big|_{Q_u} : Q_u \to \mathbb{R}^+$ is $2\tau r$-Lipschitz, i. e., the Lipschitz constant R is given by $R = 2\tau r$. And we have

$$Q_u = \{q \in \mathbb{R}^+ \mid S'(q) < 2\tau r\}\,. \tag{12}$$

Formulation 2 (Limit Settlement Speed Method)**.** Follows from postulate 1, the derivative of S at q_u is given by

$$S'(q_u) = 2\tau r\,,$$

Then we have

$$2\tau r = \tau r e^{\tau q_u}\,,$$

which implies

$$q_u = \frac{\ln 2}{\tau} \tag{13}$$

For a more conservative approach, formulation 1 and formulation 2 can be combined to obtain a more complete consideration. This combination is formally presented as follows.

Formulation 3. Follows from equation 9 of formulation 1 and equation 11 of formulation 2, it is designated that

$$S'(q_u) = \min\left\{\tau(L + r),\, 2\tau r\right\}, \tag{14}$$

where $L > 0$ is the limit settlement determined from the code in charge. And the restriction $\tilde{S} := S\big|_{Q_u} : Q_u \to \mathbb{R}^+$ is an R-Lipschitz with

$$R = S'(q_u) = \min\left\{\tau(L + r),\, 2\tau r\right\}. \tag{15}$$

Thus,

$$Q_u := \left\{q \in \mathbb{R}^+ \mid S'(q) < \tau(L + r) \wedge S'(q) < 2\tau r\right\}. \tag{16}$$

And q_u is given by

$$q_u = \sup Q_u = \frac{\ln\left(\min\left\{\frac{L}{r} + 1,\, 2\right\}\right)}{\tau} \tag{17}$$

5 Discussion

This section discusses the test on the method that has been built on earlier sections. A comparison will be made for the method constructed in this paper with an existing method called the Davisson's method (Davisson, 1973). In Davisson's method, the ultimate bearing capacity of pile is determined as a point $q_u \in \mathbb{R}^+$ such that the curve of load-settlement and a linear function $\delta : \mathbb{R}^+ \to \mathbb{R}^+$ is intersecting at q_u. The linear function δ is given in accordance with (Davisson, 1973) by

$$\forall q \in \mathbb{R}^+\,,\ \delta(q) := 0.012 D_r + 0.1\left(\frac{D}{D_r}\right) + \frac{q\ell}{A_p E_p}\,,$$

where D_r is the reference pile diameter (taken equal to 300 mm), D is the pile diameter in use (in mm), ℓ is the pile length (in mm), A_p is the pile cross section (in mm^2) and E_p is the Young's modulus of pile material (in kN/mm^2) (Das, 2011).

Given a data set of compressive SLT result from a project in Indonesia. The data set consists only the load and settlement values during loading, and is presented on table 1.

The data set is then modelled with the settlement function S. Several values are taken as follows:

$$\tilde{q} = 801.9 \text{ kN}\,, \quad 2\tilde{q} = 1603.8 \text{ kN}\,, \quad s_\nu = 2.398 \text{ mm}\,, \quad s_\xi = 5.318 \text{ mm}\,.$$

And we obtain $r = 11.000969$ and $\tau = 0.000246$. And the particular solution to S for this data set is given by

$$\forall q \in \mathbb{R}^+\,,\ S(q) = 11.000969 e^{0.000246 q} - 11.000969\,.$$

The error metric is presented to measure the accuracy of the model with the actual data, which is given in the form of mean absolute error (MAE) (Willmott and Matsuura, 2005) by

$$\text{MAE} = \frac{1}{\alpha + 1}\sum_{k=0}^{\alpha} \left|s_k - S(q_k)\right|.$$

And in this case, MAE $= 0.051474$ mm, which is considerably small. The plot of the restriction $S\big|_{[0,\,5000]} : [0,\,5000] \to \mathbb{R}^+$ is presented in figure 2.

Table 1: SLT Data from a Project in Indonesia

Load (kN)	Settlement (mm)	Load (kN)	Settlement (mm)
0.000	0.000	980.100	3.015
44.600	0.023	1024.700	3.205
88.100	0.115	1069.200	3.375
133.700	0.228	1113.800	3.568
178.200	0.398	1158.300	3.678
222.800	0.490	1207.900	3.843
267.300	0.630	1247.400	3.995
311.900	0.758	1292.000	4.153
356.400	0.913	1336.500	4.318
410.000	1.055	1381.100	4.480
445.500	1.185	1425.600	4.643
490.100	1.340	1470.200	4.818
534.600	1.465	1514.700	4.995
579.200	1.653	1559.300	5.173
623.700	1.780	1603.800	5.318
668.300	1.933	1648.400	5.490
712.800	2.080	1692.900	5.638
757.400	2.250	1737.500	5.843
801.900	2.398	1782.000	5.993
846.500	2.545	1825.500	6.195
891.000	2.685	1871.100	6.387
935.600	2.870	1915.700	6.640

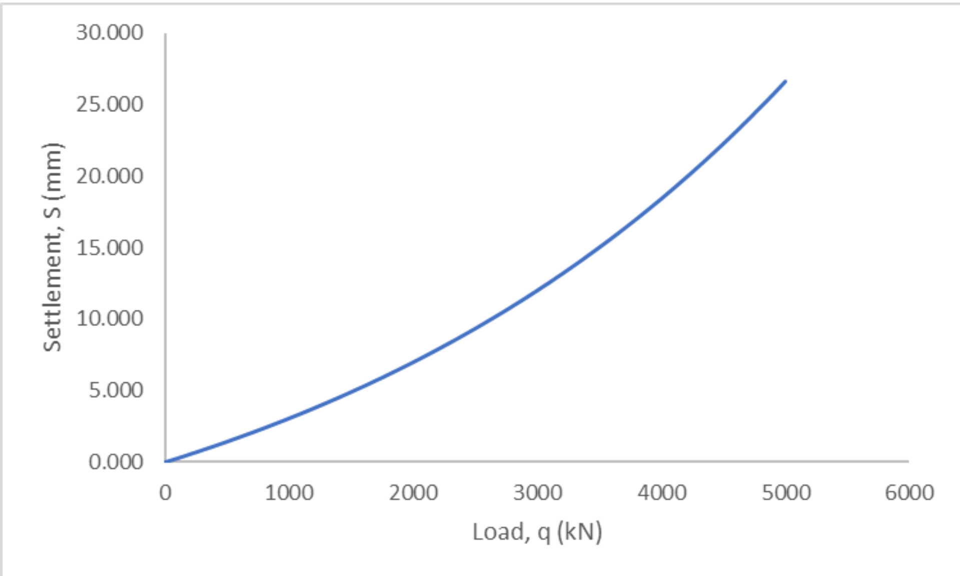

Figure 2: Plot from settlement function S restricted to [0, 5000]

5.1 Interpretation with Davisson Method

Follows from Davisson's method (Davisson, 1973), we obtain

$$S(q_u) = \delta(q_u)$$

For this SLT data, the length of pile is given by $\ell = 23000$ mm, the cross sectional area for the pile is given by $A_p = 282743.339$ mm^2 , and the concrete Young's modulus is given by 33.234 kN/mm^2 .

Thus, we obtain

$$11.000969e^{0.000246q_u} - 11.000969 = 3.8 + \frac{q_u}{0.002447}$$

The plot of the equation above is presented on figure 3. And q_u is given by $q_u \approx 2700$ kN.

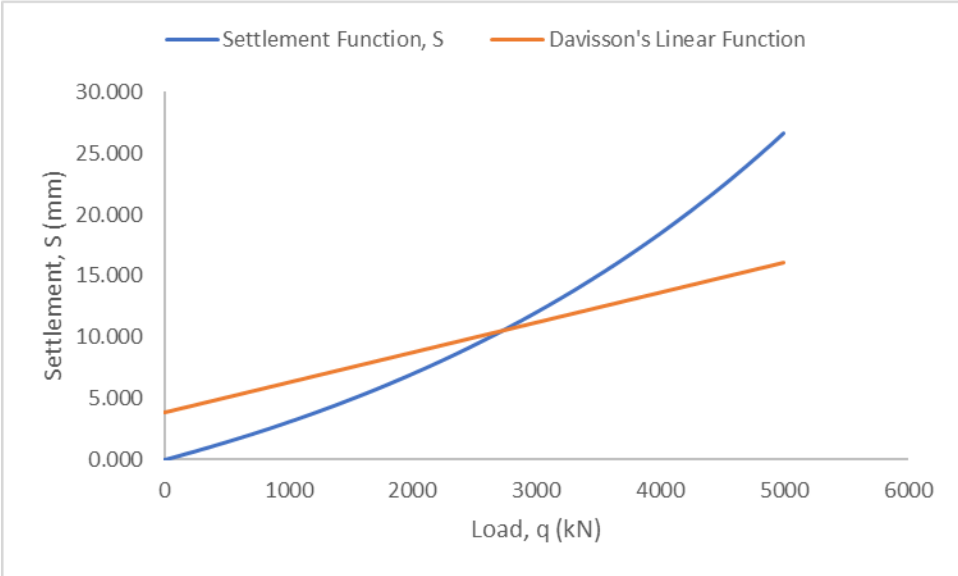

Figure 3: Davisson's Method

5.2 Interpretation with Lipschitz Condition

In the project in which the SLT data was produced, the limit for pile settlement is set to be 25 mm. Hence we designate $L := 25$ mm. And follows from equation 17, we obtain

$$q_u = \frac{\ln\left(\min\left\{\frac{25}{11.000969}, 2\right\}\right)}{0.000246} = 2819.25 \text{ kN}.$$

6 Conclusion and Future Work

As presented in section 5, it is concluded that the pile ultimate bearing capacity under the interpretation of Lipschitz condition is slightly greater than that of Davisson, given a SLT data set of a project in Indonesia. This result shows that the theory is valid.

In terms of calculation process, it is believed that the use of the method presented in this paper is simple and direct, since one can simply apply equation 6 to calculate r, apply equation 7 to calculate τ, apply equation 4 to calculate S and apply equation 17 to calculate the ultimate capacity of pile under Lipschitz condition.

Possible further research regarding this topic that can still be conducted is figuring out whether there exists a more accurate criterion to determine the ultimate bearing capacity with the Lipschitz condition based on more data sets as experimental samples.

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