Relation between Kinetic Energy and Relativistic Mass-Energy

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: Mechanical Energy is a form of energy associated with the motion and position of an object with mass. The relativistic kinetic energy is concerned with the motion of a body whose velocity approaches the speed of light. This paper presents an equation of mass-energy equivalence derived from the kinetic energy connected with the relativistic mass.

Keywords: kinetic energy, relativistic mass, speed of light

1. Introduction

The sum of kinetic and potential energy is known as a total mechanical energy. In the Einstein's theory of special relativity [1-4], the relativistic mechanics is concerned with the motion of bodies whose velocities approach the speed of light.

2. Relativistic Mass-Energy Relation

Kinetic Energy $(E) = \frac{1}{2}mv^2$.

By differentiating with respect to time, we get

$$\frac{dE}{dt} = \frac{1}{2}\left(m2v\frac{dv}{dt}\right) \Longrightarrow \frac{dE}{dt} = mv\frac{dv}{dt} = Fv = v\frac{d(mv)}{dt} = v\frac{dp}{dt},$$

$$\text{where } a = \frac{dv}{dt}, F = ma, and\ p = mv.$$

From this above expression, we conclude that

$$\frac{dE}{dt} = Fv = v\frac{d(mv)}{dt}. \qquad (1)$$

Now, let us derive an equation equivation to $v\frac{d(mv)}{dt}$ from the relativistic mass.

The equation of relativistic mas states that $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

From the relativistic mass equation, we obtain

$$m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}} \Longrightarrow m^2c^2 - m^2v^2 - {m_0}^2c^2 = 0, \text{where rest mass } m_0 \text{ and } c \text{ are constants.}$$

By differentiating with respect to time, we get

$$c^2 2m\frac{dm}{dt} - 2mv\frac{d(mv)}{dt} = 0$$

$$v\frac{d(mv)}{dt} = c^2\frac{dm}{dt} \quad (2)$$

From the equations (1) and (2), we get

$$\frac{dE}{dt} = Fv = v\frac{d(mv)}{dt} = c^2\frac{dm}{dt}$$

$$dE = c^2 dm$$

Note that the term $c^2 dm$ allows the hypothesis of variable mass as it actually occurs at high speed. Also, $c^2 dm$ is equal to the kinetic energy (dE).

By integrating the equation $dE = c^2 dm$,

$$\int_0^E dE = c^2 \int_0^m dm,$$

we conclude that $E = mc^2$, which is mass-energy equivalence.

3. Conclusion

The mass-energy equivalence along with relativistic mass, momentum, and energy play an important role in the Einstein's theory of special relativity. Also, the relativistic mechanics is concerned with the motion of bodies whose velocities approach the speed of light. In this article, the mass-energy equivalence has been derived using the kinetic energy and relativistic mass equation.

References

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