A Mathematical Approach to the Momentum Equations of Massless Photon and Particle with Relativistic Mass

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Abstract: In Einstein's theory of special relativity, the relativistic momentum is concerned with the motion of a particle of relativistic mass whose velocity approaches the speed of light, but not equal to the speed of the light. An electromagnetic photon is a massless particle and its momentum is concerned with the motion of the massless photon whose velocity is equal to the speed of light. This paper focuses on the momentum of massless photon and a particle with relativistic mass.

Keywords: relativistic motion, momentum equation, speed of light

1. Introduction

Electromagnetic radiation is described in terms of a stream of massless particles, called photons, each travelling in a wave-like pattern at the speed of light. Each photon carries a certain amount of energy in the waveform. In Einstein's theory of special relativity, the relativistic mass [1-3] plays an important role in the mass-energy equivalence. In electromagnetic spectrum, the rest mass and relativistic mass of photon are equal to zero. Thus, the velocity of any particle with rest mass or relativistic mass cannot be equal to the speed of light [4-6].

2. Einstein's Mass-Energy Equation

The equation of relativistic mas states that $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Where,

- the rest mass of the body is m_0
- the velocity of particle in motion is v
- the speed of light is c

Then, let us derive the mass-energy equivalence using the equation of relativistic mass.

$m^2 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}} \Longrightarrow m^2c^2 - m^2v^2 - {m_0}^2c^2 = 0,$ where m_0 and c are constants.

By differentiating this equation with respect to time, we get

$$c^2 2m\frac{dm}{dt} - 2mv\frac{d(mv)}{dt} = 0$$

$$v\frac{d(mv)}{dt} = c^2\frac{dm}{dt} \qquad (1)$$

The equation of kinetic energy states that $\mathrm{E} = \frac{1}{2}mv^2$, where m is mass and v is velocity.

By differentiating the equation of kinetic energy with respect to time, we get

$$\frac{dE}{dt} = \frac{1}{2}\left(m2v\frac{dv}{dt}\right) \Longrightarrow \frac{dE}{dt} = mv\frac{dv}{dt} = Fv = v\frac{d(mv)}{dt} = v\frac{dp}{dt},$$

where the acceleration $(a) = \frac{dv}{dt}$, force $(F) = ma$, and momentum $(p) = mv$.

The rate of kinetic energy with respect to time is given below:

$$\frac{dE}{dt} = Fv = v\frac{d(mv)}{dt}. \qquad (2)$$

From the equations (1) and (2), we obtain

$$\frac{dE}{dt} = Fv = v\frac{d(mv)}{dt} = c^2\frac{dm}{dt} \Longrightarrow dE = c^2 dm$$

Here, c^2dm is equal to the kinetic energy (dE).

Integrating the equation $dE = c^2dm$,

$$\int_0^E dE = c^2\int_0^m dm,$$

Now, we conclude that $E = mc^2$, which is mass-energy equivalence.

3. Momentum Equation

The momentum (p') of a massless photon is equal to the Planck constant (h) multiplied by the frequency (f)

$$p' = hf$$

The Einstein mass-energy equation is:

$$E = mc^2 = (mc) \times c = pc,$$

where momentum $(p) = mc$ and m is relativistic mass .

$p = p'$, which is a contradiction. But $p < p'$is true because the momentum (p) of a particle with relativistic mass must be less than the momentum (p') of a massless particle (photon).

4. Conclusion

In this article, the momentum of a massless particle (photon) and a particle with mass has been discussed in detail. Also, it was shown that the momentum of a massless photon is not equal to the momentum of a particle with relativistic mass.

References

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