

Technical Note

Nominal Stiffness Evaluation and Regression Analysis of GT-2 Rubber-Fiberglass Timing Belts

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Abstract

GT-style rubber-fiberglass (RF) timing belts are designed to effectively transfer rotational motion from pulleys to linear motion in small machines and mechatronic systems. One of the characteristics of belts under this type of loading condition is that the length between load and pulleys changes during operation, thereby changing their effective stiffness. It has been shown that the effective stiffness of such a belt is a function of a “nominal stiffness” and the real-time belt section lengths. However, this nominal stiffness is not necessarily constant; it is common to assume linear proportional stiffness, but this often results in system modeling error. This technical note describes a brief study where the nominal stiffness of two lengths (400 *mm* and 760 *mm*) of GT-2 RF timing belts was tested up to breaking point; regression analysis was performed on the results to best model the observed stiffness. The study was replicated three times, providing a total of six stiffness curves. It was found that cubic regression models ($R^2 > 0.999$) were the best fit, but that quadratic and linear models still provided acceptable representations of the whole dataset with R^2 values above 0.940.

Keywords: Timing belt, belt stiffness, dynamic system modeling, mechatronic systems, 3-D printers, robotics

1. Introduction

Timing belts are a common means of motion transfer between rotating motors/shafts in a machine or mechatronic system. Many small-to-medium sized mechatronic systems such as 3-D printers [1], robots [2, 3], desktop CNC machines [4], and positioners [5] use such belts, typically in the GT-style [6, 7]. The GT-style belts are specifically designed to effectively translate rotating motion from pulleys into linear motion with minimal deformation, slippage, and backlash. One of the fundamental characteristics of belts under this type of loading condition is a non-constant stiffness value; since the effective length of each belt section is changing with time during operation, the effective stiffness is also time-variant. It has been shown that the stiffness of the belts can be directly calculated as a function of a “nominal stiffness” value, the belt width, and the real-time length. For a toothed belt running on two equally-sized gears, the time-dependent stiffness of a belt section i is [8]:

$$k_i(t) = C_{sp} \frac{b}{L_i(t)} \quad (1)$$

where k_i is the effective stiffness as a function of time, C_{sp} is the nominal stiffness, b is the belt width, and $L_i(t)$ is the length of the belt section at time t . The most commonly-used GT-style belt is the GT-2; Figure 1 shows the fundamental geometry and specifications for this type of belt.

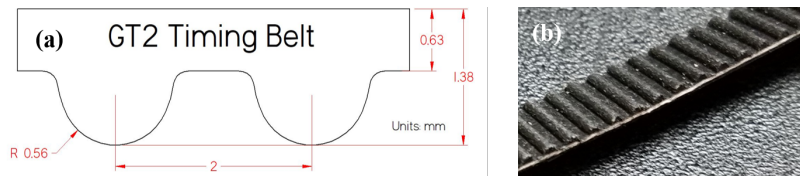


Figure 1: GT-2 belt (a) specifications and (b) basic geometry

Figure 2 shows a common application, where a GT-style belt is used to transfer motion from a stepper motor to drive a linear positioning system. Also shown is a 2-D dynamic model representation of such a system (Figure 2b), where the differences in stiffness, based on belt length, in the belt sections is clearly evident. The sections L_1 and L_2 change in stiffness as a function of time, while section L_3 stays constant during use [9, 10].

The work described in this note explored the nominal stiffness C_{sp} and the best way to model it in dynamic systems where belt length is not constant. Several previous studies have assumed that rubber-based timing belts

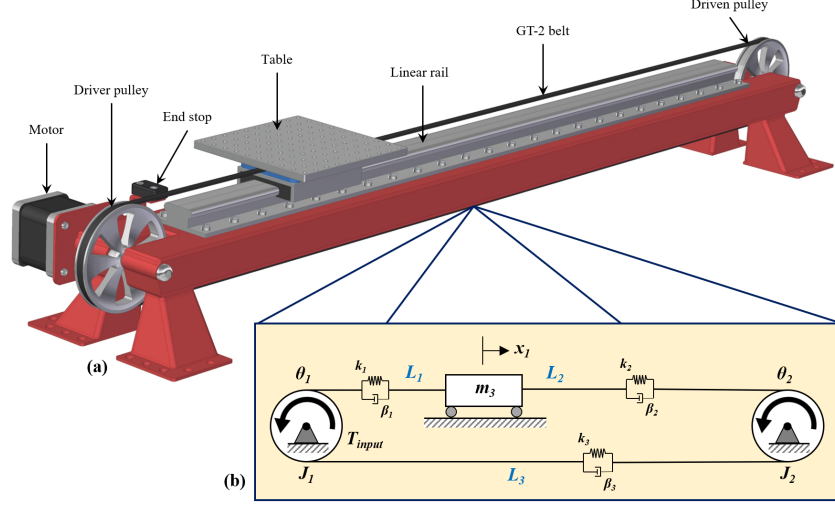


Figure 2: (a) Simple positioning system which utilizes a GT-type belt to drive the table and (b) its representative dynamic model

have a linear nominal stiffness [5, 11, 12, 13, 14, 15, 16, 9, 10]. However, it is vital for designers and engineers working with dynamic systems which use belts for energy transfer to understand the true effects of the belt stiffness [17, 18]. Therefore, experimental data was collected and used to derive conclusions on the true stiffness behavior of the GT-2 belts during use. The collected data was subjected to regression analysis to see which type of model best fit, allowing the comparison of models for the same dataset. The information in this study will prove useful, both in choosing k stiffness values for dynamic models and for judging expected model error if linear stiffness assumptions are used.

2. Procedure and Results

Two lengths of new GT-2 belts, 400 mm and 760 mm, were subjected to a simple tensile test at a rate of 0.8 mm/s until they ruptured, taking a force-deflection reading every 1.6 mm or every 2 seconds; the purpose of the discrete time measurements was to produce a moderately-sized dataset to be used for curve fitting. This was repeated twice to obtain a set of 6 different curves, three from each length. The ruptured belts were observed to fail suddenly and to show tearing of the glass fibers inside, as shown in Figure 3. The GT-2 belts used were a composite of neoprene (synthetic

rubber) and glass fibers, where the fibers appeared to drive the failure point of the belts.



Figure 3: Belt break interface, showing broken fibers

The collected data, in terms of force-deflection curves, is shown in Figure 4a, while the equivalent stress-strain curves for the tests are shown in Figure 4b. The length of the belts clearly had an effect on the force-deflection curves, but this largely disappeared when the length was accounted for in the stress-strain curves. Note that the most of the curves show hyper-elastic behavior, i.e. there is no region in the curve where the stiffness is constant.

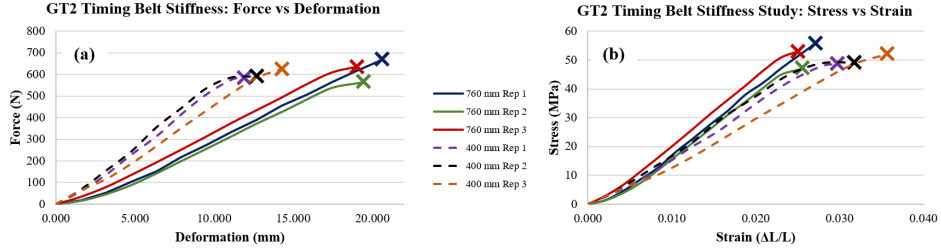


Figure 4: Belt stiffness curves: (a) force-deflection curve and (b) stress-strain curve

As the nominal compliance of the belts was clearly found to be non-linear, a regression analysis was performed to model the curves and find the level of unexplained variance in these curves. One of the most common polynomial regression models [19, 20] used for hyper-elastic materials is the cubic polynomial. The basic model used for this study began with the following polynomial model:

$$\sigma_{belt} = A\varepsilon_{belt}^3 + B\varepsilon_{belt}^2 + C\varepsilon_{belt} + D \quad (2)$$

where a cubic model includes all of the variables, a quadratic model can be generated by setting $A = 0$, and a linear model can be used with $A = B =$

0. These curve fits, completed using Microsoft ExcelTM (*Microsoft Corp, Redmond, Washington, USA*) are shown in Figure 5a, and the fits for each of the variable and the resulting R^2 values are shown in the first six cases in Tables 1 and 2.

Table 1: Cases under study

Case		Plot Reference
1	760 mm (cubic model) - $R1$	Figure 5a
2	760 mm (cubic model) - $R2$	Figure 5a
3	760 mm (cubic model) - $R3$	Figure 5a
4	400 mm (cubic model) - $R1$	Figure 5a
5	400 mm (cubic model) - $R2$	Figure 5a
6	400 mm (cubic model) - $R3$	Figure 5a
7	Full dataset (cubic model)	Figure 5b
8	Full dataset (quadratic model)	Figure 5b
9	Full dataset (linear model)	Figure 5b
10	Low strain (linear model)	Figure 5c

Table 2: Belt stiffness model curve fit data

Case	A	B	C	D	R^2
1	-2.00×10^6	94,969	958.80	-0.5658	0.9996
2	-3.00×10^6	144,204	445.86	-0.1163	0.9996
3	-2.00×10^6	95,693	1,281.60	-0.0723	0.9997
4	-2.00×10^6	81,219	849.99	0.2296	0.9993
5	-2.00×10^6	95,332	935.37	0.2840	0.9995
6	-922,283	50,993	810.95	0.4965	0.9994
7	-2.00×10^6	98,091	937.22	-0.1758	0.9672
8	-	-19,340	2,408.9	-3.3656	0.9552
9	-	-	1821.1	-0.6140	0.9431
10	-	-	2013.8	-2.3275	0.9573

After fitting the cubic models to each of the six sets of experimental data, the cubic model, a quadratic model, and a linear model were then fit to entire set at once, as shown in Figure 5b. A significant drop in the R^2 value was noted for all of the models fit to the dataset, but difference between the cubic, quadratic, and linear models were observed to be small, as shown in Tables 1 and 2.

It was observed that the low-strain region of the dataset (Figures 5b and c) conforms better to a linear model when the entire dataset is used. In actual use, it is most likely that the belts will not reach more than 20 – 30% of the belt breaking strength during normal use [21, 22, 12], so this is a valid

assumption for many systems; this will, of course, need to be determined by the modeler or designer before using a linear belt model. If the low-strain assumption can be used, then the data fits a linear model with a slightly greater R^2 value than a quadratic model for the entire dataset and is certainly superior to a linear model for the entire dataset. The linear model for this case is shown in the last row of Table 2.

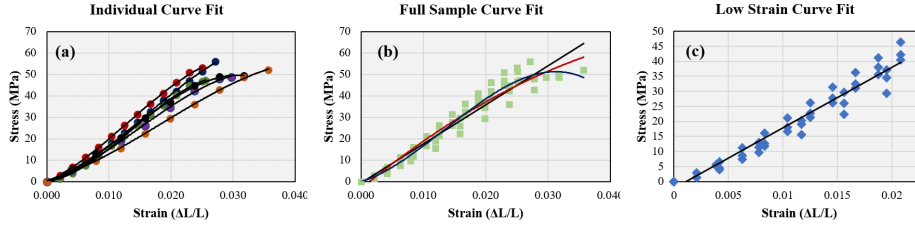


Figure 5: Curve fits for (a) individual belts (cubic model), (b) full sample curve fit (cubic, quadratic, and linear models), and (c) low-strain linear curve fit

3. Summary and Conclusions

This short technical note presents the results of a brief exploratory study on modeling the nominal stiffness of GT-2 timing belts; this information can be used to more accurately determine the true, time-dependent, stiffness behavior of common GT-2 belts when the effective length of belt sections changes with time. It was observed that these belts do not behave in a linear way, as expected for belts with a hyper-elastic base material, but that a linear model can provide a reasonable approximation of the behavior under some conditions, particularly low-strain conditions. When possible, the cubic stiffness model should be used, but this would often be impractical for systems with many components, as it can cause a simple model to become non-linear. When practical and necessary for problem tractability, a linear model may be used with a reasonable degree of accuracy. The modeler or designer should keep in mind that some uncertainty will exist with any belt model and choose the model which best balances accuracy with computational cost.

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