Technical Report for the Erratum of:

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Abstract


Erratum Description

- $\tau_d$ is wrongly used in equations (11), (12) and (13) as well as in Figure 8 under "Allocation Law". The correct symbol is $\tau_{IP,\text{ref}}$.

- Subsection "3.2 Flywheel Aerodynamics and Actuation" presented an incorrect value for air density of $\rho_g = 41.6 \text{ kg/m}^3$, which correspond to the air density per molar mass in mol/m$^3$ compared to the correct value $\rho_g = 1.181 \text{ kg/m}^3$. In addition, estimated bearing friction did not account for tribological phenomena. Given the considerably effect these values have in the estimation of the maximum flywheel speed, corrections on the analysis of aerodynamic drag and bearing friction are given below as revised sections. Moreover, it is important to note that reported experimental results were unaffected by the erroneous estimations.

Revised Sections

Revised sections below include section 3. Control Moment Gyroscope Hardware Design (particularly subsections 3.2 Flywheel Aerodynamics and Actuation and 3.3 Flywheel Stress Analysis), 6. Discussion and 7. Conclusions of the original paper.
Control Moment Gyroscope Hardware Design

Flywheel Aerodynamics and Actuation

An analysis of the different friction sources was performed to estimate the maximum flywheel speed. The power required to reach a given $\Omega$ can be calculated as $P = P_a + P_b$, where $P_a$ and $P_b$ are the power required to overcome aerodynamic drag and bearing friction respectively. The power to overcome aerodynamic drag for a rotating disc can be expressed as \[2, 5\]

$$P_a = \rho g \Omega^3 R_o^5 C_{m,a} + \frac{\pi \rho g \Omega^2 R_o^4 h}{2} C_{m,r}$$ \hspace{1cm} (6)$$

where the non-dimensional drag torque coefficients $C_{m,a}$ and $C_{m,r}$ depend on the drag produced by the lateral planar surfaces of the disc and the drag produced by the cylindrical face of the rim respectively.

Estimation of the axial torque coefficient $C_{m,a}$ was based on the relationships determined by Daily and Nece \[4\] as

<table>
<thead>
<tr>
<th>Regime</th>
<th>Flow Type Conditions</th>
<th>$\delta$</th>
<th>$K_a$</th>
<th>$K_b$</th>
<th>$K_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$Re_a &lt; 3 \times 10^5$, $s &lt; \delta$</td>
<td>5.5 $\left(\frac{\nu}{\Omega}\right)^{0.5}$</td>
<td>$2\pi$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>II</td>
<td>$s &gt; \delta$</td>
<td>3.7</td>
<td>0.1</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>$Re_a &gt; 3 \times 10^5$, $s &lt; \delta$</td>
<td>$\frac{3R_o^2}{\delta} \left(\frac{\nu}{\Omega}\right)^{0.2}$</td>
<td>0.08</td>
<td>-1/6</td>
<td>-0.25</td>
</tr>
<tr>
<td>IV</td>
<td>$s &gt; \delta$</td>
<td>0.0102</td>
<td>0.1</td>
<td>-0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Axial frictional torque coefficient regimes \[4\]
Regime | Conditions | $K_u$ | $K_v$
---|---|---|---
Laminar | $Re_r \leq 64$ | 10 | -1
Transitional Flow | $64 < Re_r \leq 500$ | 2 | -0.6
Turbulent | $500 < Re_r \leq 1 \times 10^4$ | 1.03 | -0.5
| $Re_r > 1 \times 10^4$ | 0.065 | -0.2

Table 2: Radial frictional torque coefficient regimes [2, 1]

\[
C_{m,a} = K_a G_a^{K_b} Re_a^{K_c}
\]

being a function of the axial rotational Reynolds number $Re_a$ and the axial gap ratio $G_a = \frac{s_a}{R_o}$, with $s_a$ the axial clearance between the flywheel and the casing [2]. $K_a$, $K_b$ and $K_c$ are constants determined by any of the four flow regimes that can be present [4, 2] as shown in Table 1. These regimes are categorized depending on the flow type and the nature of the boundary layers [4, 2]. To this end the radial rotational Reynolds number is calculated as

\[
Re_a = \frac{R_o^2 \Omega}{\nu} = \frac{\rho g R_o^2 \Omega}{\eta}
\]

where $\rho_g = 1.181 \text{ kg/m}^3$, $R_o = 0.1 \text{ m}$ and $\eta = 1.84 \times 10^{-5} \text{ Pa s}$, are the air density, flywheel’s rim outer radius and the air dynamic viscosity, respectively. Subsequently the behavior of the boundary layers (i.e. whether the boundary layers are merged or separate) was determined. Here, the boundary layer thickness $\delta$ was compared with the axial clearance $s$. Table 1 summarizes the constants used to estimate the axial frictional moment $C_{m,a}$.

Estimation of the radial torque coefficient $C_{m,r}$ was based on the equations reported by Bilgen and Boulos [1] as

\[
C_{m,r} = K_u G_r^{0.3} Re_r^{K_v}
\]

being a function of the circumferential rotational Reynolds number $Re_r$ and the radial gap ratio $G_r = \frac{s_r}{R_o}$, with $s_r$ the radial clearance between the flywheel and the casing [2]. $K_u$ and $K_v$ are constants determined by any of the four flow regimes that can be present [1, 2] as shown in Table 2. These regimes are categorized based on the circumferential rotational Reynolds number

\[
Re_r = \frac{R_o \Omega s_r}{\nu} = \frac{\rho g R_o \Omega s_r}{\eta}.
\]

The power required to overcome bearing friction was estimated based on The SKF model for calculating frictional moment [7]. This model estimates the bearing frictional moment as

\[
P_b = (M_{rr} + M_{sl} + M_{seal} + M_{drag}) \Omega
\]
where $M_r$ and $M_d$ are rolling and sliding frictional moments, and $M_{\text{seal}}$ and $M_{\text{drag}}$ are frictional moments caused by the bearing seals and drag due to external lubrication respectively. Given that the formulas used in this model are rather large and complex, we refer the reader to the complete model in [7] for a more detailed description.

Given size constraints in the flywheel’s hub, a 120 W Maxon EC 4-pole 22 brushless motor and high-precision hybrid angular contact ball bearings (GMN HY KH 61910 TA P4 L 252) were selected as flywheel actuator and suspension respectively. A mechanical power of 108 W was estimated taking into account the losses due to the motor efficiency $\eta_{\text{FM}} = 90\%$. Bearing friction in Eq. (11) was estimated using the grease viscosity range reported by the manufacturer (26 mm$^2$/s @ 40 °C and 6 mm$^2$/s @ 100 °C for Turmogrease HS L252). Due to the complexity of the expressions to calculate both aerodynamic and bearing friction from Eqs. (6) and (11), a script was created to estimate the maximum flywheel speed $\Omega$ based on total power $P = P_a + P_b$.

Figure 4 shows the power $P$ required to overcome both aerodynamic drag and bearing friction against flywheel speed $\Omega$, where a maximum flywheel speed of 5250 rpm was estimated for the selected flywheel motor and bearings.

**Flywheel Stress Analysis**

To evaluate the mechanical strength of the flywheel under nominal operation, the maximum flywheel angular rate $\Omega = 5250$ rpm in addition to the maximum designed gyroscopic moment of 90 Nm were used. Due to the complexity of the flywheel geometry Finite Element Method (FEM) analysis was performed using ANSYS Workbench and its static structural module. As the CMG is not used in reaction wheel mode (i.e. the flywheel angular rate remains constant in nominal operation), the imposed condition over the model is constant rotational speed. To
validate the FEM results, a simplified model keeping the same overall dimensions but using only centrifugal loads was simulated and compared with analytical results. FEM analysis of the simplified model showed good agreement with the analytic radial and tangential stresses with Root Mean Square Error (RMSE) values of $\sigma_{r,\text{RMSE}}=0.07\ \text{MPa}$ and $\sigma_{\theta,\text{RMSE}}=0.13\ \text{MPa}$.

Using the same simulation parameters and boundary conditions and including the load due to the gyroscopic moment, the flywheel geometry as shown in Fig. 3 was used to perform the FEM analysis. Figure 5 shows the equivalent (von Mises) stress, where the maximum of 50 MPa is much lower than the yield strength of the flywheel material (Aluminum 7075 - T6, $S_y = 503\ \text{MPa}$), giving a safety factor of 10 against the combined centrifugal and gyroscopic loads.

In addition to the stress analysis, it is necessary to assess the critical speed, since the flywheel can experience several types of vibrations being shaft whirl, lateral and torsional vibrations the most important [6]. This critical speed must be avoided, since the resulting deflections might cause stresses beyond the strength of the material. The ANSYS Workbench modal analysis module was used to perform the critical speed analysis given the complexity of the flywheel geometry. Shaft whirl and lateral vibration effects were neglected as the flywheel was thoroughly balanced after manufacturing and the deflection of the shaft is very small at the flywheel location during bending due to the gyroscopic moment. Thus, Torsional vibrations were analyzed using the same parameters and boundary conditions as the static structural analysis, where the modal analysis results showed a minimum critical speed of 200 778 rpm, giving a margin of 38 when compared with the intended rotational speed ($\Omega = 5250\ \text{rpm}$).
Discussion

The implemented controller successfully tracked the reference virtual stiffnesses, while keeping the inverted pendulum within its equilibrium position $\phi = 0$. Even though no damping was emulated in the controller, dry friction was present due to the inherent mechanical construction of the pendulum hinge joint. This resulted in a dead band behavior in the vicinity of the equilibrium point where the pendulum angular velocity was very low. This effect can be seen in Fig.9 where the colored regions showing the tracked stiffness seem to have a slight offset compared to the reference stiffness represented by the solid line. Note that the offset slightly changes as the stiffness tracking passes the equilibrium position showing the effect of the dead band caused by the dry friction in the hinge joint.

Despite the rather heavy weight of our device, it showed a substantially better gyroscopic-torque to weight ratio against the only other comparable device reported in literature (7 N m/kg vs. 3.4 N m/kg by Chiu and Goswami [3]), although the target gyroscopic moment of 90 N m was not reached in this specific experiment. To improve the generated gyroscopic moment, we presume that implementation of a partial vacuum flywheel chamber could considerably diminish the effect of the aerodynamic drag as lower air density can be achieved, decreasing even more power consumption and weight, and increasing top speed, thus producing higher angular momentum. Further improvements in weight reduction can be made using different material and geometry selection for the flywheel, where higher moment of inertia can be achieved with a smaller wheel size using materials with higher density (e.g. steel, iron, tungsten, etc). This could have a direct impact on the size and weight of the gimbal structure as well as on aerodynamic drag in the flywheel given that it increases rapidly with the peripheral speed. This reduction in aerodynamic drag implies at the same time a reduction in the power requirement for the flywheel motor.

Experimental results showed that our estimations were rather conservative. Measured maximum flywheel speed was higher ($\Omega = 5400$ rpm @ 118.56 W of electrical power), although in the same order of magnitude, than the estimated maximum speed ($\hat{\Omega} = 5250$ rpm). This can be explained from (i) the effect of the bearings running-in period and increase of running temperature [7] and (ii) due to assumptions in flywheel geometry and air properties in the aerodynamic drag estimation. Frictional moment in the bearings could be either similar or lower than the estimated values, once grease distributes itself within the free space of the bearing and lubricant temperature rises over our assumed temperature lowering its viscosity. Aerodynamic drag are affected by changes in the air properties. Air density and viscosity were calculated assuming constant temperatures and pressure which can vary in the closed flywheel chamber. However, given the close agreement with our results we presume that our assumptions of turbulent flow, merged boundary layers in addition to and estimated consumed power were correct.

Although, special care must be taken in the aerodynamic analysis if a more complex flywheel geometry is used, such as variable thickness flywheels with spokes or conical constructions, the estimations presented here could be used as reference for flywheel actuator selection during early design stages. It is recommended that these estimations are validated using CFD or experimentally.
As our prototype was conceived as wearable device, safety requirements were set to comply with directives for medical devices. Thus, pilot tests could be conducted to assess how humans react to transmitted moments in the upper body.

Finally, the setup used to evaluate the capabilities of the CMG as balance assistance device is rather simplistic, as it is based on a overly simple human model. Future research should involve tests with more realistic experimental platforms, for example emulating falls in all directions using a set of 2 or more CMGs in a 2 DoF inverted pendulum.

Conclusions

By including the bearing friction and aerodynamic behavior of the spinning wheel and the induced dynamics of the wearer, we demonstrated that, with the proposed design methodology, a CMG-based human balance assistance device can be built to comply with the designed specifications given a proper selection of the actuators. We showed that our device is capable of producing up to 70 Nm with a total weight of approximately 10 kg.

References


