Supplementary Material A

The parameter identification method

In the electrical ECN model shown in Figure S 1, \( v \) and \( i \) represent the ECN terminal voltage and current, respectively. Denote \( n_{RC} \) as the number of resistor-capacitor (RC) branches and define \( v_j, i_j \) \( (j = 1, 2 \cdots n_{RC}) \) as the voltage drop and current through resistance \( R_j \), and \( v_j = R_j i_j \). The time constant is \( \tau_j = R_j C_j \). The current \( i \) and the RC parameters are assumed to be constant between two measurement samples so that

\[
v_j(k + 1) = a_j v_j(k) + R_j (1 - a_j) i(k), \quad j = 1, 2, \ldots, n_{RC},
\]

where

\[
a_j = \exp(-t_s/\tau_j),
\]

and \( i_j(k) \) stands for current \( i_j \) at the \( k \)-th sampling time. \( t_s \) is sampling interval of time (unit: second). Using the widely employed coulomb counting method [1], [2], the battery State of Charge (SoC) is written as

\[
SoC(k + 1) = SoC(k) + \frac{t_s}{3600 C_n} i(k),
\]

where \( C_n \) is the battery nominal capacity at 25 °C (unit: Ampere-hour). The battery terminal voltage can be expressed as

\[
v(k) = OCV(SoC(k)) + R_0 i(k) + \sum_{j=1}^{n_{RC}} v_j(k)
\]

Figure S 1 Battery equivalent circuit network (ECN)
The batteries were tested under the full SoC range (from 100% to end of discharge), at 5 temperature levels ([15, 25, 35, 45, 55] °C) and two current rates ([1C, 2C]). The RC time constants were set as constant. This can noticeably reduce the parameter optimisation complexity without sacrificing the model accuracy [3]. On the other hand, the dependency of battery internal resistance on SoC, temperature (T) and current rates (I), is captured using a 3D look-up table.

The breakpoints of SoC, temperature and current of the 3D look-up table are set as follows,

\[ 0 \leq \text{SoC}_1 < \text{SoC}_2 < \ldots < \text{SoC}_{n_{\text{soc}}} \leq 100\%, \] (4)

\[ T_1 = 15^\circ\text{C}, T_2 = 25^\circ\text{C}, T_3 = 35^\circ\text{C}, T_4 = 45^\circ\text{C}, T_5 = 55^\circ\text{C} \]

\[ I_1 = 5A, I_2 = 10A \]

The resistor dependency on \( \text{SOC}, T \) and \( I \) is described as follows,

\[ R_j(\text{SoC}, T, i) = \sum_{k_{\text{SoC}=1}}^{n_{\text{SoC}}} \sum_{k_{T}=1}^{5} \sum_{k_{I}=1}^{2} R_{j,k_{\text{SoC}},k_{T},k_{I}} f_{k_{\text{SoC}},k_{T},k_{I}}(\text{SoC}, T, i), j = 0, \ldots, n_{RC}. \] (6)

Where \( R_{j,k_{\text{SoC}},k_{T},k_{I}} \) is the value of \( R_j \) at the node of the 3D look-up table (\( \text{SoC} = \text{SoC}_{k_{\text{SoC}}}, T = T_{k_{T}}, i = I_{k_{I}} \)), and \( f_{k_{\text{SoC}},k_{T},k_{I}}(\text{SoC}, T, i) \) is the base function,

\[ f_{k_{\text{SoC}},k_{T},k_{I}}(\text{SoC}, T, i) = F_{S_{k_{\text{SoC}}}}(\text{SoC})F_{T_{k_{T}}}(T)F_{I_{k_{I}}}(i) \]

Where the SoC-based look-up function is defined as follows,

\[ F_{S_{1}}(\text{SoC}) = \begin{cases} 1, & \text{if } \text{SoC} \leq \text{SoC}_1 \\ \frac{\text{SoC}_2 - \text{SoC}}{\text{SoC}_2 - \text{SoC}_1}, & \text{if } \text{SoC}_1 < \text{SoC} < \text{SoC}_2 \end{cases} \]

\[ F_{S_{m}}(\text{SoC}) = \begin{cases} \frac{\text{SoC} - \text{SoC}_{m-1}}{\text{SoC}_m - \text{SoC}_{m-1}}, & \text{if } \text{SoC}_{m-1} \leq \text{SoC} < \text{SoC}_m \\ \frac{\text{SoC}_{m-1} - \text{SoC}_{m+1}}{\text{SoC}_m - \text{SoC}_{m+1}}, & \text{if } \text{SoC}_m \leq \text{SoC} < \text{SoC}_{m+1} \end{cases}, \quad m = 2, \ldots, n_{\text{soc}} - 1 \] (7)

\[ F_{S_{n_{\text{soc}}}}(\text{SoC}) = \begin{cases} \frac{\text{SoC} - \text{SoC}_{n_{\text{soc}}-1}}{\text{SoC}_{n_{\text{soc}}-1} - \text{SoC}_{n_{\text{soc}}}}, & \text{if } \text{SoC}_{n_{\text{soc}}-1} \leq \text{SoC} < \text{SoC}_{n_{\text{soc}}} \\ 1, & \text{if } \text{SoC}_{n_{\text{soc}}} \leq \text{SoC} \end{cases} \]

The temperature-based look-up function is defined as follows,

\[ F_{T_{1}}(T) = \frac{T_2 - T}{T_2 - T_1}, \text{if } T \leq T_2 \]

\[ F_{T_{m}}(T) = \begin{cases} \frac{T - T_{m-1}}{T_m - T_{m-1}}, & \text{if } T_{m-1} \leq T < T_m \\ \frac{T_m - T_{m+1}}{T_{m+1} - T_m}, & \text{if } T_m \leq T < T_{m+1} \end{cases}, \quad m = 2, \ldots, n_{T} \] (8)
\[
FT_5(T) = \frac{T - T_4}{T_5 - T_4}, \quad \text{if } T_4 \leq T
\]

Finally, the current based look-up function is defined as follows,

\[
F_{I_1}(i) = \frac{i - l_2}{l_2 - l_2}, \quad \text{if } i \leq l_2
\]

\[
F_{I_2}(i) = \frac{i - l_1}{l_1 - l_1}, \quad \text{if } l_1 \leq i
\]

Substitute Eq(1), Eq(6) into Eq(4), yielding

\[
v_j(k + 1) = a_jv_j(k) + \sum_{k_{SoC}=1}^{n_{SoC}} \sum_{k_T=1}^{5} \sum_{k_l=1}^{2} R_{j,k_{SoC},k_T,k_l} f_{k_{SoC},k_T,k_l}(SoC, T, i) (1 - a_j)i(k), \quad j = 1, 2, \ldots, n_{RC}.
\]

\[
v(k) = OCV(k) + \sum_{k_{SoC}=1}^{n_{SoC}} \sum_{k_T=1}^{5} \sum_{k_l=1}^{2} R_{0,k_{SoC},k_T,k_l} f_{k_{SoC},k_T,k_l}(SoC, T, i) i(k) + \sum_{j=1}^{n_{RC}} v_j(k)
\]

In order to ensure smooth parameter transition between temperature levels, constraints are applied to ensure that the resistor values decrease monotonously with temperature increasing.

\[
R_{j,k_{SoC},k_T,k_l} \geq R_{j,k_{SoC},k_T,k_l+1,k_l}, \quad j = 0, 1, \ldots, n_{RC}, k_{SoC} = 1, 2, \ldots, n_{SoC}, k_T = 1, 2, 3, 4, k_l = 1, 2, 3, 4, 5
\]

Further, according to the Butler-Volmer equation, the battery resistance also decreases as the current increases. Therefore, the following constraints are introduced,

\[
R_{1,k_{SoC},k_T,1} \geq R_{1,k_{SoC},k_T,2}, \quad k_{SoC} = 1, 2, \ldots, n_{SoC}, k_T = 1, 2, 3, 4, 5
\]

Note that once the values of the RC time constants \( \tau_j, j = 1, 2, \ldots, n_{RC} \) are set, the resistor values of the 3D look-up table, \( R_{j,k_{SoC},k_T,k_l} \) are all linear parameters. These linear parameters can be optimized using least squares method which guarantees global optimum and high efficiency. On the other hand, the nonlinear parameters \( \tau_j \), can be optimised using a global optimisation algorithm. In this paper we use three RC networks, i.e., \( n_{RC} = 3 \). With only three nonlinear parameters to optimize, the chance of finding global minimum is greatly increased compared with optimizing all the model parameters (\( \tau_j \) and \( R_{j,k_{SoC},k_T,k_l} \)) together using e.g., Genetic algorithm [3].

The rest steps to optimise the resistor values and the time constants are similar to that shown in [3], [4]. Therefore, it is not elaborated here. The final parameter optimisation procedure is shown in Figure S2. The Matlab function ‘lsqlin’ was used as the least squares solver to optimise the linear parameters \( R_{j,k_{SoC},k_T,k_l} \), and the Matlab function ‘fmincon’ was used to optimise the nonlinear parameters \( \tau_j \).
Test data

The test data used for model parameter optimization are summarized in Table S 1. The test data are illustrated in figures below.

<table>
<thead>
<tr>
<th>Current profile</th>
<th>Temperature levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1C pulse discharge test (pulse width 4% SoC)</td>
<td>[15, 25, 35, 45, 55] °C</td>
</tr>
<tr>
<td>1C CC discharge test</td>
<td>[25, 35, 45] °C</td>
</tr>
<tr>
<td>2C pulse test (pulse width 4% SoC)</td>
<td>[25, 35, 45] °C</td>
</tr>
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</tr>
</tbody>
</table>
Figure S3 Battery pulse discharge test data at 1C under 25 °C, and the voltage simulation results of the model.
Figure S 4 Battery pulse discharge test data at 2C under 25 °C, and the voltage simulation results of the model.
Figure S5 Battery CC discharge test data at 1C under 25 °C, and the voltage simulation results of the model.
Figure S 6 Battery CC discharge test data at 2C under 25 °C, and the voltage simulation results of the model.
Battery pulse discharge test data at 2C under 25 °C. Each pulse reduces the battery’s SoC by about 20%.

Reference


