# Game-Theoretic Frameworks for Autonomous Driving Decision Making: A Literature Review

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**Abstract**

The field of autonomous driving has witnessed significant advancements in recent years, reshaping the landscape of transportation and mobility. Autonomous vehicles, equipped with an array of sensors, processors, and sophisticated algorithms, have shown the potential to revolutionize road safety, traffic efficiency, and accessibility. In addition to deep learning techniques, game theory approaches have found valuable applications in the development of autonomous driving algorithms, addressing various challenges related to coordination, interaction, and decision-making in intricate traffic scenarios. By conceptualizing driving scenarios as strategic interactions among vehicles, game theory offers a framework to optimize actions, improve efficiency, and ensure safety. This paper offers an overview of the background, techniques, and challenges associated with integrating game theory into autonomous driving decision-making. By delving into perception, decision-making, control strategies, and human-machine interaction, this review synthesizes the interdisciplinary nature of research in autonomous driving.

## 1 Introduction

The advent of autonomous driving technology stands as one of the most transformative developments in the realm of transportation. Characterized by vehicles capable of operating without direct human intervention, autonomous driving promises to redefine mobility, reshape urban planning, and reduce road accidents. Employing AI techniques in autonomous driving development revolves around leveraging artificial intelligence to enhance the capabilities of autonomous vehicles. This involves utilizing sensors, machine learning algorithms, and sophisticated data processing to enable self-driving cars to perceive their environment, make real-time decisions, and navigate safely and efficiently. AI techniques play a pivotal role in addressing the complex challenges of autonomy, such as object detection, path planning, and human-machine interaction, ultimately paving the way for safer and more efficient autonomous transportation systems. Nevertheless, autonomous driving systems may incorporate game theory and human-like thinking to navigate the intricacies of traffic scenarios. Game theory aids in modeling cooperative and competitive interactions among autonomous and human-driven vehicles, resolving conflicts, and optimizing traffic flow for efficiency. On the other hand, human-like thinking improves predictability when interacting with humans, enhances adaptive decision-making in complex ethical situations, and ensures robustness in uncertain environments. These approaches collectively contribute to safer, more efficient, and socially aware autonomous driving systems, making them better equipped to handle the nuances of real-world driving.

Game theory, a discipline examining strategic interactions among rational decision-makers, traces its roots to the early 20th century but gained formal prominence in the mid-20th century through the groundbreaking work of mathematicians and economists. Over time, it has branched into various specialized areas, each with its own focus. *Classical Game Theory*, pioneered by luminaries like John von Neumann and Oskar Morgenstern in the 1940s, centers on zero-sum games and two-player contests, laying the foundation for essential concepts like strategies and Nash equilibria. *Non-Cooperative*

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*Game Theory*, championed by John Nash in the 1950s, explores scenarios where players make independent decisions, introducing notions like the Nash equilibrium. *Cooperative Game Theory*, developed in parallel, studies the equitable division of gains among cooperating players. Additionally, *Repeated Games* analyze how strategic interactions evolve when a game is played multiple times, while *Evolutionary Game Theory*, advanced by biologists like John Maynard Smith, models evolutionary processes, particularly in biological populations. *Mechanism Design Theory*, dating to the 1970s, focuses on designing incentive-compatible mechanisms and rules. *Bayesian Games*, introduced in the 1980s, extend game theory to situations with incomplete information. *Algorithmic Game Theory*, emerging in the late 20th century, combines game theory with computer science, addressing computational aspects of game-theoretic problems. Finally, *Behavioral Game Theory* incorporates insights from psychology and behavioral economics to study real-world decision-making, deviating from the rational actor model assumed in classical game theory. In conclusion, these specialized branches have enriched our understanding of strategic decision-making and found practical applications across a spectrum of fields, from economics to biology, political science, computer science, and beyond.

In the following section, we provide an overview of how game theory is applied to decision-making in autonomous driving. We summarize these studies in Table 1.

Table 1: Game Theoretic Planning for Autonomous Driving.

| **Reference** | **Driving Scenario** | **Game Model** | **Approach** |
|---|---|---|---|
| Kedia et al. [2023] | Rare but risky events | 2-player zero-sum (adversarial forecaster and safe-planner) | Train a Maximum Likelihood Estimator and compute an $\epsilon$-equilibrium |
| Wang et al. [2020] | General | General-sum dynamic game with nonlinear cost functions | Use an iteratively linearized approximation of the system dynamics and a quadratic approximation of the cost function in solving a backward recursion for finding feedback Nash equilibria |
| Chandra et al. [2022] | General | Same as Wang et al. [2020] | Learn a mapping from a data-driven human driver behavior model to a driver's entropic risk preference and fit it into the risk-aware dynamic game solver in Wang et al. [2020] |
| Chiu et al. [2021] | General | From adversarial to cooperative over a time horizon | Optimal control and differential game theory |
| Wang et al. [2015] | Lane-changing | Dynamic game (cooperative and non-cooperative) | Optimal control theory |
| Hang et al. [2021] | Lane-changing | 2-player Normal form game and Stackelberg competition | Compute the Nash equilibrium and Stacelberg equilibrium and compare them |
| Smirnov et al. [2021] | Lane-changing | Dynamic non-cooperative game (Normal form and extensive form) | Validate the model by relying on the Co-AutoSim simulator |
| Fisac et al. [2019] | General | Stackelberg competition | Hierarchically decompose the underlying dynamic game into a long-horizon "strategic" game and a short-horizon "tactical" game |

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**Table 1 – continued from previous page**

| Reference | Driving Scenario | Game Model | Approach |
|---|---|---|---|
| Yu et al. [2018] | Lane-changing | Stackelberg game | Tested in Simulink and dSPACE, outperform fixed rule-based controllers |
| Huang et al. [2023] | General | Hierarchical game-theoretic modeling via Transformer-based encoder and decoder | The level-$k$ decoder incorporates a self-attention module to model the future interactions at level-$(k-1)$ and appends this information to the scene context encoding. Test on the Waymo interaction prediction task and the nuPlan planning benchmark |
| Wang et al. [2015] | Lane-changing and car-following | Dynamic game theory and optimal control | A problem decomposition technique is employed to reduce the dimensionality of the original problem by introducing a finite number of sub-problems and an iterative algorithm based on Pontryagin's Principle is used to solve sub-problems efficiently |
| Zhang and Fisac [2021] | Safe Occlusion-aware | 2-player zero-sum differential game (pursuer and evader) | Yield optimal strategies for both players as well as the set of initial conditions from which the autonomous vehicle is guaranteed to avoid collisions. Develop a novel trajectory planning framework for autonomous driving that provides worst-case safety guarantees while minimizing conservativeness by accounting for the vehicle's ability to actively avoid other road users as soon as they are detected in future observations. |

# 2 Game-Theoretic Planning for Autonomous Driving

## 2.1 Risk Attitude

The study by Wang et al. [2020] delves into modeling interactions among risk-aware agents using a dynamic game framework. In this context, each agent's objective is to minimize an entropic risk measure associated with their underlying cost function. To tackle the challenge of finding equilibrium solutions in such a dynamic and risk-sensitive environment, the authors introduce an iterative algorithm. This algorithm aims to approximate the elusive feedback Nash equilibria. To validate their approach, the researchers conduct experiments in a range of demanding scenarios, including complex situations like roundabouts.

In the study conducted by Chandra et al. Chandra et al. [2022], a comprehensive examination of various human driver behaviors on the road was undertaken. These behaviors encompassed a spectrum ranging from assertive actions like speeding and overtaking to more cautious tendencies such as driving at lower speeds and adhering to the right-most lane. Their methodology involved the development of a data-driven human driver behavior model known as the CMetric. This model was employed to establish a connection between a driver's entropic risk preference and their behavior. Subsequently, this risk preference was integrated into a game-theoretic risk-sensitive planner, facilitating the modeling of risk-aware interactions between human drivers and autonomous vehicles across diverse traffic scenarios. The integration of human driver behavior models with the risk-aware dynamic game solver proposed by Chandra et al. Wang et al. [2020] was accomplished using a straightforward linear regression approach. This facilitated the establishment of a correlation between driver behavior and their tolerance for risk. The findings of this study indicated that aggressive human drivers exhibit a higher frequency of lane-changing maneuvers. Additionally, it was observed that conservative drivers tend to yield to aggressive drivers while maintaining a greater following distance. These insights were further substantiated through a comprehensive user study. During this study, participants were able to discern discernible differences in the final trajectories generated by the risk-aware planner, highlighting the emergence of distinguishable behaviors between aggressive and conservative drivers.

The study by Hang et al. [2021] explores lane-change decision-making in the context of different driving styles (aggressive, normal, and conservative). They take into account three key performance metrics: driving safety, ride comfort, and travel efficiency. To capture these aspects, they incorporate them into the decision-making cost function for the ego car. This cost function considers factors such as lane speed limits, longitudinal velocities, longitudinal gaps, and relative velocities with respect to adjacent vehicles.

Subsequently, the researchers employ both Nash equilibrium and Stackelberg game theory for the non-cooperative decision-making process. In the Nash equilibrium, both the ego car and adjacent vehicles are regarded as equal and independent players. During the solving process, each entity strives to minimize its own cost function for decision-making, with no consideration for the choices made by the opponent.

On the other hand, in the context of Stackelberg equilibrium, the ego car takes on the role of a leader, while adjacent cars become followers. In this scenario, the behavior of adjacent vehicles directly influences the decision-making process of the ego car.

The study by Smirnov et al. [2021] follows a similar approach. However, a notable distinction from the previously mentioned papers lies in the ad-hoc definition of cost and payoff functions. To validate their model, the authors utilized the Co-AutoSim simulator and conducted comparisons between the model's predictions and the real decisions made by participants. Specifically, they examined whether participants permitted the autonomous vehicle to take the lead.

## 2.2 Non-cooperative and Cooperative Games

Drawing inspiration from the principles of defensive driving, Chiu et al. [2021] introduce a unique approach that explicitly characterizes other agents as adversarial only during an initial segment of the overall planning timeframe. Within this adversarial phase, they employ a general-sum game framework suitable for N-player scenarios. Subsequently, the approach transitions into a cooperative phase, where non-ego agents are expected to revert to their typical cooperative behavior. This concept mirrors defensive driving, in which a driver navigating a busy roadway remains vigilant against momentarily distracted fellow drivers who might exhibit brief, unsafe behaviors.

The method used in Chiu et al. [2021] shares some parallels with adversarial techniques in Hamilton-Jacobi-Isaacs (HJI) optimal control, drawing upon earlier research in differential game theory. However, instead of mandating that the ego vehicle avoids all possible trajectories of non-ego agents, they introduce a piecewise-defined game cost structure. This approach imbues the ego vehicle with the perspective that other agents are temporarily distracted, leading to significantly less conservative behavior compared to purely adversarial methods.

Instead of aiming to maximize the payoff function for autonomous vehicles (AVs), as presented in Wang et al. [2015], the approach taken involves defining the AV's cost function and minimizing it through the utilization of optimal control theory. This strategy entails framing the challenge as a dynamic game, where controlled vehicles make decisions based on their expectations regarding the behavior of other vehicles. Initially, they conceptualize the controller problem as non-cooperative control systems for autonomous vehicles. Subsequently, this framework is extended to cooperative control for connected vehicles, where controlled vehicles share information and collaborate via V2V communication.

The primary objective within the context of Lane-changing and Car-following Control Systems (LCCS) for fully automated vehicles at time $t_0$ is to determine the optimal control, denoted as $u^*$, over a prediction horizon of $T_p$. This optimal control should guide the system along a trajectory that minimizes the predicted cost function $J$. To address this complex task efficiently, a problem decomposition technique is introduced. This technique reduces the dimensionality of the original problem by introducing a finite number of sub-problems. Subsequently, an iterative algorithm, rooted in Pontryagin's Principle, is applied to efficiently solve these sub-problems.

In the realm of autonomous driving system development, a critical challenge lies in grappling with occlusions that impose limitations on a vehicle's perception capabilities. Navigating the landscape of potential hazards lurking beyond the vehicle's immediate field of view stands out as one of the most formidable hurdles. Occlusions cast an initial shroud of uncertainty over the pursuer's whereabouts, which may subsequently be unveiled through the vigilant sensors of the evading vehicle. In their work, Zhang and Fisac [2021] delve into the intricacies of occlusion-awareness and safety considerations from a worst-case perspective. Their approach involves modeling the problem as a two-player zero-sum differential game, pitting the pursuer against the evader. The outcome of their rigorous analysis furnishes us with optimal strategies for both players, accompanied by a delineation of the initial conditions under which the autonomous vehicle can confidently steer clear of collisions. A key innovation of their work lies in the development of a novel trajectory planning framework for autonomous driving. This framework not only provides stringent worst-case safety guarantees but also mitigates unnecessary conservativeness by factoring in the vehicle's proactive ability to evade other road users as soon as they are detected in future observations. To substantiate the efficacy of their algorithm, the authors conducted comprehensive testing across challenging urban and highway driving scenarios, employing the open-source CARLA simulator as their testbed.

## 2.3 Stackelberg competition

The actions of an autonomous vehicle on the road have a profound impact on and are influenced by the actions of other drivers. Whether it's overtaking, merging, or avoiding accidents, these interactions create a mutual dependency that can be best understood through the lens of dynamic game theory. This complex interplay between the vehicle's planning and its predictions of other drivers' behavior represents an ongoing challenge with significant implications for the safety and feasibility of autonomous driving technology. Fisac et al. [2019] introduce an innovative trajectory planning algorithm rooted in game theory for autonomous driving. This algorithm not only enables real-time performance but also breaks down the underlying dynamic game into two distinct components. The first is a "strategic" game with simplified dynamics and a comprehensive information structure that spans a longer horizon. The second is a "tactical" game characterized by full dynamics and a simplified information structure, covering a shorter horizon. The authors model the dynamic game using a feedback closed-loop information framework, where both players' actions can depend on the current state but not on the state history. This approach allows human drivers to adapt their choices based on the autonomous vehicle's current actions at each time step, resulting in a Stackelberg-style dynamic game, akin to a leader-follower scenario. In the optimization of the autonomous car's trajectory at the tactical level, they employ an iterated local best response strategy to seek a local open-loop Nash equilibrium among the short-term trajectories of the vehicles involved.

In the study by Yu et al. [2018], lane-changing is conceptualized as a Stackelberg game, and a solution is provided for the associated bilevel optimization problem. The total payoff function for the autonomous vehicle is expressed as a linear combination of two factors: safety and space considerations. This combination is contingent on the driver's level of aggressiveness. To assess its effectiveness, the game theory-based controller underwent testing using Simulink and dSPACE. The test scenarios were designed to facilitate interactions between a vehicle controlled by the game theory-based system and those operated by both robotic and human drivers. The results of the tests indicate that the game theory-based controller is capable of executing lane changes in a manner resembling human behavior and surpasses the performance of fixed rule-based controllers.

The study by Kedia et al. [2023] delves into the realm of forecasting and planning, with a particular focus on addressing rare but high-risk events. The research scrutinizes the limitations of maximum likelihood estimation (MLE) forecasting, which predominantly emphasizes improving the likelihood of recorded human motion. In contrast, the authors adopt an approach that involves the generation of adversarial counterfactuals. They achieve this by refining the performance gap between generated plans and observed demonstrations, all while taking into consideration the forecasts generated by their trained forecasting mechanism. As a result, their framework stands as a defense against infrequent yet perilous occurrences, as it produces plans that prioritize safety in light of the opposing factor.

## 2.4 Level-$k$ Game Theory

The paper by Huang et al. [2023] tackles the challenge of interaction prediction by introducing a novel approach rooted in hierarchical game theory. Their proposed framework, known as GameFormer, serves as the vehicle for implementing this approach. In particular, the authors introduce a unique Transformer decoder structure that leverages predictions from the preceding level, along with a shared environmental context, to iteratively refine the interaction process. Furthermore, they introduce a learning mechanism designed to guide an agent's behavior at the current level, allowing it to respond effectively to the behaviors of other agents observed at the previous level. To validate the effectiveness of their model, extensive experiments were conducted using large-scale real-world driving datasets. These experiments showcased the model's outstanding accuracy in handling the Waymo interaction prediction task, establishing it as state-of-the-art. The authors also demonstrated the model's ability to concurrently reason about the motion planning of the ego agent and the behaviors of multiple agents in various testing scenarios, including open-loop and closed-loop planning tests. In these evaluations, the GameFormer framework consistently outperformed various baseline methods. Furthermore, the model's performance was assessed on the nuPlan planning benchmark, where it excelled, attaining a leading position in terms of performance metrics.

# 3 Liability

Liability issues in autonomous driving refer to the legal and ethical questions surrounding who is responsible when an autonomous vehicle (AV) is involved in an accident or causes harm. Unlike traditional human-operated vehicles, where the driver is typically responsible for their actions, the introduction of autonomous vehicles raises complex issues due to the division of control between the human driver and the vehicle's automated systems.

Addressing liability issues requires a combination of clear legal frameworks, robust testing and validation processes, advanced safety technologies, and collaborative efforts among manufacturers, policymakers, legal experts, and the technology industry. As autonomous driving technology advances, these issues will continue to evolve, and the legal landscape will need to adapt accordingly.

In the study by Di et al. [2019], game theory is employed as a means to ascertain responsibility in self-driving car accidents. The necessity for such strategic analysis becomes evident through a recent ruling from the National Transportation Safety Board (NTSB) concerning a tragic incident involving an Uber autonomous vehicle (AV) and a pedestrian in Arizona. In this case, the NTSB apportioned blame among Uber, the AV manufacturer, the safety driver, the victim, and the state of Arizona. The central focus of their investigation is twofold: to assess the impact of AVs on road safety and to formulate liability rules that optimize societal welfare for both AVs and human drivers. To achieve this goal, the researchers construct a unified game model that encompasses various elements. Within this model, they incorporate a Nash game involving human drivers, a Stackelberg game pitting the AV manufacturer against human vehicle (HV) operators, and another Stackelberg game involving legislators and other road users. Furthermore, they establish the existence and uniqueness of equilibrium within this intricate game structure. Subsequently, they simulate the game using numerical examples, enabling an examination of key aspects such as the emergence of moral hazard among human drivers, the role of AV manufacturers in enhancing traffic safety, and the role of lawmakers in crafting liability frameworks. The game-theoretic framework introduced in this paper serves as a valuable analytical tool for policymakers involved in shaping regulations for autonomous vehicles. By offering this tool, the research aims to alleviate uncertainty within the current regulatory landscape concerning AV technologies.

# 4 Challenges

Applying game theory modeling to enhance the safety and efficiency of autonomous driving encounters several formidable challenges. These include the intricate nature of real-world traffic scenarios with multiple variables, the need to account for unpredictable human behavior, concerns related to privacy and data sharing, complex regulatory and legal issues surrounding liability and ethical decision-making, difficulties in achieving coordination and cooperation among various stakeholders, the demand for real-time adaptability, scalability concerns as the number of autonomous vehicles increases, the necessity for accurate prediction of behavior, the need for smooth human-autonomous interaction, and the rigorous validation and testing required for real-world implementation. Addressing these challenges necessitates a holistic, multidisciplinary approach involving experts in game theory, engineering, policy development, and legal frameworks to ensure the effective and ethical integration of autonomous driving technology into our transportation systems.

# 5 Conclusion

In conclusion, this survey paper has delved into the burgeoning field of game theory applications in autonomous driving decision-making. We've explored the promising potential of game theory to revolutionize how autonomous vehicles navigate complex traffic scenarios and interact with human-driven counterparts. As highlighted throughout this review, the challenges are significant, ranging from the intricacies of real-world traffic environments to privacy concerns and regulatory intricacies. However, the rewards are equally compelling, as game theory opens doors to safer, more efficient, and ethically sound autonomous driving systems. It is clear that ongoing research, collaboration across disciplines, and the development of sophisticated algorithms will play pivotal roles in overcoming these challenges and realizing the full potential of game theory in shaping the future of autonomous mobility. With continued efforts, we can expect game theory to be a driving force in advancing the safety and efficiency of autonomous vehicles, ultimately transforming our transportation landscape.

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