

Numerical Modelling of Seabed Impact Effects on Chain and Small Diameter Mooring Cables

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Abstract

Catenary mooring lines experience liftoff from and grounding on the seabed when undergoing large dynamic motions. Numerical line mooring models account for this interaction using various seabed models and it is known that the action of liftoff and grounding may lead to large dynamic tension fluctuations. These fluctuations may be spurious due to the inability of discretised mooring models to adequately account for the effect of the seabed on the mooring line. In this work, the root cause and conditions that lead to the production of the large dynamic tension fluctuations is determined. The effect of line discretisation and seabed model on the tension fluctuations is investigated using the widely used spring-mattress approach and a modified seabed reaction force model. An in-house mooring code was developed to perform these investigations. For code validation and benchmarking, and to illustrate the existence of the tension fluctuations problem due to nodal grounding in existing mooring line simulation codes, comparisons are made to a commercial software.

Keywords:

Mooring, Seabed impact, Snap load, Tension fluctuations, Lumped-mass

1. Introduction

Catenary mooring lines provide the restoring force necessary for station-keeping of floating structures primarily by varying its suspended weight in

response to the tension applied at the fairlead connection to the floating platform. The seabed has a significant effect on mooring line motions and loads and consequently the dynamics of the connected floating structure as well.

One description of the seabed forces is the spring-mattress model, or variously referred to as the elastic seabed model, such as that proposed by Webster [1]. Similar variations of this approach are used in Refs. [2, 3, 4, 5] and is implemented in commercial codes such as Orcaflex [6] and aNyMOOR [7]. The main advantage of this approach is that the entire line length remains active over a simulation. Hence, the effect of the grounded section on the suspended part, and vice versa, is fully accounted for. In contrast to the spring-mattress model, in which the dynamics of the grounded section are calculated, Chatjigeorgiou and Mavrakos [8] proposed a model which calculates the touchdown point from a quasi-static solution, and truncating it from the touchdown point onwards. Consequently, the effects of the line liftoff and touchdown forces are neglected. Thomas [9] proposed a seabed model in which the mass of the first two suspended nodes adjacent to the seabed are gradually reduced as they approach the sea-bed. The motivation for the development of this model was to eradicate the fluctuations in line tension associated with nodal grounding. However, a limitation of the method is that the mass modifier coefficients used to perform the nodal mass reduction have to be determined, by trial and error, for individual grounding nodes and recalibrated for each specific fairlead excitation time history.

Wang et al. [10] used the lumped mass approach to model a mooring line in conjunction with a seabed model based on rigid body collision analysis. It was noted by the authors that, upon line impact with the seabed, there were fluctuations in the fairlead tension which they attributed to the spatially discrete nature of the line structural model. Triantafyllou et al. [11] showed analytically that tension shocks may occur during both the loading and unloading phases of a dynamic mooring cable motion period and derived a condition for its occurrence. Gobat and Grosenbaugh [12] experimentally verified the condition, noting however that the occurrence of unloading shocks may not affect the fairlead tension.

In gist, there have been a variety of seabed modelling methods proposed. For time-domain analysis, the spring mattress method is the most widely used due to its flexibility, simplicity, and completeness of analysis for the entire mooring line. However, previous studies have suggested that it is prone to numerical errors associated with line contact with the seabed.

The occurrence of line tension fluctuations due to nodal grounding in dis-

crete, lumped mass mooring models is studied using an in-house code and Orcaflex [6] in this work. As noted by Yang et al. [13], such irregularities in the tension results have a direct impact on the assessment of the fatigue life of mooring lines. The spurious nature and cause of the fluctuations is determined, and the effectiveness of reducing the fluctuations using a modified spring-mattress model is evaluated.

2. Lumped mass Mooring Line Model

The applicability of the lumped mass approach to mooring line modelling has been proven in many implementations [2, 4, 14]. In a lumped mass model, a mooring line is discretised into nodes and elements, and a typical configuration of a discretised catenary line is presented in Figure 1a. Figure 1b shows the connectivity between nodes and elements in the present model. The Orcaflex theory manual [6] documents the force calculation procedures used in that software, while the methods in the current in-house code is presented in this section.

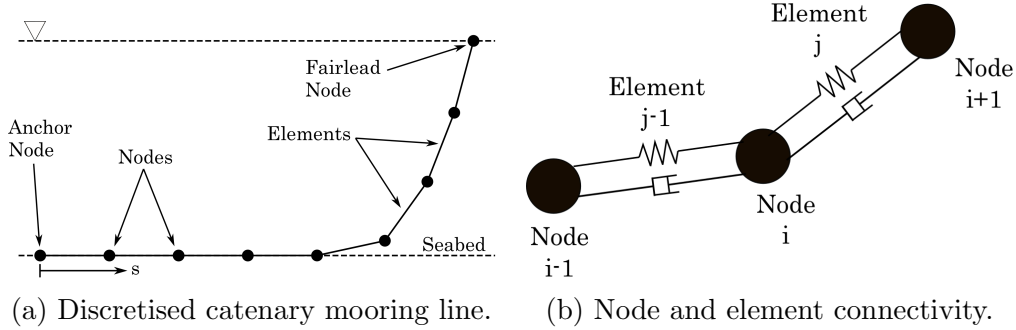


Figure 1: Typical lumped mass mooring line discretisation.

2.1. Distribution of Mass

The equations of motion are solved for the nodes. The mass of each element is distributed equally to the adjacent nodes by way of a diagonal mass matrix.

$$\mathbf{M}_i = \frac{m_{j-1}L_{j-1}}{2}\mathbf{I}_3 + \frac{m_jL_j}{2}\mathbf{I}_3 = \sum_{\xi=j,j-1} \frac{m_\xi L_\xi}{2}\mathbf{I}_3 \quad (1)$$

where m_ξ and L_ξ , $\xi = \{j, j-1\}$ are the mass per unit length and lengths of the elements j and $j-1$, and $\mathbf{I}_3$ is a 3x3 identity matrix.

2.2. Line Tension

The tension in a section of the line is represented as the tension, T_j , in the associated element j given by

$$T_j = \begin{cases} K_j \epsilon_j, & \epsilon_j > 0 \\ 0, & \epsilon_j \leq 0 \end{cases} \quad (2)$$

where ϵ_j is the strain and K_j is the linear stiffness of the element. The element strain is calculated as

$$\epsilon_j = 1 - \frac{|\mathbf{r}_{i+1} - \mathbf{r}_i|}{L_j} \quad (3)$$

where $\mathbf{r}_{i+1}$ and $\mathbf{r}_i$ are the positions of the nodes bounding element j , and L_j is the unstretched element length. With reference to Figure 1b, the resultant tension force vector acting on a node is the sum of the tension forces from its connected elements given by

$$\mathbf{T}_i = T_j = K_j \epsilon_j \hat{\mathbf{e}}_j - K_{j-1} \epsilon_{j-1} \hat{\mathbf{e}}_{j-1} \quad (4)$$

where the element unit direction vectors are given by $\hat{\mathbf{e}}_j = (\mathbf{r}_{i+1} - \mathbf{r}_i)/L_j$ and $\hat{\mathbf{e}}_{j-1} = (\mathbf{r}_i - \mathbf{r}_{i-1})/L_{j-1}$.

2.3. Seabed Forces

Figure 2 shows the seabed coordinate system and an element j in contact with the seabed. The seabed nominal elevation is $z^{B,0}$, while the seabed force cutoff elevation is $z_{B,c}$; the elevation at which the element section is not in contact with the seabed. The unit vector normal to the seabed is $\hat{\mathbf{n}}_B$.

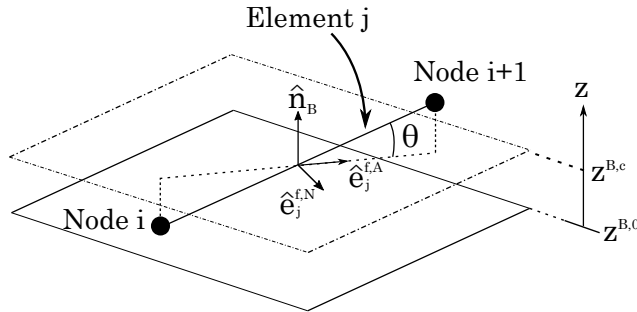


Figure 2: Seabed coordinate system definitions.

The unit vector $\hat{\mathbf{e}}_j^{f,A}$ is a unit vector that is the projection of the element direction unit vector $\hat{\mathbf{e}}_j$ on the seabed tangent plane, while the unit vector $\hat{\mathbf{e}}_j^{f,N}$ is the unit vector orthogonal to both $\hat{\mathbf{e}}_j^{f,A}$ and $\hat{\mathbf{n}}_B$.

$$\hat{\mathbf{e}}_j^{f,A} = \frac{\hat{\mathbf{e}}_j - (\hat{\mathbf{e}}_j \cdot \hat{\mathbf{n}}_B) \hat{\mathbf{n}}_B}{|\hat{\mathbf{e}}_j - (\hat{\mathbf{e}}_j \cdot \hat{\mathbf{n}}_B) \hat{\mathbf{n}}_B|} \quad (5)$$

$$\hat{\mathbf{e}}_j^{f,N} = \frac{\hat{\mathbf{e}}_j^{f,A} \times \hat{\mathbf{n}}_B}{|\hat{\mathbf{e}}_j^{f,A} \times \hat{\mathbf{n}}_B|} \quad (6)$$

2.3.1. Spring Mattress Reaction Force Model

In the usual seabed spring mattress model as described by Webster [1] and Gobat and Grosenbaugh [5], a seabed reaction force is directly applied on a node as a function of its own vertical elevation,

$$\mathbf{F}_i^{B,r} = k_i^B (z_i^{B,c} - z_i) \hat{\mathbf{n}}_B \quad (7)$$

where k_i^B is the spring constant, $z_i^{B,c}$ is the seabed force cutoff elevation and z_i is the nodal elevation. A damping force proportional to nodal velocity may also be included [5]. The nodal seabed stiffness coefficient k_i^B is given by

$$k_i^B = \frac{W_i}{N^{B,c} D_i - z^{B,0}} \quad (8)$$

where W_i is the nodal weight, $z^{B,0}$ is the nominal seabed elevation, $N^{B,c}$ is the seabed thickness coefficient and D_i is the line diameter at the s -coordinate of Node i .

2.3.2. Modified Spring Mattress Reaction Force Model

As will be discussed in Sections 4 and 5, the spring mattress model can be modified. We present a modified seabed spring mattress model in which an element can be in four states with respect to the seabed, as shown in Figure 3.

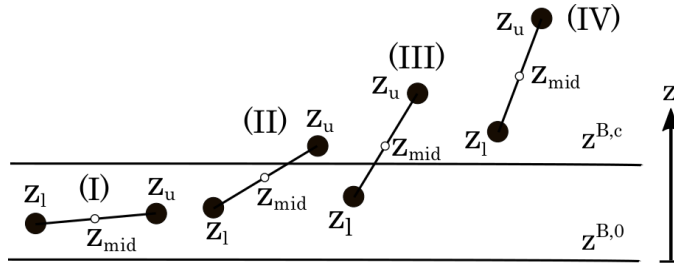


Figure 3: Element embedment states.

States I and IV respectively describe the element in fully embedded and suspended states with respect to the seabed. State II refers to an element in the state of partial embedment with the center of the element below the seabed force cutoff elevation, $z^{B,c}$. State III describes a partially embedded element with the element center above $z^{B,c}$. The four states determine how the seabed forces are calculated and distributed to the bounding nodes.

The seabed forces acting on an element, $\mathbf{F}^B$, are comprised of forces that are tangential and normal to the seabed surface, $\mathbf{F}^{B,\tau}$ and $\mathbf{F}^{B,\nu}$,

$$\mathbf{F}^B = \mathbf{F}^{B,\tau} + \mathbf{F}^{B,\nu} \quad (9)$$

In the present model, the forces that are normal to the seabed surface are comprised of the seabed reaction, damping, and added mass components, respectively represented as $\mathbf{F}^{B,r}$, $\mathbf{F}^{B,d}$ and $\mathbf{F}^{B,a}$, in Equation 10.

$$\mathbf{F}^{B,\nu} = \mathbf{F}^{B,r} + \mathbf{F}^{B,d} + \mathbf{F}^{B,a} \quad (10)$$

The seabed reaction force on element j , $\mathbf{F}_j^{B,r}$, is given by,

$$\mathbf{F}_j^{B,r} = \left(\int_{s_{1,j}}^{s_{2,j}} k_j^B \left[z_j^{B,c} - z_j(s) \right] ds \right) \hat{\mathbf{n}}_B \quad (11)$$

where $\hat{\mathbf{n}}_B$ is the seabed normal direction unit vector, s is the line axial coordinate, as shown in Figure 1a, and z is the vertical coordinate, as shown in Figure 3. The quantity, $z_j^{B,c} - z_j(s)$, in Equation 11 is the per unit length embedment depth.

The element seabed spring stiffness is given by

$$k_j^B = \frac{W_j/L_j}{z_j^{B,c} - z^{B,0}} \quad (12)$$

where W_j and L_j are the element weight and unstretched lengths respectively. The relationship between the seabed force cutoff elevation $z_j^{B,c}$ and $z^{B,0}$ is given by

$$z_j^{B,c} = N_j^{B,c} D_j + z^{B,0} \quad (13)$$

where $N_j^{B,c}$ is the seabed thickness coefficient and D_j is the diameter of the element, or equivalently taken to be the physical outer diameter of the line section. The parameter $N_j^{B,c} D_j$ denotes the distance over which the seabed force is increased linearly from zero to the element weight per unit length.

Equation 14 shows that when the entire element is resting on the seabed at the seabed nominal elevation, $z^{B,0}$, the seabed reaction force on element j is equivalent to its weight.

$$\begin{aligned}\mathbf{F}_j^{B,r} &= \left(\int_{s_{1,j}}^{s_{2,j}} \frac{W_j/L_j}{z_j^{B,c} - z^{B,0}} \left[z_j^{B,c} - z^{B,0} \right] ds \right) \hat{\mathbf{n}}_B \\ &= \left(\int_{s_{1,j}}^{s_{2,j}} \frac{W_j}{L_j} ds \right) \hat{\mathbf{n}}_B = W_j \hat{\mathbf{n}}_B\end{aligned}\quad (14)$$

Since each element is bounded by two nodes, with reference to Figure 3, the elevation of any point along an element is described with a linear interpolation between the elevations of its bounding nodes,

$$z_j(s) = z_{l,j} + \frac{z_{u,j} - z_{l,j}}{L_j} s = z_{l,j} + \frac{\Delta z_j}{L_j} s \quad (15)$$

where z_u and z_l refer to the z-coordinates of the upper and lower nodes. Differentiating Equation 15 with respect to z allows a change of integration variable from s to z in Equation 11,

$$\mathbf{F}_j^{B,r} = \frac{k_j^B L_j}{\Delta z_j} \int_{z_{1,j}}^{z_{2,j}} \left(z_j^{B,c} - z_j \right) dz \quad (16)$$

Each element can be split into two half-elements of equal lengths, and the force on each half-element is assigned to its adjoining node. With reference to Figure 3, for an element in State I, fully embedded in the seabed, the seabed reaction force contribution of element j on the adjoining node of lower elevation can be calculated by integrating Equation 16 with the limits of integration $z_{l,j} = z_l$ and $z_{2,j} = z_{mid,j} = (z_{l,j} + z_{u,j})/2$, which gives

$$\mathbf{F}_l^{B,r} = \frac{k_j^B L_j}{\Delta z_j} \left[\left(z_j^{B,c} (z_{mid,j} - z_{l,j}) \right) + \frac{z_{l,j}^2 - \left(z_j^{B,c} \right)^2}{2} \right] \quad (17)$$

For the node with the higher elevation, the limits of integration are $z_{1,j} = z_{mid,j}$ and $z_{2,j} = z_u$. Integrating Equation 16 with these limits leads to the reaction force on the upper node being given by

$$\mathbf{F}_u^{B,r} = \frac{k_j^B L_j}{\Delta z_j} \left[\left(z_j^{B,c} (z_{u,j} - z_{mid,j}) \right) + \frac{z_{mid,j}^2 - z_{u,j}^2}{2} \right] \quad (18)$$

For a partially embedded element in State II, the seabed reaction force on the lower node is as shown in Equation 17. To obtain the seabed reaction force on the upper node, Equation 16 is integrated with the limits $z_{1,j} = z_{mid,j}$ and $z_{2,j} = z_j^{B,c}$, which leads to

$$\mathbf{F}_u^{B,r} = \frac{k_j^B L_j}{\Delta z_j} \left[\frac{\left(z_j^{B,c}\right)^2 + z_{mid,j}^2}{2} - z_j^{B,c} z_{mid,j} \right] \quad (19)$$

For a partially embedded element in State III, the seabed reaction force on the upper node is taken to be zero as $z_{mid,j} > z_j^{B,c}$. For the lower node, using the integration limits $z_{1,j} = z_{l,j}$ and $z_{2,j} = z_j^{B,c}$ in Equation 16 leads to

$$\mathbf{F}_l^{B,r} = \frac{k_j^B L_j}{\Delta z_j} \left[\frac{\left(z_j^{B,c}\right)^2 + z_{l,j}^2}{2} - z_j^{B,c} z_{l,j} \right] \quad (20)$$

An element in State IV is fully suspended and does not experience any seabed forces. For Node i , the total seabed reaction force, as with all other forces, are contributed by elements j and $j - 1$,

$$\mathbf{F}_i^{B,r} = \mathbf{F}_j^{B,r}(\chi_j, z_i, z_{i+1}) + \mathbf{F}_{j-1}^{B,r}(\chi_{j-1}, z_{i-1}, z_i) \quad (21)$$

where the contributions from the elements are dependent on the elemental grounding state, $\chi = \{I, II, III, IV\}$, and the relative elevations of nodes $i + 1$, i and $i - 1$. In this modified spring mattress model, a node experiences a seabed reaction force when either of the half elements adjacent to it is in contact with the seabed, when $z_{mid,j} \leq z_j^{B,c}$ or $z_{mid,j-1} \leq z_{j-1}^{B,c}$.

2.3.3. Vertical Seabed Damping Force

The seabed normal damping force on node i , $\mathbf{F}_i^{B,d}$, is given by

$$\mathbf{F}_i^{B,d} = \mathbf{F}_j^{B,d} + \mathbf{F}_{j-1}^{B,d} = \sum_{\xi=j,j-1} \mathbf{F}_\xi^{B,d} \quad (22)$$

with the contributions from elements j and $j - 1$ as given by

$$\mathbf{F}_\xi^{B,d} = \begin{cases} -C_\xi^{B,d} \dot{z}_i \hat{\mathbf{n}}_B, & \dot{z}_i < 0, \quad |\mathbf{F}_\xi^{B,r}| > 0 \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

where $\dot{z}$ is the vertical velocity of node i . The damping force is active only when a node has downward velocity and is experiencing a reaction force from the seabed contributed by the elements attached to it. The damping force coefficient, $C_\xi^{B,d}$, following the theory of vibrations [15], is given by $C_\xi^{B,d} = 2\zeta^{B,d}\sqrt{k_\xi^B m_\xi}$, where m_ξ are the masses per unit length of their respective elements, and $\zeta^{B,d}$ is the damping ratio [15].

2.3.4. Seabed Added Mass Model

Borrowing from the theory of hydrodynamic added mass, in addition to the reaction and damping forces, an added mass force is introduced as a vertical seabed force. To the best knowledge of the authors, existing time domain seabed models typically model the seabed normal forces on a mooring line as a spring and damper system. The seabed added mass force on node i is given by

$$\mathbf{F}_i^{B,a} = \mathbf{F}_j^{B,a} + \mathbf{F}_{j-1}^{B,a} = \sum_{\xi=j,j-1} \mathbf{F}_\xi^{B,a} \quad (24)$$

where the force contribution from the adjacent elements are

$$\mathbf{F}_\xi^{B,a} = \begin{cases} -C_\xi^{B,a} \ddot{z}_i \hat{\mathbf{n}}_B, & \ddot{z}_i < 0, |\mathbf{F}_\xi^{B,r}| > 0 \\ 0, & \text{otherwise} \end{cases} \quad (25)$$

and where $C_\xi^{B,a}$, $\xi = \{j, j-1\}$ represent the added mass coefficients of elements j and $j-1$. The added mass force is active only when the acceleration of the node $\ddot{z}_i$ is downwards, and the node experiences a seabed reaction force. The added mass coefficients are prescribed as $C_\xi^{B,a} = (\zeta^{B,a} m_\xi L_\xi)/2$, where $\zeta^{B,a}$ is the added mass coefficient, and m_ξ and L_ξ , $\xi = \{j, j-1\}$ represent the structural masses per unit length and lengths of the elements j and $j-1$.

2.3.5. Seabed Friction Force

The friction force formulation is based on the Coulomb friction model. The friction force acts tangent to the seabed surface and is the sum of the components in the element axial and normal directions, $\mathbf{F}^{B,f,A}$, $\mathbf{F}^{B,f,N}$,

$$\mathbf{F}_i^{B,\tau} = \mathbf{F}_j^{B,f,A} + \mathbf{F}_j^{B,f,N} + \mathbf{F}_{j-1}^{B,f,A} + \mathbf{F}_{j-1}^{B,f,N} = \sum_{\kappa=A,N} \sum_{\xi=j,j-1} \mathbf{F}_\xi^{B,f,\kappa} \quad (26)$$

where the axial and normal elemental contributions $\mathbf{F}_\xi^{B,f,\kappa}$, $\kappa = \{A, N\}$ are given by

$$\mathbf{F}_\xi^{B,f,\kappa} = \begin{cases} -C_\xi^{B,f,\kappa} \dot{\mathbf{r}}_\xi^{f,\kappa}, & C_\xi^{B,f,\kappa} |\dot{\mathbf{r}}_\xi^{f,\kappa}| < \mu^{B,f,\kappa} |\mathbf{F}_\xi^{B,r}| \\ -\mu^{B,f,\kappa} |\mathbf{F}_\xi^{B,r}| \frac{\dot{\mathbf{r}}_\xi^{f,\kappa}}{|\dot{\mathbf{r}}_\xi^{f,\kappa}|}, & C_\xi^{B,f,\kappa} |\dot{\mathbf{r}}_\xi^{f,\kappa}| \geq \mu^{B,f,\kappa} |\mathbf{F}_\xi^{B,r}| \end{cases} \quad (27)$$

where friction coefficients $C_\xi^{B,f,\kappa}$ are defined as $C_\xi^{B,f,\kappa} = \zeta^{B,f,\kappa} \sqrt{k_\xi^B m_\xi}$, $\kappa = \{A, N\}$, and $\mu_\xi^{B,f,\kappa}$ are the static friction coefficients in the $\dot{\mathbf{e}}_\xi^{f,\kappa}$, $\kappa = \{A, N\}$, directions. The vectors $\dot{\mathbf{r}}_\xi^{f,\kappa}$, $\kappa = \{A, N\}$, are given by $\dot{\mathbf{r}}_\xi^{f,\kappa} = (\dot{\mathbf{r}}_i \cdot \hat{\mathbf{e}}_\xi^{f,\kappa}) \hat{\mathbf{e}}_\xi^{f,\kappa}$.

2.4. Weight and Buoyancy

The present mooring line model also takes into consideration the weight, buoyancy and hydrodynamic forces on the line. The weight of a line section submerged in water is given by

$$\mathbf{W} = -(w - \rho A)L \hat{\mathbf{k}} \quad (28)$$

where w is the weight per unit length of the section and A is the mean cross sectional area.

2.5. Hydrodynamic Forces

The hydrodynamic forces are evaluated using Morison's equation, which consists of the added mass, Froude-Krylov and viscous drag components [16].

$$\begin{aligned} \mathbf{F}^H &= \mathbf{F}^{H,\tau} + \mathbf{F}^{H,\nu} \\ &= \mathbf{F}^{A,\tau} + \mathbf{F}^{A,\nu} + \mathbf{F}^{I,\tau} + \mathbf{F}^{I,\nu} + \mathbf{F}^{D,\tau} + \mathbf{F}^{D,\nu} \\ &= \rho \frac{\pi}{4} D^2 [(C_a^\tau + 1) \dot{\mathbf{u}}^\tau - C_a^\tau \ddot{\mathbf{r}}^\tau] + \frac{1}{2} \rho D C_d^\tau |\mathbf{u}_{rel}^\tau| \mathbf{u}_{rel}^\tau \\ &\quad + \rho \frac{\pi}{4} D^2 [(C_a^\nu + 1) \dot{\mathbf{u}}^\nu - C_a^\nu \ddot{\mathbf{r}}^\nu] + \frac{1}{2} \rho D C_d^\nu |\mathbf{u}_{rel}^\nu| \mathbf{u}_{rel}^\nu \end{aligned} \quad (29)$$

where $\mathbf{F}^{A,\tau}$ and $\mathbf{F}^{A,\nu}$ are the added mass, $\mathbf{F}^{I,\tau}$ and $\mathbf{F}^{I,\nu}$ are the Froude-Krylov and $\mathbf{F}^{D,\tau}$ and $\mathbf{F}^{D,\nu}$ are the drag forces in the line tangential and normal directions. C_a^τ , C_a^ν and C_d^τ , and C_d^ν are the sectional tangential and normal added mass and drag coefficients, D is the hydrodynamic diameter, ρ is the water density, $\dot{\mathbf{u}}$ is the acceleration of the water, and $\mathbf{u}_{rel}$ is the relative velocity between the water and the line structure.

2.6. Time Integration of Equations of Motion

The equations of motion are assembled as

$$\mathbf{M}_i \ddot{\mathbf{r}}_i = \mathbf{T}_i + \mathbf{F}_i^S + \mathbf{F}_i^B + \mathbf{W}_i + \mathbf{F}_i^H = \mathbf{F}_i^{tot} \quad (30)$$

A state vector, $\mathbf{Y}$, containing the nodal positions and velocities, $\mathbf{r}$ and $\dot{\mathbf{r}}$, is defined and its time derivative can be written as

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} \dot{\mathbf{r}} \\ \ddot{\mathbf{r}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{r}} \\ \mathbf{M}^{-1} \mathbf{F}^{tot} \end{bmatrix} \quad (31)$$

Time integration is performed integrating the first order ordinary differential equation system in Equation 31 with the fourth-order Runge-Kutta algorithm [17].

3. Validation of Mooring Line Model Without Seabed Contact

In this section, the tension results from Orcaflex and the current in-house code are compared without considering seabed contact so as to compare the results from the two codes involving all other forces including weight, buoyancy, hydrodynamic and tension forces. The two test configurations consisting of fully suspended mooring chains, one with a fixed end and the other a free end, are shown in Figure 4.

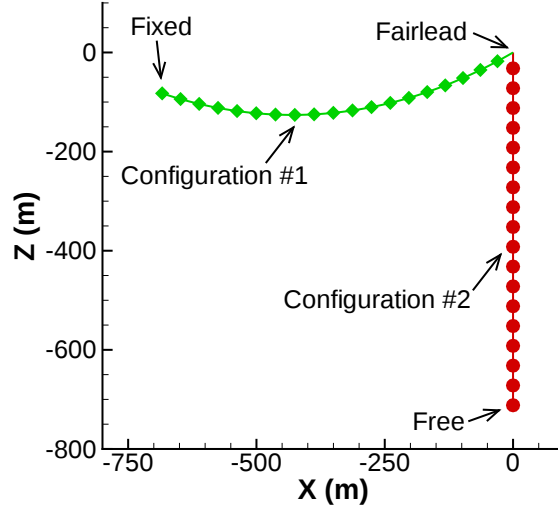


Figure 4: Suspended line configurations for validation study.

The line is assumed to be fully submerged in water for the entire duration of the simulation. The water density is 1025 kg/m^3 . The structural parameters of the mooring line are listed in Table A.1. For static analysis, there are no external loads except for weight, hence in both configurations the entire line is in the X-Z plane. The static line tensions for both configurations are shown in Table 1. For Configuration 1, the line end tensions at both ends of the line are in agreement up to 6 significant figures. For Configuration 2, the discrepancy in the fixed and fairlead end tensions are both 0.021%.

Table 1: Initial position and tension at line ends.

Config	Line End	(X,Y,Z)-coordinates	Tension- Orcaflex (kN)	Tension- Inhouse (kN)
1	Free	(0.0, 0.0, 711.3)	0.00	0.00
	Fairlead	(0.0, 0.0, 0.0)	2275.58	2275.58
2	Fixed	(-683.74, 0.0, -82.5)	2542.82	2542.28
	Fairlead	(0.0, 0.0, 0.0)	2806.61	2806.01

To validate the dynamic response of the mooring line, prescribed position and velocity are imposed at the fairlead position according to the functions,

$$\mathbf{r}(t) = R(t) \sum_{i=1}^3 \left[r_i^{WF} \sin \left(\frac{2\pi t}{T_i^{WF}} + \phi_i^{WF} \right) + r_i^{LF} \sin \left(\frac{2\pi t}{T_i^{LF}} + \phi_i^{LF} \right) \right] \hat{\mathbf{e}}_i \quad (32)$$

$$\dot{\mathbf{r}}(t) = R(t) \sum_{i=1}^3 \left[\frac{2\pi r_i^{WF}}{T_i^{WF}} \cos \left(\frac{2\pi t}{T_i^{WF}} + \phi_i^{WF} \right) + \frac{2\pi r_i^{LF}}{T_i^{LF}} \cos \left(\frac{2\pi t}{T_i^{LF}} + \phi_i^{LF} \right) \right] \hat{\mathbf{e}}_i \quad (33)$$

where the index $i = \{1, 2, 3\}$, refers to the surge (X), sway (Y) and heave (Z) directions, and $(.)^{WF}$ and $(.)^{LF}$ refer to the wave and slow-drift frequency fairlead components for the quantity in $(.)$, r_i are the motion amplitudes, T_i are the motion periods, and ϕ_i are the phase angles. $R(t)$ is a ramp-up function implemented in Orcaflex [6].

Three prescribed motions profiles, as presented in Table B.1, are used for validation of dynamic line response. Figure 5 presents the fairlead tensions time histories for Configurations 1 and 2 with the imposed fairlead motions included in Table B.1. The results are shown for time windows that are slightly wider than one period of the tension fluctuations, and when the

fluctuations are fully developed; the ramp up time region is excluded. The results show that the time histories of the fairlead tensions from both codes are in good agreement, despite the difference in the time integration methods used for the two codes. In particular, the tension peaks and troughs predicted are consistent.

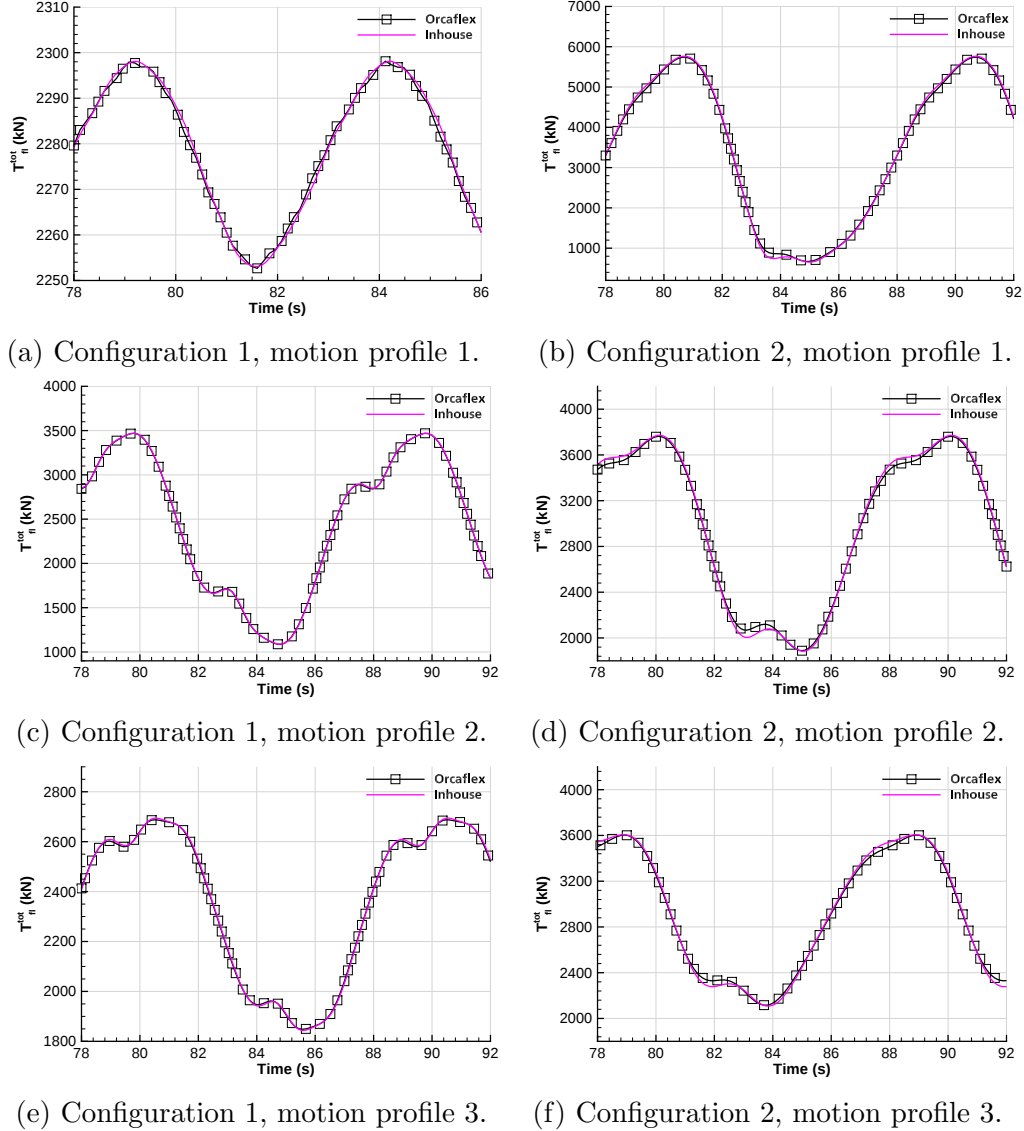


Figure 5: Fairlead tensions for fully suspended line from Orcaflex and current in-house code.

4. Effects of Nodal Grounding on Line Tension

Having validated the fairlead tension response for a fully suspended line without considering seabed model effects, in this section we examine the effects of nodal grounding on line tensions with different fairlead excitations and line spatial discretisations. As the line material specified is a studlink mooring chain, the axial length of a chain link is typically six times that of its diameter. For the line specified in Table A.1, the length of one of its chain links is thus 0.84 m. However, for numerical simulations element sizes are typically much larger than the size of a discrete chain link. For a catenary mooring line, the grounding and liftoff of the individual chain links is the primary factor that determines the fairlead tension. When element sizes that are much larger than the physical chain links are used, a discretisation error is introduced.

4.1. Test Parameters

Two single-harmonic fairlead motion profiles from Ref. [18], respectively surge motions in the wave frequency and slow-drift frequency ranges, are used as test cases. The water depth is 82.5 m. The fairlead excitation functions for position and velocity follow Equations 32 and 33. The values for the input parameters Cases 1 and 2 are shown in Table B.2. The initial line geometry is shown in Figure 6.

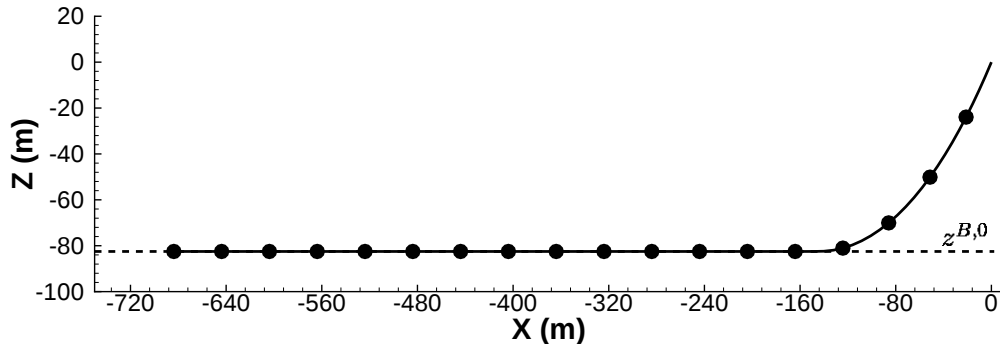


Figure 6: Configuration 1 and fairlead motion 1.

The seabed parameters used in Orcaflex are presented in the shallow water coefficient set shown in Table C.1.. The spring mattress (SM) seabed model described in Section 2.3.1, using the SM seabed force coefficient set presented in Table C.2, was used to define the seabed forces.

4.2. Influence of Element Size

Eight uniform line discretisations with element sizes of 11.86 m, 9.48 m, 8.0 m, 7.11 m, 4.74 m, 2.85 m, 2.03 m and 0.84 m are used for each test case. The finest discretisation using an element size of 0.84 m, equivalent to the size of a chain link for the present model, most closely represents physical reality. For all the test cases used in this study a uniform timestep size of 0.00025 s was applied to satisfy the stability limits of the time integration scheme applied to the 0.84 m-element discretization.

4.2.1. Case 1: Pure Surge Motion at Wave Frequency

The following discussion focuses on a time window for one period of fairlead tension oscillation in Case 1 during which a segment of the line undergoes grounding and liftoff from the seabed. Figure 7 shows the location of the touchdown point, s_{TDP} , as a function of the s-coordinate (see Figure 1a) and indicates that the line is experiencing grounding in the time period from 82.25 s to 87.0 s and the length of line in contact with the seabed increases from approximately 520 m to 560 m. Between 81.0 s to 82.25 s and 87.0 s to 91.0 s, the line is lifting off from the seabed. The instantaneous locations of the touchdown point tracked by all discretisations are in reasonably close agreement.

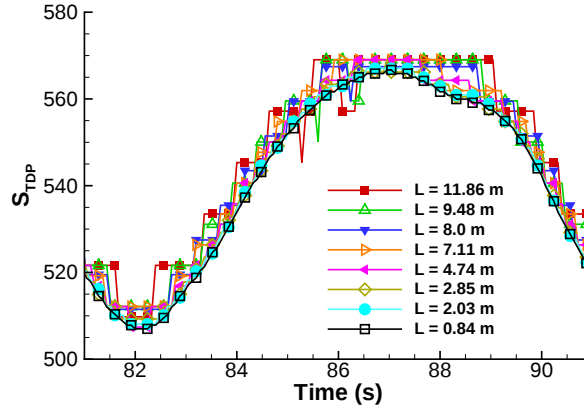
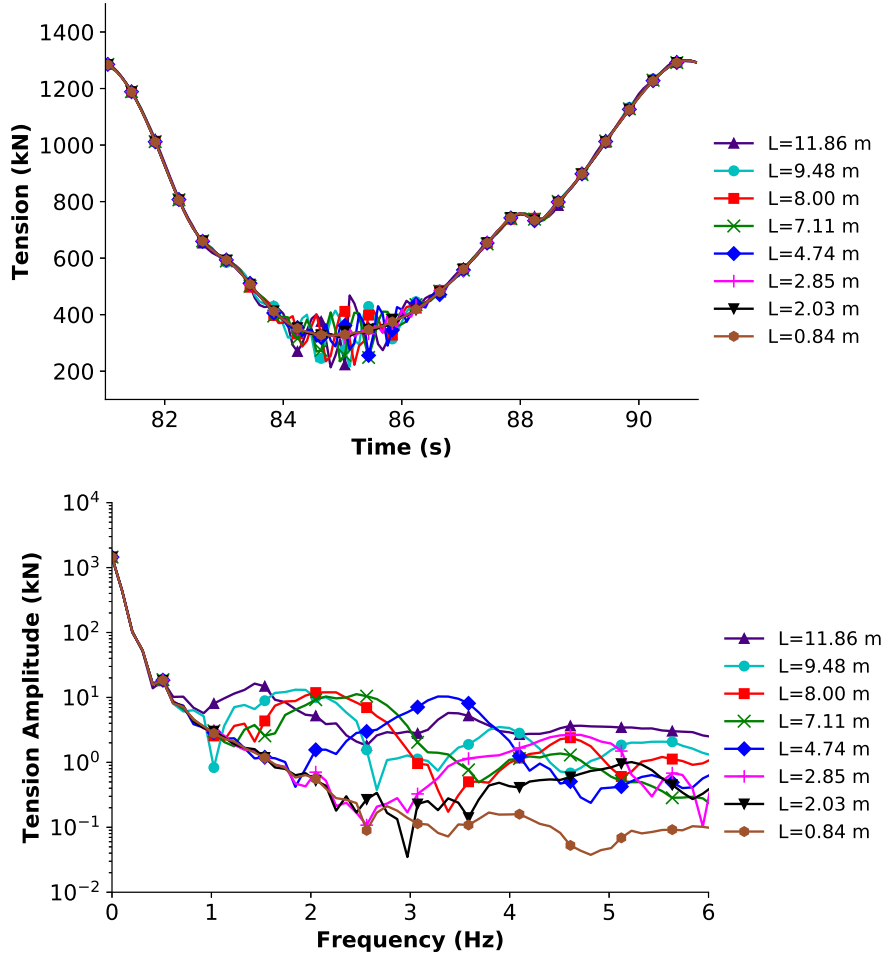


Figure 7: Time history of touchdown point location, s_{TDP} , for Case 1 from in-house code.

Figures 8a and 9a present the fairlead tension time histories for Case 1, from the in-house code and Orcaflex respectively, using the eight element lengths mentioned and show that the tension time series from both codes are

smoothly varying and in good mutual agreement from 81.0 s up to approximately 83.5 s and from 87.0 s to 91.0 s. Between 83.5 s and 87.0 s, significant fluctuations develop, coinciding with the line grounding time period as shown in Figure 7.

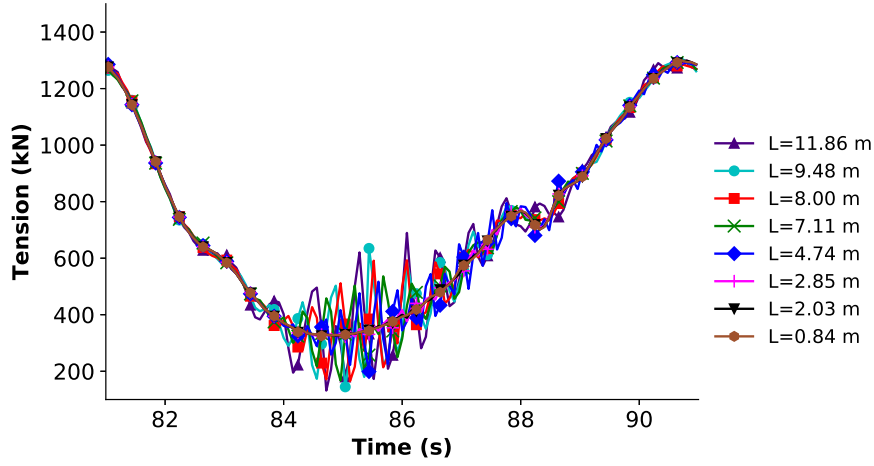


(b) Fairlead tension frequency spectra for Case 1 (current in-house code).

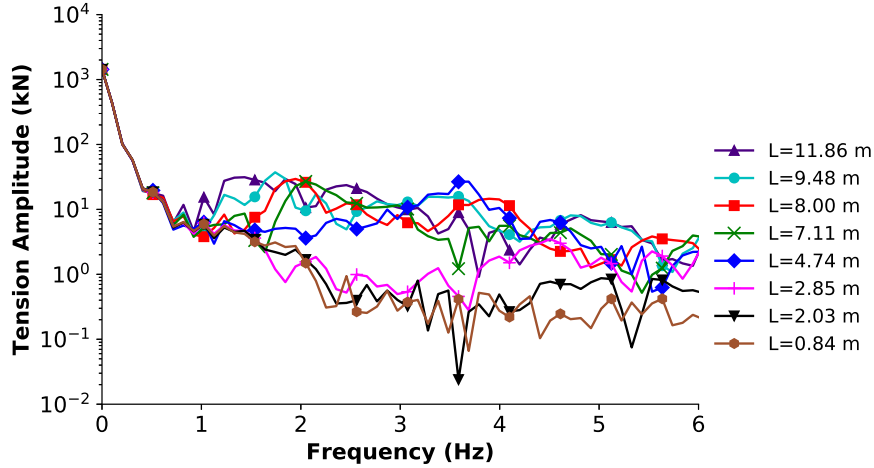
Figure 8: Fairlead tension time history and frequency spectra for Case 1 from current in-house code during nodal grounding.

Figures 8b and 9b present the frequency spectra plots for the fairlead tension time histories from the in-house code and Orcaflex respectively, and show that at lower frequencies of approximately 1 Hz and below, the magnitude of the frequency components in the tension time histories generate

by both codes, for the varying element sizes, are in good mutual agreement. However, the frequency spectra begin to diverge at frequencies above 1 Hz. Figures 8b and 9b further illustrate that as the element size is reduced, the magnitudes of the higher frequency components are correspondingly reduced as well; the tension time histories for the 0.84 m-element discretization are smoothly varying.



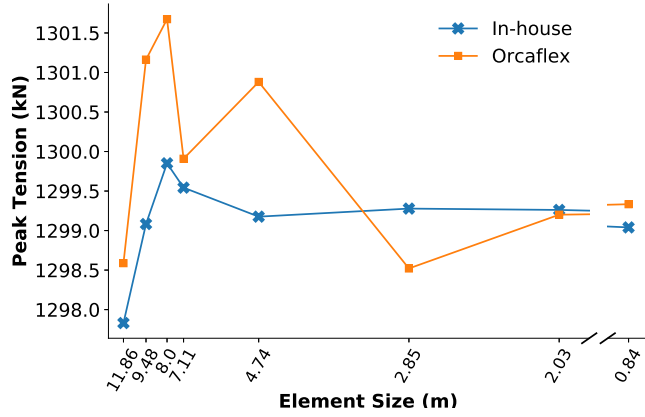
(a) Fairlead tension time history for Case 1 (Orcaflex).



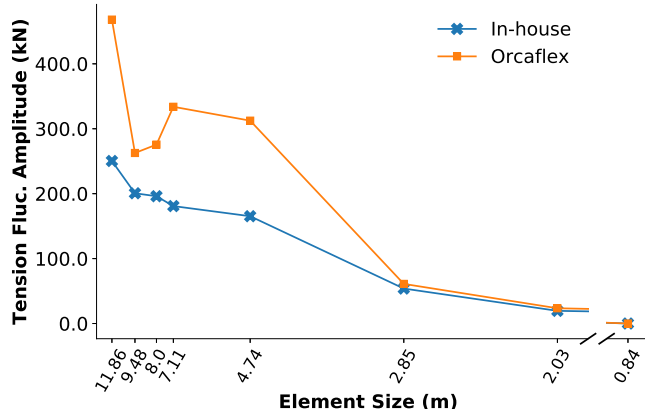
(b) Fairlead tension frequency spectra for Case 1 (Orcaflex).

Figure 9: Fairlead tension time history and frequency spectra for Case 1 from Orcaflex during nodal grounding.

Using the results for the 0.84 m-element size as a baseline and defining the deviation from this benchmark as the magnitude of the fluctuations, the maximum peak-to-peak tension fluctuation amplitude for each discretisation can be quantified and are plotted in Figure 10a, where it is illustrated again that the fluctuation amplitudes are reduced along with a reduction in the element size. The peak tensions from both Orcaflex and the in-house code with the various element sizes are presented in Figure 10a and show that as element size is reduced the peak tension converges towards to the benchmark values of the 0.84 m-element discretisation.



(a) Peak fairlead tension with varying discretisations.

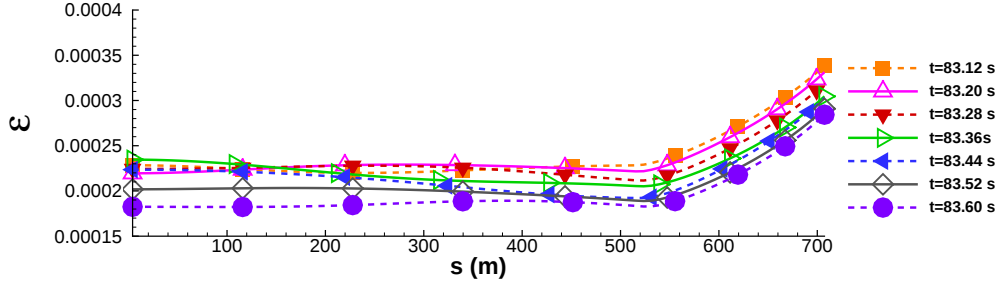


(b) Fairlead tension fluctuation amplitude with varying discretisations.

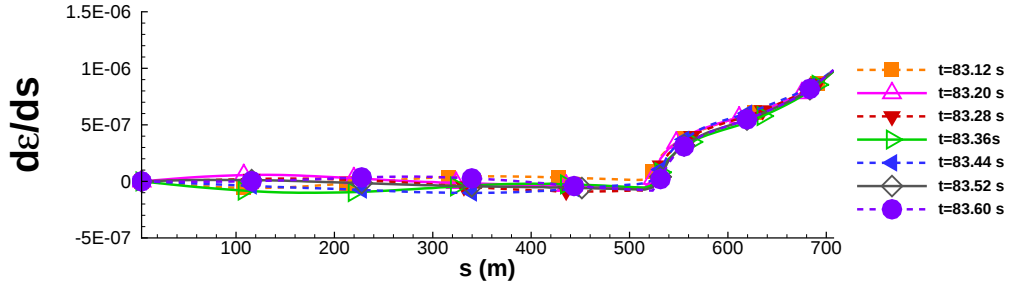
Figure 10: Peak fairlead tension and fairlead tension fluctuation amplitudes for Case 1 with uniform line element sizes ranging from 11.86 m to 0.84 m.

The deviation of the peak tension values given by the various discretisations, from their respective benchmark peak tension values of the 0.84 m-element discretisation, are generally small. In contrast, the amplitudes of the tension fluctuations, particularly for the coarser discretisations, are large in comparison to the smoothly varying tension benchmark results and degrade the quality of the solution.

To investigate the cause of the tension fluctuations, the discussion is now focused on the results for the 8.0 m and 0.84 m discretisations. The 0.84 m discretisation is chosen because the tension oscillations in this discretisation are the most subdued among all the discretisations and thus provides the conditions of the desired outcome to compare against. The 8.0 m discretisation is chosen because the magnitude of the tension fluctuations, as shown in Figure 10b, are significant and thus facilitates ease of identification of the onset of the tension fluctuations while, as will be discussed, the nodal density is sufficiently high such that the case provides a mixture of nodal grounding occurrences that do and do not generate tension fluctuations.



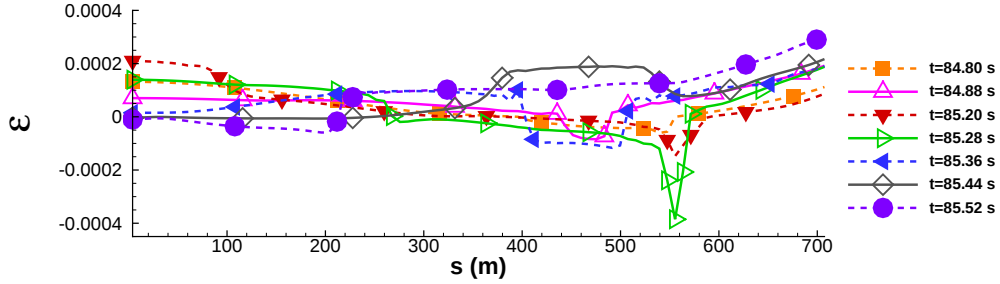
(a) Strain during grounding of Nodes 66 and 67 (Case 1) with 8.0 m-element line.



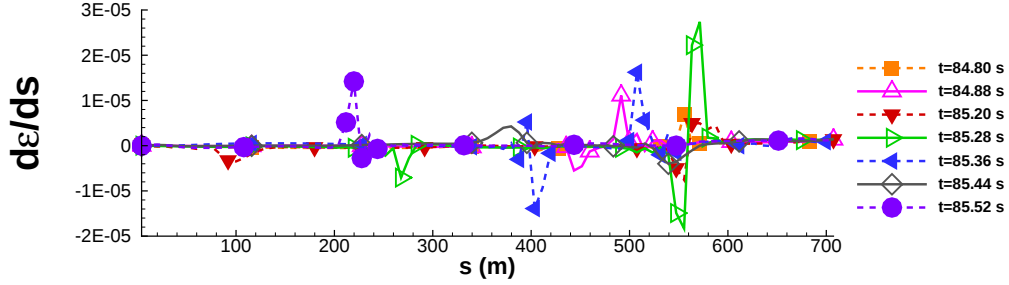
(b) Strain spatial gradient during grounding of Nodes 66 and 67 (Case 1) with 8.0 m-element line.

Figure 11: Strain and strain spatial gradient distribution with nodal grounding time window ($83.12s \leq t \leq 83.60s$) for Case 1 with 8.0 m-element line.

Figure 7 shows that line grounding starts at approximately 83.1 s. However, the tension fluctuations appear at approximately 84.0 s, coinciding with the grounding of Node 68. The grounding of Nodes 66 and 67, at 83.12 s and 83.64 s respectively, do not cause any tension fluctuations. The strain, ϵ , and strain spatial gradient, $d\epsilon/ds$, within the associated time window are shown in Figures 11a and 11b respectively. It is noted the temporal and spatial gradients of strain are smoothly varying.



(a) Strain during grounding of Nodes 68, 69, 70 and 71 in Case 1 with 8.0 m-element line.

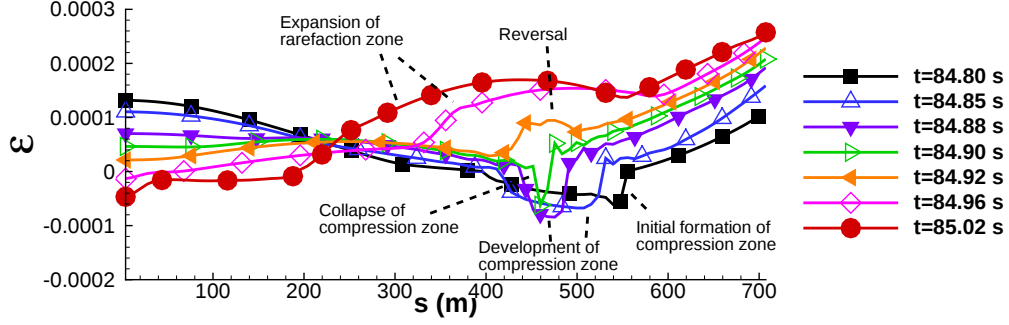


(b) Strain spatial gradient during grounding of Nodes 68, 69, 70 and 71 in Case 1 with 8.0 m-element line.

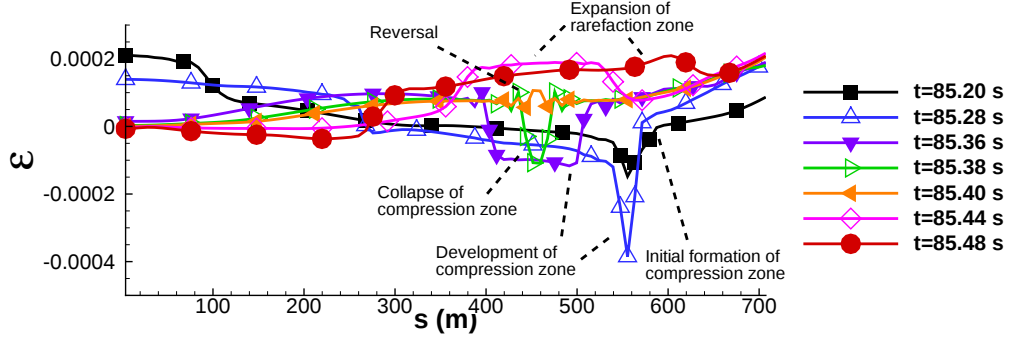
Figure 12: Strain, strain spatial gradient during nodal grounding time window ($84.80s \leq t \leq 85.60$ s) for Case 1 with 8.0 m-element line.

In contrast, as Nodes 68, 69 and 70 are grounded between 84.08 s and 84.97 s, Figure 12a illustrates that within the time window $84.88 \leq t \leq 85.52$ s the strain curves are highly irregular and large spatial gradients are observed in Figure 12b. Figure 12a also reveals the formation of localised low and negative strain zones in the strain curves of 84.20 s, 85.28 s, and 85.36 s at locations close to the touchdown point ($s \approx 500$ m). As shown in Equation 2, a mooring cable does not support compressive loading hence negative

strain indicates a slack region where line is slack. Equation 3 shows that element strain is determined from the distance between its two bounding nodes, hence, a low strain region is a compression zone where the separation distances between nodes are low. Conversely, a high strain region is a rarefaction zone where the nodes are spaced further apart.



(a) Development and reversal of low strain zone after grounding of Node 69.

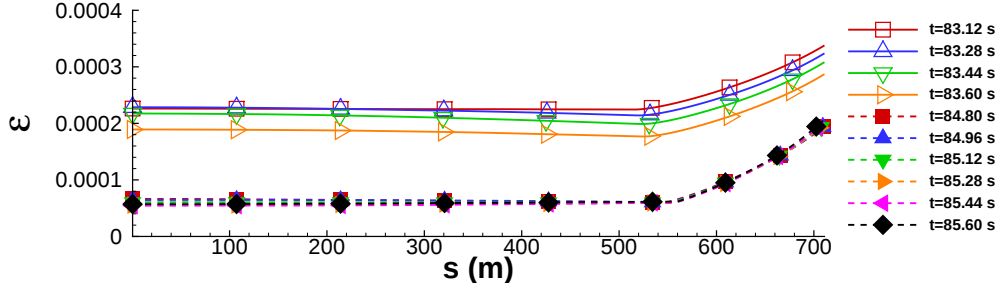


(b) Development and reversal of low strain zone after grounding of Node 70.

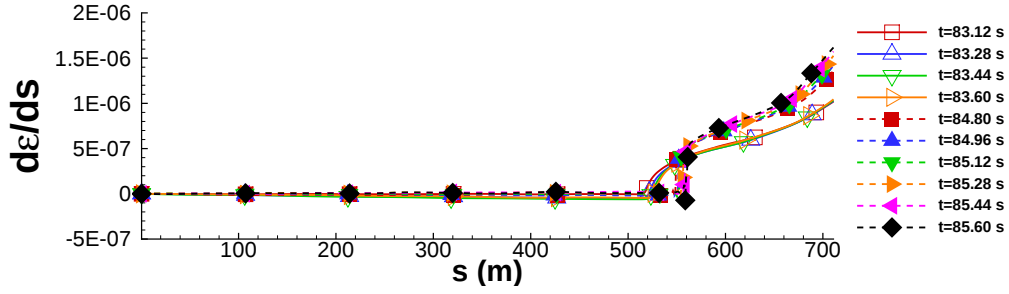
Figure 13: Development and reversal of low strain zones post-nodal grounding (Case 1).

Figure 13 provides a closer look at the development and time evolution of the compression and rarefaction zones. Figure 13a presents the strain curves of the 8.0 m-element line after the grounding of Node 69 at 84.48 s, and illustrates the development of a slack region in the region of $400 \leq s \leq 550$ m. The spatial extent of the slack zone is time-varying. At 84.96 s, the slack zone has become a high-strain, rarefaction region. Similarly, Figure 13b illustrates that after the grounding of Node 70 at 84.97 s, at 85.20 s a slack region spanning $390 \leq s \leq 590$ m develops in the vicinity of the touchdown point ($s \approx 560$ m). At 85.28 s, the elements in the vicinity of the touchdown point have undergone further compression and the maximum negative strain

has increased further. At 85.44 s, the slack span has become a high tension zone. The high tension zone then expands, as shown by an enlarged high strain region in the strain curve at 85.48 s relative to 85.44 s.



(a) Strain during nodal grounding time window ($83.12s \leq t \leq 85.60$ s) in Case 1 with 0.84 m-element line.



(b) Strain spatial gradient during nodal grounding time window ($83.12s \leq t \leq 85.60$ s) in Case 1 with 0.84 m-element line.

Figure 14: Strain, strainrate and strain spatial gradient distribution during nodal grounding time window ($83.12s \leq t \leq 85.60$ s) for Case 1 with 0.84 m-element line.

The ϵ and $d\epsilon/ds$ curves for the 0.84 m-element line are shown in Figures 14a and 14b respectively. The time period shown in these plots span the period prior to ($t < 84.08$ s) and during ($84.08 \leq t \leq 85.60$ s) the development of the tension fluctuations presented in Figure 12. Figure 14b illustrates that the $d\epsilon/ds$ curves during the period of occurrence of tension fluctuations are smoothly varying with the exception of a kink that occurs consistently in the vicinity of the touchdown point.

During the grounding of Nodes 66 and 67, the $d\epsilon/ds$ curves presented in Figure 11b have similar profiles compared to the $d\epsilon/ds$ curves for 0.84 m-element line presented in Figure 14b. Both situations were free of tension fluctuations. In contrast, the $d\epsilon/ds$ curves during the grounding of Nodes

68 to 70 for the 8.0 m-element line, presented in Figure 12b, exhibit large gradients which are up to an order of magnitude larger than in Figures 11b and Figure 14b at s -coordinates that vary widely in time.

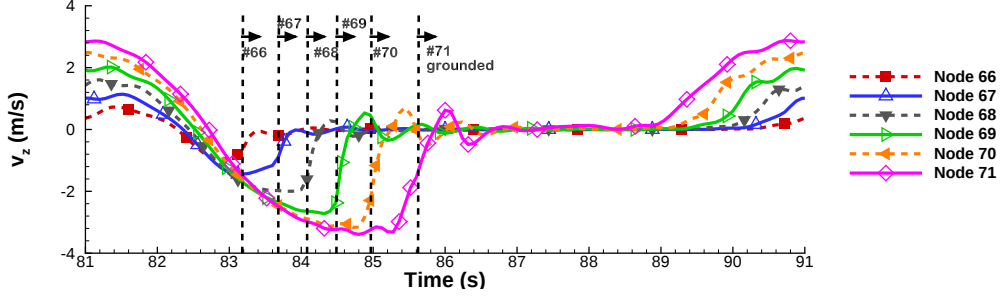
The nodal vertical and horizontal velocities as well as the vertical coordinate for the 8.0 m and 0.84 m-element lines are shown in Figures 15 and 16 respectively. The times of grounding of each node are also shown to demarcate the pre and post-grounding time regions for each node. The selected grounding nodes shown in Figure 16 for the 0.84 m-element line were chosen at s -coordinates similar to the grounding nodes for the 8.0 m-element line, as shown in Table 2, in order to compare the grounding velocities at the same s -coordinates with different discretisations. The fairlead forcing of Case 1 is a single-harmonic surge motion in the X (surge) direction, and a negative v_x at the fairlead corresponds to the unloading phase of the motion.

Table 2: Grounding nodal indices and s -coordinates (Case 1.)

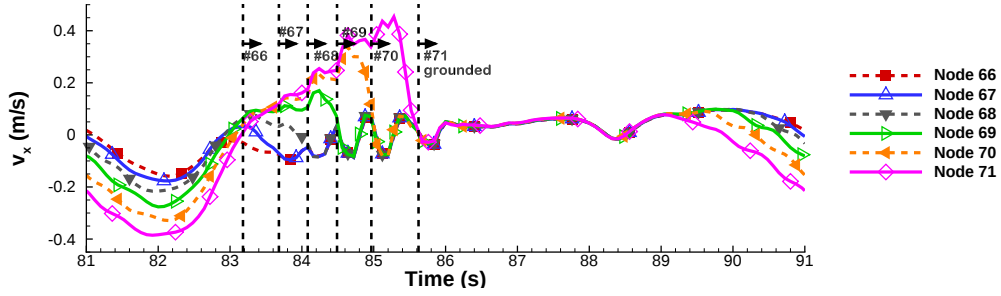
Node index	8.0 m	66	67	68	69	70	71
	0.84 m	619	629	639	648	658	667
TDP s - coord (m)	8.0 m	519.48	527.48	535.47	543.47	551.46	559.45
	0.84 m	519.60	528.00	536.41	543.98	552.39	559.95

As the fairlead velocity magnitude increased from 1.289 m/s to 3.088 m/s, the vertical velocities of the grounding nodes in the 8.0 m-element and 0.84 m-element discretisations respectively increased from 1.127 m/s to 3.348 m/s and 1.085 m/s to 3.227 m/s. Comparing the grounding velocities shown in Figures 15a and 16a with the corresponding fairlead velocities shown in Figures 15d and 16d respectively, it is observed that the magnitudes of the nodal vertical grounding velocities are positively correlated with the fairlead velocity. Comparing Figure 15a with 16a, Figure 15b with 16b, and Figure 15c and 16c, it is noted that the post-grounding velocity and positional fluctuations for the 8.0 m-element line are more pronounced than that of the 0.84 m-element line.

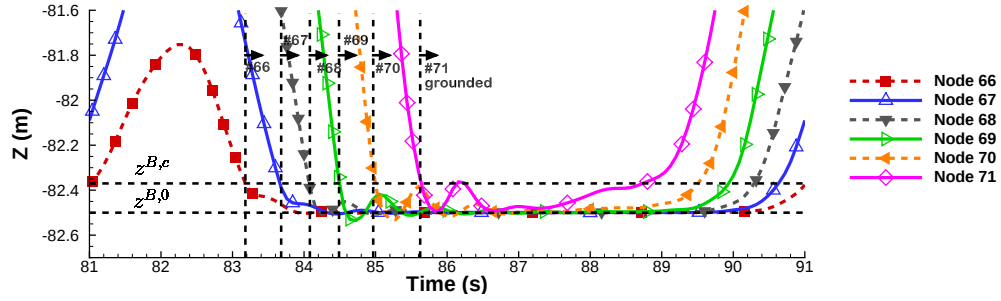
Figures 15a, 15b, 16a, and 16b illustrate that the post-grounding nodal velocities in Case 1 eventually settle at magnitudes close to zero. Comparing Figures 15a with 16a, and 15b with 16b, and 15d with 16d, it is observed that the post-grounding velocities and vertical positions of the 8.0 m-element discretisation are underdamped and experience several periods of oscillations, while the post-grounding velocities and vertical positions are critically damped.



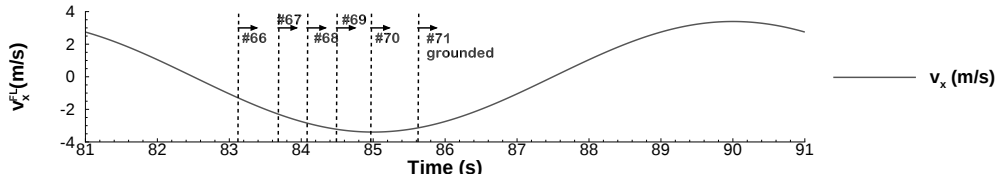
(a) Nodal vertical velocities, v_z , in Case 1 for 8.0 m-element line.



(b) Nodal horizontal velocities, v_x , in Case 1 for 8.0 m-element line.

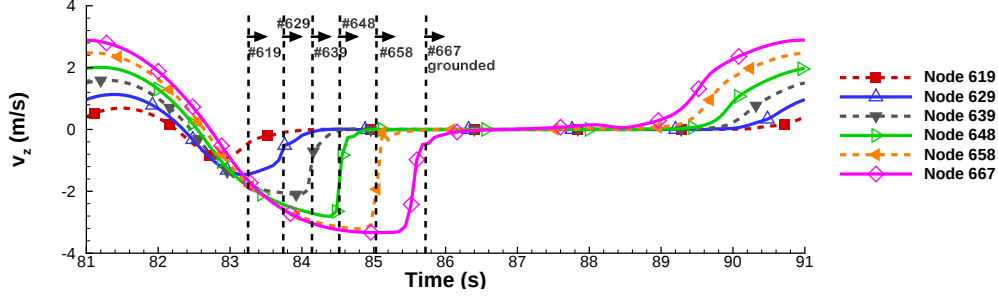


(c) Nodal vertical coordinate, Z , in Case 1 for 8.0 m-element line.

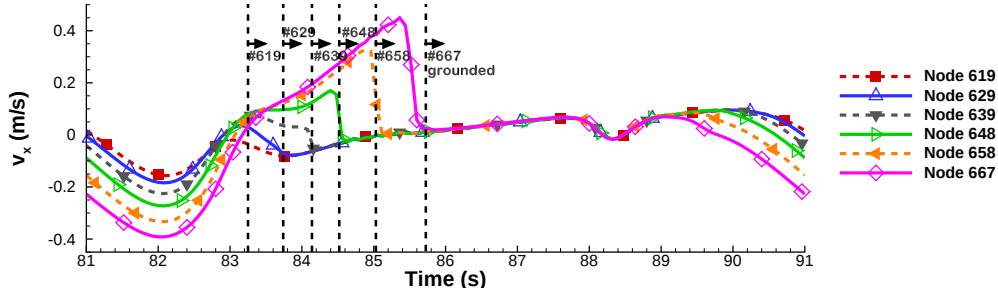


(d) Fairlead horizontal velocity v_x in Case 1 for 8.0 m-element line.

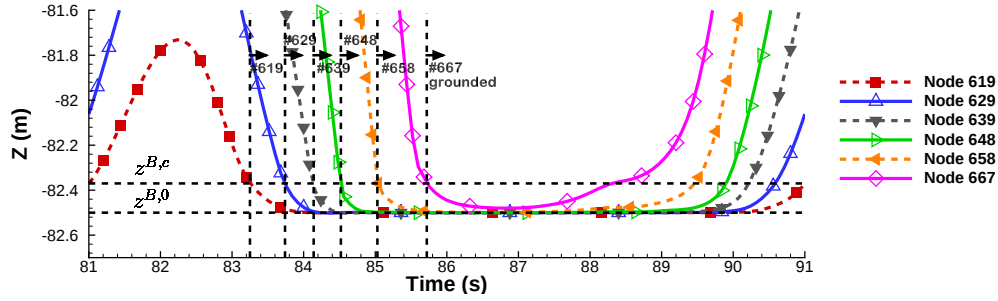
Figure 15: Nodal vertical coordinate and velocities in Z and X directions of grounding nodes in Case 1 with element length of 8.0 m.



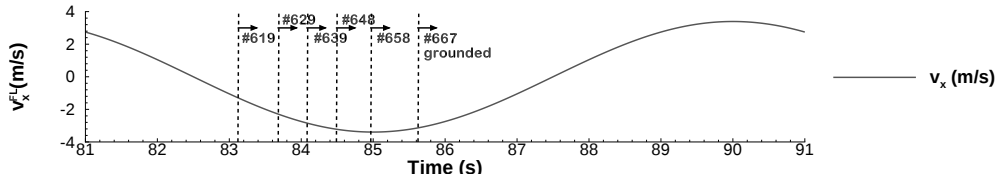
(a) Nodal vertical velocities, v_z , in Case 1 for 0.84 m-element line.



(b) Nodal horizontal velocities, v_x , in Case 1 for 0.84 m-element line.



(c) Nodal vertical coordinates, Z , in Case 1 for 0.84 m-element line.



(d) Fairlead and horizontal velocity v_x in Case 1 for 0.84 m-element line.

Figure 16: Nodal vertical coordinate and velocities in Z and X directions of grounding nodes in Case 1 with element length of 0.84 m.

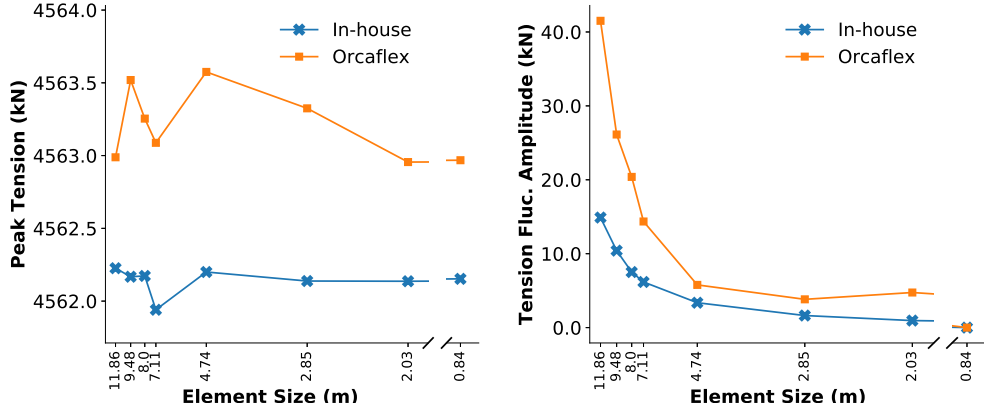
From Figure 15b, an interesting observation of the oscillatory behavior of the horizontal nodal velocities v_x in the vicinity of the touchdown zone, for the 8.0 m-element discretization, is that after a node is grounded, for a short period of time the slope of the horizontal velocity curves (dv_x/dt), or equivalently the horizontal acceleration, for that node and the nearby nodes on the side of the line that is grounded is negative, while that of the nodes that are suspended are positive.

For example, when Node 67 is grounded at 83.68 s, the slopes of the v_x curves for itself and Node 66, which is on the grounded side of the line, are negative, while slopes of the v_x curves for Nodes 68 to 71 on the suspended side of the line are positive. When Node 68 grounded at 84.08 s, the slopes of the v_x curves for the grounded Nodes 66 and 68 are negative, while the slopes of the v_x curves for the suspended Nodes 69 to 71 are positive. Similarly, when Node 69 grounded at 84.48 s, the slopes of the v_x curves for the grounded Nodes 66 to 69 are negative, while the slopes of the v_x curves for the suspended Nodes 70 and 71 are positive. Finally, when Node 70 grounded at 84.97 s, the slopes of the v_x curves for grounded Nodes 66 to 70 are negative, while slope of the v_x curve for the suspended Node 71 is positive. This behavior suggests that after a node is grounded, the nodes on the grounded side of the line as well as the recently grounded node itself, accelerate towards the anchor node while the nodes on the suspended side accelerate towards the fairlead node. However, as the horizontal velocities are oscillating, the accelerations of the grounded nodes are soon reversed until the grounding of the next node.

4.2.2. Case 2: Pure Surge Motion at Slow-Drift Frequency

Figure 17a presents the peak tensions for Case 2 with the range of element sizes and shows that the variation in the peak tension associated with varying element size is small compared to the peak tension value. Figure 17b presents the peak-to-peak tension fluctuation amplitude for each discretisation and reveals that the tension fluctuations, in the range between 3 kN to 41.5 kN, are an order of magnitude smaller than in Case 1 as presented in Figure 10b.

The grounding time window for Case 2 is $650 \leq t \leq 740$ s, as shown in Figure 18, during which the length of line in contact with the seabed increases from approximately 250 m to 600 m. The number of nodes on the seabed resting on the seabed from 32 to 77 for the 8.0 m-element line, and 291 to 720 for the 0.84 m-element line.



(a) Peak fairlead tension with varying discretisations. (b) Fairlead tension fluctuation amplitude with varying discretisations.

Figure 17: Peak fairlead tension and fairlead tension fluctuation amplitudes for Case 2 with uniform line element sizes ranging from 11.86 m to 0.84 m.

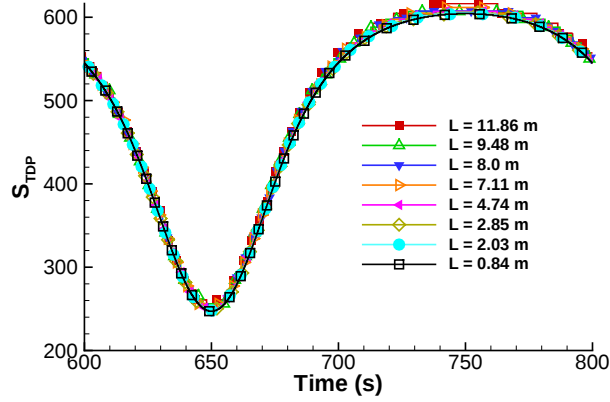
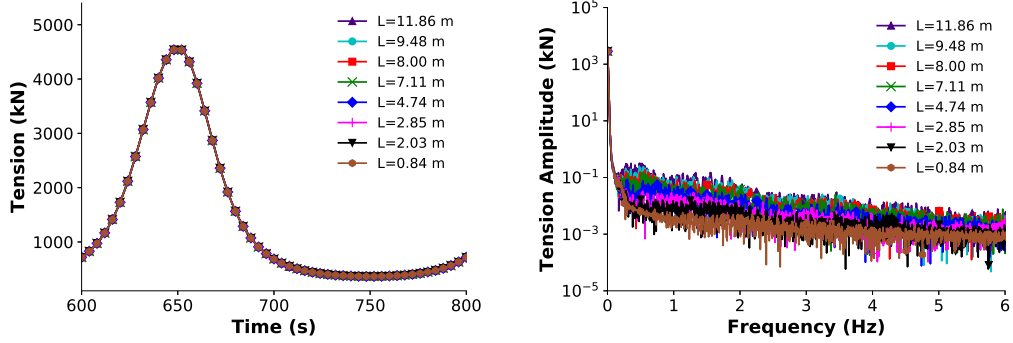


Figure 18: Time history of touchdown point location for Case 2 from in-house code.

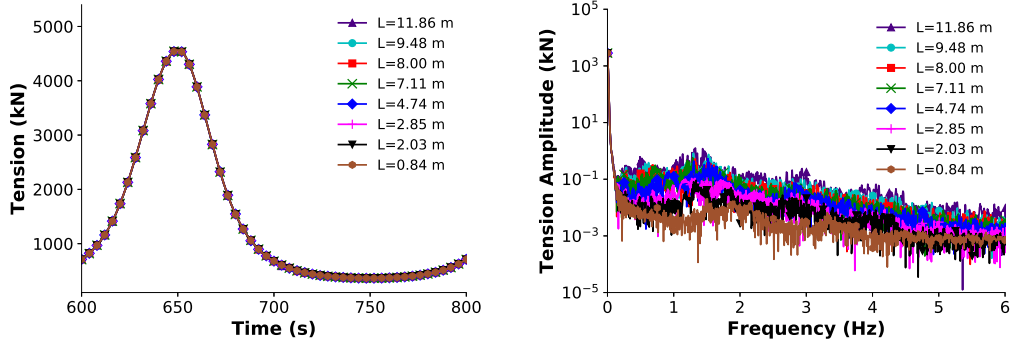
Figures 19a and 19b respectively present the fairlead tension histories and frequency spectra for Case 2 from the current in-house code, while Figures 20a and 20b show the fairlead tension history and frequency spectra for Case 2 from Orcaflex. Figures 19b and 20b confirm that the frequency contents of the tension time histories are dominated by the lower frequencies below approximately 0.1 Hz, which are in good agreement.



(a) Fairlead tension time history for Case 2 (current in-house code).

(b) Fairlead tension frequency spectra for Case 2 (current in-house code).

Figure 19: Time history and frequency spectra of fairlead tension from current in-house code during nodal grounding for Case 2.

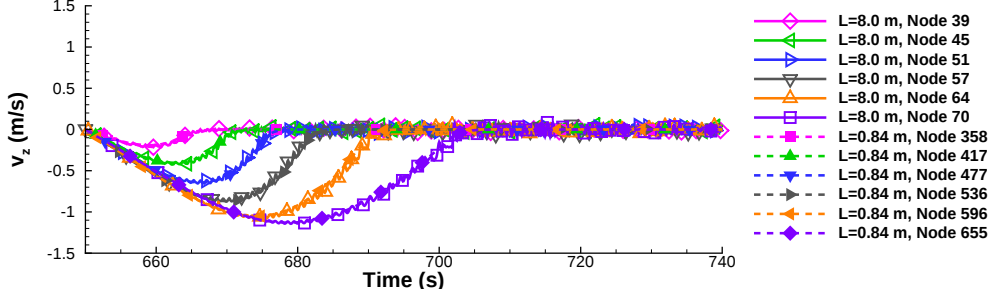


(a) Fairlead tension time history for Case 2 (Orcaflex).

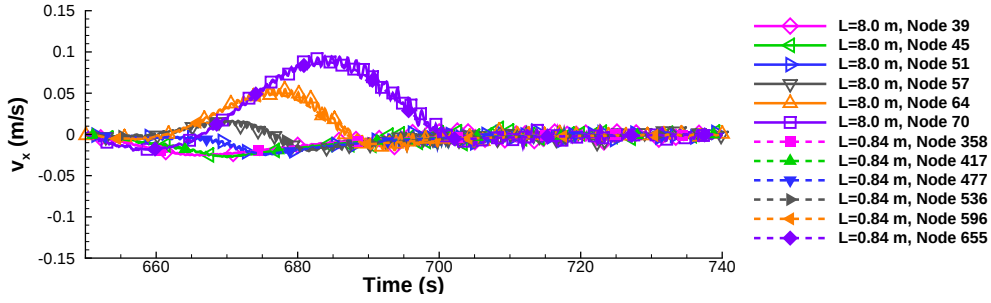
(b) Fairlead tension frequency spectra for Case 2 (Orcaflex).

Figure 20: Time history and frequency spectra of fairlead tension from Orcaflex during nodal grounding for Case 2.

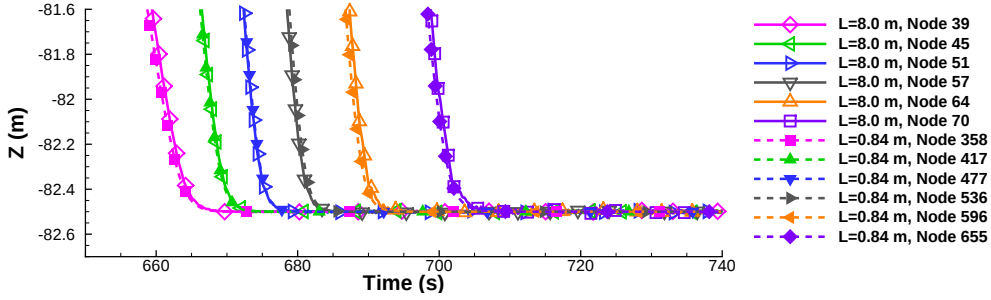
The vertical and horizontal velocities of selected grounding nodes for Case 2 with the two discretisations are presented in Figures 21a and 21b respectively. The fairlead horizontal velocity v_x is shown in Figure 21d. Figures 21a, 21b and 21c show that the velocities and vertical coordinates of the grounding nodes of the 8.0 m-element and 0.84 m-element discretisations, at comparable s -coordinates, are in close agreement. In contrast, the positional and velocity fluctuations of the grounding nodes in Case 1 with the 8.0 m-element discretisation are higher than with the 0.84 m-element discretisation.



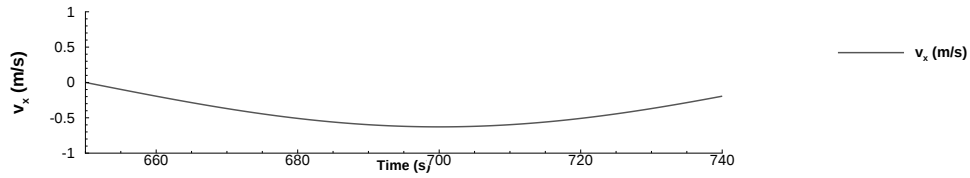
(a) Nodal vertical velocities, v_z for 8.0 m-element and 0.84 m-element line.



(b) Nodal horizontal velocities, v_x for 8.0 m-element and 0.84 m-element line.



(c) Nodal vertical coordinate, Z for 8.0 m-element and 0.84 m-element line.



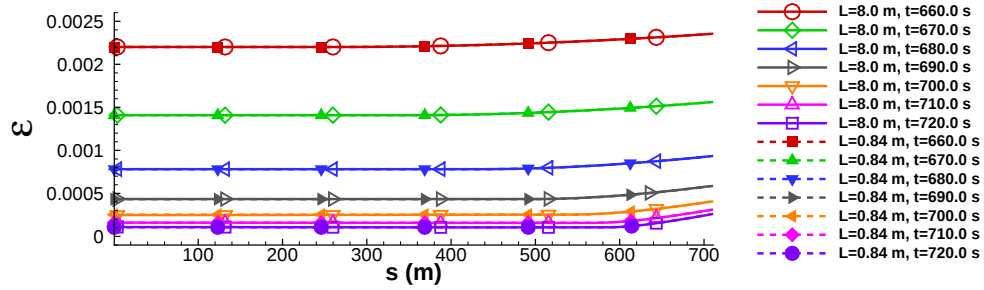
(d) Fairlead and horizontal velocity v_x .

Figure 21: Nodal vertical coordinate and velocities in Z and X directions of grounding nodes in Case 2 with element lengths of 8.0 m and 0.84 m.

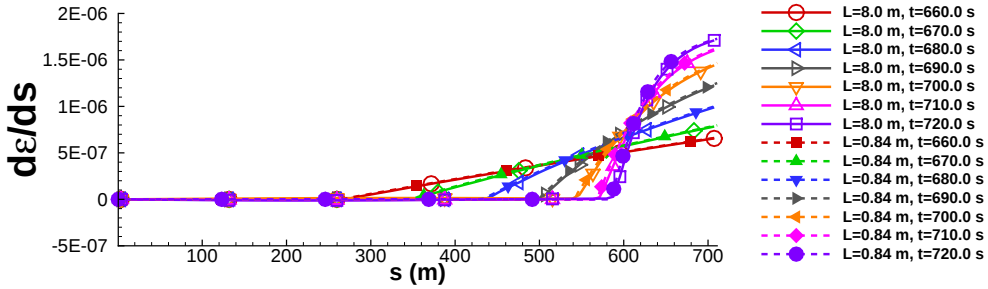
Table 3 shows the s -coordinates of the selected nodes whose velocities and vertical coordinates within the grounding time window are illustrated in Figure 21 and shows that the maximum vertical grounding velocity magnitudes in Case 2 are 1.12 m/s and 1.11 m/s for the 0.84 m-element and 8.0 m-element discretisation respectively, which are lower than the grounding velocities in Case 1. The maximum fairlead velocity magnitude in Case 2 is 0.628 m/s compared to 3.392 m/s in Case 1.

Table 3: Grounding nodal indices and s -coordinates (Case 2.)

Node index	8.0 m	39	45	51	57	64	70
	0.84 m	358	417	477	536	596	655
TDP s-coord (m)	8.0 m	303.70	351.65	399.61	447.56	503.50	551.46
	0.84 m	300.15	349.76	400.21	449.81	500.26	549.87



(a) Strain for 8.0 m-element and 0.84 m-element lines with time window $660.0 \leq t \leq 720.0$ s.



(b) Strain spatial gradient for 8.0 m-element and 0.84 m-element lines with time window $660.0 \leq t \leq 720.0$ s.

Figure 22: Strain, strain rate and strain gradient distribution for 8.0 m-element and 0.84 m-element lines for Case 2 between 660.0 s and 720.0 s.

Figures 22a and 22b respectively present the ϵ and $d\epsilon/ds$ curves of the 8.0 m-element and 0.84 m-element lines within the period of nodal grounding ($660.0 \leq t \leq 720.0$ s) for Case 2, and confirm that the strain distributions for the two discretisations are in good agreement at all the presented timesteps, and that there are no large strain spatial gradients.

4.3. Effects of Varying Seabed Force Coefficients

The choice of seabed force coefficients is arbitrary and selected typically reduce the nodal penetration depth on the seabed, and post-grounding positional fluctuations [2, 4]. Gobat and Grosenbaugh [5] stated that the choice of coefficients has little influence on the gross response of the mooring line. This section examines the effects of varying the coefficients of the spring mattress model on line tension in Case 1. This section examines the effects of applying different seabed force coefficients sets presented in Table C.2 with the 4.74 m-element discretisation. The attention is focused on the coefficients for the vertical reaction and damping forces in the spring mattress model, respectively $N^{B,c}$ (see Equation 8) and vertical damping $\zeta^{B,d}$ (see Section 2.3.3).

The 4.74 m-element discretisation is chosen because no significant change in the peak tension result is achieved with further refinement of the nodal density; the difference in the peak tension result between the 4.74 m and 2.85 m-element discretisations is 0.0015% and thus the solution for peak tension is considered to be converged. The applied time step, 0.00025 s, is significantly smaller than the maximum stable time step of 0.00328 s, and is sufficient for time step convergence for Case 1.

Figure 23a shows the fairlead tension time history for Case 1 with the SM coefficient set, which produced the tension fluctuations discussed in Section 4.2.1, compared with the tension time histories using a $\zeta^{B,d}$ value of 1.0 and $N^{B,c}$ values of 5 (SM₁), 10 (SM₂), 15 (SM₃), and 20 (SM₄). The effect of increasing $N^{B,c}$ is that the seabed vertical spring stiffness, k^B , is reduced and the seabed reaction force cutoff elevation, $z^{B,c}$ (see Figure 2), is increased. It is evident from Figures 23a that the tension fluctuations are progressively reduced as $N^{B,c}$ is increased. However, the peak tensions for the SM₁, SM₂, SM₃ and SM₄ coefficient sets, at 1285.18 kN, 1264.57 kN, 1250.44 kN, and 1232.86 kN respectively, are lower than the 1298.69 kN peak tension for the SM coefficient set.

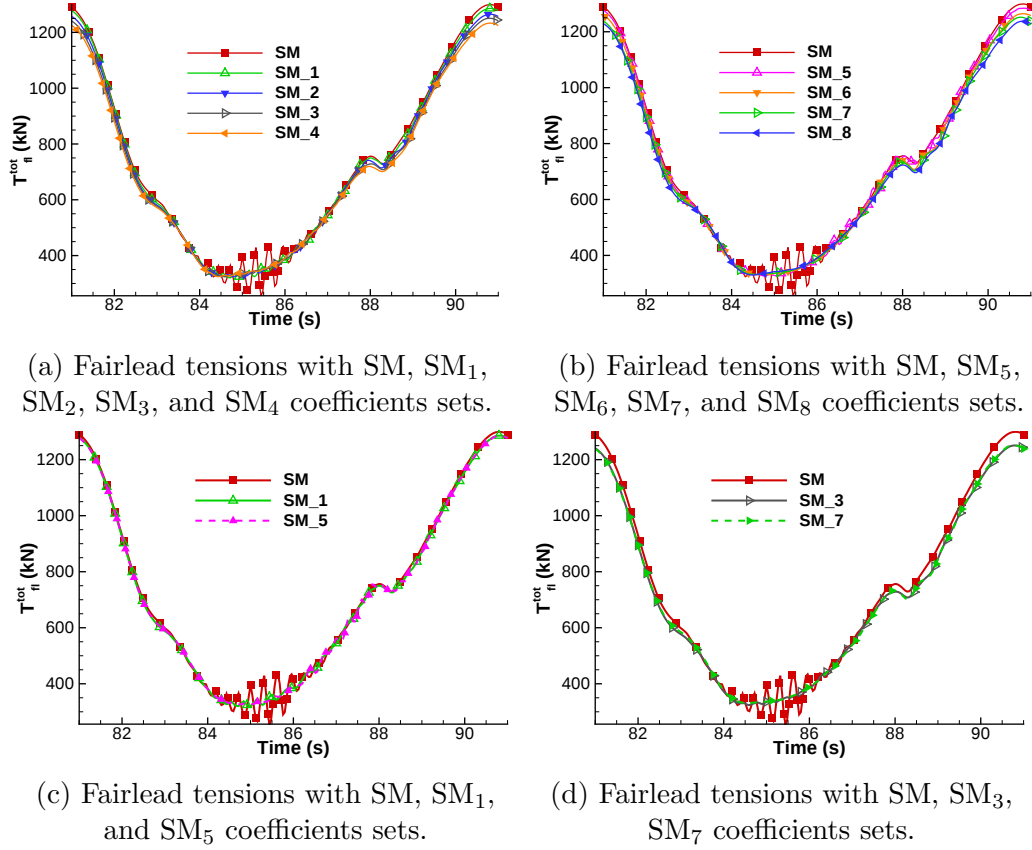
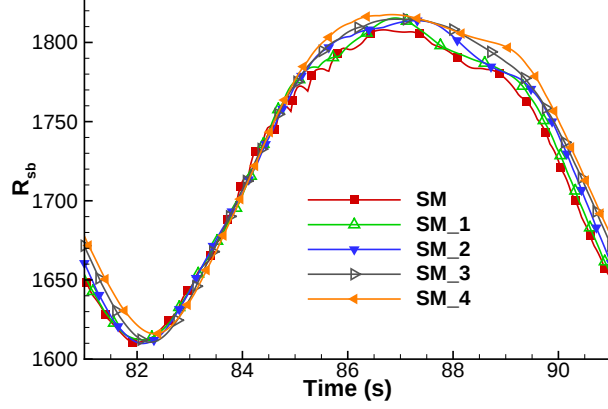
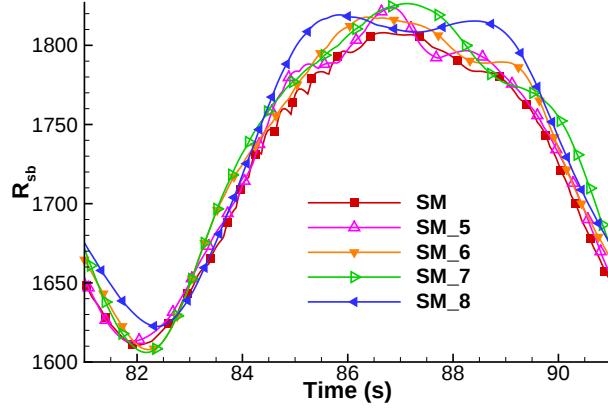


Figure 23: Fairlead tensions for Case 1 with spring mattress seabed model and SM, SM₁, SM₂, SM₃, SM₄, SM₅, SM₆, SM₇ and SM₈ coefficient sets.

Figure 23b shows the fairlead tension time history using the SM coefficient set compared with the fairlead tension time histories using a $\zeta^{B,d}$ value of 0 and the same $N^{B,c}$ values of 5, 10, 15 and 20, corresponding to the SM₅, SM₆, SM₇, and SM₈ coefficient sets respectively, and illustrates that the tension fluctuations are progressively reduced as $N^{B,c}$ is increased. The peak tensions are 1283.81 kN, 1264.29 kN, 1251.66 kN and 1237.44 kN respectively for the SM₅, SM₆, SM₇, and SM₈ coefficient sets. With a $N^{B,c}$ value of 5, the magnitude of the tension fluctuations are reduced as the value of $\zeta^{B,d}$ was changed from 1.0 to 0.0, however, the peak tension was also lowered slightly by 1.1%. With a value of 15 for $N^{B,c}$, the tension fluctuations are further reduced slightly as the value of $\zeta^{B,d}$ was changed from 1.0 to 0.0, while the peak tension experienced a small increase of 0.1%.



(a) Seabed reaction force with SM, SM₁, SM₂, SM₃, and SM₄ coefficients sets.



(b) Seabed reaction force with SM, SM₅, SM₆, SM₇, and SM₈ coefficients sets.

Figure 24: Seabed reaction force for Case 1 with spring mattress seabed model and SM, SM₁, SM₂, SM₃, SM₄, SM₅, SM₆, SM₇ and SM₈ coefficient sets.

The cause for the reduction in fairlead tension is the increased seabed reaction force on the line as a result of increasing $N^{B,c}$. Figures 24a and 24b respectively present, for $\zeta^{B,d}$ values of 1.0 and 0.0, the total seabed reaction force on the grounded line section for $N^{B,c}$ values of 5, 10, 15 and 20, as well as the seabed reaction force for the SM coefficient set, and illustrate that the seabed reaction force on the line, which reduces the suspended line weight, generally increases together with $N^{B,c}$ over a period of oscillation. Figure 25b also illustrates that for different values of $\zeta^{B,d}$, the vertical nodal coordinates begin to deviate at elevations closer to the seabed due to the increasing downward velocities of the grounding nodes as they approach the seabed (see Figure 15a) and leads to a larger disparity in the vertical damping

force arising from differing values of $\zeta^{B,d}$.

Figure 25a shows the Z -coordinate of the grounding nodes when the SM and SM₄ coefficients are used, and it is evident that the penetration depths of the grounding nodes below $z^{B,0}$ increased with a larger $N^{B,c}$ value and, correspondingly, a lower seabed spring stiffness. Figure 25b presents the Z -coordinate of the grounding nodes when the SM₃ and SM₇ coefficients are used, which respectively apply the values of 1.0 and 0.0 for $\zeta^{B,d}$ and the same value of 15 for $N^{B,c}$, and illustrates that the nodal penetration depth increases with a reduction in the value of $\zeta^{B,d}$.

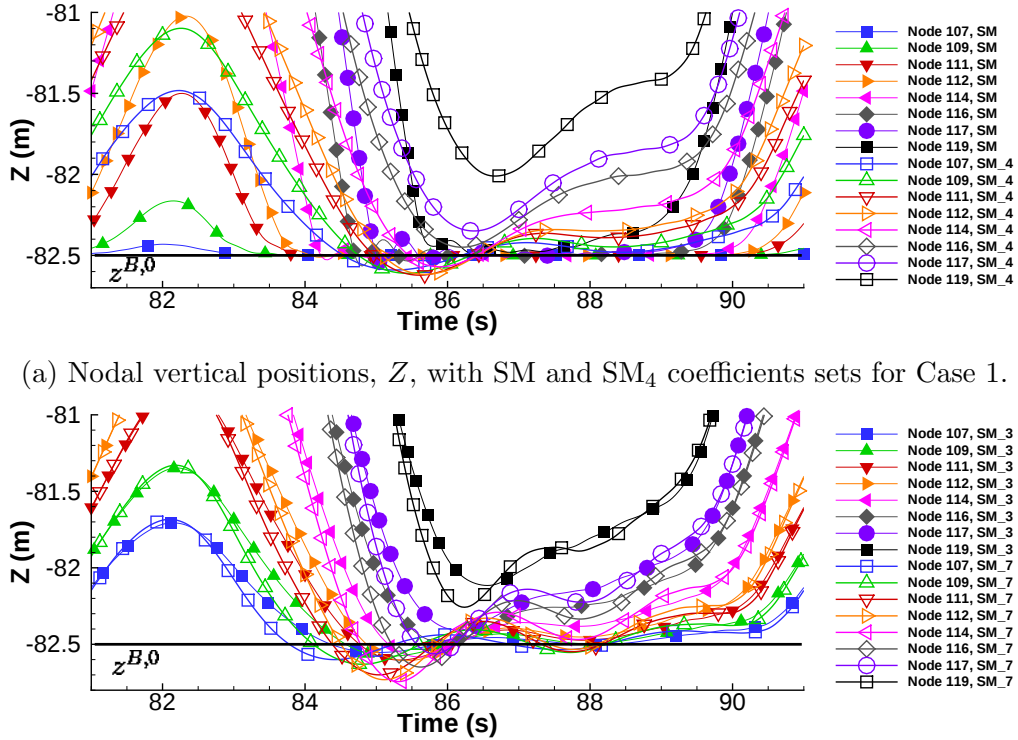
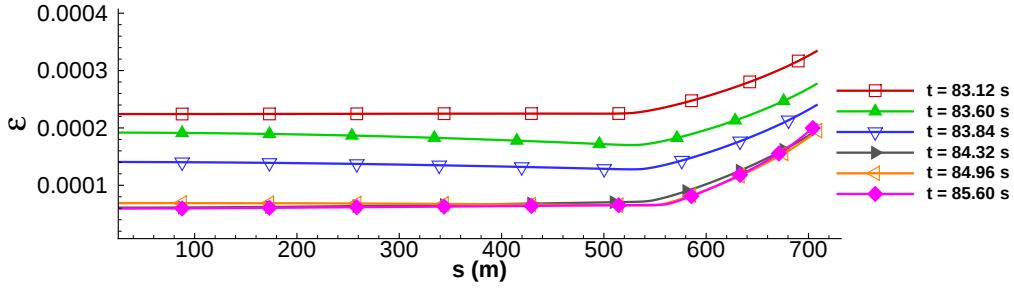


Figure 25: Nodal vertical positions, Z , with SM, SM₃, SM₄ and SM₇ coefficients sets for Case 1.

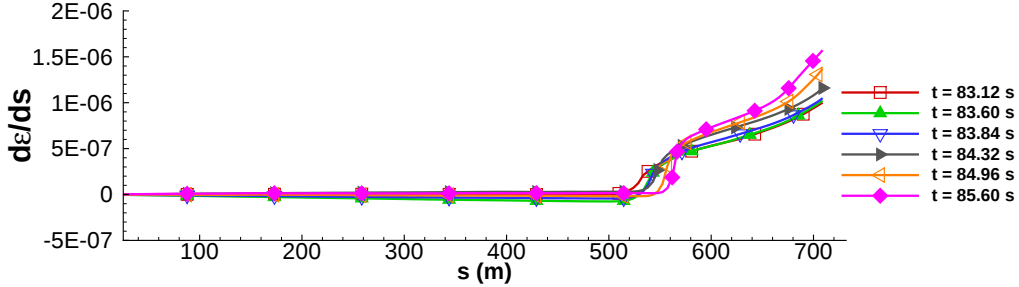
Figures 26a and 26b respectively present the strain and strain spatial gradient distributions with the 4.74 m-element discretisation and the SM₇ coefficient set during the nodal grounding time window, $83.12 \leq t \leq 85.60$ s, and illustrates that the both the strain and strain gradient curves are free of

the large spatial gradients that are observed in Figures 12a and 12b. In relation to the fairlead tension time histories, the smoothly varying strain curves and strain gradient curves, presented in Figures 26a and 26b respectively, correspond to the SM₇ fairlead tension time history presented in Figure 23d, while the undulating strain and strain gradient curves in Figures 12a and 26b respectively correspond to the irregular fairlead tension time series for the 8.0 m-element line shown in Figure 8b.

The key findings from this series of test cases are, firstly, the seabed spring stiffness has a significant effect on the fairlead peak tension and increasing it, while beneficial from the standpoint of reducing tension fluctuations, leads to the underestimation of the peak tension results. Secondly, the effect of the vertical damping coefficient is mainly on the tension fluctuations and its effect on peak tension is small, and that a lower damping coefficient has the effect of reducing the fluctuations.



(a) Strain with 4.74 m-element line, SM₇ coefficient set.



(b) Strain spatial gradient with 4.74 m-element line, SM₇ coefficient set.

Figure 26: Strain and spatial gradient of strain with 4.74 m-element line, SM₇ coefficient set and time window ($83.12 \leq t \leq 85.60$ s) for Case 1.

4.4. Effect of Grounding Element Orientation

Figure 2 shows an element that is in the grounding state, with the vertical angle between the element axis and the seabed defined as θ , the grounding angle of an element. For a catenary line, the θ of an element closer to the anchor is usually lower than for an element closer to the fairlead. Hence, to investigate the effect of θ on the production of the tension fluctuations, the 4.74 m-element line in a water depth of 82.5 m is specified together with three pretension values, respectively 706.9 kN, 1248.4 kN and 3136.5 kN; henceforth referred to as the low, mid-range and high pretension cases. The pretension value changes the s_{TDP} range, as shown in Figure 27. The fairlead excitation amplitude and frequency of 5.4 m and 10 s (Case 1) is uniformly applied for all three lines.

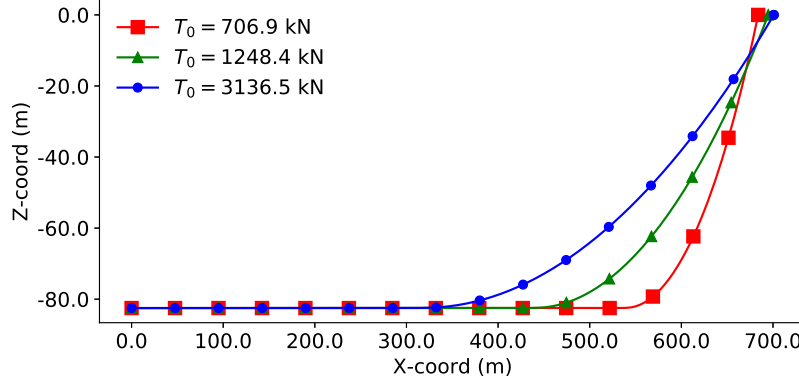


Figure 27: Initial line geometries of 4.74 m-element discretisation with pretension values, T_0 , of 706.9 kN, 1248.4 kN and 3136.5 kN.

Figure 28 presents the location of the touchdown point s_{TDP} for the lines with three pretension values in the time period for three oscillation cycles from 61 s to 91 s. As pretension is increased, s_{TDP} is shifted towards the anchor as a longer segment of the line is suspended. For the low, mid-range and high pretension value cases, the s_{TDP} range for the same fairlead excitation specified here are 161.23 m to 396.53 m, 346.16 m to 497.91 m, and 507.39 m to 569.04 m respectively.

The fairlead tension time histories are shown in Figure 29, where the disparities in the magnitudes of the tension fluctuations between the tension time histories for the three pretension cases are clearly presented. The fluctuation magnitude is largest for the low pretension case, which also experienced the highest s_{TDP} range, and smallest for the high pretension case, with the

lowest s_{TDP} range. The magnitude of the tension fluctuations for mid-range pretension case lies in between the high and low pretension cases.

The velocity of the grounding node, which is the first suspended node of a grounding element (Node $i+1$ in Figure 2), for the three pretension cases, are presented in Figure 30. The grounding node velocities for the two higher pretension cases are similar, and attained a maximum value of approximately 3.8 m/s, whereas that for the lowest pretension case achieved a maximum of 2.96 m/s.

The grounding angle θ time histories are presented in Figure 31. Comparing Figures 31 and 29, it is evident that the onsets of the tension fluctuations appear in tandem with higher grounding θ angles during the unloading phase of the fairlead excitation motion. The magnitude of the grounding angles achieved by the three pretension cases vary in the same order of the pretension values. The highest pretension case experienced the highest grounding angles, followed by the mid-range and low pretension cases. This variation in the grounding angles is due to the geometry of the catenary cable, and is also evident in the static line geometries, as seen in Figure 27.

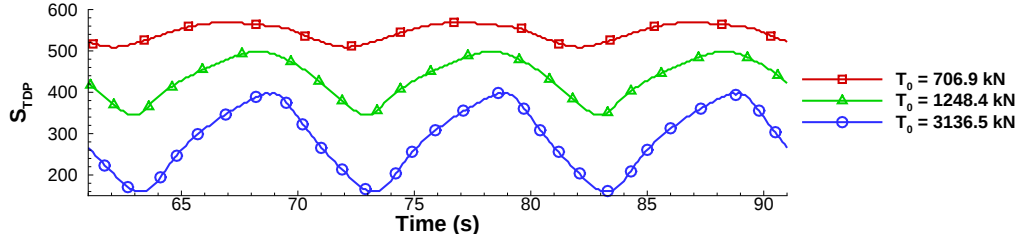


Figure 28: Touchdown location s_{TDP} of 4.74 m-element line with pretension values, T_0 , of 706.9 kN, 1248.4 kN and 3136.5 kN for Case 1 fairlead excitation parameters.

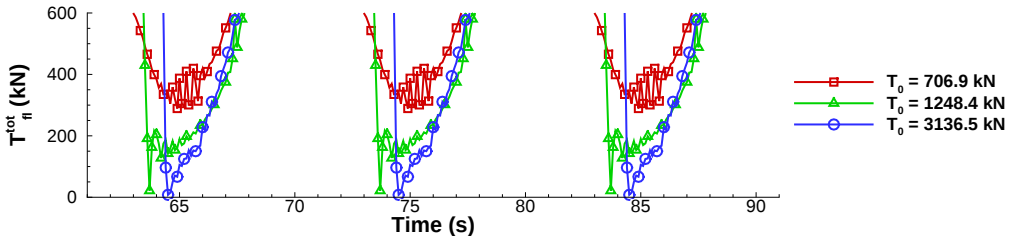


Figure 29: Fairlead tension time histories of 4.74 m-element line with pretension values, T_0 , of 706.9 kN, 1248.4 kN and 3136.5 kN for Case 1 fairlead excitation parameters.

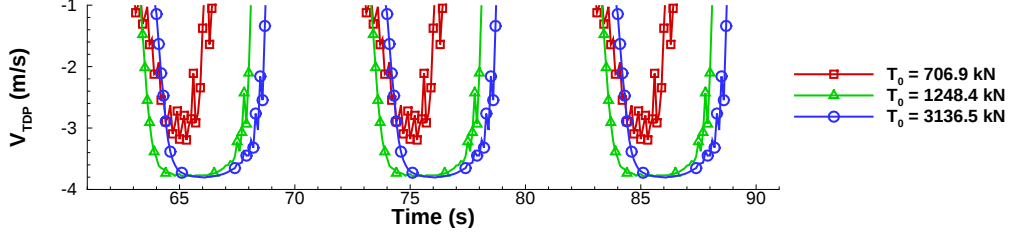


Figure 30: Vertical velocity of touchdown node of 4.74 m-element line with pretension values, T_0 , of 706.9 kN, 1248.4 kN and 3136.5 kN for Case 1 fairlead excitation parameters.

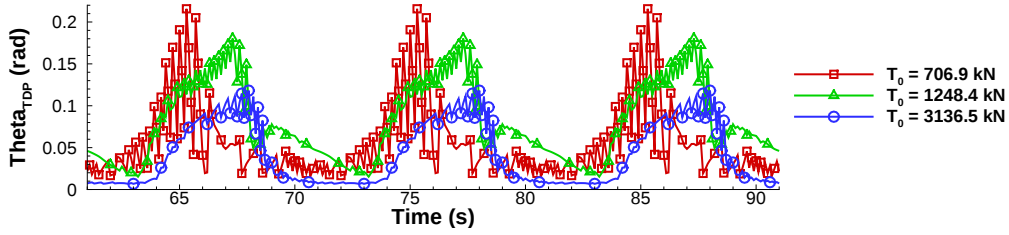


Figure 31: Grounding element angle θ of 4.74 m-element line with pretension values of 706.9 kN, 1248.4 kN and 3136.5 kN for Case 1 fairlead excitation parameters.

With the same dynamic excitation applied to the three pretension cases, the difference between them are in the pretension values and the s_{TDP} ranges. A s_{TDP} range close to the anchor, in general, leads to lower θ angles, due to the natural geometry of the catenary. Comparing the grounding velocities and θ angle profiles of the mid-range and high pretension cases, it is noted that the tension fluctuations are more significant in the mid-range case, even though the grounding velocity profiles in the two cases are similar. Furthermore, the grounding velocities in the low-pretension case is the lowest among the three cases, while also experiencing the highest θ angles, and it is notable that the tension fluctuation magnitude experienced in this case is larger than in the mid-range and high pretension cases. Therefore, the magnitudes of the tension fluctuations are positively correlated with θ .

Consider a grounding element with the asymptotic values of θ at 0 and $\pi/2$, corresponding to the perfectly vertical and horizontal orientations, as shown in Figure 32. The seabed reaction force acting in the axial direction of the element is thus $(\mathbf{F}_j^{B,r} \cdot \hat{\mathbf{e}}_j) \hat{\mathbf{e}}_j = F_j^{B,r} \sin \theta \hat{\mathbf{e}}_j$. Hence the maximum value of the seabed reaction force acting in the axial direction of the grounding

element is $F_j^{B,r}$ when θ is $\pi/2$ and the element is vertical, and acts to compress the element. Conversely, when the grounding element is horizontal and θ is 0, an isolated element will not experience compression due to seabed contact.

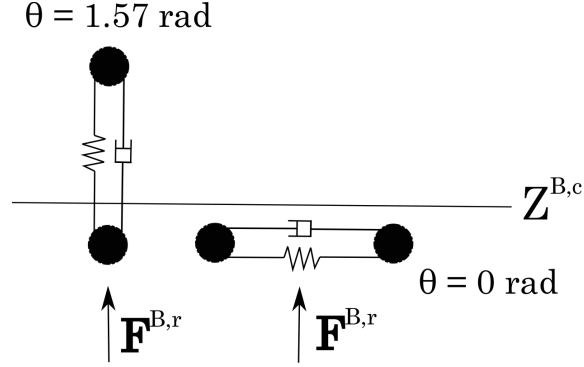


Figure 32: Grounding element with angle θ of 0 and $\pi/2$ rad.

It was evident in Section 4.2.1 that the effect of compression on a particular segment of the line gives rise to large strain gradients which in turn lead to the production and propagation of stress waves. High θ values leads to a larger effective seabed force component in the grounding element axial direction, which compresses the grounding element and leads to the rapid creation of a low strain region and large strain gradients in the line, and is therefore associated with the creation of the propagating stress waves.

4.5. Discussion of Results from Nodal Grounding Study

Reducing the element size leads to a reduction of the magnitude of the tension fluctuations. Taking Case 1 as an example, Figure 10b shows that for the in-house code an element size of 2.03 m is required to reduce the amplitude of the spurious tension fluctuations to approximately 20 kN or 1.5% of peak tension. However, the peak tension value achieved convergence with a coarser discretisation comprising of 4.74 m length elements. The use of a finer discretisation necessarily entails longer computing times, and for the in-house code, the increase was approximately 1.93 times comparing the 4.74 m (150 elements) to the 2.03 m (350 elements) discretisations. Presently, the same time step was used for both discretisations, and considering that a larger time step may be applied for the coarser discretisation, the disparity in computing times can potentially be wider.

Higher grounding velocities are associated with the development of tension fluctuations. In Case 2, the maximum fairlead velocity attained was

lower than Case 1. When considering only the grounding velocities in Case 1, Figures 15a and 16a show that the grounding velocities of Nodes 66 and 67 for the 8.0 m line and Nodes 619 and 629 for the 0.84 m-element line were lower than that for Nodes 68 to 71 and Nodes 639 to 667 for the two discretisations respectively. The same is observed in Case 2, where a comparison of Figure 21a and Figure 21d shows that the grounding velocities of the nodes increase in tandem with fairlead velocity. In Case 1, the grounding of Nodes 66 and 67 in Case 1 with the 8.0 m-element line did not lead to the development of tension oscillations, while the grounding of Nodes 68 to 71 did, due to comparatively higher grounding velocities. The relatively low grounding velocities in Case 2 also did not lead to tension fluctuations. The cause of the tension fluctuations is the formation of localised low strain or slack regions in the vicinity of the touchdown point. The process is initiated by high speed nodal grounding situations and the formation of a localised low tension or compression zone, followed by the development and expansion of the compression zone. The compression zones are transient and eventually collapse, to be replaced by a local rarefaction, high strain zone, which then expands.

High strain gradients are generated due to the presence of the compression and rarefaction zones. The large strain gradients represent unbalanced tension forces and are responsible for driving the propagation of stress waves which travel along the length of the line. The transition from a compression to a rarefaction zone is due solely to the nodal motions. When the grounding of Node 70 produced a low tension zone at the vicinity of the touchdown point, this resulted in a net acceleration of the nodes outwards relative to the low strain region. The contraction and collapse of the compression zone is a result of the outward motion of the nodes relative to it; as the nodes at the high strain gradient interfaces experience an increase in the resultant tension force due to the high strain gradient (see Equation 4), they begin to accelerate in the direction of the resultant tension forces. As the nodes move outwards from the compression zone, the separation between the nodes within it and consequently the strain, increases again. At the boundaries, namely the anchor and fairlead nodes, the stress waves are reflected back into the line, as shown by Yang et. al [19]. The inertial and tension forces on the nodes give rise to oscillatory motions. Successive nodal groundings may occur and contribute to the creation of multiple stress waves that propagate back and forth along the length of the line, as was observed in Case 1.

To accurately resolve high strain gradients close to the touchdown zone,

the discretisation has to be sufficiently high. If the number of elements is equivalent to that would be present in a physical line, the strain spatial gradient can be expected to follow that shown in Figure 14b. The kinks in the strain and strain gradient curves close to the touchdown point is characteristic of static catenary lines. In contrast, when the discretisation is insufficiently refined to capture a smooth spatial strain variation, localised strain discontinuities and low strain regions may form due to rapid nodal grounding. Of particular importance is the $d\epsilon/ds$ curve, because large spatial strain gradients deviating from the smoothly varying profiles as shown in Figures 14b and 11b indicate that there are large resultant nodal tension forces. This leads to the creation and propagation of stress waves following large nodal accelerations, velocities and motions; manifesting at the fairlead as tension fluctuations.

In addition to the grounding velocity, the grounding element angle θ has an effect on the production of the tension fluctuations and higher θ angles increase the likelihood of the creation of low strain regions and large strain gradients, particularly if the grounding velocities are significant. Locally refining the element size in such regions is expected to be helpful in reducing the effects of the propagating stress waves created due to large strain gradients. Larger elements can be used in regions where θ is expected to be lower.

Reducing the seabed spring stiffness and damping coefficients, as discussed in Section 4.3, have the effect of reducing the magnitude of the fairlead tension fluctuations. The mechanisms that produce this effect is the increased seabed penetration depths and the higher seabed force cutoff elevation, $z^{B,c}$. Consider the motion of two successive grounding nodes, as the first node is allowed to penetrate deeper into the seabed and the downwards vertical speed of the second node is reduced due to a higher $z^{B,c}$, the separation between the two nodes, which determines the element strain (see Equation 3) is less likely transition rapidly into a slack state, causing the large strain gradients that subsequently produce the propagating stress waves. However, reducing the linear spring stiffness and damping tends to lower the peak tensions in an oscillation period.

5. Application of Modified Spring Mattress Model

In this section the effectiveness of the modified spring mattress (MSM) model in mitigating the fairlead tension fluctuations is evaluated for the

shallow and deep water depths of 82.5 m and 914.0 m respectively. The features of the MSM model that differ from the spring mattress (SM) model are the modified reaction force and added mass force models described in Sections 2.3.2 and 2.3.4.

5.1. Shallow water environment

A single segment mooring line with structural parameters enumerated in Table A.1 is used for the shallow water environment. Test cases 1, 3 and 4, as shown in Table B.2, provide the fairlead excitation profiles. Comparisons are made between the results from the SM, SM₇ and modified spring mattress (MSM) seabed models using the relevant coefficient sets shown in Table C.2. Comparisons are made between the results from the SM, SM₇ and modified spring mattress (MSM) seabed models using the relevant coefficient sets shown in Table C.2. The SM₇ coefficient set is chosen because it was shown to be effective at reducing the tension fluctuations in Case 1 (see Section 4.3).

The maximum tension results from the present code are compared to that from Ref. [18], shown in Table 4 as $\bar{T}$ [18], as a means of benchmarking. A second qualitative indicator for the quality of the results is the existence of spurious oscillations, which can be evaluated from the tension time series plots for Cases 1, 3 and 4 as shown in Figures 33, 36 and 43 respectively.

For Case 1, the peak tension results, presented in Table 4, using the SM, SM₇ and MSM seabed models with the two line discretisations agree well with $\bar{T}$ [18] in general. The SM model with the 4.74 m and 0.84 m-element discretisations (SM^{4.74} and SM^{0.84}) respectively underpredict $\bar{T}$ [18] by 0.08% and 0.09%, while the MSM (MSM^{4.74}) and SM₇ (SM₇^{4.74}) coefficient sets, both using the 4.74 m-element discretisation, respectively underpredict $\bar{T}$ [18] by 0.43% and 3.72%.

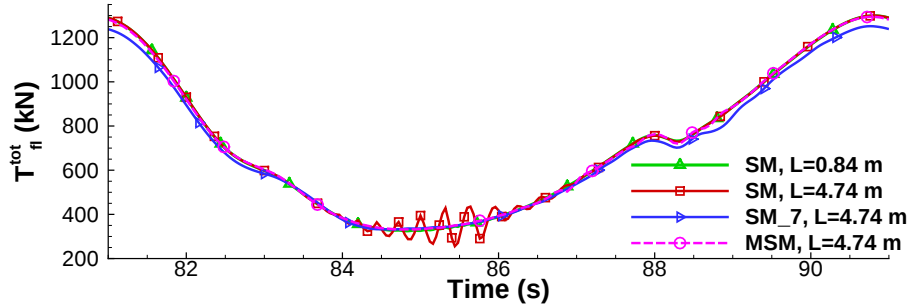
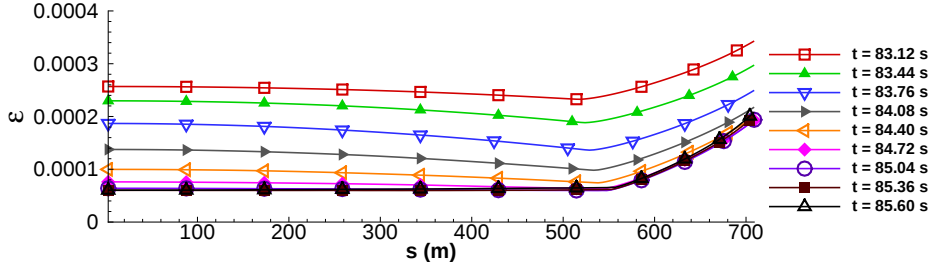


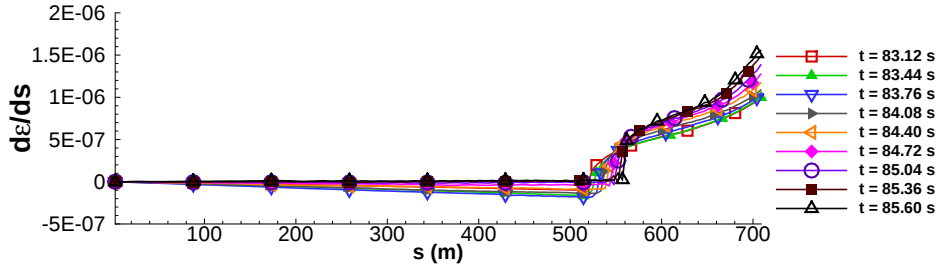
Figure 33: Fairlead tension time history for Case 1 with 0.84 m and 8.0 m-element lines and SM, SM₇ and MSM models.

Table 4: Fairlead peak tensions with SM, SM₇ and MSM models compared to benchmark tension $\bar{T}$ [18] for Cases 1 to 4.

Model	Case 1	Case 3	Case 4
Fairlead Tension (kN)			
$\bar{T}$ [18]	1300	6777	7488
$\bar{T}_{4.74m}^{SM}$	1298.98	6876.02	7384.98
$\bar{T}_{0.84m}^{SM}$	1298.87	6860.27	7414.92
$\bar{T}_{4.74m}^{SM_7}$	1251.68	6752.64	7337.61
$\bar{T}_{4.74m}^{MSM}$	1294.38	6858.60	7355.22
Percentage deviation of tension from $\bar{T}$ [18]			
$\bar{T}_{4.74m}^{SM}$	-0.079%	1.461%	-0.975%
$\bar{T}_{0.84m}^{SM}$	-0.087%	1.229%	-1.376%
$\bar{T}_{4.74m}^{SM_7}$	-3.717%	-0.359%	-2.008%
$\bar{T}_{4.74m}^{MSM}$	-0.432%	1.204%	-1.773%



(a) Strain distributions for 8.0 m-element line using MSM model.



(b) Strain spatial gradient distributions for 8.0 m-element line using MSM model.

Figure 34: Strain and strain gradient distribution for Case 1 during nodal grounding time window ($8.12 \leq t \leq 85.52$ s).

The tension time series in Figure 33 show that the tension fluctuations

are significantly reduced in the results for $SM^{0.84}$, $MSM^{4.74}$ and $SM_7^{4.74}$. The ϵ and $d\epsilon/ds$ distributions in a line with 4.74 m elements and applying the MSM model are presented in Figures 34a and 34b, and illustrate that the MSM model is effective in preventing severe strain gradients from occurring during nodal grounding.

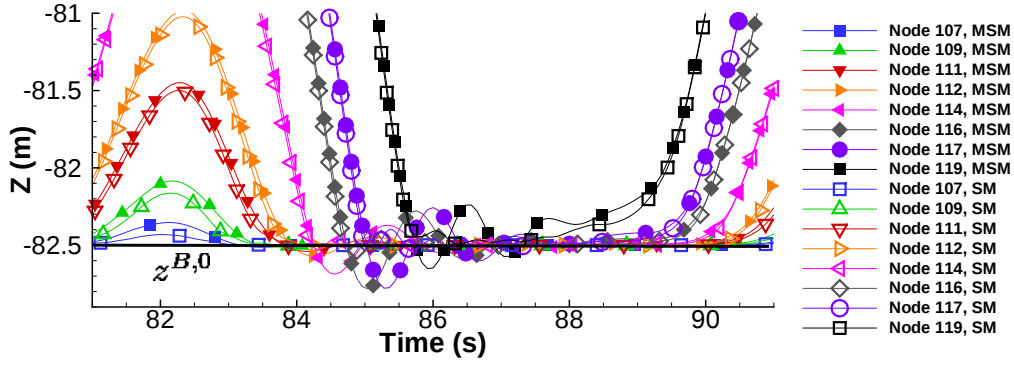


Figure 35: Nodal vertical coordinate, Z , results for 8.0 m-element line using SM and MSM models during nodal grounding time window ($8.12 \leq t \leq 85.52$ s).

Figure 35 illustrates that the penetration depth of the nodes has increased when the MSM model is applied, compared to the nodal penetration depths of the SM model, due to a more gradual application of the vertical seabed force by the MSM model as well as the addition of the artificial grounding added mass force which increases the mass of the nodes when they make contact with the seabed. The maximum penetration elevation in Case 1 with the MSM model is -82.84 m compared to that of -82.74 m with the SM_7 coefficient set.

Case 3 is a biharmonic fairlead excitation in the surge and heave directions with a maximum displacement of 25.4 m of the fairlead node from the initial position. From the tension peak values for Case 3 presented in Table 4, the results for $SM^{4.74}$ and $SM^{0.84}$ overpredicted $\bar{T}$ [18] by 1.46% and 1.23% respectively, while the $MSM^{4.74}$ results overpredicted $\bar{T}$ [18] by 1.20%. The $SM_7^{4.74}$ results underpredicted $\bar{T}$ [18] by 0.36%.

Figure 36 presents the fairlead tension time history for one oscillation period ($600 \text{ s} \leq t \leq 800 \text{ s}$) in Case 3. The tension fluctuations do not occur between 600 s to 665 s for all models and discretisations, as presented in Figure 37a, due to the low nodal grounding velocities and grounding angles shown respectively in Figures 38a and 39a. Between 665 s and 800 s, the fluctuations appear at various points in time, as presented in Figure 37b,

with some variability between the models. From 665 s to 750 s, fluctuations appear at the troughs as well as loading cycle (positive slope) of the tension results for all models, whereas between 750 s and 800 s, some fluctuations occur at the troughs of the $SM^{4.74}$ fairlead tension results and are subdued in the results for the $SM_7^{4.74}$, $SM^{0.84}$, $MSM^{4.74}$ models.

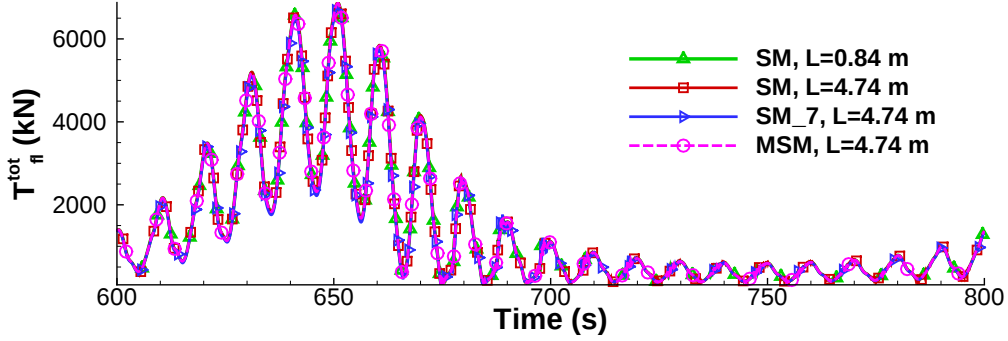
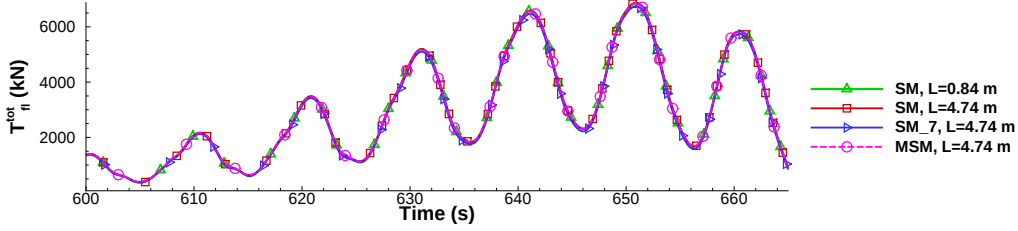
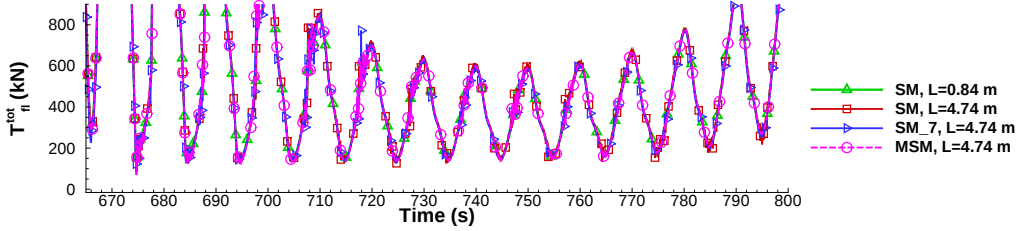


Figure 36: Fairlead tension time histories for Case 3 with time window ($600 \leq t \leq 800$ s).

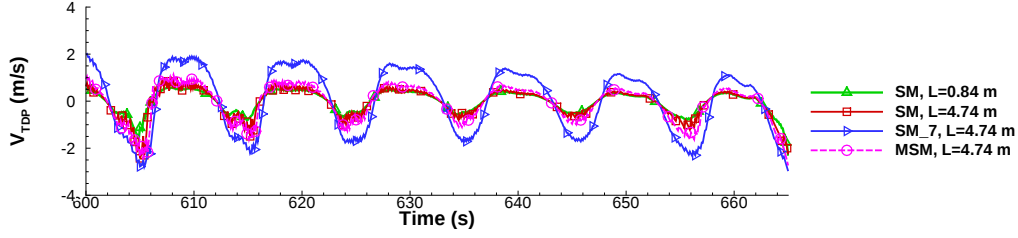


(a) Fairlead tension time histories for Case 3 with time window ($600 \leq t \leq 665$ s).

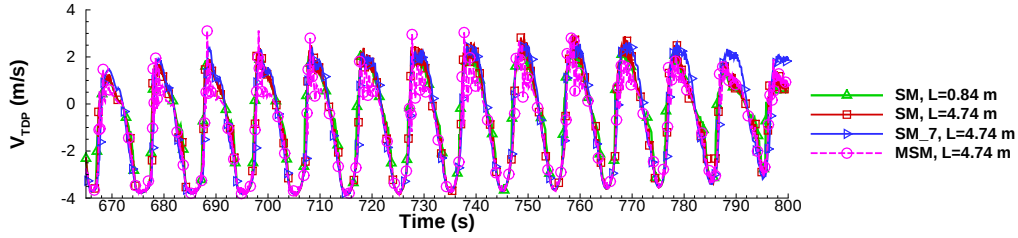


(b) Fairlead tension time histories for Case 3 with time window ($665 \leq t \leq 800$ s).

Figure 37: Fairlead tension time histories for Case 3 with SM, SM_7 , MSM seabed models and 4.74 m and 0.84 m-element discretisations.

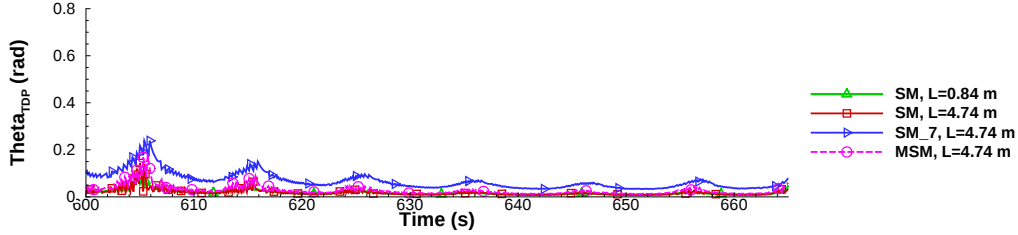


(a) Touchdown node vertical velocities for Case 3 with time window $(600 \leq t \leq 665 \text{ s})$.

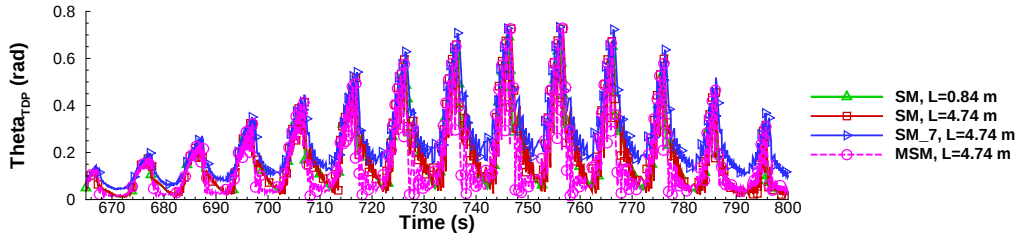


(b) Touchdown node vertical velocities for Case 3 with time window $(665 \leq t \leq 800 \text{ s})$.

Figure 38: Touchdown node vertical velocities for Case 3 with SM, SM₇, MSM seabed models and 4.74 m and 0.84 m-element discretisations.



(a) Grounding element θ angles for Case 3 with time window $(600 \leq t \leq 665 \text{ s})$.



(b) Grounding element θ angles for Case 3 with time window $(665 \leq t \leq 800 \text{ s})$.

Figure 39: Grounding element θ angles for Case 3 with SM, SM₇, MSM seabed models and 4.74 m and 0.84 m-element discretisations.

The characteristics for the some of the tension fluctuations in Case 3 differ from Case 1 in that these fluctuations are observed during the loading

cycle, and are preceded by significant fluctuations at the troughs of the time series. The tension at the trough is close to zero, indicating that the fairlead tension is close to being slack. The strains in the line during the development of the tension fluctuations at one of the troughs at $t = 726.0$ s is presented in Figure 40, and illustrates that a large segment of the line is under significant compression for all the models and discretisations used.

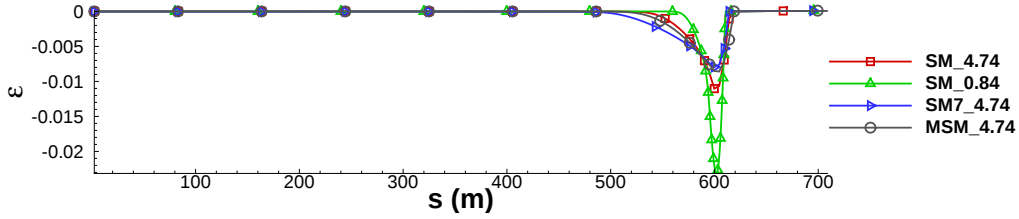


Figure 40: Strain distributions at $t = 726.0$ s for Case 3 during development of tension fluctuations.

At this point, a distinction is made between the tension fluctuations that occur in all the models between 665 s and 750 s, and those that occur in the $SM^{4.74}$ model from 750 s to 800 s. The former occurs due to the occurrence of snap loading, and is characterised by two waves of tensions fluctuations or shocks; the first occurs during rapid line unloading when line tension falls precipitously and parts of the line goes slack in the vicinity of the touchdown point, followed by a second wave of fluctuations that occur when the line is loaded again and the part of the line under compression returns to a tensioned state, which leads to the propagation of shock waves.

The occurrence of snap events has been demonstrated experimentally by Hsu et al.[20] and Azcona et al.[2]. The analytical shock condition proposed by Triantafyllou and Blik [11] and experimentally verified by Gobat and Grosenbaugh[12] states that cable unloading and loading shocks develop when the condition in Equation 34 is met,

$$\left| \frac{dx_0}{dt} \right| \leq \sqrt{\frac{T_0}{m}} \quad (34)$$

where x_0 is the position of the touchdown point, T_0 is the touchdown point tension, and m is the mass per unit length of the line, and the left and right hand sides respectively refer to the touchdown point and transverse wave speeds. In the present study, the touchdown point speed is approximated by the time rate of change of s_{TDP} , while T_0 is approximated with the horizontal component of the fairlead tension following [12].

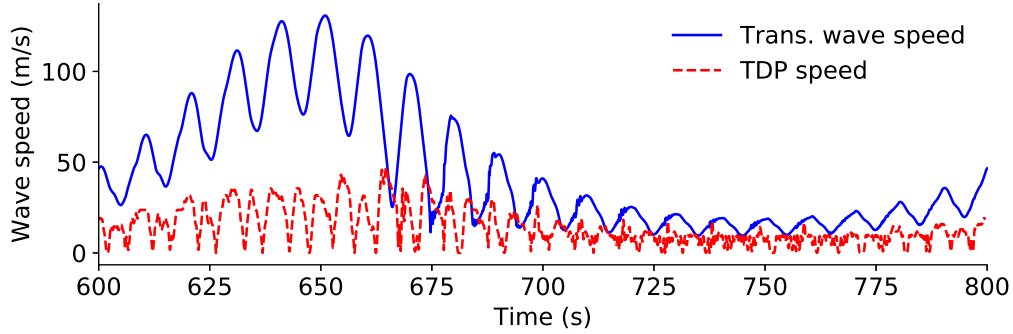


Figure 41: Touchdown point and transverse wave speeds for Case 3 during time window $600 \leq t \leq 800$ s.

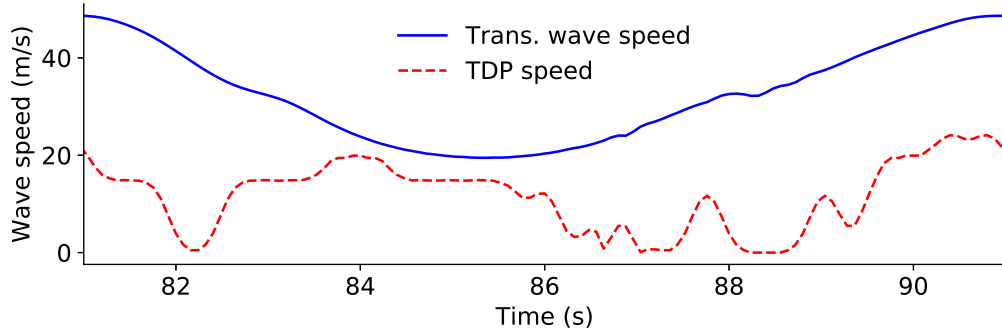


Figure 42: Touchdown point and transverse wave speeds for Case 1 during time window $81 \leq t \leq 91$ s.

The touchdown point and transverse wave speeds in Cases 3 and 1 are shown in Figures 41 and 42 respectively. Figure 41 shows the occurrence of the touchdown point speed exceeding the transverse wave speed between 665 s and 750 s, which corresponds to the occurrence of snap load tension fluctuations that comprise of line unloading and loading shocks. Between 750 s and 800 s, the touchdown point speed is lower than the transverse wave speed in Case 3. Hence, the tension fluctuations that occur in the SM^{4.74} tension results (see Figure 33) are not due to snap loading shocks. This is further illustrated in Figure 42 which shows that the touchdown point speed never exceeds the transverse wave speeds and hence not fulfilling the shock condition, even though there are fluctuations occurring in the tension.

The tension fluctuations occurring in the SM^{4.74} model in Case 1, and between 750 s and 800 s in Case 3, are caused by nodal grounding. Unlike the

snap loading shocks, which may occur when touchdown point velocities and θ angles are relatively low, as shown in Figures 38a and 39a respectively, and the fairlead tension approaches zero, the nodal grounding tension fluctuations occur when grounding velocities and θ angles are comparatively higher, as presented in Figures 38b and 39b respectively, and the line does not become slack. Parts of the line in the vicinity of the touchdown point may become slack during nodal grounding, which generates strain discontinuities, however, the magnitude of the negative strains are lower than what occurs in snap loading situations. The MSM model is effective in mitigating the nodal grounding tension fluctuations, but not the snap load tension shocks.

Case 4 is a biharmonic fairlead excitation in the surge direction with a maximum displacement of 18.0 m of the fairlead node from the initial position. For Case 4, Table 4 shows that the peak tensions with SM^{4.74} and SM^{0.84}, and MSM models underpredict $\bar{T}$ [18] by under 1.8%, while the SM₇^{4.74} model underpredicts $\bar{T}$ by approximately 2%.

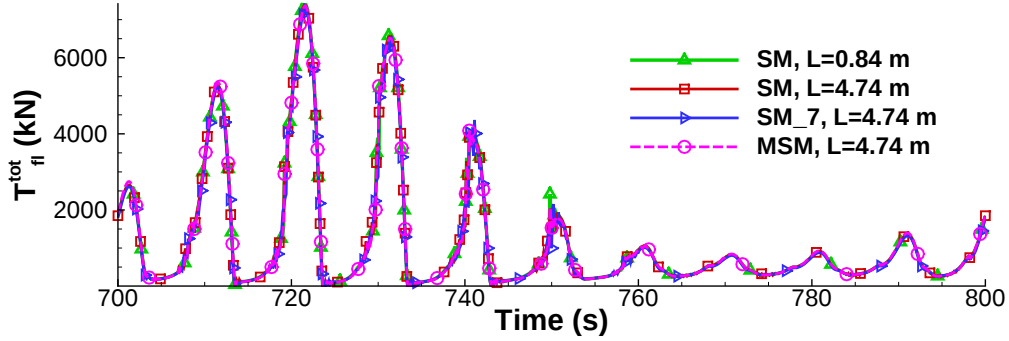


Figure 43: Fairlead tension time histories in Case 4 during ($700 \leq t \leq 800$ s).

Figure 44 presents the touchdown point and transverse wave speeds in Case 4 and illustrates that the snap loading shock condition [11] is met between 700 s and 760 s. From 760 s to 800 s, the touchdown point speed is lower than the transverse wave speed. Figures 43 and 45a respectively show the tension time histories for Case 4 for one period of fairlead motion and a close up view during the time window ($700 \leq t \leq 760$ s) when snap load shocks occur. During $700 \leq t \leq 760$ s, even though the shock condition is not met, tension fluctuations appear in the results for the SM^{4.74} model, but are significantly reduced in the SM^{0.84} and SM₇^{4.74} and MSM^{4.74} models. As was in Case 3, the tension fluctuations occurring during $760 < t \leq 800$ s are due to the occurrence of nodal grounding. As shown in the tension histories

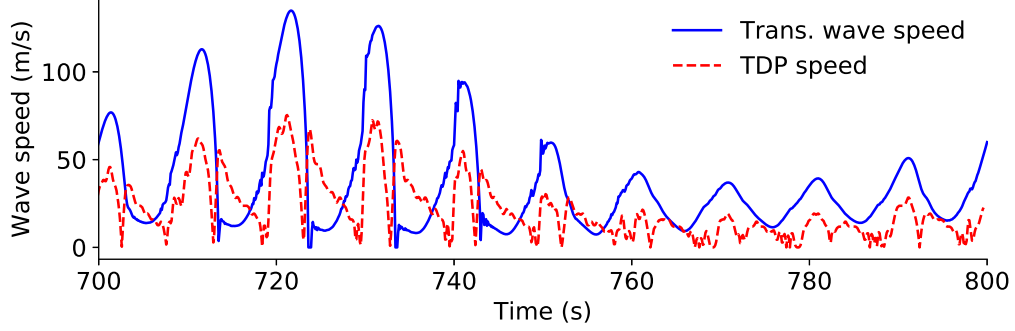
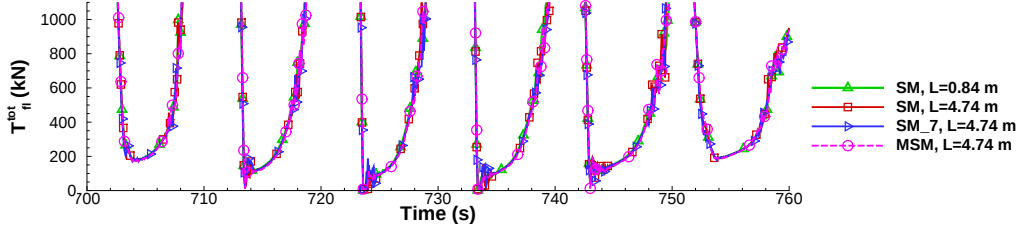
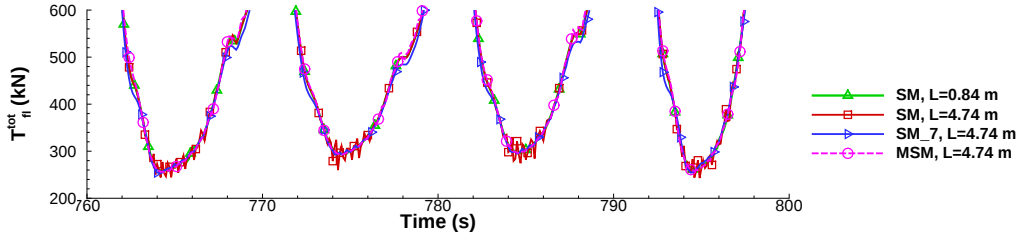


Figure 44: Touchdown point and transverse wave speeds for Case 4 during ($700 \leq t \leq 800$ s).

for the $SM^{0.84}$, $SM_7^{4.74}$ and $MSM^{4.74}$ models, this type of fluctuations can be mitigated by refining the nodal density and judiciously modifying the seabed force coefficients.



(a) Fairlead tension time histories for Case 4 with time window ($700 \leq t \leq 750$ s).

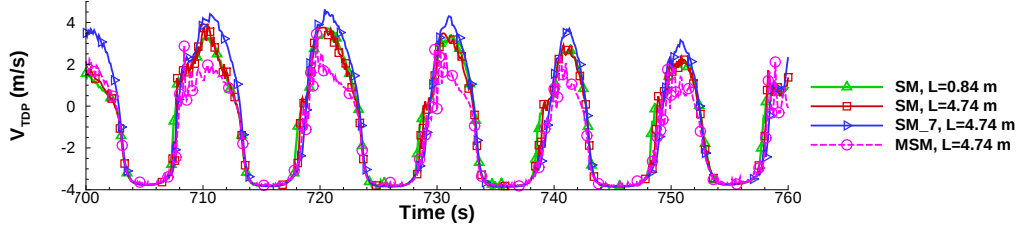


(b) Fairlead tension time histories for Case 4 with time window ($750 \leq t \leq 800$ s).

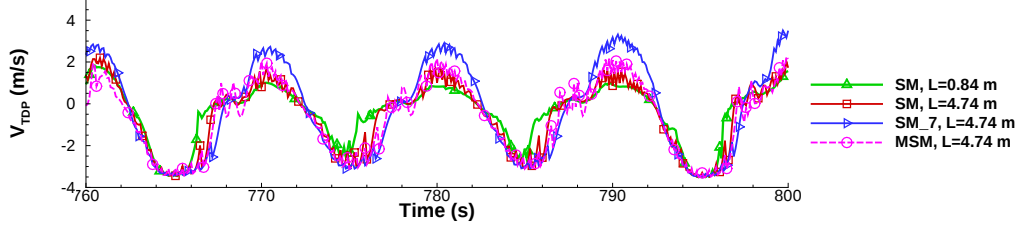
Figure 45: Fairlead tension time histories for Case 4 with SM, SM_7 , MSM seabed models and 4.74 m and 0.84 m-element discretisations.

Figures 46 and 47 respectively show the touchdown node vertical velocity and θ angles during the two aforementioned time windows in Case 4. It is observed from Figure 47a that in the time window ($700 \leq t \leq 760$ s) during which snap loading fluctuations occur, the θ angles were under 0.3 rad, and in general lower than the θ angles in the time window ($760 < t \leq 800$ s) during which nodal grounding fluctuations occur, presented in Figure 47b.

An exception for this trend occurred at 756 s, when the θ angle exceeds 0.3 rad. Comparing Figures 46a and 46b, it is observed that the maximum grounding velocities in the time windows ($700 \leq t \leq 760$) and ($760 < t \leq 800$) are of comparable magnitudes approaching 4 m/s.

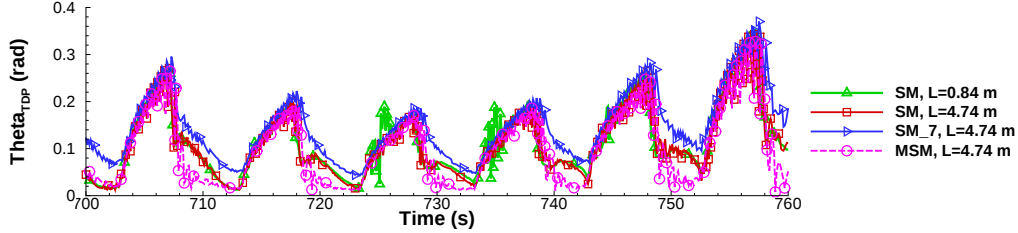


(a) Touchdown node vertical velocities for Case 4 during ($700 \leq t \leq 750$ s).

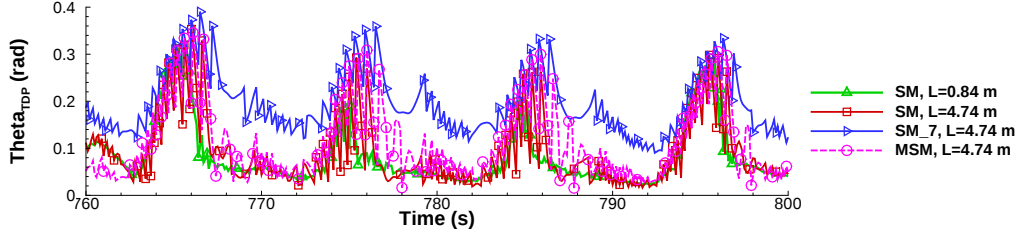


(b) Touchdown node vertical velocities for Case 4 during ($750 \leq t \leq 800$ s).

Figure 46: Touchdown node vertical velocities for Case 4 with SM, SM₇, MSM seabed models and 4.74 m and 0.84 m-element discretisations.



(a) Grounding element θ angles for Case 4 during ($700 \leq t \leq 750$ s).



(b) Grounding element θ angles for Case 4 during ($750 \leq t \leq 800$ s).

Figure 47: Grounding element θ angles for Case 4 with SM, SM₇, MSM seabed models and 4.74 m and 0.84 m-element discretisations.

The grounding velocity and θ angle at $t = 786$ s, which are approximately 3 m/s and 0.3 rad respectively, are compared with the grounding velocities and θ angles at $t = 756$ s, respectively 3.75 m/s and 0.35 rad, and are therefore higher than necessary to induce nodal grounding fluctuations. However, due to the snap load shock condition being met at 756 s, the more severe fluctuations arising from the alternating slack and taut tension states were preeminent.

5.2. Deep water environment

A multi-segment, chain-wire rope-chain line with structural parameters enumerated in Figure A.2 is used in the present deep water case, with a water depth of 914.0 m. The pretension is 1200 kN, and the initial line geometry is shown in Figure 48. The fairlead excitation for Case 4 with parameters listed in Figure B.2 is used in this case. Comparisons are made between the SM, SM₇ and MSM models for the in-house code and Orcaflex (using the deep water seabed coefficient set presented in Figure C.1), with element sizes of 8.0 m and 4.0 m.

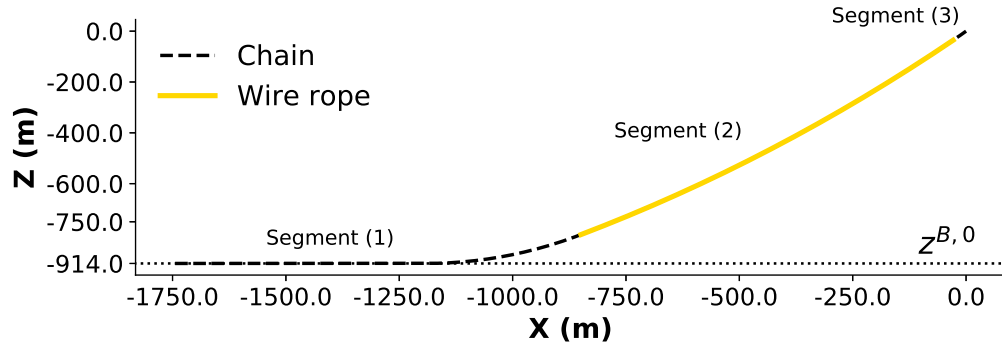


Figure 48: Initial static line geometry for multisegment mooring line.

The fairlead tension time series for one low frequency oscillation period of 100 s is included in Figure 49, while the touchdown point and transverse wave speeds are plotted in Figure 50. The results from Orcaflex and the in-house code are in good agreement, as evident in Figure 49, with a maximum difference in the peak tension of 30 kN, or 1.2%, with no noticeable phase difference in the time histories. The peak tensions from the in-house code using the SM, SM₇ and MSM models for the 8.0 m and 4.0 m element discretisations differ by a maximum of 24 kN or 0.95%.

Figure 50 shows that the tension shock conditions are met in the time window $700 \leq t \leq 800$ s. However, in contrast to the shallow water cases presented in Section 5.1, the tension fluctuations in the deep water case are less pronounced. There are spikes in the SM₇ tension results at the local tension peaks at 730.5 s, 740.5 s, 750.5 s, 760.5 s and 770.5 s but these are small in magnitude.

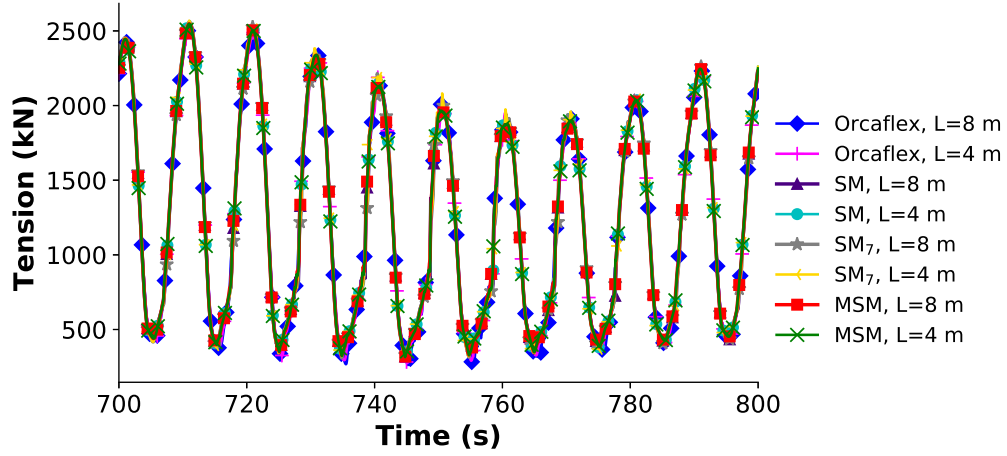


Figure 49: Fairlead tension time histories for Orcaflex and in-house code using SM, SM₇ and MSM seabed models for multisegmented line in deep water with Case 4 fairlead excitation during time window ($700 \leq t \leq 800$ s).

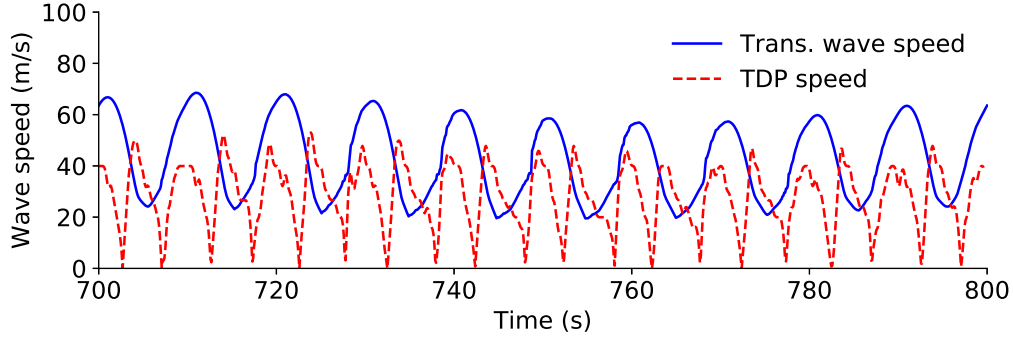


Figure 50: Touchdown point and transverse wave speeds for deep water case during time window ($700 \leq t \leq 800$ s).

Figure 51 shows the strain distribution in the mooring line over one such tension shock cycle ($744.67 \leq t \leq 748.38$ s) during which a section of the line goes slack and subsequently returns to a tensioned state. The location

of line section in compression is in the vicinity of the touchdown point. As is typical of slack line tension shocks, the magnitude of the compressive strain during the snap load cycle in the present case is significantly more severe than the negative strains associated with tension fluctuations arising from nodal grounding (see Figure 12a). The large strain gradients present thus generate propagating stress waves. The absence of large fluctuations in the fairlead tension is likely due to the significant physical separation between the source of the stress waves at the touchdown point and the fairlead. In the present deep water case, the distance between the fairlead and the line section under compression is approximately 1550 m, whereas in the shallow water Cases 1 and 3, the distances is considerable shorter at 160 m and 100 m, as shown in Figures 12a and 40 respectively. It is likely due to this larger separation distance that the travelling stress waves have been attenuated significantly due to fluid and line structural damping when they arrive at the fairlead.

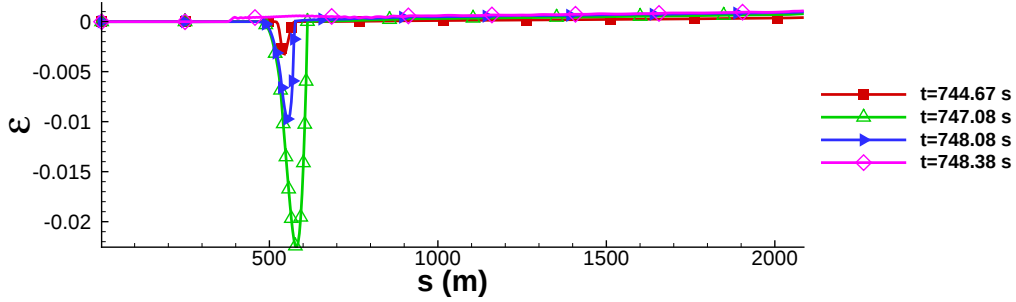


Figure 51: Strain distribution during slack and snap loading with time window ($700 \leq t \leq 800$ s).

5.3. Effect of Line Discretisation on Modified Spring Mattress (MSM) Seabed Model Coefficients

In Section 5.1, the proposed Modified Spring Mattress (MSM) model, with coefficients for $N^{B,c}$ and $\zeta^{B,a}$ both set to 1.0 for the 4.74 m element line, was shown to be effective at reducing the severity of the tension fluctuations caused by nodal grounding. The fairlead tension results for Cases 1, 3, and 4 of the shallow water cases, using the same coefficient set of the MSM model with element sizes of 8.0 m, 4.74 m, 2.85 m and 2.03 m, are shown in Figures 52, 53, and 54 respectively. For the time periods during which tension fluctuations have occurred, $84.0 \leq t \leq 87.0$ s in Case 1, $750.0 \leq t \leq 800.0$ s in Case 3 and $760.0 \leq t \leq 800.0$ s in Case 4, disparities in the severity of the

reduced tension fluctuations are small between the different discretisations. Hence, the same coefficient set is shown to be applicable and effective for a range of discretisations, which may be required for nodal density convergence studies.

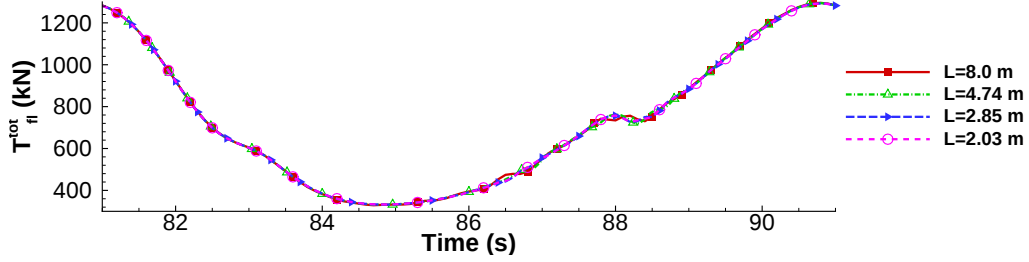


Figure 52: Fairlead tension time histories with MSM model ($N^{B,c}=\zeta^{B,a}=1.0$) and element lengths of 8.0 m, 4.74 m, 2.85 m and 2.03 m for Case 1.

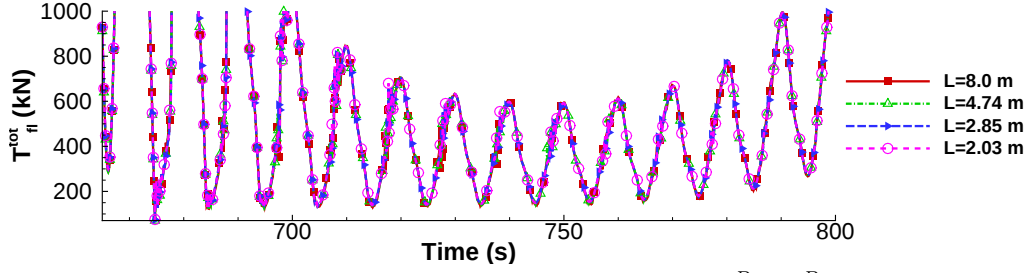


Figure 53: Fairlead tension time histories with MSM model ($N^{B,c}=\zeta^{B,a}=1.0$) and element lengths of 8.0 m, 4.74 m, 2.85 m and 2.03 m for Case 3.

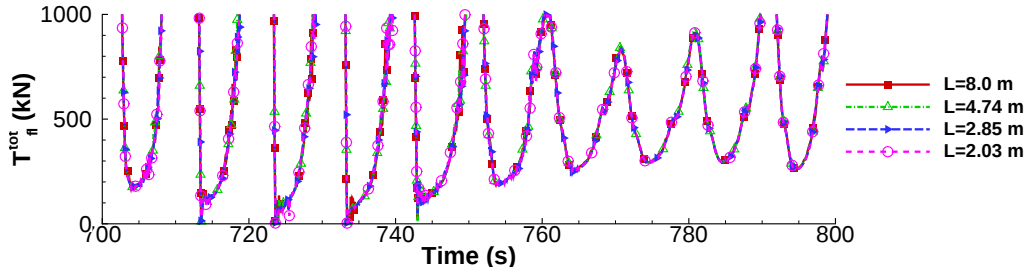


Figure 54: Fairlead tension time histories with MSM model ($N^{B,c}=\zeta^{B,a}=1.0$) and element lengths of 8.0 m, 4.74 m, 2.85 m and 2.03 m for Case 4.

5.4. Summary of Findings from Shallow and Deepwater Test Cases

Tension fluctuations arise from either line impact with the seabed and slack-snap load cycles. In both situations, negative strain zones may form

in the vicinity of the touchdown point, and is the source of the propagating stress waves that appear as fluctuations in the fairlead tension time history. The tension fluctuations are more pronounced in shallow water environments due to the close proximity of the touchdown zone to the fairlead in comparison with deep water cases. For deeper water environments, where distances from the touchdown point to the fairlead are further, the stress waves have to travel longer distances and thus undergo more damping before they arrive at the fairlead.

The tension time histories of the three test cases presented in Figures 33, 36, and 43 suggest that reducing the element size is an effective way to reduce the spurious tension fluctuations due to line-seabed interactions. The element size required depends on the environmental excitation and the grounding velocities of the nodes.

Increasing the seabed thickness coefficient $N^{B,c}$ and thereby reducing the vertical reaction spring stiffness, as well as reducing the vertical damping coefficient $\zeta^{B,d}$ in the SM model are also effective methods to mitigate the nodal grounding tension fluctuations. However, the peak tension results shown in Table 4 suggest that using required $N^{B,c}$ and $\zeta^{B,d}$ values tend to reduce peak tensions, leading to less conservative results for line tensions. This has an effect of line overload failure predictions.

The deviations of peak tension from $\bar{T}$ for Cases 1, 3 and 4, with the SM₇ coefficient set, are between -3.73% and -0.36% (underprediction), while the peak tension deviation from $\bar{T}$ with the SM coefficient set is between -0.98% (underprediction) and 1.46% (overprediction). With the MSM model, the peak tension deviation from $\bar{T}$ is between -1.77% and 1.2%. Hence, the SM coefficient set provides the most conservative line tension predictions, followed by the MSM model and SM₇ coefficient set.

The MSM model, while providing slightly less conservative maximum tension predictions compared to the SM model, is able to significantly reduce the tension fluctuations caused by line-seabed impact. The peak tensions calculated by the MSM model is also more conservative compared to the SM₇ coefficient set, while providing similar effectiveness in the mitigation of the tension fluctuations. The MSM model achieves this primarily through a higher-order formulation of the vertical reaction force that calculates the reaction force based on the embedment depth of the half elements bounding a node. The application of an added mass force also lowers the natural frequencies of line section that is in contact with the seabed and further reduces the tension fluctuations. However, a drawback is that the embedment

depth of the grounding nodes may be overestimated.

The proposed Modified Spring Mattress (MSM) model uses the coefficient set given in Table C.2 and its effectiveness in reducing the severity of the tension fluctuations arising from nodal grounding for a range of fairlead excitation frequencies and magnitudes, particularly in shallow water conditions, was shown in Section 5.1.

Finally, the recommended values for the artificial added mass, $\zeta^{B,a}$, seabed thickness $N^{B,c}$ coefficients in the MSM model, are both 1.0, and are generally applicable for different line materials and discretisations.

6. Conclusion

This work presents an investigation into the cause of the spurious tension fluctuations that affect lumped mass mooring line models due to line-seabed interactions. It was shown that both the commercial code Orcaflex as well as an in-house code may experience the tension fluctuations under moderately severe line-seabed impact conditions.

The typical strain and spatial strain gradient profiles of situations which would lead to or preclude the development of tension fluctuations were presented in Section 4. The tension fluctuations develop due to the formation of low strain regions close to the touchdown point when nodal grounding velocities are relatively high. The rapid formation of the localised low strain regions after nodal grounding leads to the creation of large strain gradients close to the touchdown point. The strain gradients represent unbalanced tension forces and lead to the propagation of the elastic stress waves along the length of the line.

A modified spring mattress model that calculates the vertical reaction force based on the orientation and elevation of the elements bounding a node was presented. This allows a more gradual application of the seabed vertical force without defining a large seabed force cutoff elevation as required in the conventional spring mattress model. The definition of an artificial added mass force also acts to reduce the natural frequencies of the line section in contact with the seabed. The model was able to avoid the fairlead tension fluctuations for the severe excitation test cases when used together with a relatively coarse line discretisation, while also providing more conservative estimates of the peak line tensions compared with the spring mattress model applied with the force coefficients required to adequately mitigate the tension fluctuations.

The recommended values for the force coefficients in the proposed model were also given and are generally applicable for different line materials, discretisations and environments. Studies are underway on applying the approach used in the modified spring mattress model to account for line bending and torsional effects so that it can be applied to pipeline and riser models.

Acknowledgements

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Appendix A. Mooring Line Physical Properties

Table A.1: Single-segment line structural parameters.

Parameter	Data
Material Type	Chain
Line length (m)	711.3
Stiffness, EA (MN)	1690
Mass per unit length (kg/m)	365.72
Wet weight per unit length (kN/m)	3202
Normal Drag coefficient, C_d^ν	3.2
Axial Drag coefficient, C_d^τ	0.6
Normal added mass coefficient, C_a^ν	1.6
Axial added mass coefficient, C_a^τ	0.2
Diameter, D (m)	0.14
Structural damping ratio, ζ^S	0.08
Element size (m)	8.0

Table A.2: Multi-segment line structural parameters.

Parameter	Seg. 1	Seg. 2	Seg. 3
Material Type	Chain	Wire	Chain
Segment length (m)	914.4	1127.8	45.7
Stiffness, EA (MN)	794	690	794
Mass per unit length (kg/m)	147.08	41.38	147.08
Wet weight per unit length (kN/m)	1379.80	343.23	1379.80
Normal Drag coefficient, C_d^ν	2.4	1.2	2.4
Axial Drag coefficient, C_d^τ	1.15	0.008	1.15
Normal added mass coefficient, C_a^ν	1.0	1.0	1.0
Axial added mass coefficient, C_a^τ	0.5	0.0	0.5
Diameter, D (m)	0.089	0.089	0.089
Structural damping ratio, ζ^S	0.08	0.08	0.08

Appendix B. Fairlead Motion Profiles

Table B.1: Prescribed fairlead motion profiles for validation cases without seabed contact.

Case	r_1^{WF} (m)	T_1^{WF} (s)	ϕ_1^{WF} (rad)	r_2^{WF} (m)	T_2^{WF} (s)	ϕ_2^{WF} (rad)	r_3^{WF} (m)	T_3^{WF} (s)	ϕ_3^{WF} (rad)
1	5.4	10.0	0.0	-	-	-	-	-	-
2	-	-	-	-	-	-	5.4	10.0	0.0
3	3.0	10.0	1.4	3.0	10.0	3.8	3.0	10.0	5.4

Table B.2: Prescribed fairlead motion profiles for evaluation for cases with nodal grounding.

Case	r_1^{WF} (m)	T_1^{WF} (s)	ϕ_1^{WF} (rad)	r_3^{WF} (m)	T_3^{WF} (s)	ϕ_3^{WF} (rad)	r_1^{LF} (m)	T_1^{LF} (s)	ϕ_1^{LF} (rad)	r_3^{LF} (m)	T_3^{LF} (s)	ϕ_3^{LF} (rad)
1	5.4	10.0	0.0	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	10.0	100.0	0.0	-	-	-
3	-	-	-	20.0	200.0	0.0	5.4	10.0	0.0	-	-	-
4	8.0	10.0	0.0	-	-	-	10.0	100.0	0.0	-	-	-

Appendix C. Seabed Model Force Coefficients

Table C.1: Seabed model coefficients for Orcaflex.

Parameter	Shallow water	Deep water
Water depth (m)	82.5	914.0
Vertical stiffness (kN/m/m ²)	131.95	348.00
Shear stiffness (kN/m/m ²)	131.95	348.00
Horizontal friction coefficient	0.74	0.74

Table C.2: Seabed model coefficients in spring mattress (SM) and modified spring mattress (MSM) models for in-house code.

Model	$\zeta^{B,d}$	$\zeta^{B,f,t}$	$\zeta^{B,f,N}$	$\mu^{B,f,t}$	$\mu^{B,f,N}$	$N^{B,c}$	$\zeta^{B,a}$
SM	1.0	1.0	1.0	0.74	0.74	1.0	0.0
SM ₁	1.0	1.0	1.0	0.74	0.74	5.0	0.0
SM ₂	1.0	1.0	1.0	0.74	0.74	10.0	0.0
SM ₃	1.0	1.0	1.0	0.74	0.74	15.0	0.0
SM ₄	1.0	1.0	1.0	0.74	0.74	20.0	0.0
SM ₅	0.0	1.0	1.0	0.74	0.74	5.0	0.0
SM ₆	0.0	1.0	1.0	0.74	0.74	10.0	0.0
SM ₇	0.0	1.0	1.0	0.74	0.74	15.0	0.0
SM ₈	0.0	1.0	1.0	0.74	0.74	20.0	0.0
MSM	0.15	1.0	1.0	0.74	0.74	1.0	1.0