Integrating Aerodynamic Properties into a Catapult's Calibration

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Abstract

All machines, no matter how precisely they were built, harbor inconsistency in the task they are supposed to perform. Science Olympiad's Trajectory event is one of many youth competitions that encourages testing for this inconsistency. Here, participants are required to build a device that can launch a projectile to a target anywhere between 2 meters and 8 meters in 1-meter increments and at heights of 0.5 meters or 1 meter. During the few months I participated in this event, I tested for inconsistency in my catapult's launch distance. Firstly, I identified three aerodynamic-related problems and fixed them with my best modification. To avoid confounding results, I only used the 2-meter-far target at a 0.5-meter-elevation for testing. Results from running a probability significance test on the modified catapult against my goal of 98% accuracy in the catapult's launch distance gave a p-value of 0.31. Because 0.31 was greater than my significance level (α) of 0.05, the catapult having 98% accuracy at the 2-meter-far, 0.5-meter-high target cannot be disproven. The results of this significance test were constant for every other possible target distance and elevation. The overarching purpose of this research was not just to create a competition device, but to also make a contribution in the use of catapults on aircraft carriers. These steam-based catapults are considered dangerous and inefficient, so I created and calibrated this catapult using a bungee cord launch force as a better alternative.

Keywords

Linear Air Resistance, Probability Statistics, Catapult, Aircraft Carrier, Final Angle, Projectile Velocity, SOLIDWORKS Design

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Introduction

Although operating on an aircraft carrier is becoming relatively safer, the job is still considered one of the most dangerous in the world. For example, from 2019 to 2021, there have been 67 non-fatal accidents on American aircraft carriers (1). While many of these incidents happen due to the high levels of activity on the relatively small aircraft carrier, there are also cases caused by a malfunction in the aircraft launcher. Aircraft launchers are catapults that provide the necessary lift for a plane to get off the runway of a carrier. The type most used today is the steam catapult, which uses a piston to drag the plane off the runway from built-up steam pressure. Steam catapults, however, are far too massive for the job they need to perform, making it difficult for precise adjustments. If too much steam is used, it "will rip off the nose-wheel landing gear which is attached to the catapult" (3). Conversely, too little steam "would not provide adequate speed for take-off" (3). Another problem caused by the inability of precision control is transients, or the energy released by the catapult, to be too large and "damage or reduce the life of the airframe" (2). Both situations can, and have, caused death on carriers.

The purpose of this research was to create and calibrate a catapult that has precision control. Instead of using steam pressure and a piston, a far more flexible launch force was used: the bungee cord. With the bungee cord, the distance launched and power output can be easily manipulated through how far it is stretched back. However, the precision control provided by the bungee cord also causes it to be highly inconsistent in the distance launched. So, aerodynamic data sets (the projectile's final angle relative to the target and final projectile velocity) were studied in this research to show how to examine a bungee cord's variability using linear air resistance calculations and probability statistics. These probability statistics were used to verify that the modifications were meaningful by comparing the unmodified catapult to the modified catapult, and the modified catapult to the theoretically ideal catapult. Ultimately, this research models the calculations needed for a bungee cord catapult system to become more accurate and replace steam-based catapults on aircraft carriers.

This catapult also had another purpose – to compete in Trajectory, a Science Olympiad event. In Trajectory (specifically the 2022-2023 event version), participants must build a device that can launch a projectile to a target anywhere between 2 meters and 8 meters with 1-meter increments. Targets may also be at 0.5-meter or 1-meter elevations. Furthermore, this catapult was built with a bungee cord as the launch force not just to research aircraft catapults, but also because it was one of the only allowable forces participants could use. So, much of the testing and building of the catapult uses elements from this competition. However, Trajectory does not contradict the overarching purpose of creating a better catapult for aircraft carriers – it only sets the boundaries for the build.

SOLIDWORKS Design for the Catapult

Before any materials or tools were used, a preliminary design was made for the catapult in SOLIDWORKS, a commonly used CAD (Computer Automated Design) software. Figure 1 shows the catapult still in the “model” mode, where it could still be worked with by the user, and Figure 2 shows the drawing mode of the model, which holds the basic measurements used to make the catapult.

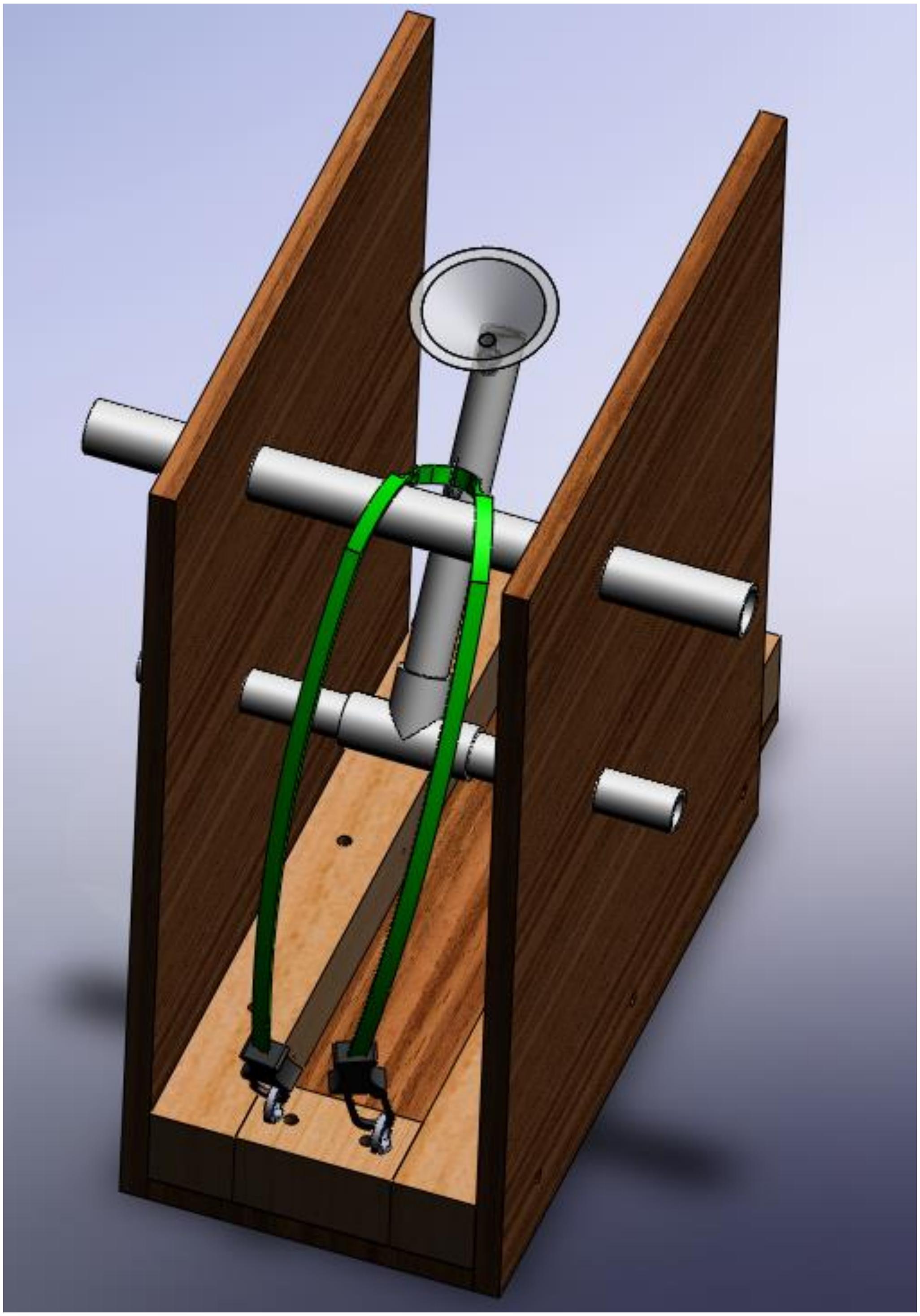

Figure 1: Preliminary SOLIDWORKS Design of Catapult (Model Mode). This design is a rough version made of the final design, so it is not completely accurate to the actual build. Note that modifications for the bungee cord's design were made after the catapult was built for readers to understand the CAD model more easily.

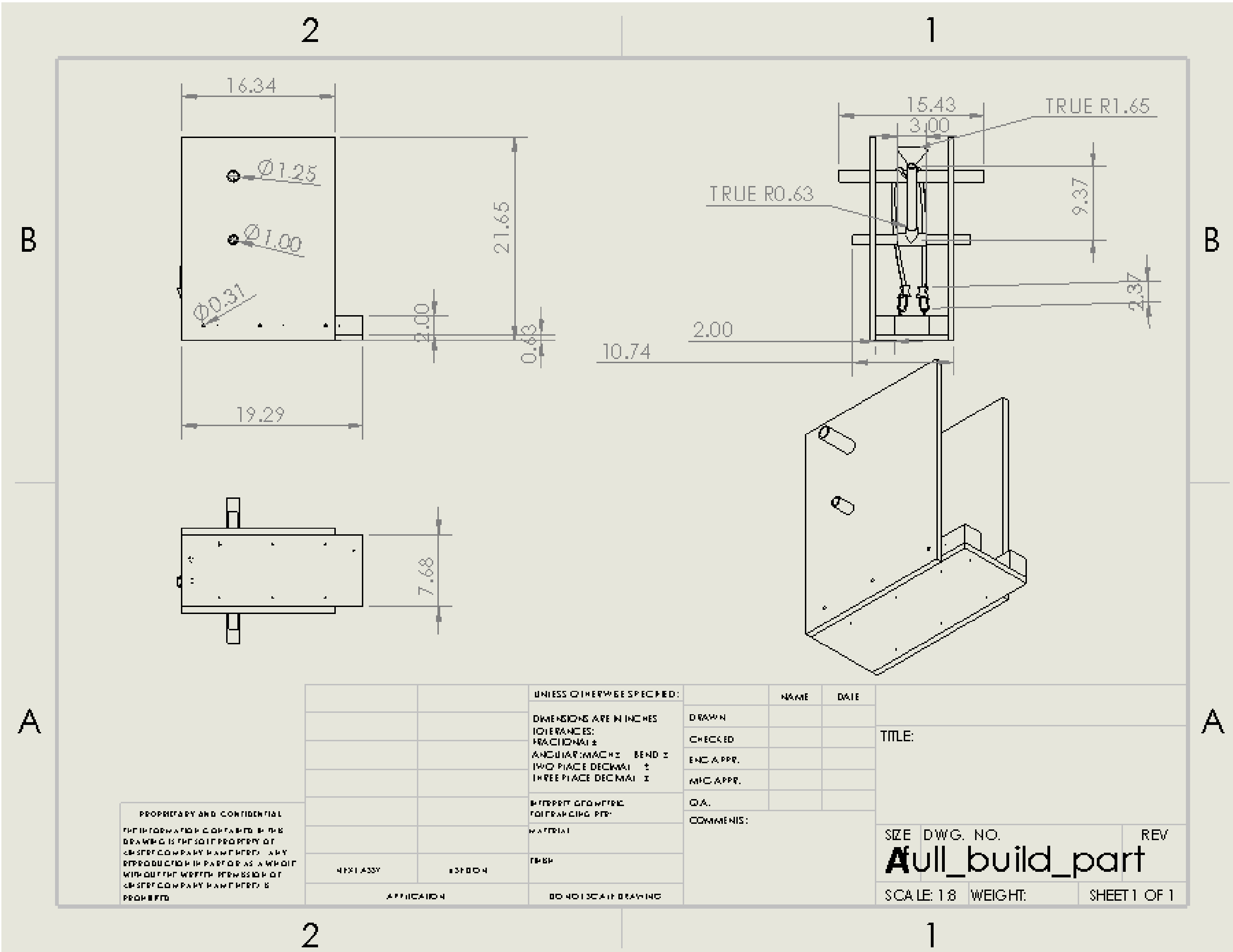

Figure 2: Preliminary SOLIDWORKS Design of Catapult (Drawing Mode). This drawing shows different angles of the catapult along with the different basic measurements used to build it.

Building the Catapult (Materials)

When the catapult was originally built, 2 wooden boards, each 0.61-by-0.61 meters, were cut down to 0.55-by-0.415 meters with a horizontal cutting saw, making the sideboards. Then, a third wooden board, also 0.61-by-0.61 meters, was cut down to 0.49-by-0.195 meters, forming the base. Additionally, 2 planks, each 0.035-by-0.93 meters, had to be cut down with a hand saw to make 3 planks – 2 0.035-by-0.49-meter "long" planks and 1 0.035-by-0.113-meter "short" plank. Then, all these parts were attached together using a DeWalt drill and wood screws. (Note that the DeWalt drill, wood screws, and hand saw were used throughout the rest of the build process even if not explicitly stated). From here, long planks were reinforced to the base, and the side boards were reinforced to the long planks using 90-degree screw brackets. 2 I-bolts were also attached on the front of the base that the bungee cord would latch on to. Afterwards, a 0.03-meter-diameter hole was drilled on both the sideboards so that a 0.03-meter-diameter PVC pipe "stopping rod" could slide in between them.

After the frame of the catapult was built, the arm was made using 1 0.03-meter diameter, 0.2-meter-long PVC pipe and 2 0.03-meter diameter, 0.1-meter-long PVC pipes inserted into the slots of a three-way, 0.03-meter diameter PVC pipe connector. The stopping rod was made to cut off the motion of the arm when firing. From there, a hole was drilled at the top of upper PVC pipe and a funnel was attached there to hold and launch the projectile, which was a tennis ball. A hook was attached to the upper arm to pull on the bungee cord from the I-bolts on the base over the stopping rod. Figure 3 shows the finished, unmodified catapult.

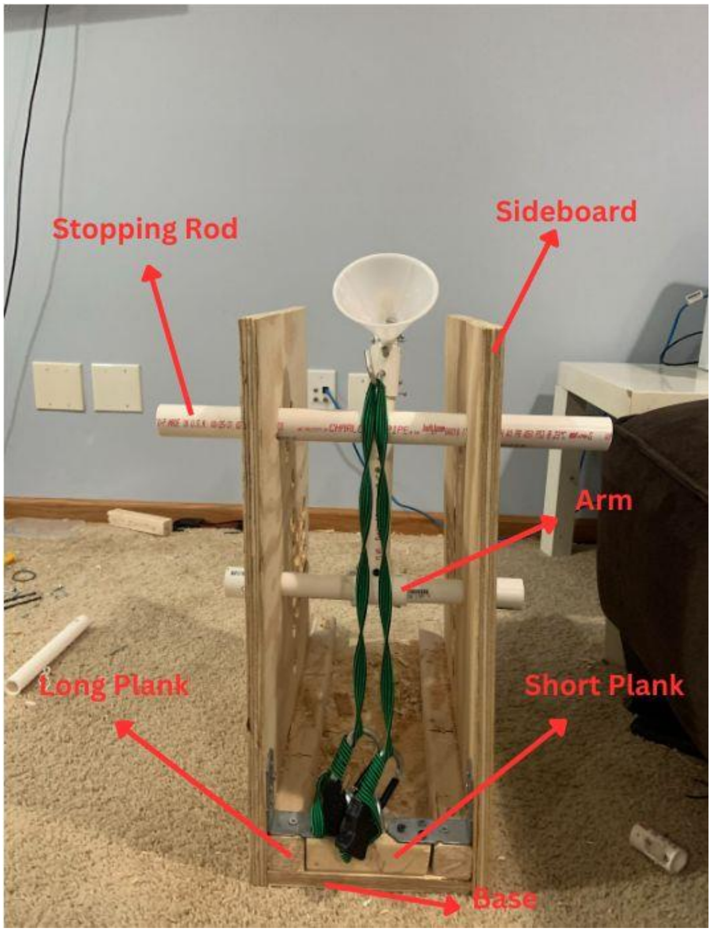

Figure 3: The Finished, Unmodified Catapult. This was the first version of the catapult built labeled with the parts that are unconventional. Conventional parts would be I-bolts or a funnel.

The Stopping Mechanism

After the arm was built, the actual catapult was completed, but a stopping mechanism still needed to be implemented. In Trajectory, participants must use a stopping mechanism that allows the device to be launched from at least 0.75 meters away. For this catapult, the stopping mechanism attempted to preserve the projectile's final angle relative to the target (Refer to Figure 4). So, as a test, 2 0.03-meter holes were drilled into the side boards, and 2 small holes were drilled on the back side of the upper arm. Then, a 0.03-meter PVC pipe was put through the holes in the sideboard, and a small hole was drilled in its center.

From there, I-bolts were manually screwed in on the openings on the PVC pipe and the upper arm. The idea is that, when the arm is pulled back, the 2 I-bolts on the arm would line up before and behind the I-bolt on the PVC pipe, and a metal "stopping rod" can slide through that opening to stop the arm's motion, as shown in Figure 5. This preserves the launch angle, which causes the final angle to be fixed. So, from taking several pictures of the arm pulled back to the same point and comparing them with each other, it was safely concluded that the modification preserved the final angle since the launch angle was constant. Because the modification worked, 17 more holes were drilled on each of the sideboards (Shown in Figure 6) and 6 more were drilled on the upper arm to provide more options for calibration (Shown in Figure 7).

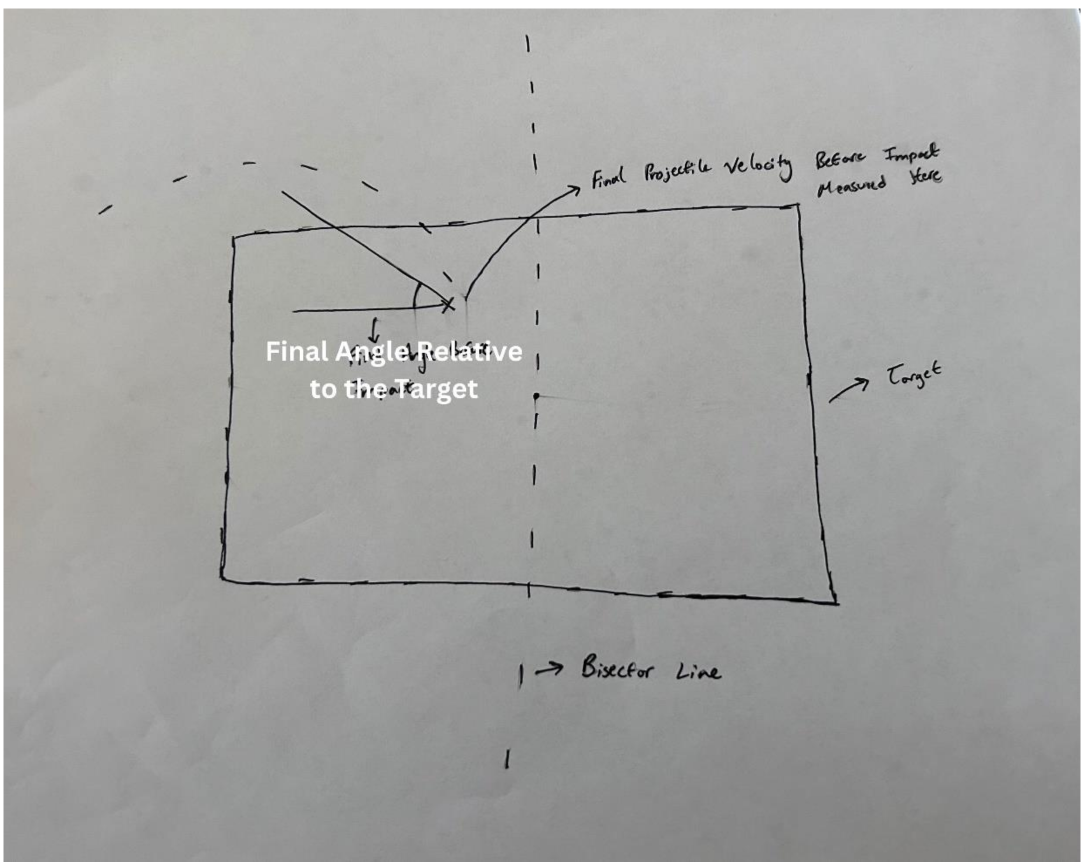

Figure 4: Final Angle and Final Projectile Velocity Diagrams. This diagram illustrates the 'Final Angle Relative to the Target (also known as just 'Final Angle') and the 'Final Projectile Velocity Before Impact' (also known as just 'Final Projectile Velocity') as they are on the target. The imaginary 'Bisector Line' divided the target into equal halves.

Figure 5: The Stopping Mechanism. This contraption attempted to preserve the final angle for any specific calibration of the catapult by fixing the launch angle.

Figure 6: Arm Holes. These are the 8 small holes on the arm part of the stopping mechanism. The numbers in red are used to identify each hole. For example, this image has the arm holes set to 8 and 4. Note that the bigger number is always put first when identifying each hole.

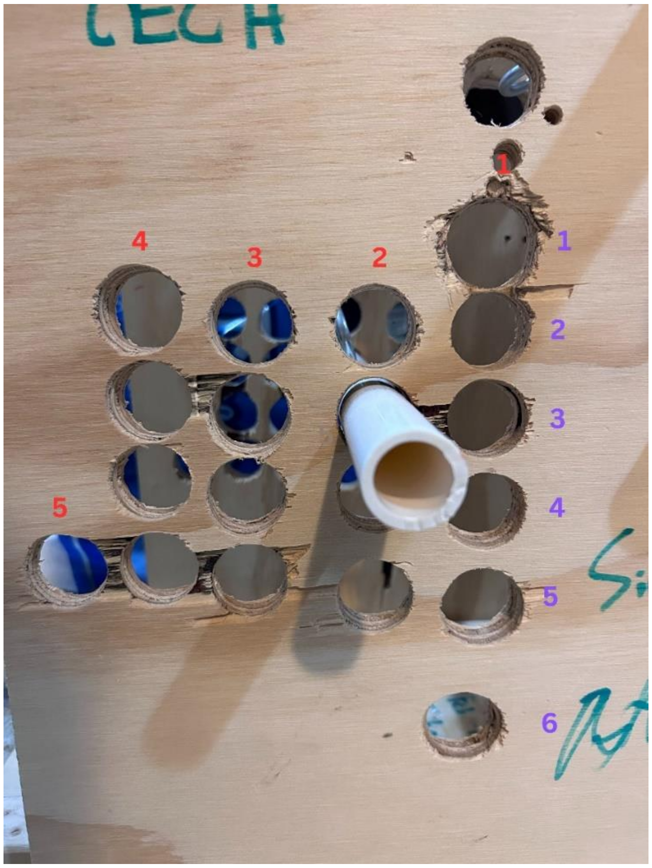

Figure 7: Sideboard Holes. This image shows the 19 holes on one of the sideboards the PVC pipe would go through. The red and purple numbers are used to identify each hole, where the format goes ‘purple, red’. For example, in this image the sideboard holes are set to 3, 2.

The Launch Process

Using this stopping mechanism, the catapult was tested and calibrated to account for every possible target distance and elevation in the Trajectory competition after fixing the three aerodynamic-related problems discussed later on. (Refer to Figure 8 for the launch set-up of the tests at a 2-meter-far and 0.5-meter-high target). Outside of the stopping mechanism, though, 2 additional factors were also manipulated for calibration. Firstly, because the bungee cord has belt-buckle hooks, it can be looped through the hooks to give the catapult more force, and therefore launch the projectile a further distance or higher elevation, when pulled back. Additionally, the catapult can be calibrated by manipulating a value called “setback”, which is described on the diagram in Figure 9. Table 1 shows the calibration measures for every possible target distance and elevation.

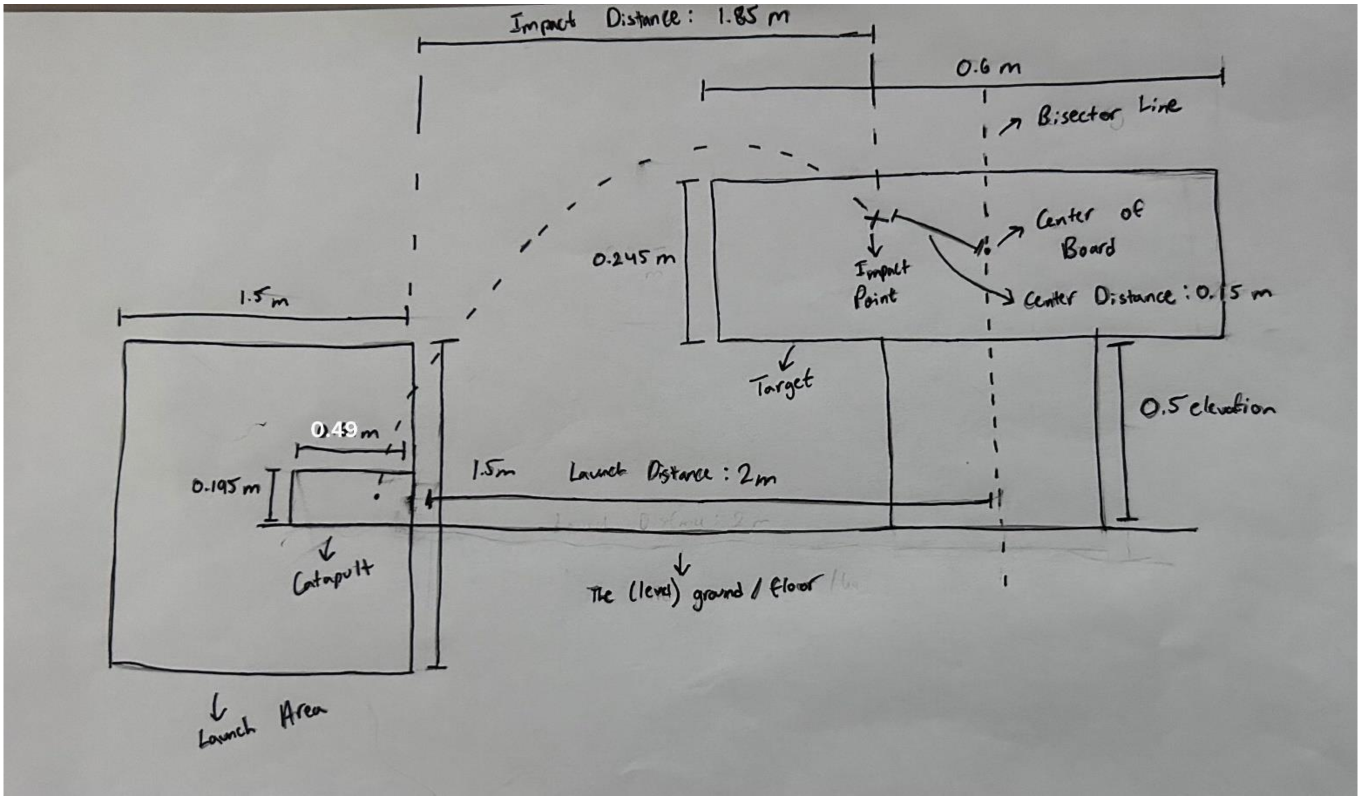

Figure 8: Launch Set-Up Diagram. This diagram describes the launch set-up of the catapult for a 2-meter-far and 0.5-meter-high target as regulated by the Trajectory competition for Science Olympiad. Here, the catapult is launched from the launch area with no "setback". The launch distance is measured by the distance from the front of the 'Launch Area' to the midpoint of the target, which is designated using a 'Bisector Line'. When the catapult is launched, the projectile goes in an arc until it hits the target. The 'Impact Point' is the point on the target where the projectile strikes. Then, the 'Center Distance' is the distance from the 'Impact Point' to the 'Center of the Board', and the 'Impact Distance' is the distance from the catapult to the 'Impact Point'. In this particular example, the 'Center Distance' is 0.15 meters, and the 'Impact Distance' is 1.85 meters.

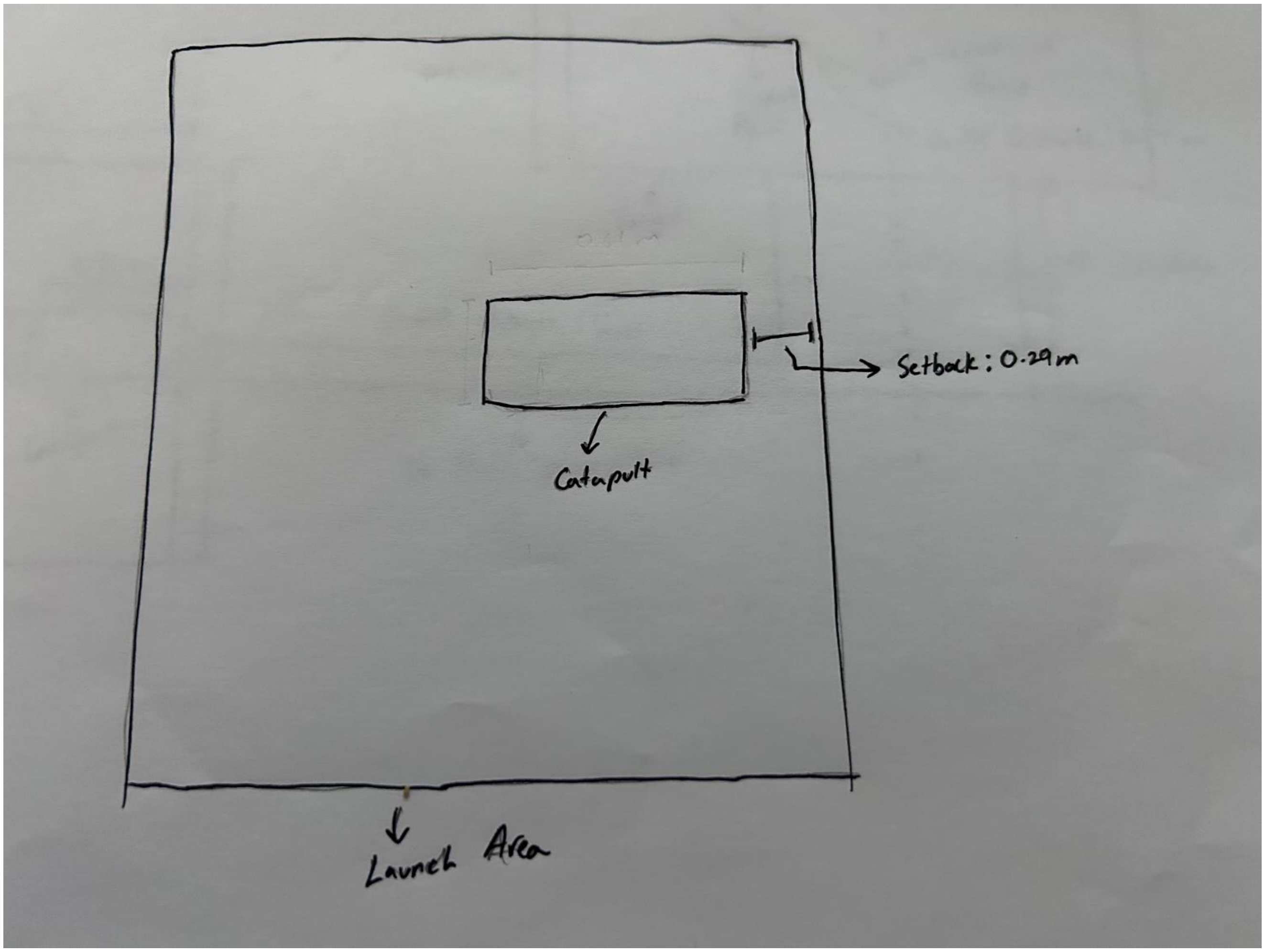

Figure 9: Diagram Demonstrating Setback. The catapult can be calibrated by being moved back from the front of the launch area. The amount the catapult is moved back is called "setback". In this diagram, an example setback of 0.29 m is used.

Table 1: All Calibrations for Every Possible Target

Launch Distance (m)	Arm Holes	Setback (m)	Sideboard Holes	# Bungee Cord Ties
2 m, 0.5 m	4 and 1	0.00	3, 2	1
2 m, 1 m	4 and 1	0.50	3, 2	1
3 m, 0.5 m	4 and 1	0.35	4, 2	2
3 m, 1 m	4 and 1	0.50	4, 2	2
4 m, 0 m	4 and 1	0.38	4, 2	1
4 m, 0.5 m	4 and 1	0.00	4, 2	2
4 m, 1 m	4 and 1	0.20	4, 2	2
5 m, 0 m	4 and 1	0.33	4, 2	2
5 m, 0.5 m	8 and 4	0.40	2, 4	3
5 m, 1 m	8 and 4	0.50	2, 4	3
6 m, 0 m	8 and 4	0.00	2, 4	3

6 m, 0.5 m	8 and 4	0.00	2, 4	4
6 m, 1 m	8 and 4	0.30	2, 4	4
7 m, 0 m	8 and 4	0.00	2, 4	4
8 m, 0 m	8 and 4	0.00	2, 4	5

Examine Figures 6 and 7 for further detail on the ‘Arm Holes’ and ‘Sideboard Holes’, respectively. Additionally, examine Figures 8 and 9 for further detail on what the ‘Launch Distance (m)’ and ‘Setback (m)’ columns refer to, respectively. Finally, Table 1 displays the final calibration of the catapult after solving the aerodynamic problems that, again, will be discussed later on.

Accounting for Errors in the Build Process

Throughout the process of building this catapult, all tools were used in the most efficient way. For example, wood being sawed was always on a table with the saw angled at 45°. Additionally, screws were also always drilled on a table, with both hands pushing down on the drill. Because of these precautions, along with having many tough materials pre-cut, any operational errors made with the catapult were minimized.

Preliminary Testing

Although the catapult operated properly, the distance it launched for a particular setting seemed to vary largely. To get a qualitative answer for this inconsistency, the catapult was launched 50 times at a 2-meter-far and 0.5-meter-high target, and the center distance was recorded. (Please note that all future analyses in this experiment will also use the 2-meter-far and 0.5-meter-high target to minimize confounding results). This distance was used because it is measured in the Trajectory competition for points. The closer the impact point is to the center of the target, the higher the score. Then, a slow-motion video was used to find the projectile’s impact point. Additionally, the target was marked with distances up to 0.2 meters from the center in 0.1-meter increments for reference in the slow-motion video. Figure 10 shows the marked-up target in closer detail. Then, refer to Figure 8 for further detail on the ‘center distance’, ‘impact point’, and ‘center of the target’ terms.

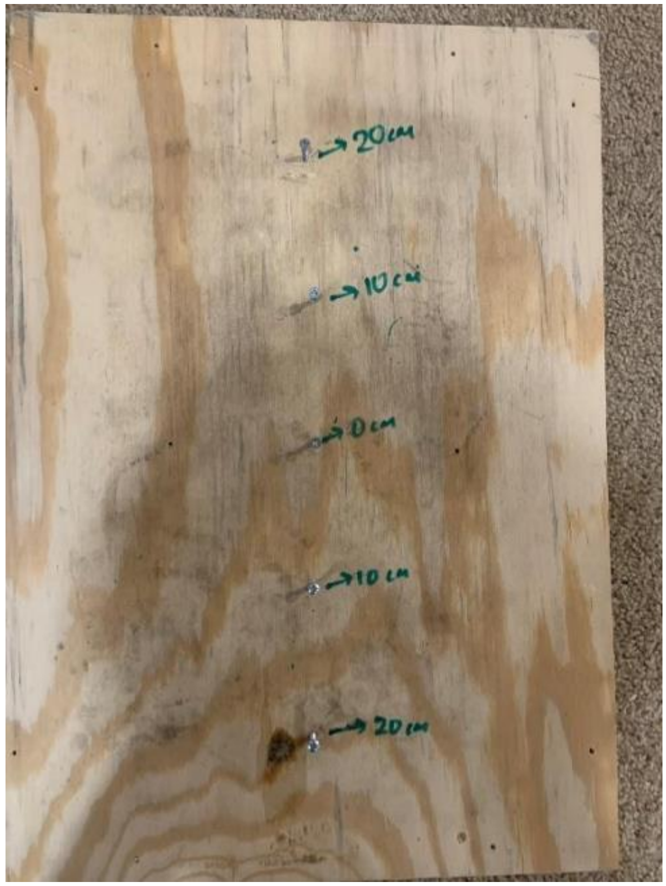

Figure 10: The Marked-Up Target. This target allowed x-value measurements to be made in the testing phase. As shown in Figure 8, this target is 0.6 meters by 0.245 meters.

The Significance Test (Method #1)

In the preliminary testing, the times where the projectile's center distance was at or within 0.2 m of the target's center were recorded down as successes, and the rest as failures. This data is shown in Table 2. 0.2 m was chosen because that amount of variance is marginal for both the Trajectory competition and for aircraft catapults. After the testing, a significance test was run to determine if the probability of successes observed in the 50 trials is equal to 98% when launching the catapult randomly at the 2-meter far and 0.5-meter high target. Here, the hypothesis format was used to model this test. The null hypothesis (H_0) was the probability of the ball hitting at or within 0.2 m from the target p = 0.98, the alternative hypothesis (H_a) was $p \neq 0.98$, and the significance level (α) = 0.05. The hypothesis format is used to determine, the chance, or the p-value, that H_a appears in a distribution assuming that the H_0 was true. So, before solving for the p-value, H_0 must first be verified to be expressed as a binomial distribution. A binomial distribution is a frequency distribution for the number of successes in a given number of trials, and it is commonly used when discussing the different probabilities of an event happening in a set number of trials. In this significance test, because there are a fixed number of 50 trials, each trial is independent from the other, each observation represents either the projectile hitting a maximum of 0.2 m from the target's center or not doing so, and the probability of success is the same for each outcome, H_0 can form a binomial distribution.

From here, a normal distribution can be used to determine the p-value of the hypothesis test. However, it is important to note that this value is only an estimate because a binomial distribution, which is what H_0 is being modeled as, is discrete and has a finite number of events, but a normal distribution is continuous and has an infinite number of events. With that in mind, the operation normalcdf(32, 66, 49, 0.99) is used to find the p-value. Here, 32 is the observed amount of the desired outcome and the lower bound, 66 is the upper bound, 49 is the expected mean amount for a 98% success rate out of the 50 trials, and 0.99 is the standard deviation of the distribution. Running this operation on a TI-84 resulted in 1, the probability that the desired outcome will happen between the upper and lower bounds. To find the 2-tail p-value for $p \neq 0.98$, 1 was subtracted from 1, resulting in a number close to 0. If a 1-tail p-value needed to be found for $p > 0.98$, the operation normalcdf(32, 100000, 49, 0.99) would be used. Additionally, this number is an approximation of 0 because the TI-84's screen is not large enough to fully display it.

Ultimately, because the p-value of approximately 0 is less than $\alpha = 0.05$, the H_0 was rejected. There is convincing evidence that, for any random launch of the unmodified catapult at a 2-meter far and 0.5-meter elevated target, the projectile's center distance would be at or less than 0.2 m with 98% accuracy. An inference about any random launch at a 2-meter far and 0.5-meter-high target can be made here because many trials were run with the unmodified catapult. (As a side note, there will be unconventional words in the future sections to describe the different states of the catapult. Those unconventional words will be listed in Table 3 along with their definitions).

Table 2: Center Distance Data for the Unmodified Catapult

Trial Number	Center Distance (m)
Trial 1	0.13
Trial 2	0.04
Trial 3	0.09
Trial 4	0.19
Trial 5	0.18
Trial 6	0.18
Trial 7	0.04
Trial 8	0.15
Trial 9	0.20
Trial 10	0.30
Trial 11	0.21
Trial 12	0.35
Trial 13	0.34

Trial 14	0.28
Trial 15	0.26
Trial 16	0.13
Trial 17	0.37
Trial 18	0.16
Trial 19	0.14
Trial 20	0.33
Trial 21	0.24
Trial 22	0.12
Trial 23	0.24
Trial 24	0.29
Trial 25	0.44
Trial 26	0.11
Trial 27	0.20
Trial 28	0.34
Trial 29	0.10
Trial 30	0.36
Trial 31	0.21
Trial 32	0.17
Trial 33	0.45
Trial 34	0.10
Trial 35	0.34
Trial 36	0.16
Trial 37	0.17
Trial 38	0.14
Trial 39	0.01
Trial 40	0.13
Trial 41	0.12
Trial 42	0.07
Trial 43	0.08
Trial 44	0.17
Trial 45	0.19
Trial 46	0.02
Trial 47	0.19
Trial 48	0.02
Trial 49	0.30
Trial 50	0.20

Table 3: Index of Unconventional Catapult Descriptors

Word	Definition
Modified Catapult	The catapult with the first set of modifications.
Unmodified Catapult	The before the first set of modifications, as seen in Figure 3.
Newly Unmodified Catapult	The catapult after removing the first set of modifications.
Funnel-Modified Catapult	The catapult with only the modification from 'The Inconsistency of Using a Funnel' section isolated. Note that this modification is the second, improved version from the 'Modified Catapult'.
Newly Funnel-Modified Catapult	The catapult with only the modification from 'The Inconsistency of Using a Funnel' section isolated. Note that this modification is the third, improved version from the 'Modified Catapult'.
Bungee-Cord-Modified Catapult	The catapult with only the modification from 'The Bungee Cord's Twisting' section isolated. Note that this modification is the second, improved version from the 'Modified Catapult'.
Moving-Funnel-Modified Catapult	The catapult with only the modification from 'The Moving Funnel' section isolated. Note that this modification is the second, improved version from the 'Modified Catapult'.
Fully Modified Catapult	The catapult with all of the final modifications added, as seen in Figure 11.

The Three Aerodynamic-Related Problems

There are 3 aerodynamic-related problems with the catapult that could help explain the ball's differing trajectory from the significance test. They are as follows – the projectile leaving the funnel at different points, the bungee cord twisting over the stopping rod, and the funnel being improperly attached to the arm. These problems were specially chosen from the catapult because they directly impact the 2 main factors of projectile motion – the projectile's final angle relative to the target and the final projectile velocity. Of course, there will aways be inconsistency with the center distance due to the stretching of a bungee cord, but fixing these problems would account for that variability. So, certain modifications were made to fix these problems to calibrate the catapult (These modifications will be covered in detail later on).

From here, the catapult was tested 50 times at the 2-meter-far and 0.5-meter-high target again using the same data gathering methods, and the number of successes and failures were recorded down in Table 4. Then, a significance test was run to determine if, for any random launch of the catapult at a 2-meter-far and 0.5-meter-high target, the modified catapult showed a greater chance of success compared to the unmodified catapult. (Refer to Table 3 for further detail on the "modified catapult" and "unmodified catapult"). So, H_0 was $p = 0.64$, the probability that success happened with the unmodified catapult, H_a was $p > 0.64$, and the $\alpha = 0.05$. Note that $p = 0.64$ was taken the 50 trials run prior in this section, and it can be inferred for any random launch of the unmodified catapult because, again, many trials were run to find this probability. Moving on, results from running this significance test gave a p-value of 0.0000028. Because 0.0000028 was less than $\alpha = 0.05$, the H_0 was rejected. There was convincing evidence that, for any random launch at the 2-meter-far and 0.5-meter-high target, the modified catapult's projectile's center distance had a greater chance of being at or less than 0.2 m than the unmodified catapult's projectile's center distance.

While this result established that the modifications were meaningful, it did not determine if the catapult had reached a 98% success rate for any random launch at a 2-meter far and 0.5-meter-high target. So, another significance test was run to determine if the probability of success observed in 50 trials of the modified catapult was close to or the same as 98% for any random launch at the 2-meter far and 0.5-meter-high target. Here, the H_0 was $p = 0.98$, the H_a was $p \neq 0.98$, and the $\alpha = 0.05$. Results from running the significance test gave a p-value of nearly 0. Because 0 was less than $\alpha = 0.05$, the H_0 is rejected. There was convincing evidence that, for any random launch at a 2-meter far and 0.5-meter-high target, the modified catapult's projectile's center distance would not be at or less than 0.2 m with 98% accuracy.

So, although the modified catapult had a higher chance of success for any random launch, that chance was still not high enough to be considered 98%. To account for these results, each problem and its modification had to be further examined to determine which modification(s) did not cause a significant change in the success rate. To do this, all the modifications were removed from the catapult, and the device was tested for 50 launches at the 2-meter far and 0.5-meter high target again, and all types of aerodynamic data (final angle and final projectile velocity) were recorded along with the center distance. Table 5 shows the data recorded in the 50 launches of the now "newly unmodified catapult" (Refer to Table 3 for further detail on the "newly unmodified catapult"). Then, each modification was isolated on the catapult, launched 50 times at a 2-meter-far and 0.5-meter-high target, and relevant aerodynamic data was recorded down. Then, the isolated modification was tested against the newly unmodified catapult for significance using the relevant aerodynamic data. By using the relevant aerodynamic data to determine the successes or failures of the modification, rather than the center distance, it was easier to identify whether the modification was working as intended. In the event that a modification was not working as intended, it was changed and tested again against the newly unmodified catapult using relevant aerodynamic data.

Table 4: Center Distance Data for the Modified Catapult

Trial Number	Center Distance (m)
Trial 1	0.19
Trial 2	0.13
Trial 3	0.08
Trial 4	0.02
Trial 5	0.17
Trial 6	0.20
Trial 7	0.13
Trial 8	0.20
Trial 9	0.19
Trial 10	0.11
Trial 11	0.19
Trial 12	0.18
Trial 13	0.19
Trial 14	0.15
Trial 15	0.11
Trial 16	0.18
Trial 17	0.05
Trial 18	0.14
Trial 19	0.16
Trial 20	0.12
Trial 21	0.45
Trial 22	0.38
Trial 23	0.29
Trial 24	0.15
Trial 25	0.24
Trial 26	0.19
Trial 27	0.13
Trial 28	0.17
Trial 29	0.14
Trial 30	0.14
Trial 31	0.19
Trial 32	0.01
Trial 33	0.02
Trial 34	0.14

Trial 35	0.19
Trial 36	0.05
Trial 37	0.46
Trial 38	0.16
Trial 39	0.20
Trial 40	0.24
Trial 41	0.20
Trial 42	0.29
Trial 43	0.28
Trial 44	0.04
Trial 45	0.04
Trial 46	0.37
Trial 47	0.26
Trial 48	0.49
Trial 49	0.24
Trial 50	0.17

This is the data center distance data used for comparing the modified catapult's center distance to the unmodified catapult's center distance.

Table 5: Aerodynamic Data for the Newly Unmodified Catapult (All Modifications Removed)

Trial Number	Center Distance (m)	Impact Distance (m)	Final Angle (°)	Final Projectile Velocity v_f (m/s)
Trial 1	0.15	1.85	1.5528	22.3336
Trial 2	0.13	1.87	1.5526	22.3337
Trial 3	0.07	2.07	1.5506	22.3345
Trial 4	0.16	2.16	1.5498	22.3349
Trial 5	0.15	1.85	1.5528	22.3336
Trial 6	0.19	2.19	1.5495	22.3351
Trial 7	0.20	2.20	1.5494	22.3351
Trial 8	0.00	2.00	1.5513	22.3342
Trial 9	0.09	1.91	1.5522	22.3339
Trial 10	0.14	2.14	1.5500	22.3348
Trial 11	0.20	1.80	1.5533	22.3334
Trial 12	0.35	2.35	1.5479	22.3358
Trial 13	0.14	1.86	1.5527	22.3337
Trial 14	0.01	2.01	1.5512	22.3343
Trial 15	0.15	1.85	1.5528	22.3336

Trial 16	0.12	2.12	1.5502	22.3348
Trial 17	0.17	2.17	1.5497	22.3350
Trial 18	0.21	2.21	1.5493	22.3352
Trial 19	0.39	2.39	1.5475	22.3360
Trial 20	0.06	1.94	1.5519	22.3340
Trial 21	0.19	1.81	1.5532	22.3335
Trial 22	0.44	2.44	1.5470	22.3363
Trial 23	0.02	2.02	1.5511	22.3343
Trial 24	0.32	2.32	1.5482	22.3357
Trial 25	0.16	1.84	1.5529	22.3336
Trial 26	0.21	2.21	1.5493	22.3352
Trial 27	0.16	1.84	1.5529	22.3336
Trial 28	0.20	2.2	1.5494	22.3351
Trial 29	0.08	1.92	1.5521	22.3339
Trial 30	0.36	1.64	1.5548	22.3328
Trial 31	0.16	1.84	1.5529	22.3336
Trial 32	0.14	2.14	1.5500	22.3348
Trial 33	0.25	2.25	1.5489	22.3354
Trial 34	0.18	2.18	1.5496	22.3350
Trial 35	0.35	1.65	1.5547	22.3329
Trial 36	0.46	1.54	1.5558	22.3325
Trial 37	0.04	1.96	1.5517	22.3341
Trial 38	0.41	2.41	1.5473	22.3361
Trial 39	0.33	2.33	1.5481	22.3357
Trial 40	0.20	2.20	1.5494	22.3351
Trial 41	0.16	2.16	1.5498	22.3349
Trial 42	0.35	2.35	1.5479	22.3358
Trial 43	0.15	1.85	1.5528	22.3336
Trial 44	0.36	2.36	1.5478	22.3359
Trial 45	0.07	2.07	1.5506	22.3345
Trial 46	0.08	2.08	1.5505	22.3346
Trial 47	0.41	1.59	1.5553	22.3327
Trial 48	0.31	2.31	1.5483	22.3356
Trial 49	0.21	2.21	1.5493	22.3352
Trial 50	0.17	2.17	1.5497	22.3350

To calculate the final angle and final projectile velocity values, the x-distance the ball lands from the catapult on the target must be used. It is also important to note that when calculating the final projectile velocity here, the same final projectile velocity in the y-direction was used because the change in the y-position was constant at 0.5 m. Both of these calculations will be described in one of the next sections called ‘Derivation of Projectile Motion with Air Resistance (Method #2)’. Additionally, refer to Figure 7 for further detail on the ‘Impact Distance (m)’ .

Problem #1: The Inconsistency of Using a Funnel

One of the possible problems with the catapult was in how the projectile was launched from the funnel. Because the funnel curves outwards, the projectile may have been releasing from different points, thereby varying the distance launched. The solution proposed for this problem was to add cardboard around the outside of the funnel. Doing so would cause the launch angle to become fixed, thereby making the final angle more constant (just as with the stopping mechanism), and causing the chance of success to increase.

Derivation of Projectile Motion with Air Resistance (Method #2)

To properly test this modification, the final angle must be tracked. To find this angle, the final angle formula (Equation 1) can be used. In an ideal world, the equations to calculate v_x, the horizontal velocity, and v_{fy}, the final vertical velocity, are not complicated, but calculus must be used to account for air resistance. To simplify the process, only linear air resistance was accounted for. Linear air resistance is air resistance only dealing with 2-dimmensions. Because the calculus is practically the same between v_x and v_{fy}, the entire process can be modeled with v_x.

Equation 1: Final Angle Formula

$\tan^{-1}(v_{fy} / v_x)$

To begin with, the acceleration of v_x in the x-direction must be put in terms of air resistance. This is because acceleration is easily modeled for air resistance, and it can be integrated to solve for v_x and x, the horizontal position. So, by substituting the v_x values into the general formula for the effect of air resistance on acceleration (Equation 2), Equation 3 is found. Note here that mg was canceled out because there is no vertical acceleration, or gravity acceleration, in the horizontal direction. Furthermore, x′ and x″ can replace v and a in this formula because acceleration is the derivative of velocity, and velocity is the derivative of position.

Equation 2: General Formula for the Effect of Air Resistance on Acceleration

$-kv - mg = ma$

Equation 3: Substituting V_x values into General Formula

$-kx' = mx''$

With Equation 3, x'', the horizontal acceleration, can now be solved for. Firstly, moving the m to the opposite side results in $x'' = -kx'/m$. Because $-k/m$ is the same as x'/x'', and reducing the unit of velocity over the unit of acceleration in x'/x'' results in 1/t, $-k/m$ can be expressed as 1/T, which is a constant for time. Rewriting the equation results in $x'' = -x'/T$. Putting this into derivative notation results in $dv_x/dt = -v_x/T$. Now, separation of variables must be used to solve for v_x. So, we can rewrite the equation to $dv_x/v_x = -dt/T$. Integrating this equation results in $\ln(v_x) = -t/T + C$. To solve for C, 0 can be plugged in for t, as the launch starts at 0 seconds. That makes $C = \ln(v_x)$. Because the $\ln(v_x)$ will now cancel out from both sides we need $\ln(v_x)$ needs to be rewritten as $\ln(v_{ox})$. This is possible because, before acceleration acts on the projectile occurs, v_x and v_{ox}, the initial x-velocity, are the same value. So, the equation now becomes $\ln(v_x) = -t/T + \ln(v_{ox})$. By moving the $\ln(v_{ox})$ to the left-hand side and applying a logarithm property, the equation then becomes $\ln(v_x/v_{ox}) = -t/T$. To cancel out the natural logarithm, the entire equation can be put to the power of e. Solving for v_x here results in Equation 4.

Equation 4: Solving Equation 3 for V_x

$v_x = v_{ox}[e\text{^}(-t/T)]$

While this equation solves for v_x, it cannot be used as it is because the v_{ox} is also unknown. So, the v_x equation must be integrated once more to find an equation for v_{ox} in terms of position and time. First, the equation for v_x needs to be written as a differentiable equation for x: $dx/dt = v_{ox}[e\text{^}(-t/T)]$. By solving for the x through separation of variables in the same method described for v_x, the equation becomes $x = Tv_{ox}[1 – e\text{^}(-t/T)]$. Then, solving for v_{ox} results in Equation 5. Now, the only variable left unknown besides v_{ox} and v_x is the time constant 1/T. However, T can also stand for the equation m/b. Here, m and b are the mass and drag coefficient of the projectile, respectively. For a tennis ball, the mass is 56.0 grams and the drag coefficient is 0.55. Finally, all the components needed to solve v_x have been accounted for.

Equation 5: Solving Equation 4 for V_{ox}

$v_{ox} = x/\{T[1– e\text{^}(-t/T)]\}$

For the v_{fy} equation, the process, again, is the same as with v_x, as integrating the general formula for the effect of air resistance on acceleration in the y-direction (Equation 6) makes v_{fy} equal to Equation 7. Then, to solve for v_{oy}, the v_{fy} equation is integrated, resulting in v_{oy} equaling Equation 8. With this, all the components needed to solve v_{fy} have been accounted for.

Equation 6: Substituting V_{fy} values into General Formula

$-ky' – mg = my''$

Equation 7: Solving Equation 6 for V_{fy}

v_{fy} = -gT + {(gT + voy)[e^(-t/T)]}

Equation 8: Solving Equation 7 for V_{oy}

v_{oy} = {[y + gTt]/[T(1 – e^(-t/T))]}

As a side note, the calculations described in this section were based on Dr. Vivek Narayanan's video "GCM05: Projectile motion with air resistance" on YouTube (4).

Statistical Analysis and Results

50 trials of the catapult were run at the 2-meter far and 0.5-meter high target with this funnel modification isolated, and the final angle was recorded down each time using the derived formulas. Then, a significance test was run to determine if the final angle had a greater chance of being between 1.5494° to 1.5532° with the newly unmodified or the "funnel-modified" catapult with any random launch at the 2-meter-far and 0.5-meter-high target. 1.5494° to 1.5532° was used because that is the interval of final angles that corresponded to the success center distances observed for the newly unmodified catapult in the 50 trials shown in Table 4. Here, the H_0 was p = 0.66 (the probability that the final angle was between 1.5494° to 1.5532° with the newly unmodified catapult), the H_a was $p > 0.66$, and the $\alpha = 0.05$. Results from running the test gave a p-value of approximately 1. Because 1 was greater than $\alpha = 0.05$, the test failed to reject the H_0. There was no convincing evidence that, for any random launch at a 2-meter-far and 0.5-meter-high target, the funnel-modified catapult had a greater chance than the newly unmodified catapult of having a final angle between 1.5494° to 1.5532°.

Although the significance test failed, a p-value of 1 showed that the current modification needed some tweaking with the final angle. So, instead of having the cardboard surround the outside of the funnel, it now towered up from the inside. Doing so would cause the catapult to shoot at an even more fixed angle.

Then, 50 more trials of the newly funnel-modified catapult were run at the 2-meter far and 0.5-meter high target, and the final angle was recorded each time again. From here, another significance test was run with the same parameters as the previous one in this section, except it now tested the new funnel modification on the catapult compared to the newly unmodified catapult. Results from running this test gave a p-value of 0.001. Because 0.001 is less than α = 0.05, there was convincing evidence that, for any random launch at the 2-meter-far and 0.5-meter-high target, the newly funnel-modified catapult had a greater chance than the newly unmodified catapult of having a final angle between 1.5494° to 1.5532°. (As a side note, refer to

Table 3 for further detail on the “funnel-modified catapult” and “newly funnel-modified catapult”).

Problem #2: The Bungee Cord’s Twisting

Another problem in the newly unmodified catapult was the bungee cord getting tangled over the stopping rod between launches, as seen in Figure 3. Because the degree of the entanglement could cause the bungee cord to output different launch forces to the projectile, this factor may have also been varying the distance the projectile travels. Because the bungee cord was not getting tangled up during the launches, the solution for this problem was simple – untangle the bungee cord between each launch.

Statistical Analysis and Results

50 launches at the 2-meter-far and 0.5-meter-high target were run with the bungee cord modification isolated on the catapult. For each trial, information about the v_f, the final projectile velocity before impact, was recorded using the formula vf = $[(v_x)^2 + (v_{fy})^2]$^0.5, where v_x and v_{fy} were found using the calculations in ‘Derivation of Projectile Motion with Air Resistance (Method #2)’ to account for linear air resistance. Here, v_f was used because the speed of the projectile should be greater if the modification was meaningful. So, a significance test was run to determine if, for any random launch at the 2-meter-far and 0.5-meter-high target, the probability of v_f being between 22.3325 m/s and 22.3363 m/s was greater with the bungee-cord-modified catapult than the newly unmodified catapult. 22.3325 m/s and 22.3363 m/s were used because that is the interval of final velocities that corresponded to the success center distances observed for the newly unmodified catapult in 50 trials. Therefore, the H_0 was $p = 0.66$, the probability that the final projectile velocity was between 22.3325 m/s and 22.3363 m/s as with the newly unmodified catapult, the H_a was $p > 0.66$, and the $\alpha = 0.05$. Results from running the test gave a p-value of 0.0000000282. Because 0.0000000282 was less than $\alpha = 0.05$, the H_0 was rejected. There was convincing evidence that, for any random launch at the 2-meter-far and 0.5-meter-high target, the bungee-cord-modified catapult (Refer to Table 3 for further detail on the “bungee-cord-modified catapult”) had a greater chance than the newly unmodified catapult of having a v_f between 22.3325 m/s and 22.3363 m/s.

Problem #3: The Moving Funnel

The final aerodynamic problem observed on the catapult was the funnel moving during launches. Here, the launch angle would be affected, thereby affecting the final angle, and causing the distance the ball travels to change. To account for this inconsistency, the funnel was drilled into the PVC pipe from the back of the arm rather than the sides for better support.

Statistical Analysis and Results

50 trials of the catapult with this modification isolated were run at the 2-meter far and 0.5-meter high target, and the final angle was recorded down each time. Then, a significance test was used to determine if the final angle had a greater chance of being between 1.5494° to 1.5532° with the

newly unmodified or with the moving-funnel-modified catapult (Refer to Table 3 for further detail on the "moving-funnel-modified catapult") at any random launch at the 2-meter-far and 0.5-meter-high target. So, the H_0 was $p = 0.66$, the probability that the final angle was between 1.5494° to 1.5532° with the newly unmodified catapult, the H_a was $p > 0.66$, and the $\alpha = 0.05$. Results from running the test gave a p-value of 0.0000000282. Because 0.0000000282 was less than the $\alpha = 0.05$, the H_0 was rejected. There was convincing evidence that, for any random launch at the 2-meter-far and 0.5-meter-high target, the moving-funnel-modified catapult had a greater chance than the newly unmodified catapult of having a final angle between 1.5494° to 1.5532°.

Discussion and Conclusions

Now that any modifications not offering significant changes to the chance of success were fixed, all of the modifications were reassembled on the catapult. Figure 11 shows the catapult with all of the modifications. From here, 50 more trials at the 2-meter-far and 0.5-meter-high target were run on the now "fully modified catapult" (Refer to Table 3 for further detail on the "fully-modified catapult"), and the number of successes was recorded. This data is shown in Table 6. Then, a significance test was run to determine if, for any random launch of the catapult at a 2-meter-far and 0.5-meter-high target, the fully modified catapult showed a greater chance of success compared to the newly unmodified catapult. So, the H_0 was $p = 0.66$, the probability of success with the newly unmodified catapult, the H_a was $p > 0.66$, and the $\alpha = 0.05$. Results from running the test gave a p-value of [$1.39(10^{-31})$]. Because [$1.39(10^{-31})$] is less than $\alpha = 0.05$, the test rejects the H_0. There is convincing evidence that, for any random launch at the 2-meter-far and 0.5-meter-high target, the projectile's center distance was at or less than 0.2 m at a greater rate with the fully modified catapult than the newly unmodified catapult.

From here, another significance test was run to determine if the probability of success observed in 50 trials of the fully modified catapult was close to or the same as 98% for any random launch at the 2-meter-far and 0.5-meter-high target. Here, the H_0 was $p = 0.98$, the H_a was $p \neq 0.98$, and the $\alpha = 0.05$. Results from running the significance test gave a p-value of 0.31. Because 0.31 was greater than $\alpha = 0.05$, the H_0 failed to be rejected. There was no convincing evidence that, for any random launch at a 2-meter-far and 0.5-meter high-target, the fully modified catapult would not shoot at or within 0.2 m of the target's center with 98% accuracy.

Running the same 2 significance tests between the fully modified catapult and the newly unmodified catapult for all possible distances from the catapult to the target listed in Table 1 had the same results – the fully modified catapult had a greater success rate than the newly unmodified one, and the fully modified catapult could not be disproven to not have a success rate of 98%.

Through the use of the stopping mechanism, setback, the properties of the bungee cord, the modifications, and solving the three aerodynamic-related problems for the modifications, the catapult has now been successfully calibrated for all distances and elevations in Trajectory. Thanks to this calibration, I was ultimately able to get 3rd place at Minnesota's Regional Competition for Trajectory on the Junior Varsity (JV) Team.

Figure 11: The Fully Modified Catapult. The catapult assembled with all the final modifications.

Table 6: Center Distance for the Fully Modified Catapult

Trial Number	Center Distance (m)
Trial 1	0.15
Trial 2	0.16
Trial 3	0.14
Trial 4	0.06
Trial 5	0.18
Trial 6	0.18
Trial 7	0.14
Trial 8	0.15
Trial 9	0.14
Trial 10	0.04

Trial 11	0.20
Trial 12	0.07
Trial 13	0.12
Trial 14	0.12
Trial 15	0.12
Trial 16	0.07
Trial 17	0.16
Trial 18	0.39
Trial 19	0.02
Trial 20	0.09
Trial 21	0.03
Trial 22	0.04
Trial 23	0.12
Trial 24	0.07
Trial 25	0.05
Trial 26	0.48
Trial 27	0.05
Trial 28	0.03
Trial 29	0.12
Trial 30	0.17
Trial 31	0.11
Trial 32	0.19
Trial 33	0.07
Trial 34	0.11
Trial 35	0.05
Trial 36	0.13
Trial 37	0.18
Trial 38	0.07
Trial 39	0.08
Trial 40	0.05
Trial 41	0.20
Trial 42	0.07
Trial 43	0.19
Trial 44	0.17
Trial 45	0.06
Trial 46	0.11
Trial 47	0.06
Trial 48	0.18
Trial 49	0.11

Trial 50	0.18

Future Implications

Because the catapult is now completely calibrated, this also means it effectively accounts for any inconsistencies from using a bungee cord as a launch force. Ultimately, this study on aerodynamic data (the final angle and final projectile velocity) provides a valuable model for examining bungee cords (with linear air resistance and the probability statistics) as the launch mechanism in aircraft catapults – a replacement that is both safer and more efficient. However, because the catapult was also made for the Trajectory competition, it could not fully be modeled as an aircraft launcher. So, further investigation of this research would entail performing aerodynamic calibrations for a bungee-cord based catapult built as part of an aircraft carrier.

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