Service Performance Indicators of Bridge Support Reaction Monitoring Big Data based on Statistically Stable Traffic Flow Load Effect Analysis Model

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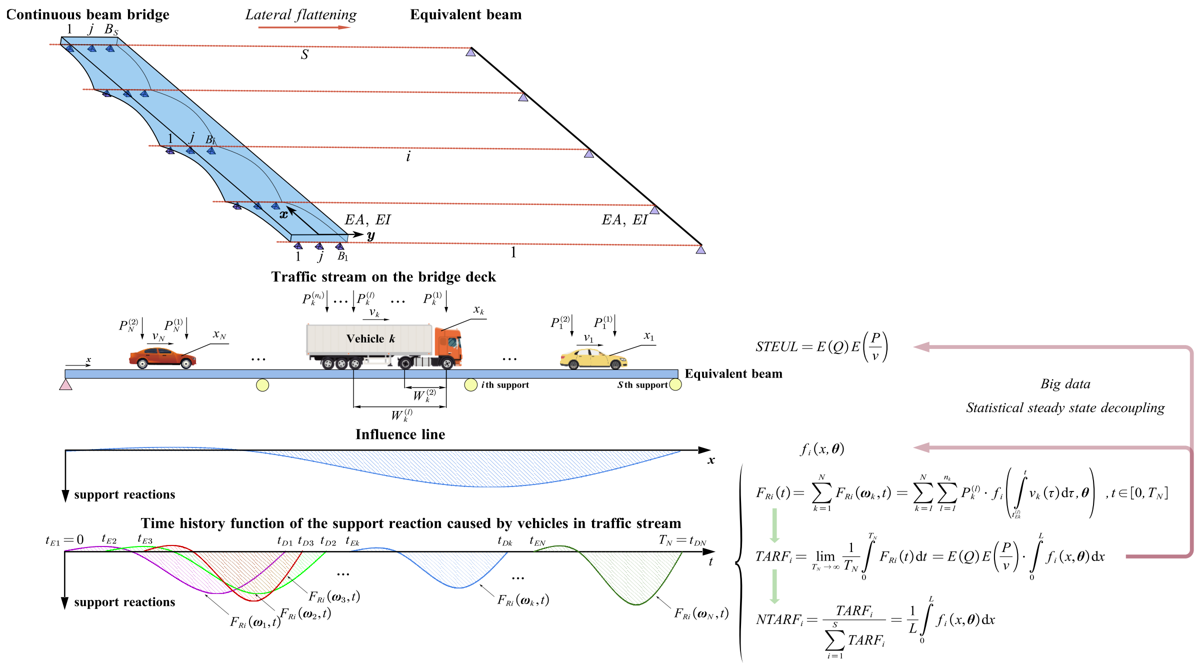
Graphical Abstract



Abstract

Monitoring information on bridge structures in service can be used to identify structural damage or to assess the service performance of the structure in general. However, it is a challenging but significant task to use the monitoring information of the structural static effects under the natural traffic conditions during the operation period to identify the structural damage or overall performance. The challenge is to define and identify indicators that characterize structural damage and service performance through monitoring information on static effects under unknown traffic loads. To this end, this paper firstly establishes an analytical model for the support reaction of continuous girder bridges under the action of arbitrary traffic stream loading, and based on this a series of new indicators that can reflect the big data mechanical behavior laws of bridges in the statistically steady state are developed, i.e., the time-average state resultant force of the support reaction and the statistically steady state traffic stream equivalent uniform line load, as well as the normalized indicator of the time-average state support reaction resultant force obtained by extension, with a view to evaluating the service performance of the bridge structure. Through the established analytical model of bridge support reaction, it is theoretically proved that this type of indicator has excellent monitorable performance under statistically steady state traffic loading, which is not only easy to monitor and perceive, but also is not affected by the specific value of immediate traffic loading, and is only related to the static properties of the structure. It can be used for service performance monitoring and damage detection of bridge structures under service conditions, which is the good monitoring indicators in the background of big data.

Keywords: Bridge structure damage detection, support reactions, monitoring big data, random traffic stream

1 Introduction

An important task in the maintenance of bridge structures in service is to understand the service performance of the bridge structure. One of the critical objectives of the structural monitoring system against this task is to utilize the monitoring system to obtain the operational parameters of the bridge structure, thus achieving the identification and monitoring of the structure's overall performance or localized damage. Among them, identifying local damage using monitoring data has been the research focus of many scholars at home and abroad in the past decade.

Currently, such damage identification methods based on monitoring data can be categorized into two main groups: frequency domain and time domain. Frequency-domain correlation methods mainly rely on modal parameters such as modal frequencies [1], vibration shapes [2], damping [3], and their derived metrics of the structure. Since they relate only to structural properties or damage, they can be used to identify structural damage or overall performance assessment. Another significant advantage of such indicators is their load-independence, which ensures that the current forces do not affect the identification results. However, the following drawbacks are the main obstacles limiting progress in the damage characterization capabilities of this class of metrics: (1) the quality of the metric measurements is interfered with by temperature, noise, etc.; (2) the indicators are insensitive to damage, especially in the face of minor damage identification tasks that need monitoring systems to play a role, and most of them perform poorly [4]. Time-domain-based methods mainly utilize the time series of static effect or dynamic response monitoring data of bridges (e.g., deflection [4][5], dynamic stress [6], strain [7], and bearing reaction force, etc.) for the identification of damage indicators, and then identify the damage. Such methods are straightforward, physically meaningful, and readily accepted by bridge engineering technicians, and thus have received as much focused attention from researchers. However, the main difficulty is how to construct an index that is independent of live loads and has an excellent ability to characterize structural damage.

Support reactions, one of the typical static responses for bridge health monitoring, can reflect the overall force performance of the bridge structure by itself. For statically indeterminate bridges under traffic loads, the magnitude of the support reactions depends not only on the load magnitude and action location but also on the overall stiffness distribution of the structure. Therefore, it is theoretically feasible to reflect the overall stiffness distribution and local damage identification of the bridge based on the monitoring information of the support reactions.

Ma et al. [8] gave damage identification indexes based on the increment of bearing reaction force in the middle support under concentrated load for two-span equal-section continuous girder bridges with single damage. Chen and Wang [9] proposed a damage location index based on the first- and second-order derivatives of the influence line change of the reaction force for two-span equal-section continuous girder bridges with single damage, and then developed a corresponding damage identification method for continuous girder bridges. Cheng et al. [10] defined a multi-information fusion damage localization index based on the second-order derivative of the influence line change of the bearing reaction force for two-span equal-section continuous girder bridges with multiple damages based on previous studies, and initially verified the feasibility of the index through model test and finite element simulation. Wang and Liu [11] proposed a damage location index (DIILSR) based on the "quadratic difference of the vertical bearing reaction line", which uses the middle bearing reaction information of two-span equal-section continuous girder bridges under heavy mobile vehicle loading, and achieved the damage identification target by comparing with its non-destructive state reference value. Li et al. [12] modeled the relationship between damage and the quadratic difference of the influence line of the middle support reaction for three-span variable-height continuous girder bridges, and proved that the damage could be located based on its slope change without the reference to the non-destructive state of the bridge.

To improve the accuracy of damage identification, the above studies require accurate measurement of the support reactions influence line, which can only be realized in the relevant special load tests. Such a load test belongs to the simplified loading method, which needs to make idealized regulations on the loading mode, weight, speed, moving route, loading times, and other conditions of the loading test vehicle. As a result, it not only affects normal traffic and cannot be used under normal service conditions, but also leads to poor accuracy of the final influence line identification due to the loading test vehicle's small weight compared to the bridge's constant load.

During the bridge service period, the working loads can be called natural traffic loads. At this point, information such as the number of vehicles, vehicle position, vehicle weight, vehicle speed, and so on is unknown, coupled with the impact loads and other disturbing loads, as well as the inevitable measurement noise, resulting in the identification of the reaction force influence lines defined in the traditional way described above becoming difficult, which also makes further damage identification difficult to achieve. The author of this paper used strain monitoring data of bridges under statistical steady-state natural traffic stream to obtain the statistically converged strain effect influence line characteristic functions by extracting, aggregating, and processing the single-vehicle effect fragments from it, and then proposed a method for constructing a class of damage indicators with large data significance [13]. However, unlike the strain monitoring data with localized effects, it is difficult to realize the extraction of single-vehicle contributing components under the action of multiple vehicles in the strut-reaction monitoring data. Therefore, it is difficult to obtain the statistical steady state support reaction influence line function corresponding to the bridge span.

In view of this, this paper defines a new class of monitoring indicators that can reflect the big data mechanical behavioral laws for bridges by time-averaging the support reaction time-course monitoring data of continuous girder bridges under traffic stream. Through the theoretical model of static analysis of bridge support reaction influence surface, the excellent characterization ability of such indicators for the service performance of bridge structures is demonstrated.

2 Influence Surface of Resultant Force of Support Reactions

Assume any S-1 span continuous girder bridge, i.e., there are S piers and abutments along the longitudinal bridge direction, and B_i bearings are arranged along the transverse bridge direction at the i-th pier, as shown in the left part of Figure 1. At this point, it is easy to construct an equivalent beam corresponding to the original width beam, as shown in the right part of Figure 1. Take the longitudinal direction as the x-axis and the transverse direction as the y-axis to establish a plane coordinate system.

The equivalent beam is a simple widthless beam that satisfies the laws of structural mechanics and has the following properties: equal length to the original beam, consistent longitudinal bridge support conditions with the original beam, the same longitudinal stiffness distribution law, and the same support reaction as the resultant force of the support reaction at the same pier and abutment of the original beam.

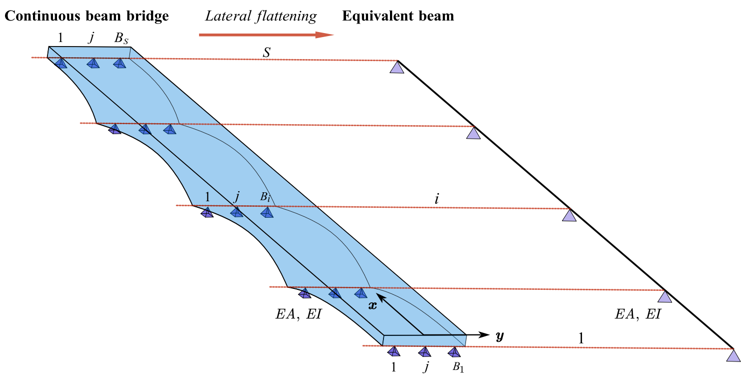


Figure 1: Diagram of a continuous beam bridge and equivalent beam

For the j-th support at the i-th pier and abutment, its reaction force is $F_{R(i,j)}$, whose influence surface $FIS_{(i,j)}$ is defined as the support reaction when the unit concentrated force is applied to any position on the bridge deck. The function of $FIS_{(i,j)}$ is denoted as $f_{(i,j)}(x,y,\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ denotes the set of parameters related to the static properties of the bridge. When x takes a fixed value x_0, the influence surface degenerates to an influence line $f_{(i,j)}(x_0,y,\boldsymbol{\theta})$ of the transverse direction, and when y takes a fixed value y_0, the influence surface degenerates to an influence line $f_{(i,j)}(x,y_0,\boldsymbol{\theta})$ of the longitudinal direction.

Define the sum of the reaction forces of all supports at the i-th pier and abutment as the support reactions resultant force F_{Ri}, which would be

$$F_{Ri} = \sum_{j=1}^{B_i} F_{R(i.j)} \tag{1}$$

According to the definition of F_{Ri}, its influence surface FIS_i can be obtained by accumulating the support reaction influence surfaces $FIS_{(i,j)}$ (j =1, …, B_i) of all supports at the i-th pier and abutment, and the function of FIS_i would be written as

$$f_i(x,y,\boldsymbol{\theta}) = \sum_{j=1}^{B_i} f_{(i,j)}(x,y,\boldsymbol{\theta}) \tag{2}$$

It is evident that the support reaction resultant force influence surface FIS_i defined above is essentially the same as the longitudinal influence line of the corresponding widthless equivalent beam. In other words, the resultant force FR_i of support reaction when the force is applied at the position (x,y) of original width beam is equal to the support reaction FR_i^* when the same magnitude of force is applied at the position x of the widthless equivalent beam. It means that the value of the influence surface FIS_i along the transverse bridge direction at any x is equal to the value of the influence line of the equivalent beam at the same x. Therefore, the function of FIS_i can be replaced by the influence line $f_i(x, \boldsymbol{\theta})$ of the equivalent beam, which can also be regarded as the FIS_i of the original beam degraded to a simplified form independent of the coordinate y.

Taking a three-span continuous girder bridge with four supports arranged in the transverse direction as an example, a solid finite element numerical analysis based on the kinematic method is carried out to obtain the reaction force influence surfaces of each support to verify the above conclusions. The kinematic method is based on the principle of virtual displacement, transforming the static problem of finding the influence line (surface) of internal force or support reactions into the geometric problem of making displacement diagram, often used in combination with the static method or finite element method, widely used in the solution of the influence line (surface).

According to the aforementioned method, the accumulation process and spatial distribution of the influence surface FIS_4 at the 4th pier and abutment of the bridge are presented in Figure 2. As shown in this figure, the FIS_4 obtained by accumulation is almost constant along the transverse bridge direction. Due to symmetry, the part of $FIS_{(4,1)}+FIS_{(4,2)}$ is omitted in Figure 2.

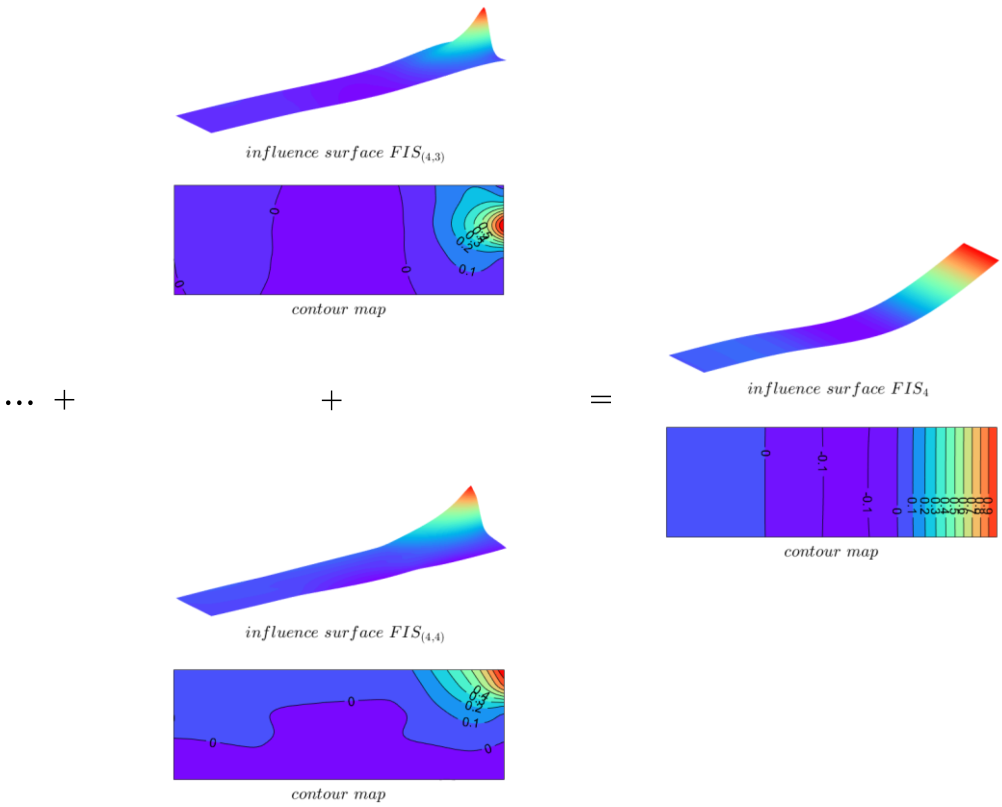


Figure 2: The accumulation process of the influence surface FIS_4

In addition, the error distribution between FIS_4 and the influence line $f_4(x, \boldsymbol{\theta})$ of the equivalent beam is given in Figure 3. The error is concentrated near the 3rd pier and abutment, and the maximum error is not more than 0.0015, which explains the reasonableness of the equivalent beam to some extent. The above deviations may originate from the computational errors in the finite elements and the fact that the equivalent beam model does not take into account the warpage deformation of the cross-section.

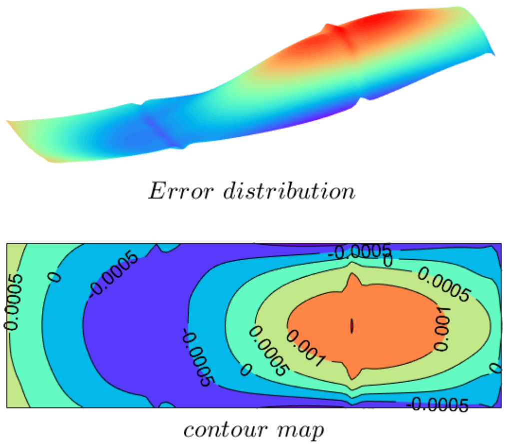


Figure 3: The error distribution between FIS_4 and the influence line of equivalent beam

Therefore, the function of influence surface FIS_i is uniformly written hereafter as $f_i(x, \boldsymbol{\theta})$. For the sake of description, the resultant force of support reactions at the i-th pier and abutment is abbreviated as the support reaction at the i-th pier and abutment in the following.

3 Service Performance Indicators for Bridge Structures

It was mentioned above that the simplified analysis can be carried out by replacing the original beam's resultant force influence surface with the equivalent beam's support reaction influence line. It requires equating the real loading mode of the original beam to the equivalent beam loading mode via the property that the influence surface FISi has equal values along the transverse bridge direction in Section 2. Specifically, the wheel loads on the same axle of a vehicle are composited into a concentrated load, which is the axle weight at the center of the axle, and the axle weights of vehicles in different lanes are translated along the transverse bridge direction to the bridge axle line.

According to the principle of influence surface loading, at the i-th pier and abutment of a continuous grider bridge, the position function of the support reaction caused by any n-axis vehicle crossing the bridge would be

$$F_{Ri}(x)=\sum_{l=1}^{n}P^{(l)}\cdot f_i\left(x-W^{(l)},\boldsymbol{\theta}\right)\quad ,x\in\left[0,L+W^{(n)}\right] \tag{3}$$

where l denotes the axle serial number, $W^{(l)}$ denotes the wheelbase between the l-th axle and the first axle of the vehicle, $P^{(l)}$ denotes the axle weight of the l-th axle, L is the bridge length, x denotes the position of the first axle of the vehicle, and $f_i(\cdot)$ is the function of the influence surface FIS_i.

Consider $P^{(l)}$, $W^{(l)}$, v, and t_E as continuous random variables; their set is denoted as $\boldsymbol{\omega}$, which is called the parameters of the vehicle crossing the bridge. Substituting the relationship between position x and time t into Eq. (3), the time history function of the support reaction would be

$$F_{Ri}(\boldsymbol{\omega},t)=\sum_{l=1}^{n}P^{(l)}\cdot f_i\left(\int_{t_E}^{t}v(\tau)d\tau-W^{(l)},\boldsymbol{\theta}\right)\quad ,t\in\left[t_E,t_D\right] \tag{4}$$

where v denotes the vehicle speed, t_E denotes the time when the first axle of the vehicle enters the bridge, t_D denotes the time when the last axle departs from the bridge, and $\boldsymbol{\omega}$ denotes parameter set of $P^{(l)}$, $W^{(l)}$, v, and t_E. t_D can be uniquely determined by the equation $\int_{t_E}^{t_D}v(\tau)d\tau=L+W^{(n)}$.

Consider $P^{(l)}$, $W^{(l)}$, v, and t_E as continuous random variables. According to Eq. (4), the vehicle-caused support reaction during the vehicle crossing bridge constitutes a random process $F_{Ri}(\boldsymbol{\omega},t)$ that varies with time. Assuming that we only consider the case of one vehicle passing over the bridge, the time history of the measured support reaction is one observation of the stochastic process $F_{Ri}(\boldsymbol{\omega},t)$. In other words, the time history of the support reaction caused by the k-th vehicle in the traffic stream during its bridge crossing can be regarded as the k-th sample function $F_{Ri}(\boldsymbol{\omega}_k,t)$ of the stochastic process

$$F_{Ri}\left(\boldsymbol{\omega}_k, t\right) = \sum_{l=1}^{n_k} P_k^{(l)} \cdot f_i\left(\int_{t_{Ek}}^{t} v_k(\tau)d\tau - W_k^{(l)}, \boldsymbol{\theta}\right) \quad , t \in \left[t_{Ek}, t_{Dk}\right] \tag{5}$$

where $\omega_k = \left\{P_k^{(1)}, \cdots, P_k^{(n_k)}, W_k^{(1)}, \cdots, W_k^{(n_k)}, v_k, t_{Ek}\right\}$ denotes the k-th sample point of the stochastic process, i.e., the set of parameters associated with the k-th vehicle in the traffic stream.

Consider that at any moment t, there may be more than one vehicle on the bridge at the same time. Therefore, Eq. (5) actually reflects only the contributing part of the k-th vehicle instead of the true time history of the support reaction at the i-th pier and abutment of the bridge.

Assuming that the sample size is N, i.e., there are N vehicles in the traffic stream, the parameters related to the k-th vehicle crossing the bridge are shown in Figure 4. It should be noted that all vehicles in the traffic stream are plotted within the bridge area for the sake of a compact layout in the figure. In fact, except for the k-th vehicle, the rest of the vehicles can be distributed outside the bridge area.

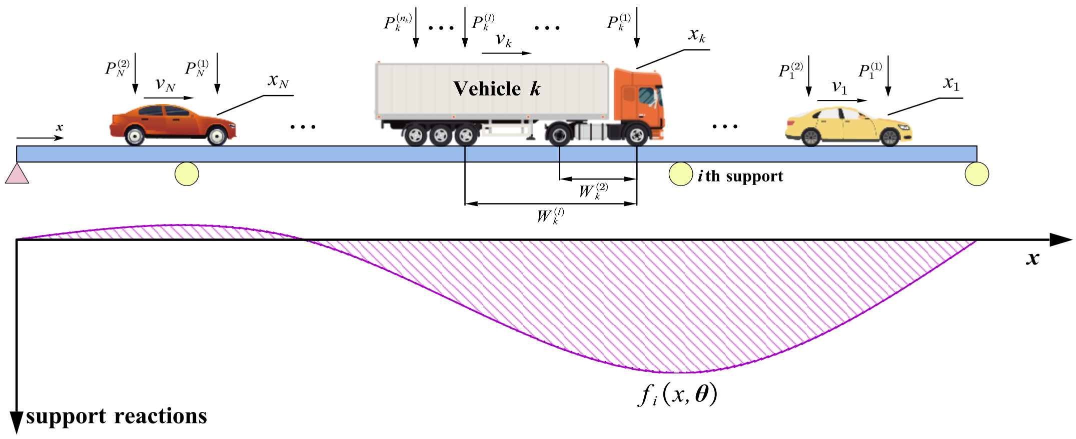


Figure 4: Diagram of the parameters for the k-th vehicle crossing bridge of traffic stream

For convenience, take the time when the first vehicle in the traffic stream enters the bridge as $t_{E1} = 0$, and denote the time t_{DN} when the last vehicle leaves as T_N. The N sample functions of the stochastic process (i.e., the N number of support reaction time-history functions caused by vehicles during the traffic stream across the bridge) are plotted within the same time axis, as shown in Figure 5.

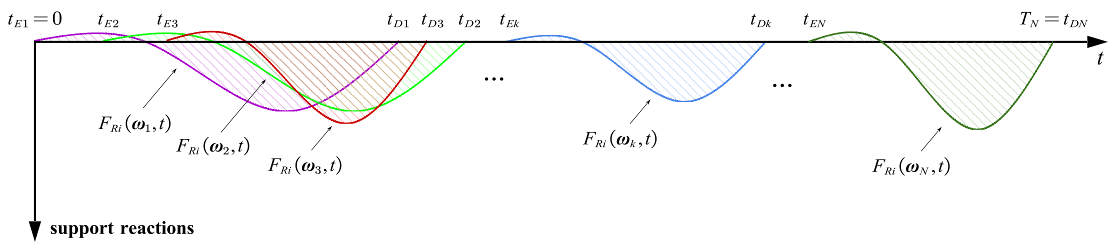


Figure 5: Time history function of the support reaction caused by each vehicle during the traffic stream across the bridge

As shown in Figure 5, during traffic stream crossing the bridge, there are periods where partial overlap of multiple sample functions occurs, periods where a single sample function exists alone, and periods where no sample function exists. They correspond to the time periods when multiple vehicles cross the bridge, a single vehicle crosses the bridge, and no vehicle passes, respectively.

To facilitate the analysis of the support reaction caused by multiple vehicles, we extend the definition domain of the function in Eq. (5) to the time range [0, T_N] when the traffic stream crosses the bridge. It should be noted that when time t lies outside the range [t_{Ek}, t_{Dk}], the function value of $F_{Ri}(\boldsymbol{\omega}_k, t)$ is constant 0, i.e., the supports do not generate the support reaction.

According to the definition, the time history function of the support reaction at the i-th pier and abutment caused by the traffic stream crossing bridge can be obtained by accumulating the N sample functions in Figure 5 in time order. From Eq. (5), this function would be written as

$$F_{Ri}\left(t\right)=\sum_{k=1}^{N}F_{Ri}\left(\boldsymbol{\omega}_k,t\right)=\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot f_i\left(\int_{t_{Ek}}^{t}v_k(\tau)d\tau-W_k^{(l)},\boldsymbol{\theta}\right)\quad ,t\in\left[0,T_N\right] \tag{6}$$

where all parameters have the same meaning as before.

The time t_{Ek} when the first axle of the k-th vehicle in the traffic stream enters the bridge satisfies the following relationship with the time $t_{Ek}^{(l)}$ when the l-th axle enters the bridge:

$$\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau=\int_{t_{Ek}}^{t}v_k(\tau)d\tau-W_k^{(l)} \tag{7}$$

Substituting Eq. (7) into Eq. (6), the time history function of the support reaction caused during the traffic stream crossing bridge would be

$$F_{Ri}\left(t\right)=\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot f_i\left(\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau,\boldsymbol{\theta}\right)\quad ,t\in\left[0,T_N\right] \tag{8}$$

Taking the time average of Eq. (8), defined as the time-averaged state support reaction resultant force at the i-th pier and abutment caused by the traffic stream crossing bridge (abbreviated as $TARF_i$):

$$\begin{aligned} TARF_i &= \lim_{T_N\to\infty}\frac{1}{T_N}\int_0^{T_N}F_{Ri}\left(t\right)\,dt \\ &= \lim_{T_N\to\infty}\frac{1}{T_N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot\int_{t_{Ek}^{(l)}}^{t_{Dk}^{(l)}}f_i\left(\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau,\boldsymbol{\theta}\right)dt \end{aligned} \tag{9}$$

where the derivation process is detailed in the Appendix.

Take the limit for T_N in Eq. (9), which corresponds to big data monitoring. At this point, we can assume that the speed of a vehicle during crossing the bridge remains unchanged, and is not lower than the idle speed, i.e., 5~10km/h. Then Eq. (9) would be simplified as

$$TARF_i=\lim_{T_N\to\infty}\bar{Q}\cdot\overline{\left(\frac{P}{v}\right)}\cdot\int_0^L f_i\left(x,\boldsymbol{\theta}\right)dx \tag{10}$$

where $\bar{Q}$ denotes the average traffic flow in the time interval [0, T_N], and $\overline{P/v}$ denotes the mean value of the vehicle weight to vehicle speed ratio, the derivation process is detailed in the Appendix.

The weight-to-speed ratios of N vehicles in the traffic stream can be formed as a sequence of random variables $\{P^{(1)}/v^{(1)},\ldots, P^{(k)}/v^{(k)},\ldots,P^{(N)}/v^{(N)}\}$, and the mean $\overline{P/v}$ is the arithmetic average of this sequence of random variables. According to the central limit theorem, the partial sum distribution of a random variable sequence asymptotically follows a normal distribution whose mean and variance are N times the mean $E(P/v)$ and variance $\sigma^2(P/v)$ of the random variable P/v. Therefore, $\overline{P/v}$ asymptotically follows a normal distribution with mean $E(P/v)$ and variance $\sigma^2(P/v)/N$, i.e., $\overline{P/v}$ converges in probability to $E(P/v)$.

Existing studies have shown that the number of vehicles passing through a certain road cross-section is a random variable that obeys a particular distribution. Therefore, the number of vehicles passing the cross-section per unit time, i.e., the traffic flow Q, is also a random variable obeying the same distribution. In general, the Poisson distribution is shown when the traffic density is not large, the binomial distribution is shown when the vehicle density is large, and the negative binomial distribution is shown in other cases.

The average flow $\overline{Q}$ can be regarded as the sample mean of the random variable Q, which is detailed in the Appendix. Thus $\overline{Q}$ also asymptotically follows a normal distribution with mean $E(Q)$ and variance $\sigma^2(Q)/N$, i.e., $\overline{Q}$ converges in probability to the mean $E(Q)$.

In summary, Eq. (10)would be written as

$$TARF_i = E(Q)E\left(\frac{P}{v}\right)\cdot\int_0^L f_i(x,\boldsymbol{\theta})dx \tag{11}$$

According to Eq. (11), $TARF_i$ is numerically equal to the support reaction resultant force (defined in Eq. (1) in Section 2 of this paper) at the i-th pier and abutment of a continuous girder bridge under a certain uniform line load. The uniform line load is related only to the random traffic stream and would be defined as the statistically steady state traffic stream equivalent uniform line load (abbreviated as $STEUL$):

$$STEUL = E(Q)E\left(\frac{P}{v}\right) \tag{12}$$

where $E(Q)$ denotes the mean value of traffic flow, and $E(P/v)$ denotes the mean value of vehicle weight to vehicle speed ratio.

Considering the time-varying characteristics of the statistical properties of the actual traffic stream, this paper uses the statistical steady state traffic stream to limit it. Such traffic stream should ensure that the sample mean of the relevant random variable can achieve sufficient statistical stability after a period of T_N and that the distribution of the random variable has not changed significantly. In other words, the time required for a significant change in the random variable distribution of the statistical steady state traffic stream is greater than the time

T_N needed for statistical stability. Thus, the mean value of the random variable in Eq. (12) actually represents the statistically stable value of the sample mean in that time period.

Substituting Eq. (12) into Eq. (11), the relationship among $TARF_i$, $STEUL$, and the bridge static characteristic parameter $\boldsymbol{\theta}$ can be obtained as follows

$$TARF_i = STEUL \cdot \int_0^L f_i(x, \boldsymbol{\theta}) dx \tag{13}$$

For an *S-1* span continuous girder bridge, the $TARF_i$ at each pier and abutment can be combined into a vector form, denoted as $\boldsymbol{TARF}$=[$TARF_1$, …, $TARF_i$, …, $TARF_S$]. From Eq. (13), $\boldsymbol{TARF}$ is a function of the bridge static characteristic parameter $\boldsymbol{\theta}$ and has the potential to identify damage conditions in continuous girder bridges, but it is also related to the $STEUL$, which characterizes the random traffic stream conditions. Therefore, when using $\boldsymbol{TARF}$ for structural performance evaluation or damage identification, $STEUL$ must also be known.

To further identify and compare bridge damage under different random traffic stream conditions, the normalized indicator vector for the time-averaged state support reaction resultant force (abbreviated as $\boldsymbol{NTARF}$) on continuous girder bridges is defined as

$$\begin{aligned} \boldsymbol{NTARF} &= [NTARF_1, \cdots, NTARF_i, \cdots, NTARF_S] \\ &= \frac{1}{\sum_{i=1}^{S} TARF_i} \cdot \left[TARF_1, \cdots, TARF_i, \cdots, TARF_S\right] \\ &= \frac{\left[\int_0^L f_1(x,\boldsymbol{\theta})dx, \cdots, \int_0^L f_i(x,\boldsymbol{\theta})dx, \cdots, \int_0^L f_S(x,\boldsymbol{\theta})dx\right]}{\int_0^L \sum_{i=1}^{S} f_i(x,\boldsymbol{\theta})\, dx} \end{aligned} \tag{14}$$

From Eq. (14), it can be seen that $\boldsymbol{NTARF}$ is a dimensionless quantity with any component less than one and the sum of parts equal to 1. As $STEUL$ is eliminated, the indicator is comparable under different random traffic stream conditions and is only related to the static characteristic parameter $\boldsymbol{\theta}$ of the continuous girder bridge. In this paper, only the definition of the indicator is given, and its specific relationship with the static characteristic parameter $\boldsymbol{\theta}$ of the continuous girder bridge will be shown in the subsequent papers.

$\sum_{i=1}^{S} f_i(x, \boldsymbol{\theta})$ in the denominator part of Eq. (14) is a function of the influence surface $\sum_{i=1}^{S} FIS_i$ obtained by accumulating the influence surfaces FIS_i of the support reaction resultant forces at each pier and abutment. Therefore, the physical meaning of $\sum_{i=1}^{S} f_i(x, \boldsymbol{\theta})$ is the resultant force of all support reactions of a continuous girder bridge caused by a unit concentrated force, and the value of $\sum_{i=1}^{S} f_i(x, \boldsymbol{\theta})$ is constant at 1. To show this fact, we also choose the three-span continuous girder bridge in Figure 2 of Section 2 and plot the superposition process of the influence surface $\sum_{i=1}^{S} f_i(x, \boldsymbol{\theta})$ in Figure 6.

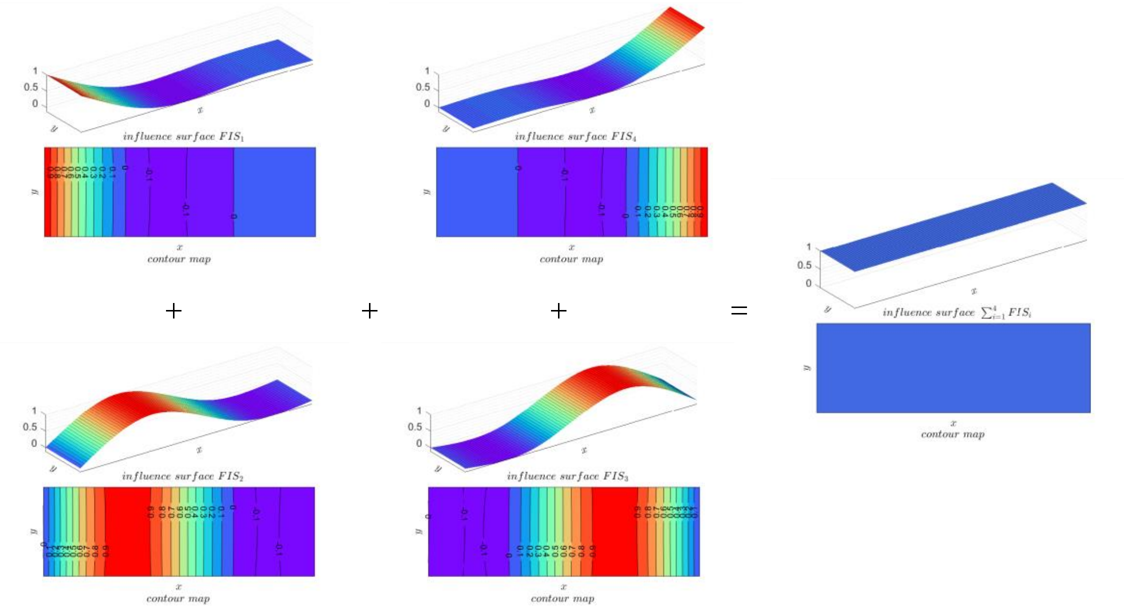


Figure 6: Superposition process and result of the influence surface $\sum_{i=1}^{S} FIS_i$

Thus, the arbitrary component of ***NTARF*** would be

$$NTARF_i = \frac{1}{L}\int_0^L f_i\left(x, \boldsymbol{\theta}\right) dx \tag{15}$$

where L denotes the length of the bridge.

From Eq. (15), ***NTARF*** is numerically equal to the support reaction resultant force at each pier and abutment of a continuous girder bridge caused by the uniform wiring load $q = 1/L$. It can be seen that when the big-data statistical stability condition is satisfied, ***NTARF*** is theoretically independent of loading and is only related to the influence surface function reflecting the static properties of the structure. Therefore, ***NTARF*** becomes a better indicator for characterizing the service performance of structures than ***TARF***.

From Eq. (13) and Eq. (15), ***STEUL*** can be directly derived from ***NTARF*** and ***TARF***, i.e.:

$$STEUL = \frac{\left\| \boldsymbol{TARF} \,./\, \boldsymbol{NTARF} \right\|_2}{L} \tag{16}$$

where $\boldsymbol{a} \,./\, \boldsymbol{b} = (a_1/b_1, \ldots, a_i/b_i, \ldots, a_n/b_n)^{\mathrm{T}}$, and $\|\cdot\|_2$ denotes the 2-norm.

Eq. (9) only gives the theoretical defining equation of TARF, which needs to measure the time-domain signal of each support reaction for practical application and calculate by the following equation:

$$\boldsymbol{TARF} = \left[\frac{1}{T_N} \int_0^{T_N} \sum_{j=1}^{B_1} F_{R(1,j)}(t)\, dt, \cdots, \frac{1}{T_N} \int_0^{T_N} \sum_{j=1}^{B_i} F_{R(i,j)}(t)\, dt, \cdots, \frac{1}{T_N} \int_0^{T_N} \sum_{j=1}^{B_S} F_{R(S,j)}(t)\, dt \right] \tag{17}$$

where $F_{R(i,j)}$ is the measured value of the reaction force sensor at support (i, j) and T_N is the duration of the measured reaction force signal.

To reduce the length of the main text, the detailed discussion about the application scope of the theory in this paper has been moved to the Appendix.

4 Parameter Analysis

The following simplifications are made to show visually the time-averaged state physical quantities and indicator in Section 3.

1) Traffic stream is free flow, which refers to the condition when traffic density is low and drivers are largely unaffected or minimally affected by other roadway users. The vehicle in this state maintains the desired speed, which is called the free flow speed, and the traffic stream basically satisfies the assumption of unchanged speed at this time. This assumption will apply to section 4.1.
2) Continuous girder bridge is taken as the simplest form - continuous girder bridge with two equal spans and constant section. This assumption will be applied to sections 4.2 and 4.3.

4.1 Effect Analysis of Statistical Steady State Traffic stream Statistical Parameters on STEUL

According to traffic stream theory, for any steady-state traffic stream, the three basic parameters describing the characteristics of the traffic stream have the following relationships:

$$Q = K \cdot \overline{V}_s \tag{18}$$

where Q denotes the traffic stream, K denotes the traffic density, and $\overline{V}_s$ denotes the space mean speed.

The space mean speed is the harmonic mean of the speeds of all vehicles passing through the roadway, i.e., the reciprocal of the arithmetic mean of the reciprocal of the speeds of all vehicles:

$$\overline{V}_s = \frac{N}{\sum_{k=1}^{N} \frac{1}{v_k}} \tag{19}$$

where v_k is the speed of the k-th vehicle and N is the total number of vehicles.

Substituting Eq. (18) and Eq. (19) into Eq. (12), *STEUL* would be written as

$$STEUL = \lim_{T_N \to \infty} K \cdot \frac{N}{\sum_{k=1}^{N} \frac{1}{v_k}} \cdot \overline{\left(\frac{P}{v}\right)} = \mathcal{K} \frac{E\left(\frac{P}{v}\right)}{E\left(\frac{1}{v}\right)} \tag{20}$$

where $\mathcal{K}$ denotes the statistically stable value of traffic density, $E(1/v)$ denotes the mean value of the reciprocal of the vehicle speed in traffic stream, and the physical meaning of the reciprocal of speed is the time spent by a vehicle to cross a unit length of the bridge deck.

According to probability theory and mathematical statistics, the mean values of two random variables satisfy the following relationship:

$$E(XY) = E(X)E(Y) + \sigma(X)\sigma(Y) \cdot \rho(X,Y) \tag{21}$$

From Eq. (21), $E(P/v)$ would be written as

$$E\left(\frac{P}{v}\right) = E(P)E\left(\frac{1}{v}\right) + \sigma(P)\sigma\left(\frac{1}{v}\right) \cdot \rho\left(P, \frac{1}{v}\right) \tag{22}$$

where $\rho(P,1/v)$ denotes the correlation coefficient between vehicle weight and reciprocal of speed, and $\sigma(P)$ and $\sigma(1/v)$ denote the standard deviation of vehicle weight and reciprocal of speed in traffic stream, respectively.

Substituting Eq. (22) into Eq. (20), the *STEUL* of simplified model would be

$$STEUL = \mathcal{K} \cdot \left[E(P) + \rho\left(P, \frac{1}{v}\right) \cdot \sigma(P) C_v\left(\frac{1}{v}\right) \right] \tag{23}$$

where $C_v(1/v)$ is the coefficient of variation of the reciprocal of the speed in traffic stream, i.e., the ratio of the standard deviation $\sigma(1/v)$ to the mean $E(1/v)$.

Eq. (23) shows the functional relationship between *STEUL* and the statistical parameters of the random variables of the traffic stream, with the introduction of an uncommon parameter, the reciprocal of the vehicle speed. When fitting parameters to real traffic, vehicles are usually classified into multiple types to improve the fitting accuracy. Therefore, the functional relationship between STEUL and common statistical parameters of any vehicle type would be given further below.

1) Statistically stable value $\mathcal{K}$ of traffic density

From Eq. (23), *STEUL* is in direct proportion to the statistical stability value $\mathcal{K}$ of traffic density.

2) Correlation coefficient $\rho(P,1/v)$ in the traffic stream

The study results of measured free-flow data on a highway [14] show that the correlation coefficient $\rho(P,1/v)$ between the vehicle weight and the reciprocal of the vehicle speed in free flow is approximately equal to 1, which shows a strong linear correlation. It is related to the fact that vehicle weight is the most important factor affecting vehicle speed in free flow. For non-free flow, the factors affecting vehicle speed are more complex, leading to a decrease in the correlation coefficient $\rho(P,1/v)$, but always greater than 0.

Therefore, *STEUL* is positive linear correlated with the correlation coefficient $\rho(P,1/v)$.

3) Mean value $E(P_r)$ and standard deviation $\sigma(P_r)$ of the vehicle weight for any vehicle type

Existing research has shown that vehicle types generally obey a uniform distribution, while the vehicle weight for any vehicle type follows a certain random distribution. Denote the proportion of each vehicle type in the traffic stream as $\boldsymbol{\alpha}$=[α_1, …, α_r, …, α_{VM}], as well as the probability density function of the vehicle weight of the rth vehicle type as $g_r(x)$. Then, the vehicle weight probability density function of the traffic stream is $g(x) = \sum_{r=1}^{VM} \alpha_r \cdot g_r(x)$.

According to the definition of the mean value, $E(P)$ is related to $E(P_r)\,|_{r=1,\,\dots,VM}$ as follows

$$E(P) = \int x \cdot \sum_{r=1}^{VM} \alpha_r g_r(x)\, dx = \sum_{r=1}^{VM} \alpha_r E(P_r) \tag{24}$$

Similarly, the following relationship exists between the vehicle weight variance $\sigma^2(P)$ of the traffic stream and the mean $E(P_r)$ and variance $\sigma^2(P_r)$ of each vehicle type:

$$\sigma^2(P) = \sum_{r=1}^{VM} \left(\alpha_r - \alpha_r^2\right) E^2(P_r) - 2\sum_{r_1=1}^{VM} \sum_{r_2=r_1+1}^{VM} \alpha_{r_1}\alpha_{r_2} E\left(P_{r_1}\right) E\left(P_{r_2}\right) + \sum_{r=1}^{VM} \alpha_r \sigma^2(P_r) \tag{25}$$

where the derivation process is detailed in the Appendix.

Substituting Eq. (24)and Eq. (25)into Eq. (23):

$$\begin{aligned} STEUL = \mathcal{K} \cdot \Bigg\{ & \sum_{r=1}^{total} \alpha_r E(P_r) + \rho\left(P, \frac{1}{v}\right) \cdot C_v\left(\frac{1}{v}\right) \\ & \cdot \left[\sum_{r=1}^{VM} \left(\alpha_r - \alpha_r^2\right) E^2(P_r) - 2\sum_{r_1=1}^{VM} \sum_{r_2=r_1+1}^{VM} \alpha_{r_1}\alpha_{r_2} E\left(P_{r_1}\right) E\left(P_{r_2}\right) + \sum_{r=1}^{VM} \alpha_r \sigma^2(P_r)\right]^{\frac{1}{2}} \Bigg\} \end{aligned} \tag{26}$$

The relationship among $STEUL$, $E(P_r)$, and $\sigma(P_r)$ has been shown in Eq. (26), and the proportion of vehicle type $\boldsymbol{\alpha}$ is the weight coefficient of $E(P_r)$ and $\sigma(P_r)$.

4) Mean value $E(v_r)$ and standard deviation $\sigma(v_r)$ of the vehicle speed for any vehicle type

According to the research results of measured free-flow data on a highway [14], the free-flow speed of any vehicle type basically obeys a normal distribution.

As the reciprocal of the speed $1/v$ tends to infinity at $v = 0$, the integral $\int_{-\infty}^{\infty} xf(x)\, \mathrm{d}x$ does not satisfy absolute convergence. Therefore, it is not possible to calculate the mean and variance of $1/v$ from the probability density function $f(x)$. In addition, the free flow speed taking values near 0 is also contrary to the definition of free flow.

By definition, $C_v(1/v)$ is related to the proportion of vehicle types in the traffic stream, the mean and standard deviation of the speed for each type of vehicle. Therefore, the measured traffic flow data of a bridge in China [15] as an example, the relationship between $C_v(1/v)$ and the mean $E(v_r)$ and standard deviation $\sigma(v_r)$ of the vehicle speed for any vehicle type is plotted in Figure 7 by the Monte Carlo method.

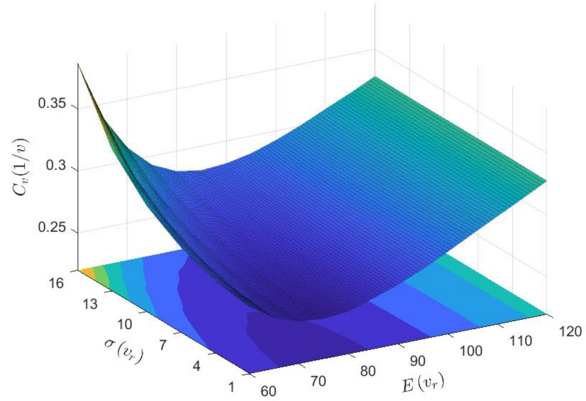


Figure 7: The relationship among $C_v\,(1/v)$, $E(v_r)$, and $\sigma(v_r)$ (unit: *km/h*)

According to Figure 7, $C_v(1/v)$ is positively correlated with $\sigma(v_r)$ and is related to $E(v_r)$ as a function of $y=a\cdot e^{b\cdot x}+c\cdot e^{d\cdot x}$, with goodness-of-fit R^2=0.9997. From Eq. (26), *STEUL* is positively linearly correlated with $C_v(1/v)$. Therefore, we can directly derive the relationship among *STEUL*, $E(v_r)$, and $\sigma(v_r)$. At this point, the proportion of vehicle type $\boldsymbol{\alpha}$ is the weight parameter of $E(v_r)$ and $\sigma(v_r)$.

Note that the coefficients a, b, c, and d are not actually constants, which are related to the proportion of vehicle types α, the mean speed $E(v_h)|_{h\neq r}$ for each vehicle type except type r, and the standard deviation $E(v_h)|_{h\neq r}$ for each vehicle type except type r. However, the functional relationship among them is so complex that it cannot be given explicitly. So, the functional relationship among $C_v(1/v)$, $\sigma(v_r)\,|_{r=1,\,\ldots,VM}$, and $E(v_r)\,|_{r=1,\,\ldots,VM}$ is denoted as

$$C_v\left(\frac{1}{v}\right)=\boldsymbol{G}_1\left[\boldsymbol{\alpha},\boldsymbol{E}\left(v_h\right)\big|_{h\neq r},\boldsymbol{\sigma}\left(v_h\right)\big|_{h\neq r}\right]\cdot e^{\boldsymbol{G}_2\left[\boldsymbol{\alpha},\boldsymbol{E}\left(v_h\right)\big|_{h\neq r},\boldsymbol{\sigma}\left(v_h\right)\big|_{h\neq r}\right]\cdot E\left(v_r\right)} \tag{27}$$

where the vector functions $\boldsymbol{G}_1(\cdot)=[g_1(\cdot), g_3(\cdot)]$, $\boldsymbol{G}_2(\cdot)=[g_2(\cdot), g_4(\cdot)]^{\mathrm{T}}$, $g_1(\cdot)$ ~ $g_4(\cdot)$ denote the functions, $\boldsymbol{E}(v_h)|_{h\neq r}=\{\ \ldots, E(v_{r-1}), E(v_{r+1}), \ldots\}$, and $\boldsymbol{\sigma}(v_h)|_{h\neq r}=\{\ \ldots, \sigma(v_{r-1}), \sigma(v_{r+1}), \ldots\}$.

Substituting Eq. (27) into Eq. (26), we obtain the final expression for *STEUL*:

$$\begin{aligned} STEUL = \mathcal{K}\cdot\Bigg\{ & \sum_{r=1}^{total}\alpha_r E(P_r)+\rho\left(P,\frac{1}{v}\right)\cdot \boldsymbol{G}_1\left[\boldsymbol{\alpha},\boldsymbol{E}\left(v_h\right)\big|_{h\neq r},\boldsymbol{\sigma}\left(v_h\right)\big|_{h\neq r}\right]\cdot e^{\boldsymbol{G}_2\left[\boldsymbol{\alpha},\boldsymbol{E}\left(v_h\right)|_{h\neq r},\boldsymbol{\sigma}\left(v_h\right)|_{h\neq r}\right]\cdot E\left(v_r\right)} \\ & \cdot\left[\sum_{r=1}^{VM}\left(\alpha_r-\alpha_r^2\right)E^2(P_r)-2\sum_{r_1=1}^{VM}\sum_{r_2=r_1+1}^{VM}\alpha_{r_1}\alpha_{r_2}E\left(P_{r_1}\right)E\left(P_{r_2}\right)+\sum_{r=1}^{VM}\alpha_r\sigma^2\left(P_r\right)\right]^{\frac{1}{2}}\Bigg\} \end{aligned} \tag{28}$$

5) Proportion of vehicle type $\boldsymbol{\alpha}$

As shown in points 3) and 4), the vehicle type proportion $\boldsymbol{\alpha}=[\alpha_1, \ldots, \alpha_r, \ldots, \alpha_{VM}]$ is the weight parameter for $E(P_r)$, $\sigma(P_r)$, $E(v_r)$ and $\sigma(v_r)$, with a constant sum of components of 1.

*4.2 Time-averaged State Support Reactions Resultant Force **TARF***

According to the force method, for a continuous girder bridge with two equal spans and constant section, the function of the support reaction resultant force influence surface FIS_i at the i-th pier and abutment would be written as

$$f_i\left(x,\boldsymbol{\theta}\right)=\frac{\int_0^L \frac{M_{P(x)}\left(s\right)\cdot M_i\left(s\right)}{EI\left(s\right)}\mathrm{d}s}{\int_0^L \frac{M_i^2\left(s\right)}{EI\left(s\right)}\mathrm{d}s} \tag{29}$$

where i denotes the serial number of the pier and abutment in the longitudinal direction, L denotes the bridge length, $EI(\cdot)$ denotes the distribution of flexural rigidity of the section along the longitudinal direction, $P(x)$ denotes the unit force acting at the coordinate x, $M_i(\cdot)$denotes the bending moment distribution of the primary structure under the action of $F_{Ri} = 1$, and $M_{P(x)}(\cdot)$denotes the bending moment distribution of the primary structure under the action of $P(x)$.

Substituting Eq. (29) into Eq. (13), the $TARF_i$ would be

$$TARF_i = STEUL\cdot\int_0^L\left[\frac{\int_0^L \frac{M_{P(x)}\left(s\right)\cdot M_i\left(s\right)}{EI\left(s\right)}\mathrm{d}s}{\int_0^L \frac{M_i^2\left(s\right)}{EI\left(s\right)}\mathrm{d}s}\right]\mathrm{d}x \tag{30}$$

From Eq. (30), ***TARF*** is related to L and $EI(\cdot)$ in addition to the relevant parameters of

STEUL in Section 4.1. By definition, the functional relationship between ***TARF*** and ***NTARF*** with respect to L and $EI(\cdot)$ is the same, and the relevant discussion would be given together in Section 4.3.

*4.3 Normalized Indicator **NTARF** for Time-averaged State Support Reactions Resultant Force*

Substituting Eq. (29) into Eq. (15), the indicator ***NTARF*** of the simplified model would be

$$NTARF_i = \frac{1}{L} \cdot \int_0^L \left[\frac{\int_0^L \frac{M_{P(x)}(s) \cdot M_i(s)}{EI(s)} \mathrm{d}s}{\int_0^L \frac{M_i^2(s)}{EI(s)} \mathrm{d}s} \right] \mathrm{d}x \tag{31}$$

From the above equation, ***NTARF*** is only related to the section flexural stiffness $EI(\cdot)$ and the bridge length L.

Since $EI(\cdot)$ appears in both the numerator and denominator of the integrated term fraction of in Eq. (30) and Eq. (31), the indicator remains unchanged when the section flexural stiffness of the whole bridge changes in equal proportion. Only when the bridge shows a non-proportional stiffness change along the longitudinal direction, or when the flexural stiffness of the cross-section changes in part of the bridge, a nonlinear change in the indicator would be caused. Such is the principle behind using ***TARF*** and ***NTARF*** indicators for bridge damage identification. Damage can be simulated by forward analysis with pre-set variations of physical parameters, which in turn investigates the ability of both to characterize the set damage.

For a continuous girder bridge with two equal spans and constant section, when one single damage occurs at the midpoint location of its left span, the damage degree β and the relative damage length γ are defined as

$$\begin{cases} \beta = \dfrac{EI_1}{EI} \times 100\% \\ \gamma = \dfrac{L_1}{L} \times 100\% \end{cases} \tag{32}$$

where EI and EI_1 denote the flexural stiffness of the section before and after the damage to the damaged part, L_1 denotes the longitudinal bridge length of the damaged part, and L denotes the bridge length.

The relationship among ***NTARF***, damage degree β, and relative damage length γ under the condition of L=100m is given by numerical analysis, as shown in Figure 8. As shown in this figure, with the increase of β and γ, $NTARF_1$ and $NTARF_3$ gradually decrease, and $NTARF_2$ gradually increases, and the change of ***NTARF*** concerning relative damage length γ is more significant.

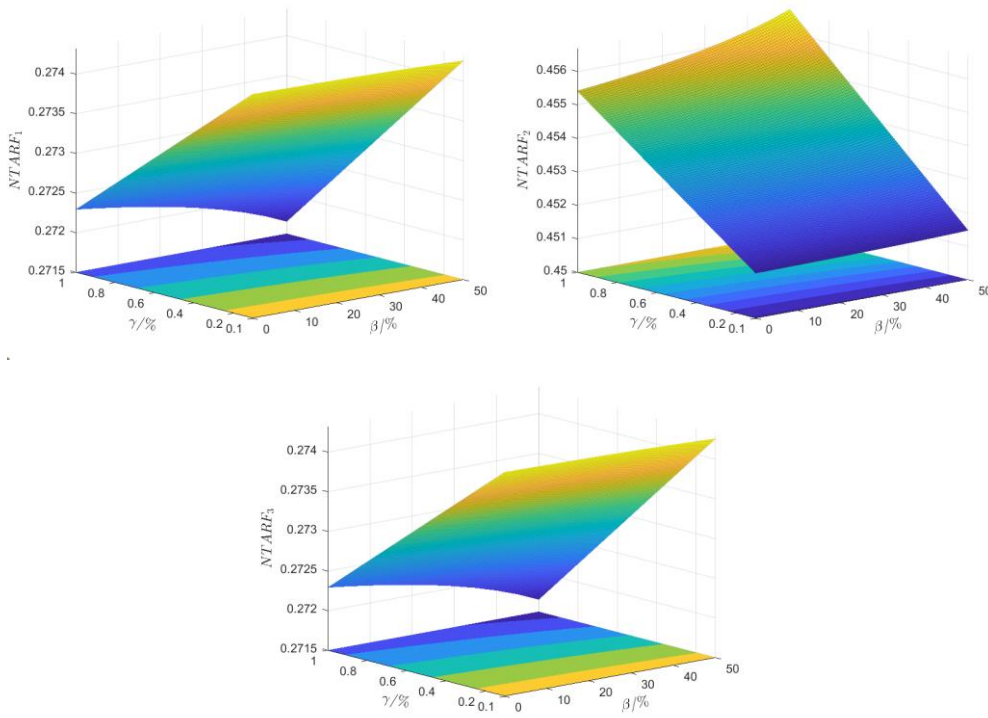


Figure 8: Relationship among components of ***NTARF***, damage degree β, and relative damage length γ

The bridge length L in Eq. (31) appears in multiple places- the upper limit of integration, the integrated term, and the divisor. Thus, the effect of L on the indicator is also nonlinear. However, for specific bridges, the bridge length is a constant value, so its variation is not considered in this section.

5 Conclusion

In this paper, we defined indicators that can characterize the overall bridge performance using the effect data for the resultant force of the support reactions in the same cross-section under traffic loading. Using the influence surface/line principle, the analytical expressions for the time-averaged state indicator ***TARF*** of the support reaction resultant force under traffic loading and its load-independent normalized indicator ***NTARF*** were given. Studies have shown that:

1) In the big data loading mode, i.e., when the traffic loading time is long enough and the number of vehicles driving over the bridge is large enough, the time-averaged state indicator ***TARF*** of the support reactions resultant force will statistically converge to a fixed value. The value consists of the product of two parts, one of which is the equivalent uniform line load *STEUL* of the statistical steady state traffic flow over the extent of the bridge deck, and the other is the integral along the bridge length of the structure's influence line after lateral reduction and merging. The further given time-averaged state normalized indicator ***NTARF*** for the support reaction resultant force is completely stripped of the influence of traffic loading. It is only related to the integral value of the influence line of each section's support reaction resultant force. In general, for a particular bridge under the action of statistical steady state traffic flow, the equivalent uniform line load *STEUL* reflects the bridge loading characteristics in the statistical sense of big data and has numerical invariance. Therefore, both of the above performance indicators can be used to assess the structure's overall performance and identify damage.
2) The time-averaged state indicator ***TARF*** of the support reaction resultant force and its load-independent normalized indicator ***NTARF*** can be easily obtained by monitoring the vehicle-caused support reaction force. Not only do the indicators have the advantage of being physically meaningful and easy to measure and identify by themselves, but the monitoring of the indicator can also be utilized to

assess the structure's overall performance and identify structure damage. On the other hand, the equivalent uniform line load *STEUL* of a bridge can also be conveniently calculated from the measured values of ***TARF*** and ***NTARF***, which provides a convenient way to evaluate the statistical steady-state traffic loads carried by a bridge.

3) The statistical steady state traffic stream equivalent uniform line load *STEUL* is positively proportional to the statistical steady values of traffic flow and traffic density, as well as to the mean value of the ratio of vehicle weight to vehicle speed. Further studies have shown that *STEUL* is related to the statistical properties of traffic stream parameters, including the vehicle type proportions, the mean and variance of vehicle weights and speeds, and the correlation coefficients reflecting the extent to which vehicle weights and speeds influence each other. For the statistical steady state traffic stream, all the above parameters converge in a statistical sense under the large data statistics condition. Therefore, the traffic stream equivalent uniform line load *STEUL* proposed in this paper will also converge statistically.

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Appendix

Discussion on the application scope of the theory in this paper:

The work in this paper is aimed at continuous girder bridges, but it can be extended by analogy to other types of statically indeterminate bridges. Meanwhile, this paper does not impose limitations on parameters such as the in-span cross-section variation of bridges, the number of spans, and the number of supports in the transverse direction, which implies that the method in this paper is generalizable. In addition, the support reactions in this paper all refer to the live load support reaction under the action of the vehicle, and the effects of other live load actions are not considered for the time being.

The influence surface of curved girder bridges is significantly different from that of straight girder bridges due to the bending-torsional coupling effect. Taking a three-span curved continuous girder bridge as an example, the spatial distribution of the influence surface FIS_4 is shown in Figure A.1. It can be seen that the isolines of this influence surface is not always along the direction of the curvature radius, so the simplified theory of influence surfaces in this paper is not applicable to curved continuous girder bridges.

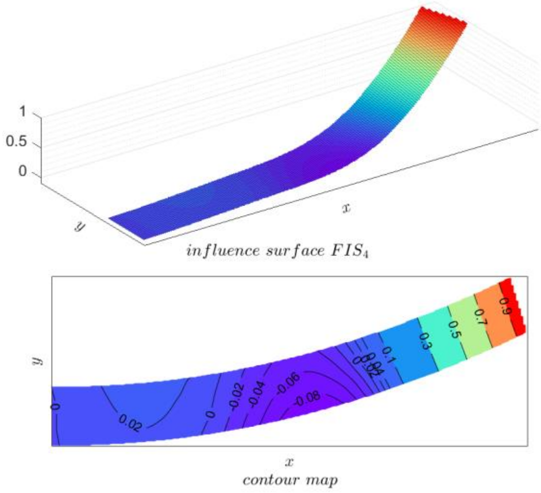

Figure A.1: Influence surface FIS_4 of curved continuous girder bridge

For the above reasons, the indicators in this paper are not applicable to curved continuous girder bridges. In addition, due to the statically determined property of a simply supported girder bridge, the support reaction cannot reflect the change in structural stiffness, i.e., $f_i\,(x,\boldsymbol{\theta})= f_i\,(x,L)$. In other words, the ***NTARF*** of a simply supported girder bridge is a constant independent of the structural stiffness EI, and it is not possible to identify damage by this indicator. According to the loading principle of influence line/surface, it is inaccurate to use influence surface loading for statically indeterminate bridges with strong nonlinearities such as cable-stayed bridges and suspension bridges, so the theory in this paper can only be used to analyze these types of bridges approximately.

The derivation process of Eq. (9):

$$\begin{aligned} TARFE_i &= \lim_{T_N\to\infty}\frac{1}{T_N}\int_0^{T_N}\sum_{k=1}^{N}F_{Ri}\left(\boldsymbol{\omega}_k,t\right)\,dt \\ &= \lim_{T_N\to\infty}\frac{1}{T_N}\int_0^{T_N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot f_i\left(\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau,\boldsymbol{\theta}\right)dt \\ &= \lim_{T_N\to\infty}\frac{1}{T_N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot\int_0^{T_N}f_i\left(\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau,\boldsymbol{\theta}\right)dt \\ &= \lim_{T_N\to\infty}\frac{1}{T_N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot\int_{t_{Ek}^{(l)}}^{t_{Dk}^{(l)}}f_i\left(\int_{t_{Ek}^{(l)}}^{t}v_k(\tau)d\tau,\boldsymbol{\theta}\right)dt \end{aligned} \tag{A.1}$$

The derivation process of Eq. (10):

$$
\begin{aligned}
TARFE_i &= \lim_{T_N\to\infty}\frac{1}{T_N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}P_k^{(l)}\cdot\int_{t_{Ek}^{(l)}}^{t_{Dk}^{(l)}} f_i\left(v_k\cdot\left(t-t_{Ek}^{(l)}\right),\boldsymbol{\theta}\right)\,dt \\
&= \lim_{T_N\to\infty}\frac{N}{T_N}\cdot\frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}\frac{P_k^{(l)}}{v_k}\cdot\int_0^{v_k\left(t_{Dk}^{(l)}-t_{Ek}^{(l)}\right)} f_i\left(v_k\left(t-t_{Ek}^{(l)}\right),\boldsymbol{\theta}\right)\,d\left[v_k\left(t-t_{Ek}^{(l)}\right)\right] \\
&= \lim_{T_N\to\infty}\frac{N}{T_N}\cdot\frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k}\frac{P_k^{(l)}}{v_k}\cdot\int_0^{L} f_i\left(x,\boldsymbol{\theta}\right)dx \\
&= \lim_{T_N\to\infty}\bar{Q}\cdot\frac{1}{N}\sum_{k=1}^{N}\frac{P_k}{v_k}\cdot\int_0^{L} f_i\left(x,\boldsymbol{\theta}\right)dx \\
&= \lim_{T_N\to\infty}\bar{Q}\cdot\overline{\left(\frac{P}{v}\right)}\cdot\int_0^{L} f_i\left(x,\boldsymbol{\theta}\right)dx
\end{aligned}
\tag{A.2}
$$

Derivation of the relationship between the average flow $\bar{Q}$ and the random variable Q:

Taking the unit time as Δt and dividing the time interval $[0,T_N]$ into $TimeN$ unit times and remaining time $[TimeN,T_N]$, the average traffic flow $\bar{Q}$ in Eq. (10) can be regarded as the sample mean of the random variable Q, i.e.:

$$
\bar{Q}=\frac{\sum_{t=1}^{TimeN}N_t+N_{[TimeN,T_N]}}{T_N}\approx\frac{\sum_{t=1}^{TimeN}N_t}{TimeN\cdot\Delta t}=\frac{\sum_{t=1}^{TimeN}Q_t}{TimeN}
\tag{A.3}
$$

where N_t and Q_t denote the number of vehicles and traffic flow through a cross-section in the t-th unit time, respectively.

The derivation process of Eq. (25):

$$
\begin{aligned}
\sigma^2(P) &= E\left(P^2\right)-E^2(P) \\
&= \int x^2\cdot\sum_{r=1}^{VM}\alpha_r g_r(x)\,dx-\left[\int x\cdot\sum_{r=1}^{VM}\alpha_r g_r(x)\,dx\right]^2 \\
&= \sum_{r=1}^{VM}\alpha_r E(P_r^2)-\left[\sum_{r=1}^{VM}\alpha_r E(P_r)\right]^2 \\
&= \sum_{r=1}^{VM}\alpha_r\left[E^2(P_r)+\sigma^2(P_r)\right]-\sum_{r_1,r_2=1}^{VM}\alpha_{r_1}\alpha_{r_2}E\left(P_{r_1}\right)E\left(P_{r_2}\right) \\
&= \sum_{r=1}^{VM}\left(\alpha_r-\alpha_r^2\right)E^2(P_r)-2\sum_{r_1=1}^{VM}\sum_{r_2=r_1+1}^{VM}\alpha_{r_1}\alpha_{r_2}E\left(P_{r_1}\right)E\left(P_{r_2}\right)+\sum_{r=1}^{VM}\alpha_r\sigma^2\left(P_r\right)
\end{aligned}
\tag{A.4}
$$

Symbol table

Indices

S	number of piers and abutments or bridge spans +1 ($i \in \{1, \ldots, S\}$)
B_i	number of bridge bearings at the ith pier ($j \in \{1, \ldots, B_i\}$)
N	number of vehicles in traffic stream ($k \in \{1, \ldots, N\}$)
n_k	number of axles for the k-th vehicle ($l \in \{1, \ldots, n_k\}$)
VM	number of vehicle types in the traffic stream ($r \in \{1, \ldots, VM\}$)

$F_{R(i,j)}$	support reaction of j-th support at the *ith* pier and abutment
F_{Ri}	support reactions resultant force at the *ith* pier and abutment
$FIS_{(i,j)}$	influence surface of $F_{R(i,j)}$
FIS_i	influence surface of F_{Ri}
$\boldsymbol{\theta}$	Parameter set related to the static characteristics of bridges
$f_{(i,j)}(x,y,\boldsymbol{\theta})$	function of $FIS_{(i,j)}$
$f_i(x,y,\boldsymbol{\theta})$	function of FIS_i
$f_i(x,\boldsymbol{\theta})$	simplified form of $f_i(x,y,\boldsymbol{\theta})$
<u>Parameters</u>	
$P_k^{(l)}$	axle weight of the l-th axle of the *kth* vehicle
v_k	velocity of the k-th vehicle
t_{Ek}	time when the first axle of the k-th vehicle enters the bridge
$t_{Ek}^{(l)}$	time when the *lth* axle of the k-th vehicle enters the bridge
t_{Dk}	time when the last axle of the k-th vehicle departs the bridge
T_N	time when the last vehicle in traffic stream departs the bridge, which is equal to t_{DN}
$W_k^{(l)}$	wheelbase between the l-th axle and the first axle of the k-th vehicle
L	bridge length
$\boldsymbol{\omega}_k$	Parameter set for $P_k^{(1)}, \ldots, P_k^{(n)}, W_k^{(1)}, \ldots, W_k^{(n)}, v_k$, and t_{Ek}
$F_{Ri}(\boldsymbol{\omega}_k, t)$	time history function of the support reactions resultant force at the i-th pier and abutment induced by the k-th vehicle crossing the bridge
TARF	vector that is composed of time-averaged state support reactions resultant force at each pier and abutment caused by traffic stream crossing the bridge
NTARF	indicator vector obtained by normalizing ***TARF***
STEUL	statistical steady state traffic stream equivalent uniform line load
K	traffic density
$\mathcal{K}$	statistically stable value of traffic density
Q	traffic flow
V	space mean speed
$C_v(1/v)$	coefficient of variation of the reciprocal of the speed in traffic stream
$\rho(P,1/v)$	correlation coefficient between the vehicle weight and the reciprocal of the vehicle speed in traffic stream
$E(v_r)$	mean value of speed for vehicle type r
$\sigma(v_r)$	standard deviation of speed for vehicle type r
α_r	proportion of vehicles in type r
$E(P_r)$	mean value of vehicle weight for type r
$\sigma(P_r)$	standard deviation of vehicle weight for type r
$EI(\cdot)$	distribution of flexural stiffness of bridge section along the longitudinal bridge direction