Computing With Spin: Cohomology And Mathematical Applications Of Holographic Satisfiability

Adewale Oluwasanmi

waleoluwasanmi@yahoo.com

ABSTRACT: In [Oluwasanmi A. https://doi.org/10.31224/3065. (2023)], we presented an (holographic) algorithmic system for finding polynomial time proofs of both satisfiable and unsatisfiable 3 SAT problems and presenting them as differential varieties. In [J.M. Landsberg et al. Linear Algebra and its Applications, Volume 438, Issue 2, Pages 782-795 (2013)], the authors have identified holographic identities as spinor varieties. In this paper, we combine these insights and represent the solution manifold as a spinor space, that is, a spin manifold, along with varieties emerging as spin valuations using a form of complex arithmetic that we develop. Unsurprisingly, the spin forms that emerge can be associated with a type of algorithmic and relativistic, quantum gravity, which is not unlike the actual gravity associated with physical particles. We explore the implied gravitational field and also show that its structure is not only convenient for studying physical force fields but also can shed proof theoretic insights on several deep mathematical problems by encoding their computations as variables associated with the spin cohomology introduced.

To obtain these results, we first examine the usual syntactical gadgetry invoked in other holography papers – variants of Pfaffian circuits - and find them to be inadequate to fully develop and associate the quite rigid notion of spin to definite numerical values in high dimensional differential space. We then introduce a fine-grained formalism that models the selection/deselection process as an algorithmization of a Lie algebra-type flow over a spin moduli stack in which the spins then arise as natural quotients of representations of the moduli group.

1. Introduction

Valiant introduced the world to holographic computation via matchgates in [3]. [2] extends Valiant's methods but shows that it is not necessary to use matchgates to compute the needed solutions. Instead, identities occurring along matchgates are shown to be generally Pfaffian and take on a Plücker-Grassman form which makes them spinor varieties constructible via other means.

We recall that in [1], we generate our holographic solutions as differential varieties, embedded within a scheme. We can show that this embedding differentially generalizes the Plücker-Grassman construction and results in a variety valued over an extended form of the complex numbers, embedded in hyper-dual space. This variety, like conventional spinor varieties, can be shown to be the double cover of a binding force which we equate to gravity. We can show a correlation to gravity by demonstrating a full unified field theory where we extend this force of gravity by spin analogy to other fundamental forces and show that this theory could be an exciting way to compute solutions to equations of physical spacetime.

We reintroduce our solution system here as a system of interacting spins governed by this force of gravity – much like an Ising model. The mathematical formalism involved is that of the notion of a flow along a derivative of a second-order function – a flow measured as a spin dual functional that drives the normed numeric value of the spin (function).

Along with spacetime and physical computations, we can also show that some other types of mathematical problems can be addressed or at least abstractly modeled with this spin-gravity formalism. These ideas will be addressed in the following sections.

2. Revisiting the Differential Manifold As An Embedding Space

2.1 A Word On Correspondences

To establish the notion of spin algorithmically and how it can be measured, we should first take a summary look at what is called the program-proof-category correspondence in computing.

 If we take it that programs evaluate types, proofs reason about propositions and categories create morphisms between objects, then it is obvious from our previous paper [1] that these three mathematical objects correspond over a topology in a particular algebraic way. Essentially these objects generalize Boolean algebra by the Stone representation theorem. In our new terminology, we can say that these objects "spin" (evaluate, reason, morph) over the space (field) of sets to converge on some particular set composition.

In other words, the points on our manifold can be associated with spinors or continuous spin particles. This spin can then be viewed as being derived from a functional mapping of a spin path to some scalar value. The spinor variety obtained can be shown to be slightly different in flavor and complexity from that described in [2] as follows: [2] realizes the varieties as linear functionals acting on fermions, but in this article, we enlarge the notion of spin as a linear functional (a line of reasoning) which is a full field theory, acting on both (using proof theoretic language), fermions (proofs/multiplicative proof fragments) and bosons (propositions/structures of propositions).

Having thus established the notion of a spinor, we can now reintroduce our manifold of (implicit) functions as a differential spinor field, and we can study the valuation of this field using some insights from spin mechanics.

2.2 The Differential Forms and the Cohomological form

We recall that in [1], we proved that certain implicit functions, represented by the dual of their coefficients, can be integrated up to a certain limit to determine satisfiability by showing that a subset of that dual space (sets of 3 depth sums and less) can either be differentiated down to a satisfying solution or not. This differentiation, for satisfiable equations, takes place over the field of sums (partial differential equations) of depth 3 and less and each variable is shown to be assignable within the bounds of a twice differentiable partial function that solves each equation. This is a 2 form σ_2, on the tangent space of our main manifold. This makes this manifold a symplectic manifold. Notice that the tangent space itself forms a second manifold, with a 1 form σ_1, of assignments, which makes this manifold, with some dimension counting which we show later, a contact manifold.

Now, we can create a template form for the cohomology of the assignment chains that live on the contact manifold, from the perspective of any one particular chain. This is because these cohomologies are differentially valued, in some sense. close to étale cohomology, which is diffeomorphically valued. At some points during our computation, some prospective cohomologies will vanish and the weight on the remaining cohomologies will adjust relativistically. Here, relativism is intrinsically tied to an idea we will introduce in the section on measures – that solutions are values as eventual distributions, that is as events, in a quantum mechanical but relativistic space. In this case, the ring of differential polynomials, itself having coefficients in time (to adjust for relativity) becomes the coefficient of its perspective-dependent or quantum relativistic cohomology. We can show that this cohomology in its kth index, for n variables, from the perspective of any assignment, should take the following form, valued as shown in [1] in direct sums:

$$H^k(C(p)) \cong \bigoplus_{j=1}^{n} H_{n-k}(C(p), D_{(t \in N)}(q))$$

where $C(p)$ is the flow classifying a single solution at every point in some direction and $D(q)$ is a twisting (in linear time) coefficient field. Here, we note that the word twisting is meant to have a similar meaning to that in which it does in Twistor theory – that this coefficient field creates a link between 2 real points in complex space – a demonstration we complete in the section on gravity. Note that the size (number of indices) of t here may be equal to n or less.

The $n\text{-}k^{th}$ homology of any particular solution (for a satisfying formula) eliminates the $k + 1^{th}$ cohomology component associated with its complement as a set of quotient operations, that is, each chain is valued at index k for some (final) direction d as:

$$H_{n-k}(C(p) \cong \bigotimes^n H^{k+1}(C(p))/ \mathbb{Q}_d(p)$$

where $\otimes^n$ denotes a "new" type of n-structural tensor product of n fields and $\mathbb{Q}_d(p)$ is the (chiral) reflexive, rational subgroup of the total group $H^n(C(p))$ that divides $H^n(C(p))$ into $n-1$ irrational left cosets and 1 rational right coset (which is our solution), in the direction d. The tensor product of cohomologies in this case is the tensor product of fields at each point in spacetime where each cohomology vector in any direction treats the collection of possible cohomologies in other directions as a co-vector form and is thus in a natural vector-covector (dualized) tensor product form. The quantities of this product form contribute to the coefficient factor and are best compared to (a differential version) the Gromov-Witten invariant formalism in standard quantum cohomology.

Here it seems a differential version of the Hodge conjecture holds as the eventual and complete cohomology associated with each solution is just a Pfaffian combination of its level homologies.

We can formalize the above in the following theorem that characterizes the entire space (like before) just in terms of the symplectic form. We refer to the process defined in [1] where we integrate over all terms up to a depth of 6 as holographic integration.

The formal proof of the below theorem and some demonstrations of its applications can be taken to be the main object of this paper:

Theorem 1

If a formula F has non-zero holographic integral H over n variables, then H is the twisting cohomology of a contact variety and is induced by the construction of a compatible symplectic manifold under a ∂-graded $\partial\bar{\partial}$-ideal for all solutions in the field of solutions of F, where chains take graded values in $\bar{\partial}$ and their coefficients take values in ∂ under the ∂-algebra fixed by the 3 dimensional ∂-ideal, with ambient model support in (the extra dimensions of) H.

This is essentially a symmetric $\partial\bar{\partial}$ theorem and we can consider the theorem proved when the notion of differentiation (split into two different symmetric operators over manifold boundaries) has been clarified and this $\partial\bar{\partial}$ relation has been demonstrated in our construction of a definite spin cohomology.

2.3 Modeling The Distinction Between Fermions And Bosons

In [1], we presented the solution manifold as a single k-dimensional manifold, where k counts just (perfect) matchings, and like we have shown above, ignores the bipartite count on the pairing of each (rewritable) clause with an assignment. We will now expand on the bipartite decomposition of this manifold into two different and related manifolds in this section.

Let us recall that matchgates are circuit elements in the original formulation of holography by Valiant and they are circuits with a fan-in of 2, taking nodes in a bipartite structure and fan-out of 1, outputting edges (perfect matches) connecting these nodes.

Let the reader imagine a 2 SAT formula encoded in the holographic formulation from [1]. We can solve it with the following linear time algorithm:

i. Generate the initial sums of unassignable statements for clauses in the input problem (as described in [1]).

ii. Combine eligible sums pairwise to form a direct sum of topologies. No need to discard sums as no sum will have a depth greater than 2. Repeat for each new sum added until no more sums can be added, or two sums contradict.

iii. If (ii) completes successfully, perform a random walk over the entire set, that is, rewrite every sum IF NOT satisfied. The reader should now see that any rewrite will immediately satisfy the underlying clause, that is, the 2 SAT satisfaction looks like a simple DIFFERENCE operation. We can say that the 2 SAT problem has definite circuits: For any assignment, the flow takes us to a definite and repeatable end state, thus we say the 2 SAT problem is completely algebraic. Note immediately that this is not the same in the 3 SAT case, which we say is only differentially algebraic and does not have this direct circuit representation.

Now, even though a direct circuit representation is not possible with the 3 SAT problem, we notice that we still have a bipartite structure that we seem to exploit. This can be described in one part as the flow of assignments and in the second part as obstructions to this flow. These obstructions are resolved differentially so the entire operation can be reread as a Lie bracket on a function fixing coordinate assignments along a flow:

$$f(p) = [p, q]$$

Where q is the variable associated with differentiating out unassignable statements and p is the flow of assignments generated by this derivation process along p axial coordinates.

The quantities p and q now live on different manifolds – one fermionic and one bosonic, characterized as follows:

i. A differentiable bosonic **contact** manifold with a differential 1-form σ_1, and a non-integrable external derivative $d(\sigma_1)$: This records the flow of the assignments and is an additive quantity (non-zero). Quantities on this manifold are common to all clauses and can be associated with an eventual probability distribution with a norm of 1. This we borrow directly from quantum mechanics where observables are probability

amplitudes, which in our case we differentiate (instead of squaring) to the final probability (1) of our selection. These differentiations are the "events" of the physical system and are so valued in an "eventual", quantum manner. Note that this manifold is symmetric and has an associated group structure (to be fully identified in the section on measures). To accord with the definition of contact structures, we will fit this manifold with an extra point at $-\infty$ (the null hypersurface from which bosons arrive). This would give our structure $2n + 1$ dimensions as needed.

ii. A differentiable fermionic **symplectic** manifold with a vanishing external derivative on its 2-form: $d(\sigma_2)$: This records the repulsion of non-selections and are a vanishing quantity. Quantities on this manifold are specific to clauses and the end quantity recorded on this manifold is assigned zero. This manifold naturally has a dimension of $2n - 1$, with first and last points subtracted from the contact manifold. This manifold is also symmetric and for satisfiable formulae, these symmetries match up with the symmetries of the contact form via a homological mirror (reflection) relation, that is, the contact manifold reflects the fermion through a change of planes via torsion. We believe this can be directly related to the homological mirror symmetry in physics.

Combining these 2 pieces of information, we arrive at the notion of the combined structure as two sheaves of groups (2-sheaf) forming a stack that computes equivalence classes (moduli) of solution paths, which is, a homotopy equivalence.

We can think of this as a reflexive stack characterizing (computing the trace and thus the accumulated spin) of a spin group as a differentially complex (p,q) differential form. We stress the term, differentially complex, as the p (bosonic) holomorphic coordinates are "differentially" wedged to the q (fermionic) obstructions. This gives the form a second-order differential form – as we differentiate the fermions in time, the differentials obtained are used to differentiate bosons in space, and the values that the bosons take further restrict how much we can differentiate along time – a classic picture of gravity.

The two manifolds we have just described can be said to form components of the tensor functional of a differential ring with one component in the reals (p forms) and another set of components (q forms) valued in the dual numbers [4]. According to [4], this is also a binding relation – indices of differentiation of both groups are paired one to one so it can be said that one group (the normal subgroup of subtractive elements to the right) witnesses (has an index) the interiority of the quotient group (group of assignments). In this case, the quotient group is the g character group in relation to the normal subgroup π and is the space of equivalence classes of group homomorphisms from π to G and contains the intersection of the spectrum of prime ideals of G.

$g(\pi, G) = \text{Hom}(\pi, G)/G$

2.4 Measures On The Manifolds

Now to associate a functional value to every point in the spectrum of points along each manifold.

We just need to examine the total form to discover that over the entire collection of initially assignable values, we have naturally defined an entropy functor that broadly follows Shannon's formulation – we are computing the expectation value of the information content for some random variable given by a log distribution.

$\mathbb{E}[-\log p(X)].$

Where $\mathbb{E}$ is the expected value operator. In the case of classical information theory, this would be computed as classical Shannon entropy, but in our case, where we have decided to adopt the notion of an eventual distribution, this amounts to a form of quantum entropy, that is, we can map every moment of the function to a quantized entropy value (we will still look at the explicit form of the function under the section of Gravity but for now let us stay with its functional valuation) as follows:

We treat all assignment values as continuous information amplitudes that vanish as we complete the information with a density of 1 (on the p form). Here, a complete set of assignments is the information desired. The character group of the assignments mentioned above now takes on the same character (pun intended) as the group character taken along a trace (traversal) of the group representation of linearly independent (tensor) components and our entropy formula takes on the form of Von Neumann Entropy with density norm ρ and total trace summation value of 1.

$\text{tr}[-\rho \log \rho] = -\text{tr}[\rho \log \rho]$

This establishes the set of assignments as a projective embedding of a number system (Grassmannian-type field projective embedding) into a witnessing field of vanishing numbers. We can see here we are performing a type of lifted reasoning where we can associate to our reasoning flow, entropy-based numbers associated with the size of the vanishing set, that is, we are measuring the differential equivalent of a (hyper) dual number.

We now formalize the first of the number systems needed to express our cohomology.

We can observe that the points on both manifolds are associated with an alternating sheaf algebra, which is differentially graded. Points on the number system swept out by this algebra can be associated with the gradient values induced by an inverted Laplacian operator ∇. Also note that this gradient goes to zero, much like squaring on a dual number does. We designate this number field as the field of dually gradient numbers and represent it as $\nabla^d_\varepsilon.$

With superscript d standing in for numbered differential depth and the subscript ε denoting dual differentiation (the symbol ε standing in for dual numbers in the normal literature). This field is symmetrized by the normal subgroup and is the coefficient field for our cohomology.

We have in essence set up a dual measure space for our bosonic functions taking place over $\mathbf{n}$ assignments and fermionic functions taking place over $\mathbf{m}$ clauses at a maximum differential depth of 6. This dual measure is extractable from just the contact manifold as measures on functions. All functions in our case belong to groups, which we will now call $\mathbf{HU(n, m)}$ for the holographic unital group on the bosonic space with function addition and multiplication on a field of characteristic ≤ 6 and $\mathbf{HO(m,\ n)}$ for the orthogonal fermion group with function composition and exponentiation. On $\mathbf{HU(n, m)}$ we have an L^n measure, differentiated down to L^0 for as the initial distance between the entire field strength/potential carried by (all) gravitons and the final strength (0). On $\mathbf{HO(m,\ n)}$, we have an L^6 to L^0 word distance measure between the partial functions satisfying each clause. All distances are between real points (we show a correspondence to the real field in the section on integer factorization) twisted through complex space (also shown in the integer factorization section).

We now just have to complete the cohomology picture by identifying the nature of the number field associated with the satisfying unit functions, that is, the $\mathbf{HU(n, m)}$ group – elements of the chain/quotient subgroup. This number field is normed at 1, but in actual computations, we can assume that this norm will be ***multiplied by a representation weight*** to yield an integer value greater than 1. At this point, $\mathbf{Theorem\ 1}$ is proved. Before we finalize the picture/concept of the described number field, we will first take a short look at the gravitational force implied by our spin field.

3   A Picture Of Gravity

Recall that in [1], we evaluate the complement of proofs by summing over complementary polynomials (to the polynomial we intend to satisfy) and they take on the form:

$$\sum f_i g_i$$

Where i is taken to be instances of time supplied by a reflexive differential, which in other words can be taken to mean that the algebra being invoked is a reflexively graded algebra where the i term comes from applying a differential operator on a quantity.

Recall from above we have two interacting manifolds, one in which we directly measure and one in which we perform asymptotic lifted reasoning. Both are co-differentiated and in [1] are represented by the following term at each index:

$$f^n\ \partial^n\ g(x)\ /\ \partial^n t$$

With each fⁿ being the algebraic coefficient at a differential level (level sets/co-vectors) and ∂-ideals split along a space (with functions on directions) and time submanifolds. Each (sub) manifold is differentiated along a boundary and the boundary operators form natural Dolbeault operators valued as the induced vector valued operator taking directed values in the difference between the scalar-valued total (exterior) derivatives taken between the preceding manifold $M(x)$ and its lower dimensional boundary $M(x+1)$.

That is: $\partial(M(x+1) = d(M(x)) - d(M(x+1))$

It can be inferred that the complementary manifold is valued in time and the other quotient manifold in space. This time must be a time that is intrinsic to the process, that is, the process contains its own regularized clock valued in natural periods. But how might we discover/construct such a clock?

In [5], Bousso describes a light sheet where light rays from infinity converge on a caustic before dispersing. We will employ a similar concept here, which we will call a gravity sheet.

The idea is that the entropy on the manifold represents an obstruction to the path of a graviton arriving from null infinity and thus records the path of the graviton around that mass. We can formalize this in the following way.

i.   All quotients are GIT as well as geometric quotients.

ii.   We can give our selection process the asymptotic geometry of a dynamic 2 sphere by imagining that for any variable, any two opposing assignments are two points on an ∞-diameter circle and all possible circles bound a 2 sphere. This makes perfect geometric sense if we once again forget the extra 3 dimensions introduced in [1] and think of our algorithm as operating only in the bulk defined by the three-dimensional core subspace – then these circles form perfect "fractional" axes on which to take measurements.

iii. Final assignments represent a direction on the sphere.

iv.   Removal of entropy in any direction reduces the gravitational potential in all opposing directions to zero and assures us that the graviton is traveling unobstructed to the chosen point.

v.   Since this is a spinor, we are working with a double cover and so at zero complementary potential, let us assume a black hole setting, where even the graviton does not travel to infinity but "spins" around to complete the cover (in a somewhat unphysical manner). The graviton is assumed bound to the particle and oscillates with it ad infinitum. Here, we do not make any assumptions about net travel in any direction. The two spin points represent the rotation phases of a fermion and are meant to adhere to the Pauli exclusion principle abstractly via the induced skew symmetry, that is, for the 2 points A and B, $AB = - BA$ AND $BA = -AB$. One can speak of this as a transposable "quantum" identity $n-1$ by $n-1$ matrix, for a problem with n variables.

Here we have the definite topology of knots in space with the indexed set of complements forming the undercrossing at each rational point and main assignments comprising the overcrossings in the link. It is now easy to see how time can be associated with one manifold and space with the other. This comes squarely from a combination of the uncertainty principle and Noether's theorem as follows:

i. Momentum and position are quantum conjugates so we can only measure one accurately.

ii. In isotropic space, even when we do not know the momentum, we know it is conserved, and we know the following hold:
momentum = mass * velocity
momentum = mass * (distance/time).

iii. Since distance is equal in all cases, different masses for a fixed momentum will produce different lengths (sizes) of time and thus distances/ lengths of paths through space.

iv. As implied in (i), we cannot measure this mass directly, but we can measure the (conformal) angular distance covered by the lifted measure.

Now we have a more complete picture of our computational process as an ensemble of information points co-spinning under the influence of mutual gravity. This lets us speak of these spins as computing something. We address the question of how much of this artificial gravity is like the real notion of gravity in future work. However, we must note here that this picture of gravity, in addition to using a bosonic sheet approach from [5] also blends insights from two physics-based theories of Holography. On one hand, we have the Celestial Holography program [6], which associates 4d gravity matrix (sums) and correlation functions in a conformal (directions given in implicit affinely presented angles) field theory living on an infinite sphere and on the other hand the ADS/CFT [7] which describes a correspondence between a theory (sums again) living in negatively curved (ADS) space and another theory living in (asymptotically in our case) flat space. In our case, for ADS/CFT, we formalize the correspondence as an intermediate theory living positively curved Riemannian space.

To give our theory the same level of physical dynamism as these two other theories, we must unbind the graviton from the point as gravitons are assumed to be massless and travel at speeds not possible by massive particles. Let us now allow the graviton to travel on to infinity until it hits another obstruction. Now we can say conclusively that spin along with mass does compute gravity, or the warping of spacetime, like in Einstein's relativity, but with quantized spin added. This is the spin of the graviton, and we must assume that anywhere there is gravitational motion in any direction of spacetime, there is a graviton (one or many?) traveling in that direction that can pick up that directional expectation and communicate it to other expectations within the field. In short, everywhere in spacetime, a graviton is traveling, or that gravitons are spacetime itself. The above would give us a non-commutative (group multiplication-wise) gravitational field theory with a mass gap/description of the underlying fermions, and we believe that the theory

should be considered seriously in the discussion of the Yang-Mills mass gap problem, which is usually defined over the 2-dimensional complex numbers.

A word on compactification here. We mentioned in [1] that the classifying manifold can discover all solutions, when they exist, via symmetric categoricity. We refine that statement here in the following form – we can discover all solutions because we have applied a symmetric compactifying form. In that paper, we introduce the first form of topological compactification, explicitly made apparent here – we compactified the $2n + 1$ geometry of the contact form into separate co-dimensional 6 sums which we then unified into a single 6-dimensional structure $+ 1$ dimension at -∞. In our visual presentation here and in restricting our algorithm to the 3-dimensional core subspace, we have introduced a second, fine-grained, geometric compactification to 3 dimensions $+ I$ dimension at -∞. This second sense is the one in which compactification is normally spoken of in the String theory literature in terms of Calabi-Yau manifolds. We hope the link with the first type of compactification will be of help here. In short, we think the whole project of Holographic Satisfiability can be viewed as an exercise in a broader set compactification theory.

4 A Completed Cohomology Definition Via A Model Of Solving Integer Factorization

Here we address how we can use our new spin model to compute arithmetic functions using multiplication as a case in point and how that helps us define the nature of the chain number field.

A standard reduction of factorization to SAT conjuncts an open multiplication circuit (see example below) to a bound equality circuit (to some number C) so that the circuit satisfactions are factors ($A \text{ and } B$) of the multiplication circuit:

$A \times B = C$

Now let us assume that this circuit has been integrated over holographically using the system in [1]. Applying any one of the integer factors will cause the function to vanish, that is, once we apply the one factor, we auto-differentiate to its cofactor(s).

The above procedure lets us think of our manifold as an implicit division algebra, that is $A/B = C$ and $A/C = B$, where C and B are each ***the representation weights applied to the contact norm***.

What of factors that do not divide evenly this way? Let us call the circuits that compute these types of results, non-integer circuits. For this, let's imagine an equality circuit D which dividing by a string of bits E, leaves the choices of assignments to the remaining variables

open. If we can represent all integer divisions via normal ABC circuits, then these new numbers must extend that field,

First, we notice that, along the lines of any computation, even over an infinite field, we can always place all points one on one with the natural/rational numbers. The fact, however, that some of these numbers are generated by non-integer circuits means in a number system, they add on to the natural numbers, allowing ANY particular combination of assignments to particular variables be able to take on an infinite set of values . In fact, the true cardinality of the allowed numerations is an infinite multiple of the cardinality of natural numbers. These associate them with the (constructively $\infty\text{-adic}$) completion of a field, isomorphic to the uncountability of the field of real numbers $\mathbb{R}^n$ and the fact that that the operation lives on a manifold which eliminates imaginaries (paths that lead to negative evaluations), identifies this field at the very least as equal in arithmetical hierarchy to the complex numbers in a hyperplane (with torsion to separate different planes of realization of the integers). In essence, one can think of each spin variety living in a different complex plane of a rational/countable cutout superposition of planes, and at each step, we multiply all path variables that we intend to cancel out by i, making them go to -1, and effectively eliminated from the multiplicative projection, which is twisting this variety over the hyperplane. The final value of the variety, which we previously associated with the value 1, multiplied by some result weight, can now also be written out like a number with many real and many imaginary terms, whose total number sums up to n, such that the completed operation appears like multiplication by i immediately followed by a conjugation which eliminates out the parts with i that have become negative and also multiplies the initial real terms by a constant term from their starting fractional values. This action can be viewed as the inverse of convolution over the set of possible functions in the space.

In fact, the hierarchy, derived from a standard model-theoretic arithmetic hierarchy scheme, can be specified as follows:

1. Δ^0_0 – Our basic arithmetic circuits that we wish to compute. These are non-alternating (0 subscripts) and operate directly on base numbers (0 superscripts).
2. $\prod^1_1$– The set of assignments that invert (multiply) the main path and take a sum over all other complementary elements. This alternates with and is dual to Σ^1_1 and operates on functions over numbers. It is directly generated by Σ^2_2 .
3. Σ^1_1– The set of complementary differentiations that sum over all impossible paths and invert (multiply) the main path. Dual to $\prod^1_1$ and alternates with it. It is indirectly generated by $\prod^2_2$. Operates on function over numbers.
4. Δ^1_1 – Our satisfiable evaluating circuit. Combines $\prod^1_1$ and Σ^1_1. Operates on function over numbers.

5. Σ^2_2– The extended sum to determine satisfiability. Second order extension of Σ^1_1. Generates $\prod^1_1$. Alternates on $\prod^1_1$ and $\prod^2_2$. Operates on second-order functionals (functions over functions) over numbers.

6. $\prod^2_2$ – Extends $\prod^1_1$ into second order – implicit. Alternates on both Σ^1_1 and Σ^2_2 indirectly generates Σ^1_1. Also operates on second-order functionals over numbers.

7. Δ^2_2 – Implicit – combines Σ^2_2 and $\prod^2_2$.

One can immediately see an arithmetic reason for which our circuit can solve the halting problem as shown in [1] – the initial problem lives in a lower hierarchy Δ^o_0 while the halting decider is in the arithmetical second order Δ^2_2 and manipulated through the duality between the Σ^1_1 and Σ^2_2 classes.

Now, if we can arithmetize this way, we can go on to designate the field in which the chain complex takes values as the field of complex gradient numbers, as they absorb their values gradually and symmetrically from the vanishing of the dual-graded numbers and can be represented (just like the dually graded numbers) with ∇^d_i.

Where i now represents differentiation via the imaginary numbers and d explicitly identifies the numbers of partial coordinate elements for which we have to eliminate imaginaries (differentiating to -1, see [1] for the full field description associated with the coefficient field) We can now easily redefine our cohomology equation on any variety at the kth index as:

$$H^k(\nabla^d_i(p)) \cong \bigoplus^n_{j=1} H_{n-k}(\nabla^d_i(p), \nabla^d_{\varepsilon\,(t\,\in\,\mathbb{N})}(q))$$

The cohomology can be said to have a twist in complex space, determined in time. A twisted Dolbeault cohomology has also been obtained in [8] for the $\mathbf{SU(n)}$ group studied in that paper. Other papers that use spin bordic and cobordic (surgery over manifolds) approaches to computing spins and their cohomology include [9, 10]. The cohomology theory itself can be viewed within a traditional algebraic topology frame as a sheaf-based topological $\mathbf{K}$ theory, which seeks group completions of fields over a differential variety with an associated twisted Galois theory. The field to be completed in this case is simply a field of compactifiable sets and as discussed previously, we seek the differential group symmetry that compactifies the valid assignments sets to the original problem. Recent work on prismatic (structures of rings paired with ideals in a p-adic context) cohomology [11] in which derivations are spread out and total in some sense, also mirrors our work here.

We can see that the differentially complex numbers are finer-grained than the relativistic natural numbers and that approaching any integer representation factors through complex steps. We can argue informally here that integer representations should take more work/energy to resolve than complex ones and any problem which naturally demands that all factors should immediately be an integer one should require more spacetime to resolve. If all satisfiable circuits represent complex division/factoring this way, then we can assume that among all running times, integer circuits will take the longest. A more formal proof may involve a type of dynamic sphere

packing argument borrowing from and/or enhancing the arithmetization/number system we have laid out here.

The way in which we have treated this is similar to the concept of complex multiplication, with multiplication by the imaginary, over some endomorphism ring and so this formulation may be interesting to those working on Hilbert's twelfth and other related problems.

5   The Riemann Hypothesis And The Hodge Conjecture

Following up on our insight about how the cohomology at every point in our manifold is a differential (Pfaffian) combination of dual space homologies, suggesting a differentially algebraic variant of the Hodge theory, we can promote this to a full algebraic version by considering the 2 SAT case, which we have shown is purely algebraic. We can reason in the following way:

i.   Any satisfiable 3 SAT problem is isomorphic to a 2 SAT problem.
ii. Since descent down to any solution is one time-step from a * space, the element is a proper dual.
iii. Any differential combination of homologies is isomorphic to an algebraic one, constructively as follows: For all satisfiable formulae, we can construct a satisfiable sub-formula that consists of **ANY** two variables from each clause. This resulting 2-SAT formula has algebraic assignment values associated to each possible variable assignment over n variables. For any 3-SAT formula, we always assign in order, and for the first 2 assignments in each 3 clause is algebraic, we can rewrite each clause in our 3 SAT formula as a "dynamic" 2-SAT formula consisting of the value of the algebraic assignment of the first two assignments in each clause (which also is a proposition) **DISJUNCTED** with the 3rd variable (also a proposition). This dynamic 2 SAT for any possible assignment order on a 3-SAT formula is also dynamically algebraic – precisely the disjunction is isomorphic to a linear difference equation between the algebraic values of the first and the second variables in each clause.
iv. The Hodge Conjecture is trivially satisfied in this finite case.

Now let us look at a natural counterpart to the Hodge conjecture in our system, the finite Riemann Hypothesis.

In the case of the finite Hodge structure, every metric can be rewritten (using the language of sheaf theory/cohomology) as a valued stalk f^p where f is the initial function germ. These powers have to be related to each other by relative number forms which need not themselves be whole numbers, which would make p a field characteristic. For the sake of initial abstraction, let us assume that these values are whole and are positive integers. The powers themselves need not be whole numbers in our case but are positively valued. In fact, if scalar values of f are taken to

be (relativistic) integers, the powers have to be (differentially) complex numbers. We can surmise that the initial sum representing any 2 SAT clause is a linear sum of these inverted powers.

$$f = 1/\ f^p + 1/\ f^{2p}$$

The entire formula is a product form and so the summations for each clause must lay side by side as product terms, by isomorphic analogy, for any k-sat formula, $k \in \mathbb{Z}^+$:

$$(1/\ f^p + 1/\ f^{2p} + \ldots + 1/\ f^{kp})(1/\ g^p + 1/\ g^{2p} + \ldots + 1/\ g^{kp}) \ldots f, g, \ldots \in P \text{ (the set of co-primes (field ideals))}$$

Since all product terms (functions of types f's and g's whose stalks are the unique p-powered stalks, where p is the complex-valued field characteristic) are relatively prime (ideals of the underlying ring), what we have here is a type of finite L function in Euler product form. We can justify the "rounding" up of factors between powers to integers as follows: Each term along any stalk is Pfaffian, which is a derivative of a previous function and can be written in terms of this function. Each power of progression along a stalk ($f, g, \ldots$) also eliminates other progression stalks (mapped to a unique power) by a Pfaffian factor (other than the unit factor being computed on the normed stalk). This means that the evaluation of every stalk can be rewritten in terms of the evaluations of every other stalk. We can in some sense, "squash" up the positions of the non-integer function derivatives (on the non-integer stalks) into the positions of the integer terms in our expansion while maintaining the integrity of the computation, thus allowing us to express the function as just a sum between integer terms. In fact, we can use this to informally support our integer sphere packing assumption we made for integer factorization – that requiring an integer factor-only circuit somehow involves physically eliminating the possibility of the intermediate rational terms in this Euler expansion. To obtain a "squashed" sum that matches the Euler product expansion, we can follow the following recipe:

i. In a linearly ordered enumeration of finite elements, every element, on the contact manifold, is the derivative of the preceding derivative, which is the function being differentiated. As such, each (Pfaffian) derivative can be written as this preceding term (differentiable function) multiplied by some constant: $dx = \Phi\ (d(x-1))$.

ii. The value of Φ is relatively fixed for any differentiation interval but the angular distance (since we are in isotropic space) covered by each total derivation is still given a norm of 1. We note that these angular distances can be weighted in such a way as to respect the order on the derivatives. These distances will correspond to the value expansion terms in the denominator of each term in our L function and act more like values on a clock since as we stated, we are working in an evenly spaced, isotropic manifold.

iii. Let us refer to the distances (weights on the norm) attached to a point x and a point $x - 1$ as ρ_x and $\rho_{x-1,}$ respectively.

iv.   Assume positions are also related by a constant c such that $\rho_x = c(\rho_{x-1})$. Note that this constant need not be fixed like Φ but can represent a choice of curvature as we show below.

v.   Assume the distances are members of a number field of 0 up to some integer n, that is, in between our rationally valued distance values, we have integer-valued distances from 0 to n.

vi.   Let us fix the value of c, the relational ratio between distances to 1/2. This essentially implies, for the convenience of our computations, that lower derivatives cover a larger distance than higher-ordered derivatives. On absorption of a previous term, the scalar value associated with a positional variable in our L function contraction, doubles. Let us also assume that we will be skipping odd-valued positions (odd numbers in the field) and only squashing into even-valued positions.

vii.   If we squash up all the terms less than the first even integer, 2, which is the first prime in the first function expansion, f, we now get the first new term as a doubled term: $2/2^p$. The necessary choice of 2 as the first index also numerically justifies fixing c at 2: if we assume a curve of constantly hyperbolic (see v. and vi. below for explicit reasoning behind this) curvature, that is, a sphere, over the numbers.

viii.   We could perform the same for the second even-valued derivative 4, except now the first integer has absorbed half its value. To obtain the same doubling effect as the first integer, we need to double the interval from $(2-0)$ to $2*(2-0)$ which is 2^2.

ix.   To obtain this integer-valued Euler product, we will need to double this interval as we progress down the chain infinitely, that is at every n^{th} index of our new operation, we will need to squash the previous 2^n elements into the current one.

x.   This rewritten product, in the end, now looks like: $(1/\ 1^p + 1/\ 2^p\ + \ldots + + 2/\ 2f_j{}^p \ldots)(\ 1/\ 1^p + 1/\ 2^p\ + \ldots\ + 2/\ 2f_j{}^p + \ldots)$ where $f_j \equiv f(j) \equiv j$ for all $j \in \mathbb{Z}^+$.

xi.   Notice that the use of 2 as the value of c is a transcendental usage (is algebraically independent) and is comparable to the use of e in the circle group with the n power mapping to $i\theta$ ($e^{i\theta}$). This now gives us a curve (of a section of the surface of a sphere with a hyperbolic generating group) which computes the above product. We notice that the first term in the second function expansion is wrong (should be 3 by definition) and so the nth term is also wrong (not a g term). Could this be fixed by altering the curvature for infinitely small sections so that the result is correct to a high numerical degree? Say, for example, to fix (remove) the first sub-term in each product term, we can fix an analog of $c = 4$. If such a valid mathematical process could be formulated, we would have a single definition of a curve that satisfies the Riemann hypothesis in an analytic/approximate sense. Another way to make this exact is to consider the original function not as just one curve but as a product of curves, infinitely countable perfect spheres with different curvature constants (one for each prime in a continuous p-adic fashion).

If like we have done, assume this to be a field of numbers and also take the valid convention of valuing satisfying solutions with positive numbers and negative solutions with negative numbers, we can conclude the following:

The L function is solved exactly on the inner product of the vector space of characteristic power vectors, which is quantitatively (half) a step from the p-norm of 1 or the smallest positive valuation, that is, where the real part of the power can be considered to have a fixed "half" value. When it is not satisfied, its solutions lay in the negatively valued space. This is especially supported by the 2 SAT conversion if we take it that the chain values are initialized at $1/2$ and the coefficients at $-1/2$ (consistent with the negative valuation of entropy), and we simply add a half value to both fields to compute final values of 0 $(\mathrm{symplectic})$ and 1 $(\mathrm{complex})$. In fact, one can formally write our nullstellensatz equation from [1] over the symplectic manifold for satisfiable formulae as:

$$-1/2(\omega) + \sum f_i g_i = 0$$

Or alternatively as:

$$-1/2(\omega) + \sum (f^n) (\partial^n \mathbf{g(x)} / \partial^n \mathbf{t}) = 0$$

Where ω is some relative unknown constant weight that is twice the sum of the multiplied (unknown and implicit) functions f and (derivations of) g.

Any finite 3 SAT formula can be rewritten as a finite 2 SAT formula and so the finite versions of both of these conjectures are true.

One can easily extend this argument to a function of infinite f's and g's, (on some infinite alphabet) that is, infinitely (sized) satisfiability formulae of infinitely many disjunctions and conjunctions, if such can be imagined as an $\infty\text{-}\mathrm{sat}$ equation. In that case, we assume the formula still has an infinite conversion to 3 SAT, and that the nature of zeroes will still hold. In that case the infinite versions of both the Riemann Hypothesis (approximate or exact depending on how we construe the function, as described) and the Hodge(exact) Conjecture then also hold.

While both of these formulations could be regarded as abstract and relativistic ***simulations*** of the original conjectures, the point is to show that computing with spin the way we do is powerful for calculating concrete values where needed (satisfying assignments) and also in providing a more homo-iconic frame of computation for better understanding the semantics of complex mathematical problems and their possible syntactic realizations and resolutions. For example, because of the arithmetical nature of our theorem and the way it mimics an étale structure, it can be used to investigate, say the Tate Conjecture in number theory, a conjecture with which it shares many other similarities like a twist in the coefficient space. In another possible use case, we can show that since we have a solid demonstration of automorphy, a morphism from a topological group onto the complex numbers, we can generally study things like automorphic forms, which generalize elliptic forms.

The approach taken here mirrors the development of Connes' work, beginning with an approach inspired by non-commutative geometry (reflected in our work) and culminating in the definition of a non-commutative scaling site along with Consani [12] (topos theory, see [1] for analogy) associated with a given arithmetic geometry[13] and associated scaling functions via this geometry[14]. This also very much ties in with Connes' et al work on Von Neumann algebras (of which ours can be considered a special complex case) and their scaling laws [15]. Connes' works very much used tropical semirings whereas in our case we use "tropicalized" rings, a benefit that comes from the (relativistic) entropy-based quantization of the fully invertible topological structure of our classifying space. As a case in point, linked to the spherical nature of these algebras, note that we also work with the same type of hyperbolic groups for which we can compute definite word distances. We hope that these correlations can spur more complete and formal work on these problems and others of similar complexity, extending our approach.

6 Conclusions

Holography promises to be an exciting juncture of fields. Understanding differential (differentially algebraic) spin will be key to this. In this paper, we have scratched the surface of what looks like a rich Differential Galois theory for what we may call a variant of automatic differentiation [16. 17, 18] implemented over multiple indices using a type of hyperdual differentiation [19]. We hope this will help boost this field as it is known to be a notoriously difficult one. Our approach to this topic follows the approach taken in [20] for strongly normal group extensions based on (differential) schemes. We add localization to this approach and also embed the scheme in a reflexively twisted quantum algebraic structure. The two groups identified, by way of their structure, can, as we stated in [1], be viewed as elements of a 2-sorted combinatorial species [21] along with the specified operations on the function space.

As elucidated in [1], another main thrust of this theory seems to be in shedding light on nature and intelligence via forms of mechanical reasoning. This is an area in which we wish to make contributions in the future using this holographic theory. We also followed up on [1] by expanding on the twisted (hyper) Kähler structure of the presented manifold. Our opinion is that the metric exposed on this form is a powerful and native form of natural perception. We also hope to take this idea further in future work.

Finally comes the question of quantum computation and how our procedure may help shed light on practical quantum computation. We hope to show in future work on information theoretic gravity that we have an equivalent of a quantum field computer [22] or rather, a quantum holographic computer, as the computation utilizes the holographic principle which studies the intersection of two quantum fields in the form of a complex scalar-tensor field theory.

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