SIZE MATTERS:
Impact Energy Absorption Across Five Decades of Length Scale

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Abstract

The Laser-Induced Particle Impact Test (LIPIT) can be used to probe projectile, target, and synergistic projectile-target responses under high strain rate deformation at the microscale. LIPIT’s advantages over other microscale launching techniques include the ability to controllably launch a single microparticle and precisely characterize the projectile momentum and kinetic energy before and after target impact. In addition, a LIPIT apparatus possesses a small laboratory footprint and is suitable for extension to high-throughput testing. Hence, LIPIT experiments have been used to study the dynamic response of many polymers, gels, and metals in different structural forms with various target thickness to projectile diameter ratios. These microscopic high-rate (>10⁶ s⁻¹) deformation behavior and impact energy absorption studies were used to infer deformation mechanisms, as well as to suggest promising materials for macroscopic applications. Geometric scale, however, can significantly influence dynamic material behavior through scale-induced changes in event time, strain rate, projectile/target material homogeneity, and more. In this study, such geometric-scale effects are intentionally investigated. Noncrystalline alumina spheres ranging five orders of magnitude in diameter (dₚ = 3 µm to 10 mm) were launched into scaled hₜ/dₚ amorphous polycarbonate targets at normal incidence using either LIPIT or a gas gun, depending on the scale. Projectile impact velocity and the projectile diameter to target thickness ratio were held constant in all experiments (vᵢ = 550 m/s and hₜ/dₚ = 0.25, respectively). Impact energies spanned from hundreds of Joules down to nanoJoules (eleven decades), representing the broadest range ever addressed in a single experimental impact study. Nominal target perforation times and strain rates varied by roughly three orders of magnitude. Length scale reduction resulted in a remarkable ~230% amplification in specific energy absorption (E_p*) and a ~240% increase in
relative impact deformation area. The fraction of target material engaged during impact more than doubled at the smallest scale. Estimated target ballistic limit velocities, scaling with $d_p^{-0.25}$, were roughly three times higher in the LIPIT experiments. The improved performance of targets at smaller scales likely stemmed from rate- and size-induced enhancements of yield and failure stress values, shifting the bulk failure mode from plugging to dishing. These findings demonstrate that material property discoveries made using emerging high-throughput methods (LIPIT, nanoindentation, laser-driven flyers, etc.) are not be directly indicative of macroscopic behavior and performance and suggest the need for scaling studies of each class of material.

**Keywords:** Laser induced particle impact test (LIPIT), Single-stage gas gun, Scanning electron microscopy (SEM), Optical microscopy, Specific energy absorption, Length scale, Strain rate, Polycarbonate, Alumina, Thin films, Profilometry, Laser confocal microscopy, Microspheres, Impact scaling, Geometric scaling, Elastic Plastic Impact Computation (EPIC) code

1. **Introduction and Motivation**

    Nature has many types of spectacular impacts, including collisions of galaxies transcending eons [1, 2], asteroid-planetary impacts triggering mass extinctions [3], biological predator events like the potent strikes of mantis shrimp against clam shells [4], and star-powering fusion of atoms lasting only zeptoseconds [5]. Similarly, human-engineered collisions play vital roles in asteroid/meteoroid redirection [6], construction and fabrication processes [7], hypersonic missile defense [8], kinetic energy weapon impacts [9], cold spray application [10], innovative drug delivery methods [11], and fusion power generation [12]. Investigations of impact dynamics and their outcomes fuel the development of ballistic armor [13], automobile collision passenger safeguards [14], and spacecraft micro-meteoroid/orbital debris shielding [15]. These impacts, driven by forces ranging from gravity to atomic interactions, release energy ranging from yottaJoules ($10^{24}$ J) to attoJoules ($10^{-18}$ J) and can transform kinetic energy into thermal, chemical, potential, and mechanical energy, activating material elastic or inelastic deformation, fracture, fragmentation, melting, vaporization, sublimation, ionization, fission, and fusion. Amplifying this complexity, the mechanisms and processes at play can change dramatically with spatial and temporal scales.

    Many impacts can be characterized by the projectile and target material properties, projectile diameter ($d_p$), target thickness ($h_t$), and impact velocity ($v_i$), provided that the target’s lateral dimensions are large enough to render sample boundary effects negligible. The following discussion focuses on a set pair of projectile and target materials. For a given target thickness to projectile diameter ratio ($h_t/d_p$), increasing $v_i$ shortens the primary event duration, which in turn raises the strain and heating rates. These rates are closely linked to the instantaneous material properties and phase [16]. Enlarging the spatial scale alone
effectively lowers strain and heating rates through a corresponding increase in event duration. Decreasing target scale has the opposite effect. For a given combination of $h_t$, $d_p$, $h_t/d_p$, there exists a critical velocity at which target perforation will occur, commonly termed $v_{50}$ [17]. Conversely, for a given combination of $v_i$, $h_t$, $d_p$, there is a critical geometric ratio, $(h_t/d_p)_{crit}$, that results in target perforation. When a rigid $d_p$ projectile strikes a target at $v_i$, a ratio $h_t/d_p \ll (h_t/d_p)_{crit}$ typically yields thin film perforation, which resembles axisymmetric membrane stretching and piercing [18]. Elevating $h_t/d_p$ to $(h_t/d_p)_{crit}$ alone causes a shift in impact behavior to ballistic limit perforation, characterized by target cratering, cracking, shear banding, spalling, bulging, dishing, petalling, and/or shear plugging [18, 19]. Semi-infinite penetration follows for $h_t/d_p \gg (h_t/d_p)_{crit}$, leading to extensive target penetration and cratering. Material hierarchical inhomogeneities, from macroscale aggregates to microscale crystal grains, result in spatial variations and orientations in material properties and failure mechanisms that complicate matters further [20].

Historically, scaled impact research has predominantly focused on ballistic impact and planetary science, with most attention given to composites, granular materials, and lightweight metals (see, e.g., [21–25]). However, these studies often rely on certain simplifying assumptions (fully hydrodynamic behavior, axisymmetry, etc.) and are only applicable to a relatively narrow range of potential impact scenarios (e.g., cratering with $h_t/d_p \gg (h_t/d_p)_{crit}$). Despite the prevalence of natural and artificial impacts, the knowledge of how energy transformation processes and dissipation mechanisms vary with length scale remains insufficient, particularly when transitioning from macro ($\sim 10^{-2}$ m) to micro ($\sim 10^{-6}$ m) material length scales. Such a gap in understanding must be addressed as high-throughput material characterization techniques transition to increasing smaller length scales [26–29].

At the microscale, executing and characterizing controlled impacts is fraught with challenges in experimental repeatability, measurement uncertainties, sample preparation, and stochastic variations in material properties. The Laser-Induced Particle Impact Test (LIPIT) has become an attractive method for probing microscale projectile, target, and synergistic projectile-target responses to high strain rate deformation [30–35]. LIPIT’s advantages over other microscale launching techniques include the ability to controllably launch a single microparticle with relatively low kinetic energy ($\sim$ nanoJoules) and precisely characterize the projectile momentum and kinetic energy before and after target impact. A LIPIT apparatus also has a small laboratory footprint ($\sim 1 \times 1$ m$^2$) and is suitable to employ in high-throughput testing (Fig. 1a) [36]. LIPIT experiments have been applied to investigate impact responses of polymers, gels, and metals, in various structural forms [35–42]. These prior studies have leveraged microscopic high-rate deformation, failure, and relative impact energy absorption results to identify promising materials for macroscopic applications. However, the specific energy absorbed in these microscopic events often exceeds macroscopic counterparts by at least an order of magnitude. For reference, Fig. 1b displays a comparison between the specific energy
absorption ($E_p^*$) values obtained from microscopic and macroscopic experiments, as reported in a previous review [36]. Some microscopically determined $E_p^*$ values [e.g., for multi-walled carbon nanotube (MWCNT) sheets] exceed macroscopic values for metals by over 2,300%! This stark difference underscores the need to exercise extreme caution when extrapolating microscopic material behavior, particularly at high rates, without a comprehensive grasp of length scale effects.

Studying scale-induced changes in impact phenomena requires careful consideration of factors such as the projectile and target materials and geometries, as well as the impact velocity. The high sensitivity of polymer mechanical properties to temperature, pressure, and test rate, combined with the extreme conditions created in ballistic experiments, results in significant gradients in temperature, strain, strain rate, and stress state. These gradients evolve both over time and location. Consequently, the ballistic deformation of polymers stands out as the most scale-sensitive scenario (temporally and spatially). For instance, at the microscale, target thickness reduction alone has been shown to enhance $E_p^*$ [43]. One systematic approach to isolating length scale influences involves the selection of macroscopically and microscopically homogeneous materials while holding the impact velocity and relative geometries of the projectile and target constant. Amorphous polycarbonate (PC) can be used to satisfy these criteria. PC is an important tough and transparent commercial thermoplastic with a broad range of applications in electronics, automobiles, construction, medicine, space exploration, and ballistic protection [44–50]. Moreover, PC inherently has just two material length scales—the root-mean-square distance between its chain ends (proportional to its
weight average molecular weight, \( M_w \)) and the average distance between its chain entanglements (inversely proportional to its entanglement molecular weight, \( M_e \)). In general, an increase in \( M_w \) for fixed \( M_e \) increases the number of entanglements per chain (\( N_e \sim M_w/M_e \)) but does not necessarily change the bulk volume density of entanglements (\( \nu_e \)).

For amorphous thermoplastic polymers such as PC, polystyrene (PS), and poly(methyl methacrylate) (PMMA), material yield stress is generally proportional to strain rate (\( \dot{\varepsilon} \)) and inversely proportional to temperature (\( T \)) [51]. These effects have been widely studied for glassy thermoplastics using various testing methods: universal tensile testing (\( \dot{\varepsilon} \sim 10^{-4} - 10^1 \text{ s}^{-1} \)), rheometry (\( \dot{\varepsilon} \sim 10^{-3} - 10^3 \text{ s}^{-1} \)), Split-Hopkinson tension/compression/torsion bar testing (\( \dot{\varepsilon} \sim 10^3 - 10^4 \text{ s}^{-1} \)), and Taylor impacts (\( \dot{\varepsilon} \sim 10^5 \text{ s}^{-1} \)) [51–53]. Test temperatures have spanned \(-150 \lesssim T \lesssim 200^\circ\text{C}\). Across this range of loading rates and temperatures, the mechanical behavior of glassy polymers can be attributed to their \( \alpha \) and \( \beta \) transitions occurring at \( T_\alpha \) and \( T_\beta \), respectively [52]. For these polymers, \( T_\alpha \) defines the glass transition temperature (\( T_g \)). For PC, \( T_\alpha \approx 147^\circ\text{C} \) and \( T_\beta \approx -100^\circ\text{C} \). In its glassy range (\( T < T_\alpha \)), PC’s tensile, compressive, and torsional yield stress all exhibit a linear increase with \( \log \dot{\varepsilon} \) at constant \( T \) and a linear decrease with \( T \) at a constant \( \dot{\varepsilon} \) (i.e., PC behavior is described by the Eyring theory) [52, 53]. These relationships deviate from linearity at \( T \approx -50^\circ\text{C} \) due to reduced backbone chain motion, leading to the observed \( \beta \) mechanical damping peak at \( T_\beta \). Polymers, unlike metals and ceramics, are quite sensitive to hydrostatic pressure, which typically raises their yield strength, bulk modulus, and \( T_g \). Post-yielding behavior in PC (\( T_g \sim 147^\circ\text{C} \)) results from a balance between adiabatic thermal softening and strain (and strain rate) hardening. At and above the yield stress, PC displays extensive visco-plastic flow, while other glassy polymers, like PMMA (\( T_g \sim 136^\circ\text{C} \)) and PS (\( T_g \sim 100^\circ\text{C} \)), undergo a ductile to brittle transition that is exacerbated by increasing strain rate [54]. PC maintains this ductility over a broad range of \( T \) and \( \dot{\varepsilon} \) due to its unusually low \( T_\beta \), contrasting with most thermoplastics that are brittle for \( T < T_g \). PC, however, can embrittle when subjected to ultra-high strain rates, ultra-low temperatures, or annealing.

For \( T < T_g \) and moderate to low strain rates, PC generally yields, flows, converts approximately 50–60% of plastic work to heat, and exhibits damage localization in the form of shear banding and crazing [55–59]. Above the glass transition, the material deforms homogeneously. Fracture occurs as crazes nucleate and break down into tensile cracks, which propagate as new crazes form at their tips. Despite its low thermal conductivity, PC’s enhanced toughness through deformation-induced molecular orientation generally prevents its fracture from heat-induced shear instability. When subjected to macroscale ballistic impact (\( \dot{\varepsilon} \lesssim 10^5 \text{ s}^{-1} \) and \( 0.1 < h_t/d_p < 1.8 \)), PC exhibits a variety of impact-driven failure modes, including dishing, petalling, cratering, cone cracking, and plugging depending on the \( h_t/d_p \) ratio [47]. The combined adiabatic heating from shocks and plastic work generates high temperatures, and this localized heating is
enhanced by PC’s low thermal conductivity. Elevated temperatures and ensuing visco-plastic flow during impact can potentially suppress or even “erase” evidence of prior, low temperature shear banding, crazing, and other failure mechanisms. Even so, under macroscale hypervelocity impact conditions (:\dot{\varepsilon} \sim 10^6–10^7 \text{s}^{-1}
 and \(0.8 < h_t/d_p < 3.2\), PC appears to fail more brittlely, hinting at a possible ductile-to-brittle transition with increasing strain rate [44]. Such a transition (if present) is influenced by projectile/target shapes, impact velocity, event duration, length scale, and more.

Although PC is extensively used and studied at the macroscale, it has been the subject of only one LIPIT study [50]. This previous work demonstrated that, for PC, increasing entanglement density (\(\nu_e\)) noticeably increases \(E_p^*\). PC, with its high entanglement density (\(\nu_e \approx 10^{26} \text{entanglements/m}^3\)), stands out from other glassy polymers like PS (\(\nu_e \approx 4 \times 10^{24} \text{entanglements/m}^3\)). A higher entanglement density (more entanglement junctions per unit volume) generally enhances material toughness and deformation resistance by forming an energy-absorbing network that restricts chain mobility, evenly distributes stress, enhances elasticity, and hinders crack growth [60–64]. Moreover, unlike other polymer glasses, PC’s high-rate mechanical behavior is less susceptible to adiabatic heating, potentially explaining its 81% higher LIPIT \(E_p^*\) value at \(v_i = 500 \text{m/s}\) compared to PS (cf. Fig. 1b) [65]. Such material features could be responsible for the nominal entanglement-driven increases in \(E_p^*\) reported in [50]. This previous study, however, did not compare LIPIT results with corresponding macroscale tests. In fact, a systematic experimental analysis of how length scale influences ballistic impact phenomena from macroscale to microscale has yet to be conducted for any material system.

In this study, rigid noncrystalline alumina spheres ranging five orders of magnitude in diameter (\(d_p = 3 \mu\text{m–}10 \text{mm}\)) were launched into PC targets of thickness \(h_t\) at normal incidence using either LIPIT or a gas gun, depending on the scale. The projectile impact velocity (\(v_i \approx 550 \text{m/s}\)) and the ratio of projectile diameter to target thickness (\(h_t/d_p \approx 0.25\)) were held fixed for all experiments. Impact energies span from hundreds of Joules down to nanoJoules, significantly expanding on previous relevant investigations [50, 66, 67]. The specific impact energy absorption (\(E_p^*\)), local plastic deformation, and deformation microstructures are compared across all considered scales. The decrease in length scale results in a significant increase in specific energy absorption and relative deformation area. Numerical predictions of PC impact behavior for short times are combined with the experimental results to show limitations in current continuum-based material modeling approaches. These preliminary observations show that length scale influences cannot be ignored when developing and employing emerging high-throughput experimental techniques that promise accelerated material discovery (e.g., LIPIT, laser-driven flyers, and nanoindentation). Moreover, macroscopically observed impact phenomena and accompanying theory/models may not translate well to the microscale, as a decrease in spatial scale for fixed \(h_t/d_p\) results in higher average strain and heating rates. In light of these
implications, this study seeks to explore the influence of geometric scale on impact phenomena.

2. Methodology: Scaling the Impact

Films of PC were perforated at $v_i \approx 550$ m/s by noncrystalline rigid alumina spheres 3, 10, 100, 500, 1,000, 4,000, 10,000 µm in diameter. Regardless of length scale, the ratio of target thickness to projectile diameter ($h_t/d_p$) was fixed at roughly 0.25. These $v_i$ and $h_t/d_p$ values guaranteed target perforation while preserving the projectile’s integrity. Maintaining $v_i$ and $h_t/d_p$ ensured consistent impact scenarios across scales. This effectively isolated the influence of key scale-dependent factors (strain rates, heating rates, projectile impact energy, target areal density, etc.) on PC’s deformation, failure, and specific energy absorption.

2.1. The Projectile Launching Techniques

The breadth of geometric scales probed in this study necessitated the use of two distinct projectile launching techniques. For the 3 µm and 10 µm diameter projectiles, a LIPIT apparatus in the Materials Microstructures and Properties Laboratory at Texas A&M University (TAMU) was employed [41, 42].1 The LIPIT technique involves using a laser pulse to ablate a gold film sandwiched between a glass substrate and a crosslinked polydimethylsiloxane (PDMS) film [32, 36, 40, 43, 68]. Before launch, microparticles were distributed on the elastomer on the side of the “launch pad” that faces away from the laser’s point of incidence (Fig. 2a). The launch pads were prepared as described in Ref. [42]. The laser pulse generates a plasma that expands and rapidly deforms the PDMS layer, propelling a projectile through lab air ($\sim 101$ kPa) at high velocity towards a target (Fig. 2b). The laser power can be adjusted to vary the projectile’s launch velocity. In general, microparticles with diameters ranging 1–50 µm can be readily accelerated to velocities surpassing 1 km/s using this technique. A dedicated study to probe how length scale and velocity influence impact phenomena together is ongoing.

The larger projectiles were launched using a single-stage gas gun (SSGG) and accompanying aeroballistic range located within the TAMU Hypervelocity Impact Laboratory (HVIL) [69].2 The SSGG launching mechanism operates using helium gas, initially contained in a high-pressure reservoir at pressures up to 34 MPa (Fig. 2c). Upon remote activation of a fast-acting valve, the He gas rapidly expands down the launch tube (barrel), accelerating a projectile towards its target (Fig. 2d). The SSGG can launch single projectiles ranging from 2.0–12.7 mm in diameter or clusters of particles down to 100 µm in diameter using a simultaneously launched distributed particle (SLDP) technique, to velocities ranging 0.1–1.5 km/s [70].

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1Material Microstructures and Properties Laboratory Website: https://elt.engr.tamu.edu/.

2TAMU Hypervelocity Impact Laboratory Website: https://telacyjr.engr.tamu.edu/facility/hypervelocity-impact-laboratory-hvil/.
Figure 2: Projectile launching devices employed in this study: (a, b) a laser-induced particle impact test (LIPIT) apparatus and (c, d) a single-stage light gas gun. Both devices are shown in (a, c) pre- and (b, d) post-launch configurations. The LIPIT schematic was adapted from Ref. [36].

An aside, launch velocities up to 8 km/s can be achieved using the HVIL two-stage light gas gun [69]. For the SSGG technique, a single projectile or collections of projectiles were loaded into a four-piece spherical cavity sabot. During free-flight through a tank containing lab air (~101 kPa), the sabot segments radially separated from the projectile(s). The projectile(s) then passed through an annular steel plate, which halted the sabot. Single 4 mm and 10 mm spheres were launched in an appropriately sized sabot, while the SLDP technique was used to launch the 100, 500, and 1,000 µm diameter particles to increase the likelihood of observable target impact (increase experimental success). The lab air conditions used to separate the sabot segments also induced radial SLDP dispersion via aerodynamic forces. This launch process resulted in a well-distributed impact site pattern on the corresponding PC thin film, with individual sites being far enough apart to be considered as separate impacts (nearest neighbor impact site spacing \( \gg 10d_p \) [70].
2.2. Materials and Sample Preparation

The PC target materials were sourced from Plaskolite, LLC. \( \text{TUFFAK®; } h_t: 1,016 \text{ µm (PC1)}^{1016} \text{ and } 2,380 \text{ µm (PC1)}^{2380} \) [71] and Rowland Advanced Polymer Films \( \text{RowTec®; } h_t: 127 \text{ µm (PC2)}^{127} \text{ and } 254 \text{ µm (PC2)}^{254} \) [72]. Differential scanning calorimetry (DSC) curves for both as-received PC1 and PC2 samples show the materials have very similar glass transitions and glass transition temperatures \( (T_g = 148–153°C) \) (Fig. 3a). Both materials had a mass density of \( \rho_t = 1.20 \text{ g/cc} \) and key quasi-static mechanical properties that differed by less than 3% (see Supplementary Information, Sect. SI.3). The polymers’ molecular weight averages \( (M_n: \text{number average, } M_w: \text{weight average, } M_z: \text{Z-average}) \) and molecular weight distribution \( (PDI = M_w/M_n: \text{polydispersity}) \) were measured using a TOSOH Ambient Temperature gel permeation chromatography (GPC) instrument with tetrahydrofuran (THF) as the solvent and polystyrene as the calibration standard. Both commercial materials had similar chromatographs: (PC1) \( M_n = 27,737 \text{ g/mol, } M_w = 55,711 \text{ g/mol, } M_z = 87,908 \text{ g/mol, } PDI = 2.01 \) and (PC2) \( M_n = 28,927 \text{ g/mol, } M_w = 55,876 \text{ g/mol, } M_z = 87,749 \text{ g/mol, } PDI = 1.93 \) (see Table S3 in Supplementary Information, Sect. SI.4).

The sample preparation technique was dependent on the film thickness. Targets with \( h_t = 127, 254, 1,016, \) and \( 2,380 \) µm were cut directly from the as-received PC1 or PC2 material and placed between two apertured steel fixture plates (Fig. 3b and 3c). The 30 µm thick samples (PC2 \( 30 \)) were created by heated compression of the PC2 \( 127 \) material. Given the small in-plane dimensions of the resulting targets (around 25.4 mm), a custom fixture with a grid pattern was created to hold multiple square thin films (Fig. 3d). This arrangement enhanced the likelihood of target impact during a given SLDP launch, thereby facilitating more extensive post-impact analysis. For LIPIT samples, the PC2 \( 127 \) material was dissolved using 50/50 dichloromethane/toluene, then deposited on a silicon wafer attached to a spin coater. High-rate rotation and solvent evaporation yielded roughly 0.75 µm (PC2 \( 0.75 \)) and 2.25 µm (PC2 \( 2.25 \)) thick films, which were placed on a transmission electron microscopy (TEM) grids serving as LIPIT target fixtures (Fig. 3e). For all macroscopic SSGG experiments, the target assembly (target plus fixture) was centered on the launch tube (impact) axis. Translation stages were used as part of the LIPIT apparatus to identify a suitable projectile on the launch pad and impact site on the target inside a TEM grid square. In all experiments, target boundaries were far enough from the impact point to prevent in-plane reflected waves from affecting penetration/perforation dynamics \( (i.e., \text{ perforation times were less than } \sim 10\% \text{ of wave travel times; see Supplementary Information, Sect. SI.2}).\)

Alumina \( (\text{Al}_2\text{O}_3) \) was chosen as the projectile material due to its relative homogeneity across the given length scales. The noncrystalline spheres were sourced from various commercial vendors depending on the diameter \( (d_p) \): Huake Scientific Research Materials Co., Ltd. \( (d_p = 3 \text{ µm and } 10 \text{ µm}), \) Corpuscular Microspheres-Nanospheres \( (d_p = 100 \text{ µm}), \) Goodfellow Cambridge Ltd. \( (d_p = 500 \text{ µm and } 1,000 \text{ µm}), \) and
Figure 3: An overview of the polycarbonate (PC) targets: (a) a representative differential scanning calorimetry curve for each as-received target material and (b) the target preparation process for each film thickness, $h_t$. Two materials were used in target preparation: Plaskolite, LLC. TUFFAK® (PC1; $h_t = 1.02$, 2.38 mm) and Rowland Advanced Polymer Films RowTec® (PC2; $h_t = 0.75$, 2.25, 30, 127, 250 µm). All material had similar densities ($\rho$) and glass transition temperature ($T_g$) values. Targets were fixed (c) between apertured plates ($h_t = 127$, 254, 1,016, 2,380 µm), (d) in an array on a custom grid ($h_t = 30$ µm), or (e) on a 200 mesh TEM grid ($h_t = 0.75$, 2.25 µm).

Kyocera Corporation ($d_p = 4,000$ µm and 10,000 µm). For reference, Fig. 4 shows representative images of the projectiles at each length scale captured with either an optical camera (Figs. 4a and 4b) or Thermo Fisher Helios NanoLab 660 dual-focused ion beam-scanning electron microscope operating with an incident beam energy and working distance of 1 kV and 4 mm, respectively (Figs. 4c–4g). In this figure, the projectile diameter ($d_p$) decreases from left to right and top to bottom: (a) 10 mm, (b) 4 mm, (c) 1 mm, (d) 500 µm, (e) 100 µm, (f) 10 µm, and (g) 3 µm. The spheres displayed consistent sphericities and diameters even at the finest length scale, maintaining relative uniformity and comparability across the impact experiments. With a density of $\rho_p = 3.95$ g/cc, the projectiles varied in mass from $m_p = 5.6 \times 10^{-11}$ g ($d_p = 3$ µm) to $m_p = 2.1 \times 10^0$ g ($d_p = 10$ mm). To underscore the extensive range of scale, the given mass values were used to calculate the projectile impact kinetic energy via $E_i = 1/2 m_p v_i^2$, assuming $v_i = 550$ m/s and neglecting rotational kinetic energy (Fig. 4h). The ballistic impacts presented in this work span eleven orders of magnitude in kinetic energy ($\sim 10^{-9} - 10^{2}$ J), representing the broadest range ever addressed in a
Figure 4: Micrographs showcase the alumina projectiles used in the impact experiments with the following diameters ($d_p$ decreases from left to right): (a) 10 mm, (b) 4 mm, (c) 1 mm, (d) 500 µm, (e) 100 µm, (f) 10 µm, and (g) 3 µm. For reference of scale, the (h) impact kinetic energy ($E_i = \frac{1}{2} m_p v_i^2$) for each of these $d_p$ values is plotted, assuming impact velocities of $v_i = 550$ m/s and projectile masses of $m_p = \frac{1}{6} \pi d_p^2 \rho_p$, where $\rho_p = 3.95$ g/cc is the projectile density.

Prior to each experiment, the target thickness was measured using either a digital caliper ($h_t > 100$ µm) or a Keyence VK-X3000 Three-Dimensional (3D) Surface Profiler laser confocal microscope ($h_t < 100$ µm).\(^3\) In addition, the Keyence instrument was used to inspect the films for any obvious defects (impurities, cracks, etc.), significant thickness variations, and excessive surface roughness. Similarly, the diameter of each projectile was measured using either calipers, optical microscopy, or scanning electron microscopy (SEM), depending on the diameter. These steps were essential to ensure that the $h_t/d_p$ ratio was largely maintained and that target inhomogeneities were minimized.

LIPIT thin films targets varied slightly in thickness due to inherent variability in the spin coating process, affecting the $h_t/d_p$ ratios. However, they remained near the intended value of 0.25. Similarly, thickness discrepancies in samples around $h_t = 30$ µm, made via compression molding, altered the $h_t/d_p$ ratio. Residual internal stresses in the PC targets caused by material manufacturing and processing might affect impact energy absorption and deformation, particularly at smaller scales. The spin-coating and compression-molding sample preparation techniques, however, did not introduce any noticeable material anisotropy, as evidenced by cross-polarizing microscopy (refer to Supplementary Information, Sect. SI.5).

\(^3\)Laser confocal and scanning electron microscopy were performed at the TAMU Small Scale Mechanical Behavior Laboratory.
A further point to consider is the potential introduction of molecular anisotropy in the LIPIT samples resulting from their notably thin structure \( (h_t = 0.75, 2.25 \, \mu m) \). Such anisotropy could alter dynamic material behavior and failure in a way that affect ballistic performance. The influence of molecular anisotropy, however, was unlikely due to the substantial size difference between the average PC molecule and even the thinnest target. For instance, the volume of the \( h_t = 0.75 \, \mu m \) target material beneath the projectile before impact \( \left( \pi/4 d_p^2 h_t \right) \) was \( \sim 100,000 \) times greater than the volume of a sphere enclosing the average PC chain as defined by its root-mean-square end-to-end distance (see Supplementary Information, Sect. SI.1) [73]. Furthermore, while surface effects can depress \( T_g \) and \( \nu_e \) as films become thinner, these effects have a negligible influence on the bulk material \( T_g \) for linear glassy polymers until film thicknesses reach \( h_t \approx 100 \, \text{nm} \) (\( \sim 87\% \) thinner than the thinnest \( h_t = 750 \, \text{nm} \) film) [74, 75]. These considerations show that the molecular length scale was much smaller than the target thickness and projectile diameter and thus did not have a significant effect target energy absorption.

2.3. Quantifying Specific Energy Absorption and Normalized Deformation Area

For the more macroscopic experiments, a high-speed camera captured high-contrast, shadowgraphic images of the impact event. SSGG experiments employed a Shimadzu HPV-X2 camera, operating at a frame rate of 250 kHz and exposures ranging between 1000–1500 ns [76]. Light from high-intensity LED arrays, passed through a diffuser box, was used to illuminate the event [77]. A 50 mm lens was fitted onto the Shimadzu camera for the 1, 4, and 10 mm diameter particle impact experiments, while an Infinity K2 Distamax long distance microscope with a CF2 objective was used for the other tests. The microscopic LIPIT experiments were captured at a rate of 1 GHz and exposure of 5 ns using a Specialized Imaging SIMX camera [78]. A collimated laser served as the source of illumination for these tests. High-speed images of each projectile, taken just before impact, were analyzed alongside OM and SEM micrographs to verify its diameter. These diagnostic setups ensured consistent, comparable, and high-contrast shadowgraphic images across all scales [41, 69]. Figure 5 showcases annotated schematics of the projectile, target, launch technique, and the diagnostic tools, sequentially arranged in descending order from left to right. This simple, high-level representation of the scaled experiments highlights their uniformity in impact scenarios and diagnostic setups.

The impact conditions and relative projectile/target material properties ensured negligible projectile deformation and no projectile fragmentation occurred for all experiments. Each alumina projectile impacted its corresponding PC target with velocity \( v_i \), remained intact during target perforation, and excited with a residual velocity \( v_r \) (Fig. 6a and 6b). Hence, the in-situ images also facilitated the measurement of \( v_i \) and \( v_r \) using open-source motion tracking softwares, such as Tracker [79] and ImageJ [80]. These velocity
measurements were then used to estimate the energy absorption of the target material, given by

\[ E_p = \frac{1}{2} m_p v_i^2 - \frac{1}{2} m_p v_r^2 - E_{\text{drag}}, \]  

(1)

where \( m_p \) is the mass of the projectile and \( E_{\text{drag}} \) is projectile kinetic energy loss due to drag, which was nonnegligible for the microsphere impacts and calculated using methods described in the supporting information of Refs. [43, 81]. The energy absorption can be normalized by the mass of an ideal plug of target material “ejected” by the projectile during target perforation (Fig. 6c) [40, 42, 50]. This nominal specific energy absorption facilitates ballistic performance comparisons across various materials and geometric scales and is given by

\[ E^* = \frac{E_p}{m_{\text{plug}}}, \]  

(2)

where \( m_{\text{plug}} = A_p \rho_t h_t \), \( A_p = \pi / 4 d_p^2 \) is the projected area of the projectile, \( \rho_t = 1.20 \text{ g/cc} \) is the mass density of the PC target material, and \( h_t \) is the thickness of the target. These calculations were instrumental in assessing a given target’s impact energy dissipation in a way that was comparable across length scales and to relevant \( E^* \) values reported in the literature (see, e.g., [50]).

After each experiment, optical micrographs of the perforations and out-of-plane deformation on both the impact and exit sides of the target were captured using a Keyence VK-X3000 3D Surface Profiler. The effective deformation diameter \( (D) \) of an in-plane circular region containing permanent deformation
Figure 6: Simplified diagrams illustrate the projectile: (a) approaching the target at time $t_0$ with velocity $v_i$ and (b) after perforating the target at time $t_f$ with a residual velocity $v_r$. Diagram (c) represents the method for normalizing the energy absorption of the target, $E_p$, using the mass of a conceptualized “ejected” material plug (highlighted with crosshatching); the plug mass is defined as $m_{plug} = \pi/4 \rho h t d_p^2$. The final diagram, (d), depicts the normalized deformation area concept, expressed as $A_d/A_p = D^2/d_p^2$, where $D$ is the diameter of the circle defined by $A_d$.

was calculated as twice the in-plane radius ($R$) from the center of impact to the radial location of $\sim 5\%$ permanent (residual) vertical ($z$) deflection of the target’s impact surface, i.e.

$$R = r : \frac{|z_t - z_p(r)|}{z_t} \approx 5\%,$$

where $z_t$ is the constant vertical height of the undeformed target impact surface, $z_p(r)$ is the height of the deformed target impact surface, and $r$ is the radial coordinate (Fig. 6d).\(^4\) This diameter was used to characterize the normalized deformation area,

$$\frac{A_d}{A_p} = \frac{D^2}{d_p^2},$$

consistent with definitions used in the literature [50]. One-dimensional (1D) deformation profiles collected at each scale were normalized by the projectile diameter allowing for direct comparisons of the relative deformation for each test. SEM imaging of the LIPIT perforations on both the target impact and exit surfaces was performed using the ThermoFisher Helios NanoLab 660 dual-focused ion beam-scanning electron beam microscope used for inspecting the alumina spheres (1 kV operating voltage, 4 mm working distance). Hence, the variations in deformation behavior and bulk failure mechanisms across length scales were probed. An overview of the experimental workflow is summarized in Fig. 7.

\(^4\)A deflection value of approximately 5% was chosen as it loosely marks the threshold for detectable deflection without considerable influence from noise in the profilometry data.
3. Results and Discussion for the Scaled $h_t/d_p$ Impacts

This section highlights essential findings from the scaled impact experiments, including specific energy absorption ($E_p^*$) and normalized deformation area ($A_d/A_p$) measurements, as well as representative optical microscopy images and profilometry scans of each perforation on both the front and back surfaces of each target. SEM micrographs of the LIPIT perforations are also presented. $E_p^*$ and $A_d/A_p$ are also compared to calculated values from the Elastic Plastic Impact Computation code (EPIC) simulations across all considered length scales (see Supplementary Information, Sect. SI.7). The constant $h_t/d_p$ impact experiments were performed at seven distinct length scales, which can be described by the diameter of the alumina projectiles: $d_p = 3, 10, 100, 500, 1,000, 4,000,$ and $10,000 \mu m$. For each scale, at least three tests were run to establish average data points and corresponding standard deviations, which tended to increase at smaller scales.

3.1. Spatial Scale Dependency of Specific Energy Absorption and Normalized Deformation Area

Regardless of length scale and launch technique (SSGG, SSGG + SLDP, or LIPIT), the projectile’s motion was captured using high-speed shadowgraphy. Figure 8 shows in descending order representative high-speed images captured before (Figs. 8a, 8c, 8e, 8g, 8i, 8k, 8m) and after (Figs. 8b, 8d, 8f, 8h, 8j, 8l, 8n) each scaled impact event by either the HPV-X2 ($d_p = 100, 500, 1,000, 4,000, 10,000 \mu m$) or SIMX ($d_p = 3$ and $10 \mu m$) camera. Yellow arrows and red dashed lines superimposed on the images highlight the direction of projectile motion and relative target thickness inside/on the corresponding fixture, respectively. The projectile impact velocity ($v_i$) and residual velocity ($v_r$) were determined for all experiments by tracking its horizontal motion across multiple shadowgraphs. The launching technique, relative length scale, and average impact and residual velocities are also shown in Fig. 8 for reference. Variations in $v_i$ at a given length scale were
relatively minor (\(<10\%\)). The key experimental parameters (\(d_p, v_i, v_r, h_t, h_t/d_p, E_p^\ast\), and \(A_d/A_p\)), as well as the launch apparatus used at each length scale, are summarized in descending order in Table 1.

The specific energy absorption \([E_p^\ast, \text{Eq. (2)}]\) of each scaled PC target was calculated for all experiments using the measured \(v_i\) and \(v_r\) values provided in Table 1. The variations in experimental \(h_t\) and \(v_r\) values, coupled with measureable deviations in \(d_p\), likely explain the standard deviations in \(E_p^\ast\) and \(A_d/A_p\) at a given scale, particularly the notable increase in standard deviations with decreasing \(d_p\). A heightened sensitivity of the target material to velocity changes occurs as \(h_t\) decreased due to increasing strain and heating rates. This could also partially explain the observed higher standard deviations. Under the assumption of constant projectile-target homogeneous materials, relative geometries, and impact conditions, \(E_p^\ast\) would be expected to remain constant in the absence of any scaling effects. Contrarily, \(E_p^\ast\) exhibited a dramatic increase with a decrease in scale, ranging from approximately 0.4 MJ/kg for the \(h_t = 2.38\, \text{mm}\) (thickest) target to as high as \(\sim 1.25\, \text{MJ/kg}\) for the \(h_t = 0.75\, \mu\text{m}\) (thinnest) target (a roughly 230% increase; cf. Table 1). The specific energy absorption is plotted in Fig. 9a for each geometric scale, demonstrating an inverse power law relationship with projectile diameter \((E_p^\ast \propto d_p^{-0.16})\). The error bars on each data point represent the standard deviation in \(E_p^\ast\) provided in Table 1. As an aside, the LIPIT \(E_p^\ast\) measurements align well with previously reported values for \(v_i \approx 500\, \text{m/s}\) PC film impacts, though with reduced \(h_t/d_p \approx 0.04\) ratios and \(26,000 < M_w < 59,000\, \text{g/mol}\) [50]. The notable rise in \(E_p^\ast\) with decreasing scale shown in Fig. 9a emphasizes that without proper physical understanding and scaling laws, microscopic impact phenomena cannot be directly extended to the macroscale or visa versa.

Post-impact characterization and measurements often help unravel in-situ material behavior and failure. The effective deformation area \((A_d; \text{cf. Fig. 6d})\) provides a simple estimation of the extent to which the target material was engaged and subsequently affected (through permanent deformation or failure) by the impact event. To establish a comparison baseline that spans various length scales, this area was normalized by the projected area of the projectile \([A_d/A_p; \text{Eq. (4)}]\). Assuming no scaling effects, the

Table 1: A summary of the scaled impact experiments performed in this study. A minimum of three experiments were performed at each length scale to quantify the effects of impact velocity and target thickness variations on specific energy absorption \((E_p^\ast)\).

<table>
<thead>
<tr>
<th>No.</th>
<th>(d_p) ((\mu\text{m}))</th>
<th>(v_i) ((\text{m/s}))</th>
<th>(v_r) ((\text{m/s}))</th>
<th>(h_t) ((\mu\text{m}))</th>
<th>(h_t/d_p)</th>
<th>Launch Apparatus</th>
<th>(E_p^\ast) (MJ/kg)</th>
<th>(A_d/A_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000±2.5</td>
<td>561±29</td>
<td>482±29</td>
<td>2,380±130</td>
<td>0.21±0.01</td>
<td>SSGG</td>
<td>0.38±0.02</td>
<td>1.33±0.00</td>
</tr>
<tr>
<td>2</td>
<td>4,000±2.5</td>
<td>537±20</td>
<td>437±20</td>
<td>1,016±25</td>
<td>0.25±0.01</td>
<td>SSGG</td>
<td>0.42±0.02</td>
<td>1.70±0.00</td>
</tr>
<tr>
<td>3</td>
<td>1,000±2.5</td>
<td>539±4</td>
<td>434±4</td>
<td>254±25</td>
<td>0.25±0.03</td>
<td>SSGG SLDP</td>
<td>0.45±0.01</td>
<td>1.82±0.01</td>
</tr>
<tr>
<td>4</td>
<td>500±2.5</td>
<td>529±4</td>
<td>385±4</td>
<td>127±13</td>
<td>0.25±0.03</td>
<td>SSGG SLDP</td>
<td>0.57±0.01</td>
<td>2.15±0.02</td>
</tr>
<tr>
<td>5</td>
<td>100±2.5</td>
<td>523±47</td>
<td>251±47</td>
<td>30±5</td>
<td>0.30±0.05</td>
<td>SSGG SLDP</td>
<td>0.77±0.16</td>
<td>2.66±0.13</td>
</tr>
<tr>
<td>6</td>
<td>10±0.80</td>
<td>522±42</td>
<td>207±42</td>
<td>2.25±0.25</td>
<td>0.23±0.04</td>
<td>LIPIT</td>
<td>1.12±0.15</td>
<td>3.92±0.62</td>
</tr>
<tr>
<td>7</td>
<td>3±0.32</td>
<td>535±34</td>
<td>38±34</td>
<td>0.75±0.05</td>
<td>0.25±0.04</td>
<td>LIPIT</td>
<td>1.25±0.24</td>
<td>4.81±1.01</td>
</tr>
</tbody>
</table>

\(d_p\), projectile diameter; \(v_i\), projectile impact velocity; \(v_r\), residual velocity; \(h_t\), target thickness; \(h_t/d_p\), target-thickness-projectile-diameter ratio; \(E_p^\ast\), specific energy absorption [Eq. (2)]; \(A_d/A_p\), normalized deformation area [Eq. (4)].
Figure 8: High-speed images capture the impact events: the alumina projectiles in images (a, c, e, g, i, k, m) are depicted prior to impact, and those in images (b, d, f, h, j, l, n) are shown after target perforation. Each row in the image array corresponds to a specific projectile diameter \(d_p\), which decreases sequentially from the top to the bottom. The target films are highlighted using red dotted lines for enhanced visibility (the dark areas around the films indicate target fixtures). Yellow arrows show the direction of projectile motion. Information provided alongside each series details the launching method (SSGG, SSGG + SLDP, or LIPTT), the impact velocity \(v_i\), residual velocity \(v_r\), and an accompanying scale.

normalized deformation area would remain largely unchanged. Interestingly, however, measurements taken from optical microscopy and laser confocal micrographs indicate a similar trend between the \(A_d/A_p\) and \(E_p^*\),
both increasing dramatically with decreasing length scale. In fact, the normalized deformation area rises by a factor of three from the largest \((d_p = 10 \text{ mm}, h_t = 2.38 \text{ mm})\) to the smallest \((d_p = 3 \mu \text{m}, h_t = 0.75 \mu \text{m})\) scale (cf. Table 1). Similar to \(E_p^\ast\), the LIPIT \(A_d/A_p\) values are consistent with the results of Chan et al. [50], who found that at the microscale, PC’s \(E_p^\ast\) rises with entanglement density and deformation area. As shown in Fig. 9b, \(A_d/A_p\) follows an inverse power law trend with projectile diameter \((A_d/A_p \propto d_p^{-0.16})\). Hence, as the length scale decreases, a larger proportion of the target material undergoes deformation and failure. Notably, the scaling exponents for \(A_d/A_p\) and \(E_p^\ast\) are virtually the same.

An alternative approach to normalizing impact energy absorption \((E_p)\) is by employing the deformation area rather than the projectile’s projected area \([i.e., E_p^{**} = (A_p/A_d)E_p^\ast]\). Interestingly, if 5% is chosen as the deflection threshold (see Eq. 3), the \(E_p^{**}\) values remain largely constant across the different scales (see Fig. 9a; dash-dot line). These findings reveal that \((i)\) relative target features resulting from impact-induced deformation and failure lack some consistency across length scale, even when the projectile/target materials and geometries and the impact velocity are unchanged, and \((ii)\) the added volume of target material undergoing deformation as length scale decreases \([A_d - A_p]h_t\) is responsible (through plastic work, heating, etc.) for the rise in \(E_p^\ast\)
3.2. Micrographs and Normalized Cross-Sectional Profiles of the Scaled Perforations

Post-experiment, the impacted PC samples were immediately sealed at room temperature to preserve the deformation and minimize contamination. Subsequent examinations of the impact and exit surfaces were conducted using OM ($d_p \geq 100 \ \mu m$) and SEM ($d_p \leq 10 \ \mu m$), depending on the length scale (cf. Fig. 7). Figure 10 offers a visual comparison of the damage from alumina sphere impacts of diameters $d_p = 10, 4, 1, 0.5 \ \text{mm}$, arranged in descending order. Figures 10a, 10c, 10e, and 10g illustrate the impact sides of the perforated films, and Figs. 10b, 10d, 10f, and 10h depict the exit sides. Each image includes a superimposed yellow dashed circle representing the projectile’s projected diameter, and a scale bar positioned in the bottom right corner. The microscopy technique is provided in the bottom left corner of each image. Micrographs of the remaining perforations ($d_p = 100, 10, 3 \ \mu m$) are displayed in Fig. 11 using the same layout.

In all experiments, the deformed target area exceeded the projectile’s projected area ($A_d/A_p > 1$; cf. Fig. 9b). The effective diameter of the perforation hole/opening was consistently smaller than that of the projectile, suggesting a degree of hole closure post-perforation, akin to material “self-healing” [82, 83]. Notably, the perforations from projectiles of 4 mm (Figs. 10a and 10b) and 100 $\mu m$ (Figs. 11a and 11b) diameters seemed completely closed, possibly due to slightly larger $h_t/d_p$ ratios. Minimal out-of-plane deformation was observed on the impact face, in contrast to the significant deformation on the exit face across all considered length scales. Signs of jetting, the ejection of material from the impactor-target contact point, were present (Figs. 10e–10h), and there was little to no evidence of large-scale crazing or cracking. The apparent absence of these features might be due to adiabatic heating elevating temperatures above $T_g$, leading to substantial visco-plastic flow that smoothed out such localized deformation features. In fact, evidence of material flow was present in all micrographs. A slightly higher degree of surface roughness was present in the $h_t = 30 \ \mu m$ target sample ($d_p = 100 \ \mu m$) due to the compression molding process (Figs. 11a and 11b); however, with nominal peak-to-valley distances being less than $0.01d_p$, the experimental results were likely unaffected. Although the geometry and regularity of the perforations appear to vary with scale, the micrographs do not show any obvious changes in primary failure mechanisms or material behavior, such as ductile to brittle transitions. Figures 10 and 11, however, do visually demonstrate the growth of the deformation area ($A_d$) relative to the projectile’s projected area ($A_p$) as the length scale decreases.

Isometric SEM images at a 48° angle from the film plane were also taken of the same $d_p = 10 \ \mu m$ (Figs. S5c and S5d) and $d_p = 3 \ \mu m$ (Figs. S5g and S5h) projectile perforations to better highlight the perforation geometry and deformation features not evident in the normal SEM images. The relatively large conical perforation geometries seen in the isometric scans were characteristic of the LIPIT samples (more later) and are in general agreement with previous LIPIT results for PC thin films [50].
Figure 10: Orthogonal micrographs (along impact axis) display perforations in the PC targets caused by impacts from alumina spheres with diameters of (a, b) \( d_p = 10 \text{ mm} \), (c, d) \( d_p = 4 \text{ mm} \), (e, f) \( d_p = 1 \text{ mm} \), and (g, h) \( d_p = 0.5 \text{ mm} \). Within the array of images, the left column (a, c, e, g) represents the impact face of the target, whereas the right column (b, d, f, h) depicts the exit face. On each micrograph, a 2D projection of the projectile is overlaid, indicated by a dashed yellow line, to serve as a point of reference. The microscopy technique is provided in the bottom left of each image.
Another objective of this study was to probe scale-induced changes in perforation geometry. The OM and SEM images in Figs. 10 and 11 offer only limited quantitative data on the cross-sectional perforation geometries, so laser confocal microscopy was employed to collect profilometry data on both the front and back surfaces of the perforations at each length scale. To facilitate comparison and visualization, the front and back side 1D profiles were normalized using the projectile diameter \( d_p \) and shifted by the appropriate average \( h_t/d_p \) ratio (cf. Table 1). This normalized representative two-dimensional (2D) cross-sectional profiles, as shown in descending order in Fig. 12. These cross sections are annotated with corresponding \( d_p \) and \( h_t \) values.
and highlighted with distinct colors. Matching microscopy images (cf. Figs. 10 and 11) are provided for reference on the right, with the imaging method noted at the top left of each micrograph. A scaled projectile profile \((d_p/d_{p0} = 1)\) moving downward at velocity \(v_r\) is depicted at the bottom of the plot, illustrating that the normalized effective deformation area increases as the length scale decreases \((A_d/A_{d0} \propto d_p^{-0.16})\).

The figure also clearly shows that the perforation openings are consistently smaller than the projectile. Interestingly, the thicker specimens exhibit signs of plugging failure behavior, while the thinner specimens seem to undergo more bulk bending or “membrane-like” deformation (dishing) [47]. This apparent transition in predominant target failure mode is consistent with the observed rise in \(E_p^\ast\) (cf. Fig. 9a). Dishing involves proportionally more target material in plastic deformation than plugging, which is typically more localized and less deforming (more later).

3.3. The Effect of Size on Target Perforation Time, Strain Rate, and \(v_{50}\)

In the impact experiments, reducing the spatial scale alone dramatically increased the nominal target strain and heating rates due to a corresponding decrease in the duration of the penetration/perforation event. Specifically, the nominal target perforation time, \(t_p \approx 2h_t/(v_i + v_r)\), decreased from about 10 \(\mu\)s to 10 ns (by \(~1,000\) times!) when transitioning from the macro- to microscale (Fig. 13a). This temporal reduction potentially inhibited some failure mechanisms from contributing to smaller-scale energy absorption, while activating or enhancing the contribution of others (through added strain/strain rate hardening, thermal softening, etc). One simple approach to estimating the nominal strain rate within the thin targets involves using an analytical expression, such as that derived by Lee et al. [32]:

\[
\dot{\varepsilon} = \frac{t_p}{2} \left( \frac{v_i}{R_c} \right)^2,
\]

where \(R_c \approx v_c t_p\) is the cone radius, \(v_c \approx 1.23c_{{\parallel}}[v_i/(\sqrt{2}c_{{\parallel}})]^{2/3}\) is the cone velocity [84], and \(c_{{\parallel}} \approx 1,400\) m/s is the in-plane wave speed. Calculated \(\dot{\varepsilon}\) values, also plotted against projectile diameter in Fig. 13a, show a roughly 1,000-fold rise \((\sim 10^5\) to \(\sim 10^8\) s\(^{-1}\)) from the largest to smallest length scale. As an aside, for \(d_p \lesssim 500\) \(\mu\)m, the \(\dot{\varepsilon}\) values exceeded the strain rate commonly used to define the transition from macroscale terminal ballistic to hypervelocity impact regimes \((\sim 10^6\) s\(^{-1}\)).\(^5\) These extreme strain rates undoubtedly affected the target’s local, instantaneous yield stress \((\sigma_{y,t})\) and failure stress \((\sigma_{f,t})\). Previous studies have shown that PC’s yield stress exhibits a bilinear relationship with \(\log(\dot{\varepsilon})\), roughly doubling as the strain rate

\(^5\)The shift from the terminal ballistic regime to the hypervelocity impact regime can be defined by criteria like sonic thresholds, initial to complete melting conditions, or characteristic strain and heating rates of the projectile or target. At the macroscale, a common nominal impact-induced target strain rate used to characterize the transition to the hypervelocity impact regime is \(\sim 10^6\) s\(^{-1}\) (i.e., \(\dot{\varepsilon}_{\text{crit}} \sim 10^6\) s\(^{-1}\)). The impact velocity corresponding to \(\dot{\varepsilon}_{\text{crit}}\) \((v_{\text{crit}})\) decreases with spatial scale, assuming the projectile-target materials and geometries are fixed (i.e., \(v_{\text{crit}} = f(d_p, h_t, h_i/d_p, \ldots)\); see Fig. S8).
Figure 12: Representative experimental one-dimensional cross-sectional profiles of the perforated PC target samples (hatched region, figure inset). All profile data was captured using laser confocal microscopy and normalized by the respective projectile diameter ($d_p$) for comparison across length scales. At each scale, impact and exit side profiles (dark solid lines) are displaced by the corresponding $h_e/d_p$ values provided in Table 1 for ease of visualization. An impact side micrograph of the target perforation is shown for each cross-sectional profile, with the microscopy technique noted in the top left corner: optical microscopy (OM) or scanning electron microscopy (SEM). Length scale decreases from top to bottom. Vertical dashed lines show the projectile diameter ($d_p/d_p = 1$) relative to the perforation regions: normalized effective deformation area [$A_d/A_p$; Eq. (4)] grows with decreasing length scale.
is increased from $\sim 10^{-4}$ s$^{-1}$ to $\sim 10^4$ s$^{-1}$ (see, e.g., [85–88]). This hardening is generally the result of a competition between loading rate and rate of polymer chain motion/alignment and entanglement dynamics. Although strain rates tested in these studies fall short of those shown in Fig. 13a, time-temperature superposition extrapolations of stress-strain data suggest PC’s $\sigma_{y,t}$ and $\sigma_{f,t}$ values, as well as its modulus, continue increasing proportional to strain rate [88]. Such dynamic material behavior likely resulted in higher specific target impact resistance ($\sigma_{f,t}/\rho_t$ or $\sim \sigma_{y,t}/\rho_t$) at the smaller scales. Enhancement of $\sigma_{y,t}$ and $\sigma_{f,t}$ may have been partially offset by added thermal softening from increased adiabatic heating rates at the microscale:

$$\dot{q} \sim \beta_p E_p/(m_{plug}t_p),$$

$\beta_p$ being the fraction of $E_p$ converted to heat ($\sim 0.5$–0.6 for PC).

Other aspects of the scaled impacts also showed size dependency. For example, the time required for the projectile to perforate the target ($t_p$), relative to some scale-specific characteristic time ($t^*$), was notably larger for the LIPIT experiments. Here, $t^*$ can be defined as the duration for a compression wave, traveling at velocity $c_t$ (PC’s bulk sound speed), to traverse a distance $h_t$. From the largest to smallest scale, $t_p c_t/h_t$ roughly doubled (Fig. 13b). Interestingly, this increase coincides with a seven-order magnitude reduction in the ratio of projectile kinetic energy ($E_t$) to target area density ($AD_t = h_t \rho_t$), as illustrated in Fig. 13b. $t_p c_t/h_t$ is also proportional to the fraction of target material subjected to rapid deformation before perforation occurs. Hence, as $d_p$ decreased, proportionally more of the target (e.g., $\sim \pi c_t^2 t_p^2 h_t \rho_t / m_{plug}$) was involved in projectile energy dissipation. These findings are consistent with the observed inverse scaling of normalized deformation area with $d_p$ ($A_d/A_p \propto d_p^{-0.16}$; cf. Fig. 9b). Furthermore, they align with the transition in bulk target failure mode from predominantly plugging to dishing as the spatial scale decreased (cf. Fig. 12). Dishing engages a relatively larger volume of target material, resulting in more extensive viscoplastic deformation (radial stretching and thinning) before it fails primarily in tension [18]. Consequently, the “relative time to material failure” ($t_p c_t/h_t$) for dishing can significantly exceed that of plugging, which is more abrupt and localized. These factors make dishing more efficient at absorbing impact energy.

Length scale reduction also resulted in a $\sim 90\%$ decrease in the projectile’s residual velocity ($v_r$; Fig. 13c), indicating that the target’s ballistic limit velocity ($v_{50}$) increased with decreasing $d_p$. For a fixed projectile-target material combination and $h_t/d_p$ ratio, a transition from bulk plugging to dishing failure of a ductile target plate has been associated with decreasing projectile residual velocity (i.e., $v_i$ approaching $v_{50}$) [18, 47]. Hence, the coupled increase of the target’s $v_{50}$ and degree of dishing with decreasing spatial scale is expected. This consistency, however, does not address why the target’s $v_{50}$ rose with decreasing $d_p$.

---

6 Experimental techniques used to determine $\sigma_{y,t}$ and $\sigma_{f,t}$ in these studies depended on the desired loading rate but included screw-driven and hydraulic tensile testing, dynamic mechanical analysis, and split-Hopkinson pressure bar (SHPB) experimentation.

7 Here, $t_p \approx 2h_t/(v_i + v_r)$ is used for simplicity. The ratio $t_p c_t/h_t$ would be more accurately estimated using, $t_p \approx 2\delta/(v_i + v_r)$, assuming $\delta$, the distance over which the projectile and target interact until complete perforation occurs, could be determined.
Figure 13: Key parameters that vary with length scale, $d_p$, for fixed projectile-target materials and relative geometries: (a) perforation time, $t_p$, decreases and nominal strain rate, $\dot{\varepsilon}$, increases, respectively, by a factor of $\sim 10^3$ from the largest to smallest length scale [32, 84]; (b) the fraction of target material engaged during impact, $t_{p,E} / h_t$, roughly doubles, while the ratio of projectile kinetic energy to target area density, $E_i/AD_t$, drops by seven orders of magnitude, for the microscale LIPT experiments; (c) the normalized projectile residual velocity, $v_r/v_i$, scales proportionally with $d_p$; and (d) the normalized target ballistic limit velocity, $v_{50}/v_i$, is enhanced by 200% at the microscale. A two-term power law was fitted through the $v_r/v_i$ measurements in (c); this $v_r/v_i \propto a + b d_p^{-0.25}$ fit was used to generate the solid red lines in (a), (b), and (d).

A simple method to estimate $v_{50}$ for chunky projectiles (length to diameter ratio near unity) impacting thin plates involves using the Recht-Ipson model [89, 90], derived from momentum and energy considerations:

$$\frac{v_{50}}{v_i} = \sqrt{1 - \left(\frac{v_i^2}{v_i^2} \left(1 + \frac{m_{plug}}{m_p} \right)^2\right)}.$$  \hspace{1cm} (6)
Average values for $d_p$, $h_t$, $v_t$, and $v_r$ (cf. Table 1) were used in Eq. 6 to approximate the PC targets’ normalized $v_{50}$ values, which saw a $\sim 200\%$ increase from the largest to smallest scale (Fig. 13d). A two-term power law fitted through the $v_r/v_t$ measurements ($v_r/v_t \approx 0.98 - 0.04d_p^{-0.25}$; Fig. 13c) was used to generate empirical predictions (solid red lines) for all calculated parameters shown in Figs. 13a, 13b, and 13d. Notably, the fitted $v_{50}$ estimates scale with $d_p^{-0.25}$, a trend likely attributable to rate and/or size effects on the target’s yield and failure stress. Evans et al. [91] recently showed that $\sigma_{f,t} \propto d_p^{-0.5}$ for a number of polymers (including PC) across LIPIT and gas gun length scales ($d_p \sim 10^{-6}-10^{-2}$ m). This relationship was linked to the dependency of a material’s failure stress on the $-1/2$ power of flaw size, as described in linear elastic fracture mechanics (LEFM) (i.e., $d_p$ is analogous to flaw size) [91–93]. A Buckingham II analysis was also employed to develop the expression $v_{50} = 1.15(\sigma_{f,t}/\rho_p)^{0.5}(h_t/d_p)^{0.5}$, which, for fixed $h_t/d_p$ and $\rho_p$, suggests that $v_{50} \propto d_p^{-0.25}$ if the $\sigma_{f,t} \propto d_p^{-0.5}$ scaling holds [91]. A similar scaling of $v_{50}$ with $d_p$ can be demonstrated using simple analytical expressions, such as that developed by Phoenix and Porwal [84] for a two-dimensional thin target: $v_{50} = (1 + m_{plug}/m_p)(2A_p h_t/\rho_p)^{0.5} \varepsilon_f^{0.25}$, where $\varepsilon_f$ is the target’s failure strain. Assuming fixed $\varepsilon_f$, $h_t/d_p$, and $\rho_t/\rho_p$, this analytical expression also predicts that $v_{50}$ scales with $d_p^{-0.25}$ if $\sigma_{f,t} \propto d_p^{-0.5}$. The consistency of these predicted $v_{50}$-$d_p$ relationships with that shown in Fig. 13d strongly suggests that the observed enhancements of the PC target’s $E_p^s$, $A_d/A_p$, $v_r$, and $v_{50}$ at smaller scales are linked to an increase in its $\sigma_y$ and $\sigma_f$ values, resulting from strain rate hardening, actual size effects, or both.

In essence, the decrease in length scale alone drastically shortened the duration of the impact event. This temporal reduction amplified impact-induced strain rates in the target, which in turn increased its instantaneous, local yield and failure stress. This strength enhancement allowed the target to better resist abrupt failure by plugging, facilitating a transition to bulk failure via dishing. Consequently, at smaller scales, a larger fraction of the target material was involved in deformation and failure and relatively more time was available for energy dissipation mechanisms to operate. This combined effect ultimately led to the observed rise in the target’s specific energy absorption (cf. Fig. 9a). These insights offer a path forward for material modeling efforts focused on capturing the observed size dependency of ballistic impact performance.

3.4. Predicted Specific Energy Absorption and Normalized Deformation Area Using EPIC

Complementary numerical impact simulations in EPIC ($v_t \equiv 550$ m/s and $h_t/d_p \equiv 0.25$) were used to approximate the projectile residual velocity ($v_r$) at each length scale. The simulation methodology, material models, and model parameters are provided in Supplementary Information, Sect. SI.7. While the models account for strain and strain rate hardening, as well as thermal softening, of the target material, they are calibrated only for strain rates up to roughly $10^4$ s$^{-1}$ due to limited experimental data available in the
literature. In addition, these simulations do not account for the effect of microstructural factors (molecular weight, number of entanglements per chain, etc.) on polymer dynamic behavior. The simulated \( v_i \) and \( v_r \) values were then used with Eq. (2) to arrive at predicted specific energy absorption \( E_p^* \) values (see Fig. 9a, dashed line). The simulations fail to reflect the experimental rise in \( E_p^* \) with decreasing projectile diameter \( (d_p) \). The EPIC simulations were also used to predict changes in normalized deformation area with scale, as shown in Fig. 9b. These predictions are based on the diameter \( D \) of a circular region encompassing all out-of-plane target deformation quantified using the same method described in Fig. 6d and Eq. (4). Similar to the \( E_p^* \) calculations, EPIC predicts negligible change in \( A_d/A_p \).

Despite the material models used [Eqs. (S5), (S6), and (S7)] being sensitive to strain rates and temperature, they do not accurately replicate the experimental target response. This disagreement exists although simulated strain rates are nominally consistent with expected experimental values (i.e., they grow by orders of magnitude). Most high-rate material models are calibrated using experimental results from Split-Hopkinson Bar compression tests at strain rates up to \( 10^4 \) s\(^{-1} \)—but this is over three orders of magnitude less than nominal LIPIT values [94]. Moreover, initial sample temperatures typically do not exceed roughly 200°C for polymers. Therefore, a likely explanation for the discrepancy between the simulated and experimental \( E_p^* \) values could be that the material models and EOS employed are not designed and/or calibrated for the specific impact conditions under investigation. Even so, calibrating existing established material models will likely be an insufficient approach: closing the gap in understanding may require the development of entirely new material models or computational approaches that accurately capture scale-induced changes in predominant material deformation behavior, phase transformations, and failure mechanisms that are negligible/overwhelmed at larger scales. These issues are discussed in more detail in the Supplementary Information, Sect. SI.8.

4. Conclusions

When the length scale decreases from centimeters to microns, the dynamic behavior of materials can dramatically change due to corresponding modifications in event duration, strain and heating rates, temperature, state of stress, material inhomogeneities, among other factors. Addressing this issue is vital to the success of emerging high-throughput material characterization techniques. The Laser-Induced Particle Impact Test (LIPIT) has proven to be a valuable method for studying high strain rate deformation of various materials at the microscale, thanks to its precision, minimal lab space requirement, and high-throughput testing capacity. Initial observations from LIPIT indicate that microscale energy absorption can exceed macroscale results by over a decade. This current study intentionally probes geometric scaling effects on
dynamic material behavior by using a LIPIT apparatus and a single-stage light gas gun to launch noncrystalline alumina spheres of 3, 10, 100, 1,000, 4,000, and 10,000 µm in diameter into scaled $h_t/d_p = 0.25$ polycarbonate (PC) targets. Impact energies ranged from hundreds of Joules to nanoJoules, covering eleven decades, the widest range in a single impact study. Perforation times decreased and strain rates increased, respectively, by about three orders of magnitude from the largest to smallest scale. Reducing the length scale increased target specific energy absorption ($E_p^*$) by approximately 230% and the impact deformation area ($A_d/A_p$) by about 240%. Interestingly, $(A_p/A_d)E_p^*$ values exhibited no apparent size dependency. The fraction of the target material engaged during impact more than doubled at the smallest scale. Target ballistic limit velocities scaled with $d_p^{0.25}$ and were about three times higher in the LIPIT experiments. The relative increase in target performance at smaller length scales corresponded to a transition in its predominant failure mode from plugging to dishing. This apparent transition was likely caused by rate- and/or size-induced enhancements in PC’s yield and failure stress values.

Complementary numerical simulations of these impacts, however, do not show any increase in specific energy absorption or normalized deformation area even though predicted strain rates reflect expected values. Such discrepancies underscore the limitations of current material models when scaling down impacts from macroscale ($10^{-2}$ m) to microscale ($10^{-6}$ m). Experimental and simulation design, implementation, and analysis would be complicated by the introduction of material hierarchical inhomogeneities, such as those present in metals, carbon nanotube mats, etc. The findings of this study strongly suggest that length scale cannot be ignored in both developing and applying established and innovative microscale material characterization techniques, particularly for advanced material discovery. For these reasons, one key goal of this study is to stimulate scientific dialogue and begin developing a framework for investigating how geometric scale affects impact phenomena.

Acknowledgments

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References


Supplementary Information for

SIZE MATTERS:
Impact Energy Absorption Across Five Decades of Length Scale

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SI. Supplementary Information

SI.1. Polycarbonate Target Molecular Anisotropy Considerations

A polymer’s unperturbed root-mean-square (RMS) end-to-end distance \( R \) is one measure for estimating its molecular size and is generally best determined through one or more experimental methods (e.g., small-angle neutron scattering and/or small-angle X-ray scattering) [1, 2]. Theoretical models, however, can also approximate it by linking the polymer’s physical properties to its dimensions. Flory [3] developed a widely used expression given by

\[
R^2 = C_\infty N l_0^2, \tag{S1}
\]

where \( C_\infty \) is the characteristic ratio, \( N = M_w/m_0 \) is the number of monomer units in the polymer chain, \( l_0 \) is the length of a single monomer, and \( m_0 \) is the average molecular weight per monomer. For polycarbonate (PC), \( C_\infty \approx 2.4, l_0 \approx 7 \, \text{Å}, \) and \( m_0 \approx 127 \, \text{g/mol} \) [4]. Taking \( M_w \approx 56,000 \, \text{g/mol} \) (Table S3), the approximate average number of monomer units is \( N \approx M_w/m_0 = 56,000/127 = 441 \). Therefore, an approximate RMS end-to-end distance for the PC used in this study is \( R \approx 23 \, \text{nm} \), which indicates that the \( h_t = 750 \, \text{nm} \) (thinnest) film thickness was roughly 32 times (3,200%) larger than the PC RMS end-to-end distance (Fig. S1).

The volume of the sphere which completely contains the PC chain \( (V_{PC}) \) can be approximated from \( R \) as

\[
V_{PC} \approx \frac{4}{3}\pi (R^2)^{3/2}. \tag{S2}
\]

Hence, the approximate pervaded volume taken up by a given PC molecule is \( V_{PC} \approx 51,000 \, \text{nm}^3 \). The volume of PC target material beneath the projectile is \( V_p = \pi/4 d_p^2 h_t \) (Fig. 6d). For the same thickness sample, this target volume was roughly \( 1.0 \times 10^5 \) times \((1.0 \times 10^5\%)\) greater than the spherical volume containing the PC chain (Fig. S1). These simple calculations demonstrate that molecular anisotropy is negligible and likely does not contribute to the observed changes in specific energy absorption or effective deformation area (cf. Fig. 9), even for the thinnest films.

SI.2. Polycarbonate Target Boundary Considerations

In-plane reflected waves from the target’s boundary, influenced by the materials, shapes, and relative impact velocity of the projectile and target, can affect the dynamics of penetration and/or perforation. If the time taken for perforation \( (t_p) \) is significantly shorter than the duration for waves to travel from the
impact point to the boundary and back ($t_b$), disruption from reflected waves is unlikely. Hence, evaluating the ratio $t_p/t_b$ offers insight into potential boundary effects.

The time for a wave to travel to the boundary and back in a given target can be estimated as

$$t_b \approx \frac{2 r_b}{c_t}, \quad (S3)$$

where $r_b$ is the in-plane radius from the impact point to the target boundary and $c_t = \sqrt{K_t/\rho_t}$, $K_t$, and $\rho_t$ are the target’s bulk sound speed, bulk modulus, and mass density, respectively. Similarly, the perforation/puncture time can be approximated as

$$t_p \approx \frac{2h_t}{v_i + v_r}, \quad (S4)$$

where $h_t$ is the target film thickness, $v_i$ is the projectile impact velocity, and $v_r$ is the projectile residual velocity after impact. For the PC used in this study, $K_t = 3,352$ MPa and $\rho_t = 1.2$ g/cc. Hence, $c_t \approx 1,700$ m/s for all samples. Using experimental $v_i$, $v_r$, and $r_b$ measurements, values for $t_b$, $t_p$, and $t_p/t_b$ were approximated for all PC samples (Table S1). These simple calculations show that for all scaled impacts, the perforation time was much less than the wave travel time ($t_p \ll 0.1t_b$), indicating reflected in-plane waves did not influence experimental perforation dynamics.
Table S1: Approximations of wave travel time \((t_b)\), perforation time \((t_p)\), and their ratio \((t_p/t_b)\) for all PC samples. Calculated values are based on the target’s bulk sound speed \((c_t)\), projectile impact and residual velocities \((v_i\) and \(v_r)\), and in-plane radius from the impact point to the boundary \((r_b)\). The analysis shows that for all scaled impacts, the perforation time is significantly shorter than the wave travel time.

<table>
<thead>
<tr>
<th>No.</th>
<th>(d_p) (µm)</th>
<th>(v_i) (m/s)</th>
<th>(v_r) (m/s)</th>
<th>(t_p) (ns)</th>
<th>(r_b) (mm)</th>
<th>(t_b) (ns)</th>
<th>(t_p/t_b) (%)</th>
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<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>561</td>
<td>482</td>
<td>4,600</td>
<td>38</td>
<td>45,000</td>
<td>10</td>
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<tr>
<td>2</td>
<td>4,000</td>
<td>539</td>
<td>437</td>
<td>2,100</td>
<td>38</td>
<td>45,000</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
<td>537</td>
<td>434</td>
<td>520</td>
<td>38</td>
<td>45,000</td>
<td>1.2</td>
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<tr>
<td>4</td>
<td>500</td>
<td>535</td>
<td>385</td>
<td>280</td>
<td>38</td>
<td>45,000</td>
<td>0.62</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>529</td>
<td>251</td>
<td>78</td>
<td>13</td>
<td>15,000</td>
<td>0.51</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>523</td>
<td>207</td>
<td>6.2</td>
<td>0.05</td>
<td>59</td>
<td>11</td>
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<tr>
<td>7</td>
<td>3</td>
<td>522</td>
<td>58</td>
<td>2.7</td>
<td>0.05</td>
<td>59</td>
<td>4.6</td>
</tr>
</tbody>
</table>

SI.3. Polycarbonate Target Material Properties

Comparative material studies require materials to be (at least) initially similar. Table S2 outlines the some key material properties for TUFFAK® (PC1) and RowTec® (PC2), including mass density, Rockwell hardness, tensile yield strength, tensile modulus, thermal conductivity, and coefficient of thermal expansion, as supplied by the manufacturers [5, 6]. Noteworthy is the minimal variation in properties between the two materials, confirming their suitability for the purposes of this investigation. This similarity is crucial, as it supports the assumption that any differences in the ballistic impact response of the materials can be attributed to the effects of length scaling effects rather than material inconsistencies/differences.

Table S2: Comparative material properties of TUFFAK® and RowTec® polycarbonates as provided by their manufacturers [5, 6].

<table>
<thead>
<tr>
<th>Property</th>
<th>TUFFAK®</th>
<th>RowTec®</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density (g/cc)</td>
<td>1.2</td>
<td>1.2</td>
<td>0%</td>
</tr>
<tr>
<td>Rockwell hardness (R scale)</td>
<td>R118</td>
<td>R118</td>
<td>0%</td>
</tr>
<tr>
<td>Tensile yield strength (MPa)</td>
<td>62</td>
<td>60</td>
<td>3%</td>
</tr>
<tr>
<td>Tensile modulus (GPa)</td>
<td>2.34</td>
<td>2.41</td>
<td>3%</td>
</tr>
<tr>
<td>Thermal conductivity (W/m/K)</td>
<td>0.19</td>
<td>0.19</td>
<td>0%</td>
</tr>
<tr>
<td>Coefficient of thermal expansion (1/K)</td>
<td>68 \cdot 10^{-6}</td>
<td>68 \cdot 10^{-6}</td>
<td>0%</td>
</tr>
</tbody>
</table>

SI.4. Gel Permeation Chromatography (GPC) Characterization of Polycarbonate Samples

Gel permeation chromatography (GPC) data was obtained for both commercial polycarbonate (PC) materials: PC1 (TUFFAK® from Plaskolite, LLC.) and PC2 (RowTec® from Rowland Advanced Polymer Films). Specifically, their molecular weight averages \((M_n: \text{number average}, M_w: \text{weight average}, M_z: \text{Z-average})\) and distribution (polydispersity \(PDI = M_w/M_n\)) were assessed using a TOSOH Ambient Temperature GPC instrument. The results for both materials were similar: \(M_n\) ranged from 27,737 to 28,927 g/mol, \(M_w\) around 55,800 g/mol, \(M_z\) approximately 87,800 g/mol, and \(PDI\) between 1.93 and 2.01 (Table S3). Figure S2a shows representative GPC chromatographs for PC1 and PC2 in millivolt (mV) versus log molecular weight \([\log (M)]\) space, shedding light on molecular size distribution. Figure S2b shows cumulative weight percent versus molecular weight \((M)\) normalized by the entanglement molecular weight \(M_e \approx 2,490\) g/mol.
Figure S2: Gel permeation chromatography (GPC) analysis of polycarbonate (PC) samples PC1 and PC2: (a) a plot of detector response in millivolts (mV) versus the logarithm of molecular weight \([\log (M)]\) and (b) cumulative weight percent of the PC as a function of molecular weight normalized by the entanglement molecular weight \([M/M_e]\), with an inset showing the lower molecular weight range.

\[^{[7,8]}\] indicating how molecular weight distribution correlates with the entanglement threshold. The comparable molecular weight distributions of the PCs confirm the materials’ similarity, supporting their comparison across length scales.

Table S3: Gel permutation chromatography (GPC) results \([M_n, M_w, M_z, \text{ and } PDI]\) for the two commercial polycarbonate materials, PC1 and PC2 (see Supplementary Information, Sect. SI.4 and Fig. S2). The mass densities \((\rho_t)\) and glass transition temperatures \((T_g)\) obtained from DSC of the target materials are also provided for reference.

<table>
<thead>
<tr>
<th>Vendor/Material</th>
<th>(M_n) (g/mol)</th>
<th>(M_w) (g/mol)</th>
<th>(M_z) (g/mol)</th>
<th>(PDI) ((M_w/M_n))</th>
<th>(\rho_t) (g/cc)</th>
<th>(T_g) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RowTec(^{(R)}) (PC1)</td>
<td>27,737</td>
<td>55,711</td>
<td>87,908</td>
<td>2.01</td>
<td>1.20</td>
<td>149.2</td>
</tr>
<tr>
<td>TUFFAK(^{(R)}) (PC2)</td>
<td>28,927</td>
<td>55,876</td>
<td>87,749</td>
<td>1.93</td>
<td>1.20</td>
<td>152.7</td>
</tr>
</tbody>
</table>

SI.5. Target and Perforation Isotropy Inspection Using Cross Polarizers

Images of representative PC samples at each length scale were obtained using cross-polarizing optical microscopy (OM) to assess their uniformity and isotropy—especially crucial for ultra-thin samples \([9]\). The in-plane orientation of the targets was determined by analyzing camera pixel intensity values within a “pixel sampling region,” identifying angles with the minimum \((\theta = 0^\circ)\) and maximum \((\theta = \pm 45^\circ)\) light transmission. This process was first applied to a representative as-received 2,380 µm thick sample (Fig. S4a and S4b), followed by an as-received 1,016 µm thick sample (Fig. S4c and S4d), an as-received 254 µm thick sample (Fig.

\[^{[8]}\] Literature values for \(M_e\) differ from 2,490 g/mol; Fetters et al. \([4]\) report \(M_e \approx 1,300\) g/mol, whereas Chan et al. \([8]\) note \(M_e \approx 1,800\) g/mol.
S4e and S4f), an as-received 127 µm thick sample (Fig. S3a and S3b), a 30 µm thick compression-molded sample (Fig. S3c and S3d), and finally a spin-coated LIPIT sample (Fig. S3e and S3f). For the $h_t \approx 0.8$ µm and $h_t \approx 30$ µm films, the difference in light intensity at $\theta = 0^\circ$ and $\theta = 45^\circ$ was negligible (<1%). Hence, the spin-coating and compression-molding sample preparation processes did not induced significant material anisotropy. As the sample thickness increased ($h_t \approx 127$ µm and above; $h_t \gtrsim 160h_t^{\text{LIPIT}}$), the transmitted light intensity at $\theta = 45^\circ$ did begin to increase. Birefringence in thicker samples might not accurately reflect the material’s inherent anisotropy, as the optical path is substantially longer. The extended optical paths in thicker samples can result in cumulative effects of light passing through the material, potentially distorting or exaggerating the anisotropy measurements. In thinner films that initially show no anisotropy, increasing thickness alone can lead to more interactions of polarized light with negligibly varying refractive indices and internal stresses, thereby amplifying birefringence effects. Material anisotropy in thicker films (if present) likely becomes less significant due to the averaging effect over the considerably larger thicknesses.
Figure S3: Cross polarizing optical microscopy images at in-plane angles $\theta = 0^\circ$ (lowest light intensity) and $\theta = 45^\circ$ (highest light intensity) for representative as-received (a, b) $h_t \approx 2.4$ mm, (c, d) $h_t \approx 1$ mm, and (e, f) $h_t \approx 0.25$ mm samples. Light intensity was sampled in a circular “pixel sampling region” to identify in-plane angle $\theta$. 
Figure S4: Cross polarizing optical microscopy images at in-plane angles $\theta = 0^\circ$ (lowest light intensity) and $\theta = 45^\circ$ (highest light intensity) for representative (a, b) as-received $h_t \approx 127 \mu m$, (c, d) compression molded, and (e, f) spin-coated samples. Light intensity was sampled in a circular “pixel sampling region” to identify in-plane angle $\theta$. 
**SI.6. Isometric SEM Images of LIPIT Perforations**

Figure S5: Orthogonal (90°) and isometric (48°) SEM images of the PC target perforations resulting from LIPIT impacts from alumina spheres with diameters of (a–d) \( d_p = 10 \mu m \) and (e–h) \( d_p = 3 \mu m \). For each viewing perspective, an (a, c, e, g) impact side and exit side (b, d, f, h) image is shown. On each normal micrograph, a 2D projection of the projectile is overlaid, indicated by a dashed yellow line, to serve as a point of reference.

**SI.7. Scaled Numerical Impact Simulations**

The scaled \( h_t/d_p \) impact experiments were complemented with simulations using the Elastic Plastic Impact Computation (EPIC) code, which couples finite element analysis with smooth particle hydrodynamics to capture large-scale deformations, fracture, and fragmentation. Impact-induced equivalent plastic strain, strain rate, and temperature distributions, as well as specific energy absorption (\( E^*_p \)) and normalized deformation area (\( A_d/A_p \)), were computed for each length scale. In the simulations, the alumina projectiles were assumed to be perfectly rigid. The \( h_t \) thick PC targets were idealized and meshed in 3D quarter-symmetry using tetrahedral finite elements, where the characteristic element size increased with radial distance from the axis of impact. Projectile and target meshes consisting of a combined total of \( N_{el} = 686,280 \) elements were utilized for all simulations, irrespective of scale; a mesh sensitivity analysis was performed to ensure a convergent solution. The finite element edge length for the projectile (\( \sim 0.08d_p \)) matched that for the target in the projectile-target contact region. The element-to-particle conversion feature of the EPIC code was employed, with the equivalent plastic strain (\( \varepsilon_{eq} \)) required for conversion in the target set to 30%, a standard value for ductile materials [10]. In essence, these projectile and target meshes were linearly scaled
such that the target thickness ($h_t$) matched that of the corresponding experiment ($h_t/d_p = \text{constant} = 0.25$; Fig. S6). Key simulation features, including time steps ($\Delta t$), run times ($t_{\text{run}}$), and number of elements ($N_{\text{el}}$), are summarized in Table S4.

The PC target material behavior was simulated using a Mie-Gruneisen EOS [11], JC constitutive model [12], and a JC fracture model [13]. A cubic form of the Mie-Gruneisen EOS was employed, i.e.,

$$P = (K_1\mu + K_2\mu^2 + K_3\mu^3)(1 - \Gamma \mu/2) + \Gamma E_s(1 + \mu),$$

(S5)

where $P$ is the pressure, $K_1$, $K_2$, and $K_3$ are polynomial coefficients, $\Gamma$ is the Gruneisen coefficient, $E_s$ is the internal energy per unit volume, and $\mu$ is given by the relationship $\mu = \rho/\rho_0 - 1$ where $\rho$ is the current density and $\rho_0$ is the initial density. The JC constitutive model was used to determine the material’s dynamic flow stress ($\sigma$).

Table S4: A summary of the scaled impact simulations performed using the EPIC code. Key impact conditions, mesh geometry features, and simulation parameters are provided for reference.

<table>
<thead>
<tr>
<th>No.</th>
<th>$d_p$ (µm)</th>
<th>$v_i$ (m/s)</th>
<th>$h_t/d_p$</th>
<th>$\Delta t$ (ps)</th>
<th>$t_{\text{run}}$ (µs)</th>
<th>$N_{\text{el}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>550</td>
<td>0.25</td>
<td>2.070</td>
<td>80</td>
<td>686,280</td>
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<tr>
<td>2</td>
<td>4,000</td>
<td>550</td>
<td>0.25</td>
<td>831</td>
<td>35</td>
<td>686,280</td>
</tr>
<tr>
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<td>0.25</td>
<td>0.62</td>
<td>0.15</td>
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</table>

$d_p$, projectile diameter; $v_i$, projectile impact velocity; $h_t/d_p$, target-thickness-projectile-diameter ratio; $\Delta t$, simulation time step; $t_{\text{run}}$, simulation run time; $N_{\text{el}}$, total number of elements.
Table S5: Mie-Gruneisen EOS [Eq. (S5)] and Johnson-Cook constitutive and fracture model [Eqs. (S6) and (S7)] parameters for the PC target.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress, $C_1$</td>
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<tr>
<td>Hardening coefficient, $C_2$</td>
<td>68.9</td>
<td>[MPa]</td>
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<tr>
<td>Strain rate coefficient, $C_3$</td>
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</tr>
<tr>
<td>Pressure coefficient, $C_4$</td>
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<td></td>
</tr>
<tr>
<td>Hardening exponent, $N$</td>
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<td></td>
</tr>
<tr>
<td>Softening exponent, $m$</td>
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<tr>
<td>Shear modulus, $G$</td>
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<td>[MPa]</td>
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<td>Strain to failure coefficient, $D_1$</td>
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<td>Exponential coefficient, $D_2$</td>
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<td>Temperature coefficient, $D_5$</td>
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<td>Bulk modulus, $K_1$</td>
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<td>Quadratic M-G coefficient, $K_2$</td>
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<td>Cubic M-G coefficient, $K_3$</td>
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<td>Gruneisen coefficient, $\Gamma$</td>
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<td>Melt temperature, $T_m$</td>
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</table>

\[
\sigma = \left( C_1 + C_2 \varepsilon_{eq}^N \right) \left( 1 + C_3 \ln \dot{\varepsilon}_{eq}^* \right) \left[ 1 - (T^*)^m \right] + C_4 P, \tag{S6}
\]

where $C_1$ is the uniaxial yield stress, $C_2$ is the hardening coefficient, $\varepsilon_{eq}$ is the equivalent plastic strain, $N$ is the hardening exponent, $C_3$ is the strain rate coefficient, $\dot{\varepsilon}_{eq}^*$ is the dimensionless total equivalent plastic strain rate (normalized by a strain rate of $1 \text{ s}^{-1}$), $T^* = (T - T_0) / (T_m - T_0)$ is the homologous temperature, $T$ and $T_0$ are the current and reference temperatures, $m$ is the softening exponent, $C_4$ is the pressure coefficient, and $P$ is the hydrostatic pressure where compression has a positive sense. The equivalent plastic strain to fracture ($\varepsilon_p^f$) under constant conditions of $\dot{\varepsilon}_{eq}^*$ [cf. Eq. (S6)] is given by

\[
\varepsilon_p^f = \left( D_1 + D_2 e^{D_3 \sigma^*} \right) \left( 1 + D_4 \ln \dot{\varepsilon}_{eq}^* \right) (1 + D_5 T^*), \tag{S7}
\]

where $D_1$-$D_5$ are fitting parameters, $\sigma^* = \sigma_m / \sigma_{eq}$ is the pressure stress ratio (stress triaxiality [13]), $\sigma_m$ is the mean normal stress, and $\sigma_{eq}$ is the equivalent stress. In the simulations, material point “damage” is defined as the ratio of the accumulated equivalent plastic strain to fracture (i.e., $\varepsilon_{eq} / \varepsilon_p^f$). An element or converted particle is considered to be fully damaged or fractured when this ratio reaches a value of unity. All material model and EOS parameters were sourced from the EPIC materials library or the literature [14]. These values, as well as other important target material parameters, are reported in Table S5.

SI.8. Simulation Considerations for Scaled Impacts

Length-scaled impact simulations necessitate careful consideration of material properties, features, and behaviors that are often negligible at the macroscale. These include material surface effects (adhesion,
friction, surface tension), microstructure and morphology, defects and imperfections, as well as potentially drastic changes in phase, adiabatic heating, deformation, failure, and more. The mechanical behavior of polymers is highly sensitive to temperature, pressure, and testing rate [15]. Since impact experiments induce extreme conditions with dynamic gradients in temperature, strain, strain rate, and stress state that evolve spatially and temporally, ballistic deformation of polymers is particularly susceptible to length scale effects. In addition, the surface-to-volume ratio is much higher at the microscale, meaning surface effects, including surface tension and surface energy, can potentially compete with or dominate over bulk properties [16, 17]. While macroscale properties assume an averaged, isotropic orientation of molecules, microscale properties can be highly anisotropic due to alignment of polymer chains arising from processing or geometric constraints (e.g., target thickness). The influence of polymer microstructural features, such as chain ends, entanglements, and voids, can have an amplified effect on local mechanical properties, such as impact resistance and ductility [18, 19]. Polymers may experience different phase transitions at the microscale due to constraints on molecular movement and phase separation processes [20, 21]. The microscale thermal conductivity of polymers can deviate from their bulk properties. This conductivity is heavily influenced by the microstructure, where features such as chain structure, crystallinity, crystal form, and the orientation of polymer chains in thin films play a more crucial role than the material’s density [22]. Such features cannot be rendered using macroscale continuum approaches. Moreover, due to the relative length scale and irregular nature of these polymer features, multi-scale continuum-based codes are likely also inadequate.

As deformation rates increase with decreasing scale, thermoplastics can exhibit significantly different viscoelastic and fracture behaviors compared to bulk materials due to a competition between loading rate and rate of polymer chain motion/alignment and entanglement dynamics. Furthermore, the validity of using conventional (bulk) material parameters at ultra-high strain rates exceeding $10^6 \text{s}^{-1}$ is questionable due to the current inability of reliably quantifying material properties at such rates. Material features and local variations in material properties that are negligible or “averaged out” at macroscales (i.e., continuum or bulk material properties) can become unavoidably magnified at the microscale. Yet, it is macroscale material properties that are used to model material elasticity, plasticity, thermal conductivity, fracture, failure, and more. Hence, the failure of continuum-based Finite Element Analysis (FEA) and/or Smooth Particle Hydrodynamics (SPH) computation to accurately simulate microscale impact behavior is not surprising. Since the issues reside with the continuum-based assumptions, they are not unique to EPIC and likely exist for other continuum codes, including LS-DYNA [23], Ansys Autodyn [24], CTH [25], and ALE3D [26]. Coarse-graining molecular dynamics (MD) might offer a more precise representation of microscale behaviors [27].

The EPIC simulations were analyzed more thoroughly to display these considerations given only changes
in the impact scenario’s length scale ($v_i \equiv 550$ m/s and $h_t/d_p \equiv 0.25$). This inspection is also critical to identifying potential areas of improvement for future modeling. One simple approach involves capturing target cross-sectional snapshots depicting elemental $\varepsilon_{eq}$, $\dot{\varepsilon}_{eq}$, and $T$ values at a certain time $t$ post-impact, such as at $t = 0.5t_p$ [where $t_p \approx 2h_t/(v_i + v_r)$], when the projectile is roughly midway through perforation. At this stage, significant deformation is observable for all scaled impacts without the onset of extensive material failure. Such snapshots were obtained for particles with $d_p = 10,000$ µm (Figs. S7a–S7c), $d_p = 4,000$ µm (Figs. S7d–S7f), $d_p = 1,000$ µm (Figs. S7g–S7i), $d_p = 500$ µm (Figs. S7j–S7k), $d_p = 100$ µm (Figs. S7m–S7o), $d_p = 10$ µm (Figs. S7p–S7t), and $d_p = 3$ µm (Figs. S7s–S7u) simulated impact perforations. These PC target cross sections are arranged in decreasing order of $d_p$, with each column representing a different variable: (left to right) $\varepsilon_{eq}$, $\dot{\varepsilon}_{eq}$, and $T$. In each snapshot, the rigid alumina projectile has been hidden for ease of visualization. The simulation time after impact ($t$) and length scale are indicated in the bottom left and right, respectively. The color bars at the bottom of each column correlate the colors used in the plots with their quantitative values. Variations in the deformation geometry are minor: the instantaneous “crater” size, penetration depth, and back-face bulging seem to be similar regardless of the scale. As $d_p$ decreases, the elements possessing peak $\varepsilon_{eq}$ values (i.e., the element-to-particle conversion strain of 30%) appear to shift outward symmetrically from directly beneath the projectile to lateral locations. The number of elements with $\varepsilon_{eq} \approx 0.3$ also seem to decrease. Reducing the length scale leads to a rise in peak calculated strain rates (up to $\dot{\varepsilon}_{eq} \sim 10^{10}$ s$^{-1}$), without significantly affecting the overall distribution. Interestingly, the predicted temperature distribution remains largely unchanged, with peak temperatures reaching just $T \approx 0.75T_m$ for the $d_p = 3$ µm impact.

In essence, the EPIC simulations fail to accurately mirror the physics seen in the impact experiments, as the simulations show little variation in strain, strain rate, and temperature, as well as overall deformation and energy absorption, with decreasing length scale. This largely invariant response not only disagrees with experimental observations but also is contrary to expectations that nominal (not just peak) strain and heating rates should rise notably with decreasing scale due to shorter perforation times. Moreover, despite a significant increase in predicted peak strain rates (from $10^6$ s$^{-1}$ to $10^{10}$ s$^{-1}$), the lack of noticeable changes in the simulation outcomes suggests that the material models do not capture changes in material strain and strain rate hardening, thermal softening, melting, etc. These clear discrepancies underline the need to develop or rethink material models to better capture the behaviors observed in scaled impacts. This study, of course, is limited to a single material, PC, which is homogeneous and amorphous at all examined scales. Introducing hierarchical material inhomogeneity and anisotropy, that are present in metals and carbon nanotube mats, into computational models would substantially increase their complexity. The simulation results included in this study simply demonstrate modeling limitations and potential areas for advancements.
Figure S7: Cross-sectional EPIC simulation snapshots for the (a–c) $d_p = 10$ mm, (d–f) $d_p = 4$ mm, (g–i) $d_p = 1$ mm, (j–l) $d_p = 500$ µm, (m–o) $d_p = 100$ µm, (p–r) $d_p = 10$ µm, and (s–u) $d_p = 3$ µm. Each column in the image array corresponds to a different spatially vary parameter: (a, d, j, m, p, s) equivalent plastic strain ($\varepsilon_{eq}$), (b, e, h, k, n, q) equivalent plastic strain rate ($\dot{\varepsilon}_{eq}$), and (c, f, i, l, o, r, u) temperature ($T$). The time after impact ($t \approx 0.5t_p \approx 0.5\frac{h}{v_i}$) is provided in the bottom right corner of each snapshot.
Figure S8: The influence of spatial scale on the transition from the terminal ballistic velocity regime to the hypervelocity regime. In the macroscale context, the transition to the hypervelocity impact regime is typically marked by a nominal impact-induced target strain rate of approximately $10^6 \text{s}^{-1}$ (i.e., $\dot{\varepsilon}_{\text{crit}} \approx 10^6 \text{s}^{-1}$). Assuming that the materials and geometrical configurations of the projectile and target are kept constant, the critical impact velocity ($v_{\text{crit}}$) associated with $\dot{\varepsilon}_{\text{crit}}$ decreases as the spatial scale is reduced [i.e., $v_{\text{crit}} = f(d_p, h_t, h_t/d_p, \ldots)$].

References for Supplementary Material