Method of the *corrected distance* for the valuation of real estate.

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Abstract

This article presents a novel valuation model by market comparison that requires a minimum application of criteria by the appraiser, thus seeking the highest degree of objectivity possible. The premise that supports the model is that the price differences observed between comparable assets of the same market must be proportional to the dissimilarities (distances) between those same assets, but it shows that not every distance implies a price gap, so a *corrected distance* is used. The solution of the model requires the use of non-linear programming, and an example shows that the results are very similar to those obtained by applying Goal Programming, which is another optimization method applicable to the valuation of real estate.

Keywords

Appraisal, real estate, property valuation, applied optimization methods.

Resumen

En este artículo se presenta un nuevo modelo de valoración por comparación de mercado que requiere una mínima aplicación de criterios por parte del tasador, buscando así el mayor grado de objetividad posible. La premisa que sustenta el modelo es que las diferencias de precios observadas entre activos comparables de un mismo mercado, deben ser proporcionales a las disimilitudes (distancias) entre esos mismos activos, pero se muestra que no toda distancia implica una brecha de precios, por lo que se utiliza una *distancia corregida*. La solución del modelo requiere el uso de programación no lineal, y un ejemplo muestra que los resultados son muy similares a los obtenidos aplicando Programación por Metas, que es otro método de optimización aplicable a la valoración de bienes raíces.

Palabras clave

Tasación, bienes raíces, valoración de propiedades, métodos de optimización aplicados.

1. Introduction

The main methods of real estate valuation are in two major categories: *capitalization methods* and *comparative methods* [1]. The techniques are completely different between these groups, in the first case they are based on the anticipation approach, where the projected return of the asset is calculated as a present value according to a certain discount rate. It is worth saying that there is considerable uncertainty when using the capitalization approach, both because of the uncertainty of a projected cash flow and because of the high sensitivity of the results to the rate used. Its use is justified when the information of similar transactions is limited and the purpose of the property is commercial, being intended for the generation of income.

Comparative methods, on the other hand, are based on the principle of substitution, wherein the value of a good must be similar to that of others with comparable characteristics, making them theoretically interchangeable. This approach is commonly chosen when dealing with houses or apartments in a mature market with transactions that can serve as references. It is within this context that the technique presented in this article is situated.

The comparison of real estate with the aim of valuing a certain property of interest admits a wide variety of procedures, but in turn it can also be subdivided into two main subcategories: *direct comparison* and *indirect comparison*.

It is said that a *direct comparison* is made (or that *a sales comparison model* is used) when the appraiser adjusts the known values of reference properties according to the similarity they present with the evaluated property (grid adjustment), to subsequently average the adjusted prices and take that figure as the value estimate. A strong adjustment in the price of a comparable, indicates a slight resemblance to the property being studied, and on the contrary, a small adjustment occurs when there is a great resemblance between the two. It should be emphasized that the degree of similarity must be established in terms of the observable signs of the buildings (attributes).

In the comparison method, the idea is to homologate the prices of each comparable, obtaining a set of adjusted values that ideally should present the least possible dispersion. As a measure of dispersion, appraisers frequently use the coefficient of variation, but recently, in [2] it was shown that the best results are obtained by minimizing the variance of the adjusted values.

Although it is a simple method, which does not require a sample size, direct comparison has the weakness of largely incorporating the subjectivity of the appraiser. The appraiser's criterion directs not only the selection of the referents that are part of the study, but also the choice of attributes, how to weight them (which would be solved if the criterion of minimum variance of Guijarro [2] is used), the comparison process itself where it is decided which property is better in a certain attribute, and how to adjust the price according to the result of the comparison.

Alternatively, *indirect comparison* involves the use of statistical/mathematical models that, following certain criteria, find patterns in the data and provide a valuation function (also dependent on the observable characteristics), and within this subcategory Multiple Linear Regression is undoubtedly the most well-known tool.

As with direct comparison, the application of regression models has weaknesses: 1- It is a technique with many assumptions (although some are flexible), which are not always fulfilled or guaranteed. 2 – To guarantee inferences about the calculated parameters, it is necessary to collect a large amount of data, 3 – It is quite sensitive to the presence of outliers, 4 – Although nonlinear relationships can be modeled (since the term “linear” refers to linearity in parameters, not in variables), a specific model of the curve (hyperplane) must always be assumed, and 5 – Strictly speaking, the results are valid only within the domain of the sample (interpolation).

As indicated, regardless of the type of comparison, the attributes of the properties are the key to applying a specific technique and reaching a result (estimation of the unknown value of a real estate asset). In this sense, any comparative analysis is hedonic, which is a generic term for models that assume value as the aggregate of the contributions of characteristics, and whose origins date back to the beginning of the twentieth century [3,4].

In this article, we will consider that determinants of value are intrinsic attributes of buildings, but that is only because it is assumed that the external factors are similar and therefore it does not matter to include them in the comparison. However, it is clarified that exogenous factors (from the environment) can play a relevant role in the value that the market assigns to a property. In fact, we find studies focused on the impact that can be attributed to a relevant factor linked to the dynamics and structure of the city; for example, noise [5, 6], road traffic [7], crime [8], or subway lines [9, 10]; just to mention a few of the topics with recent research.

The aim of this research is to present an objective method of valuation that can be used with small samples. Objectivity operates through an assignment of weights to the attributes that is independent of the appraiser, and that is completely based on the sample data.

The underlying principle is that of internal coherence, that is, the differences observed between the prices of the properties that are part of the sample of transactions in a certain market, must be proportional to the differences between the referents.

2. Literature review

The search for objectivity in appraisal procedures is a task in development, but it has already generated results of great importance. It was previously mentioned that comparative methods must in one way or another take into account the characteristics of the assets, and the idea that similar properties should have equivalent prices is not new, in this way it is reasonable to quantify similarity using a distance measure. A work of great impact in this regard was presented by Isakson [11] who used the inverse of *Mahalanobis´s distance* to construct an index of similarity or nearness between properties (nearness in the sense of having similar attributes, not in the spatial or geographical sense), this proposal is called *Nearest Neighbors Method* (NNM).

The premise of NNM is that the price of the subject evaluated will be better estimated if more relevance is given within the procedure to the prices of those referents that most closely resemble that property for which the price is unknown. The price of the subject within the sample that has the shortest distance from the property evaluated will receive a higher weighting. The NNM estimates the unknown value as a weighted average of comparables. What is not considered is whether this price is consistent with the attributes presented by the different properties that make up the sample.

On the other hand, regarding the consideration of consistency between prices and attributes, the methods based on quality points should be highlighted. For decades, work has been on the idea of using a quality variable that encompasses all the characteristics of assets in a single number. In this approach, the different factors are classified on an ordinal scale, and a linear function weighted according to the importance of each variable allows obtaining "quality points" [12,13]. Each property thus obtains an associated quality score, which must be related to the price; usually with a price-quality regression. An obvious advantage of this procedure is that what is originally a multiple regression is transformed into a much simpler model of simple regression (a single independent variable: quality).

What is interesting to rescue from the Price-quality Ranking approach is that a version of this form of analysis, known simply as *Quality Point* [14,15], does place the emphasis on the internal coherence of the sample of known properties and prices. Seeking to objectively weigh the determining factors of the price, by means of a nonlinear programming model, it is intended that the set of optimal weights is the one that complies with making the monetary value of each quality point, as similar as possible among all the comparables that make up the sample.

It is an interesting and significant proposal, which, however, retains some elements to improve:

- the subjectivity of the points assigned to the attributes (quality rating),
- working with the average value of quality points (and the well-known sensitivity of averages to extreme values), as well as
- assume a direct quality-price relationship (price ≈ coefficient x quality), in other words, two properties with the same quality score – regardless of which attributes have contributed to generating the quality indicator and in what way – must have the same price; which is not necessarily true in all scenarios.

A recent study [15] integrated the principles of Quality Point with other tools, creating an eight-step assessment protocol that deals with granting objectivity to the assessment, both in determining the relative importance of the attributes or variables that explain the value, and in their selection. The challenge that this proposal will face is to reconcile the lack of parsimony (the procedure involves two regressions and several steps with associated calculations) with the operability that appraisers require in practice. In light of the results provided by the researchers who developed this methodology, it remains to be verified whether in general the added sophistication in the appraisal is proportional to the accuracy achieved.

In the same way as the advances that have been discussed, the method developed in this article seeks objectivity in the determination of the relative importance of attributes, maintaining the premise that similar properties must have similar prices, and using a measure of similarity/dissimilarity to establish the degree to which two properties resemble each other; but in addition, the basic element is the search for coherence, in the sense that price differences should correspond to the dissimilarities of the properties. The idea of coherence or consistency has always been present in the comparison approach, and was already formalized a long time ago [16], where the following is proposed:

Let $\widetilde{P}_s$ *and* $\widetilde{P}_1$ the price estimate (value) for the building to be valued and for the comparable 1, α_1 and α_2 the weights of two attributes with the ability to explain the value, and Xij the value of characteristic *i* in property *j*, then

$$\widetilde{P}_s - \widetilde{P}_1 = \alpha_1 (X_{1S} - X_{11}) + \alpha_2 (X_{2S} - X_{21}) \tag{1}$$

That is, the difference between the price estimates of these two assets has to be explained by the differences between their attributes, according to the relative importance of each.

Which also implies that

$$\widetilde{P}_s^{\,1} = P_1 + \left(\widetilde{P}_s - \widetilde{P}_1\right) \tag{2}$$

The estimate of value that can be given for subject property based only on comparable 1, is the real price of comparable 1 plus the difference between the estimates of both properties, and which is due to the different degrees in which they possess each of the two attributes considered.

In the next section, we will operationalize these ideas, through an optimization program which estimates weights with a different criterion than the traditional reduction of the gaps between estimates and actual prices.

3. The model

3.1 - *Some preliminary insights*

In the following, bold uppercase letters will indicate matrices, while bold lowercase letters will be used for vectors.

Let us consider that we have a sample of transactions corresponding to houses, apartments (in general real estate assets) related to a certain market. We assume, as usual in appraisal work, that for each one we know the price, and that these references must be used to make a price estimate on a property that is subject to evaluation. Thus, the equation

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{X}; \boldsymbol{\alpha}) + \boldsymbol{e} \quad (3)$$

It expresses that a price vector can be explained in terms of a certain function of a set of attributes inherent to the properties (X_1, X_2,... Xn), in addition to the corresponding errors. The vector **α** describes the relative importance of each attribute, and determining this objectively is part of the purpose of the methodology described here.

Now, we have opted for a new criterion for the selection of weights (**α**). Instead of defining a loss function directly on the error terms of each case, we focus on the *internal coherence* according of the sample. The weighting values must be adjusted to the price differences among the referents, according to the values of the characteristics.

Formally, given two properties, let us say "*a*" and "*b*"; property *a* with price P_a and property *b* with price P_b. If

$$|\Delta P_{ab}| = |P_a - P_b| \quad (4)$$

then,

$$|\Delta P_{ab}| = \Phi DC_{ab} + \epsilon_{ab} \quad (5)$$

Where ϵ_{ab} is the error term, and DC_{ab} is the *corrected distance* between property *a* and property *b,* which is explained in detail in the next section.

If the features explain the prices, the price differences between referents must be proportional to the (corrected) distances between them.

The proposed method seeks

$$\text{Min} \sum\sum |\epsilon_{ij}| \mid i \neq j \quad (6)$$

Obviously

$$\epsilon_{ij} = |\Delta P_{ij}| - \Phi DC_{ij} \quad (7)$$

Minimization will require finding the appropriate values of **α** and the proportionality constant **Φ.**

3.2– *Corrected distance*

A necessary premise to develop the model is that variables are directly linked to prices. That is, if a certain variable increases, but the rest remains unchanged, the price must increase in proportion to the importance of the variable that changes, and if the variable decreases, the price will also decrease.

At this point, the reason for using a corrected distance can be presented. One property can be very different from another, according to some measure of dissimilarity/distance applied to the set of attributes they both exhibit. However, that distance (wich depends on features) does not have to translate directly into a price gap. Depending on the relative position of the referents within the attribute hyperspace, the distances can correspond to proportional price differences, but also to reduced price differences, or even no price difference at all. This is possible because the relative changes in the different variables that generate the distance between two referents can cancel each other out (compensation), or on the contrary they can be added together to generate a greater effect on the price (combination).

To illustrate this, we must refer to figure 1. In this figure we assume that the two variables that appear have the same relative importance ($\alpha_1 = \alpha_2 = 0{,}5$) and also, as intended for the rest of the article, we are going to limit ourselves to two variables or attributes to simplify the exposition.

Figure 1.a – It shows only the vector **n** that is 45° from any of the axes that represent the magnitudes of the attributes. This direction of change represents the price gradient.

Figure 1.b – In this case (combination), there are two referents (r_1 and r_2) that are separated by a distance $|\mathbf{c_1}|$. The distance separating r_1 and r_2 should correspond to a directly proportional change with respect to the prices of these referents, because in order to move the point from r_1 to r_2, both variables (X_1 and X_2) had to increase by equal magnitude. In other words, r_2 improves in every way over r_1, and each variable contributes the same to the change. Here $(X_{2r2} - X_{2r1}/X_{2r2} - X_{2r1}) = 1$.

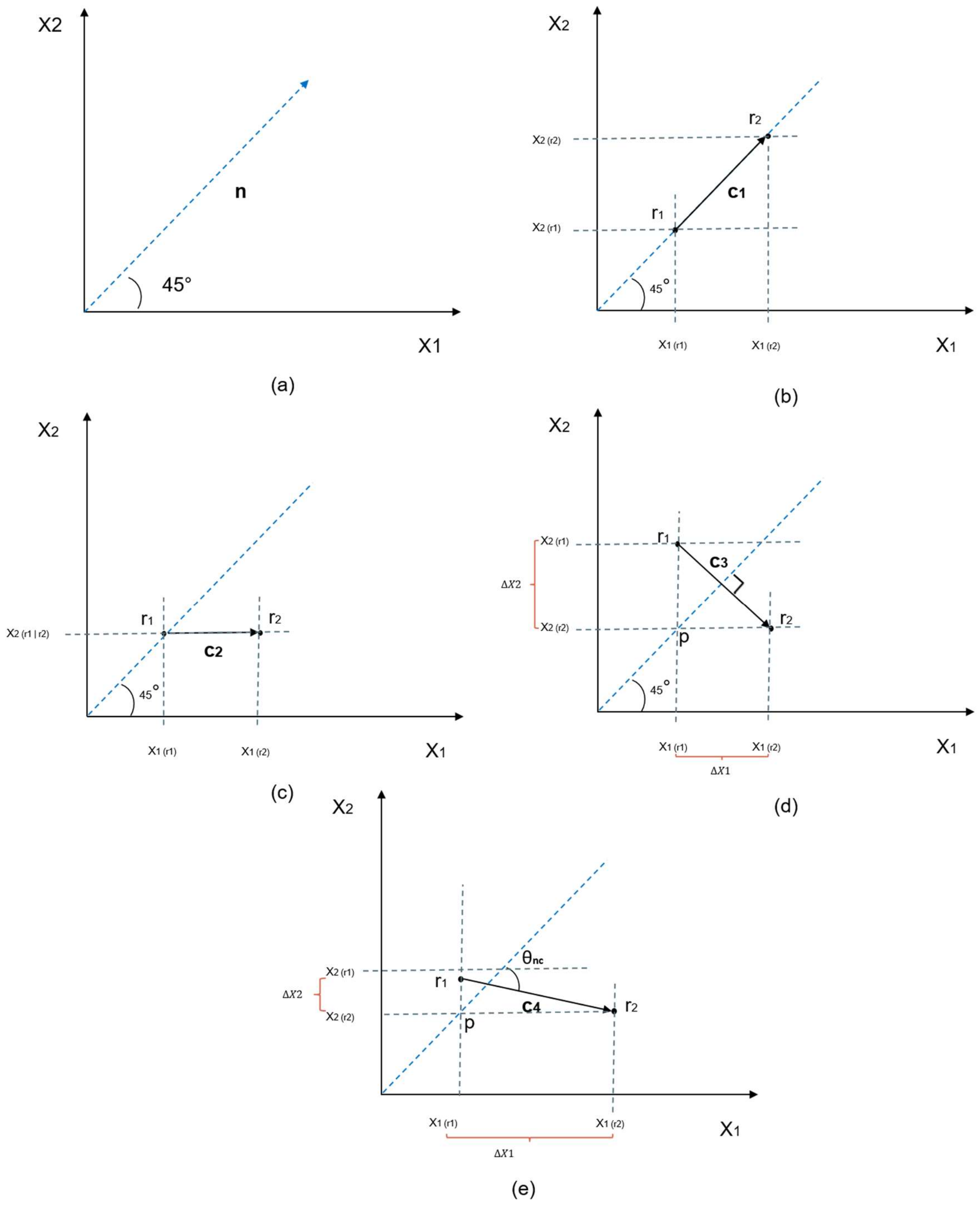

Figure 1. *Possible distance configurations*

Figure 1.c – This case is similar to 1.b, but with the difference that the change between r1 and r2 consists of a variation only of the variable X1, and since we assume that both variables

contribute in the same way to the formation of the price, the impact we should expect on the price is half that compared to scenario 1.b.

Figure 1.d – A compensation scenario is shown here. Starting from point p, we must advance ΔX_1 horizontally to reach r_2, and move ΔX_2 vertically to obtain r_1. In this case $\Delta X_1 = \Delta X_2$ and the angle formed by **c** and **n** is 90°. Again, if the variables have the same relative weight in the formation of the price, then neither referent has an advantage over the other, so the distance between them should not imply a difference in the prices of r_1 and r_2. In other words, if the relative importance assigned to the variables is correct, it follows that if there were a price difference between r_1 and r_2 it would not be attributable to X_1 and X_2.

Figure 1.e – Cases b, c and d show borderline situations. The case shown in figure 1.e is a general one, where the vector **c** forms any angle θ_{nc} with **n**.

The question now is how to incorporate what has been pointed out, into an operating model, and this is achieved by correcting the distance by cosine square of the angle between **n** and **c**, that is, by multiplying it by $(\cos \theta_{\boldsymbol{nc}})^2$.

The distance to be used (and to be *corrected*) is the Euclidean distance weighted by the relative importance of each of the variables. The distance between r_i and r_j will be

$$d_{ij} = [\sum_k \alpha_k \left(X_{ik} - X_{jk}\right)^2]^{1/2} \qquad (8)$$

K is the number of variables.

The *corrected distance* will be of interest to the model

$$DC_{ij} = d_{ij} [\cos \theta_{n(cij)}]^2 \qquad (9)$$

The choice of cosine squared is obvious for three reasons: 1- a positive value for distance is preserved, 2 – it is consistent with the scenarios shown in figure 1: it worth one in the case of combination (1.b), it takes the value 1/2 in the intermediate situation 1.c, and 0 (zero) in the compensation situation (1.d), and 3 – the discount factor increases rapidly when the ability of the distance to explain the price difference decreases.

3.3– *The model to solve*

What has been explained in sections 3.1 and 3.2 can be summarized as follows:

$$Min \sum \sum |\epsilon_{ij}| \quad (10)$$

s.t.

$$\sum \alpha_i = 1 \quad (11)$$

$$DC_{ij} = d_{ij} [\cos \theta_{n(cij)}]^2 \quad (12)$$

$$\cos \theta_{n(cij)} = \frac{\boldsymbol{n} \cdot \boldsymbol{c}}{|\boldsymbol{n}| \cdot |\boldsymbol{c}|} \quad (13)$$

The joint determination of $\boldsymbol{\alpha}$ and Φ by an iterative optimization procedure is then required, and it can be solved using any optimization program, we can even use *Solver* in Microsoft Excel.

3.4– *Price estimation using the model*

Section 3.3 describes the model for obtaining the vector of relative weights for the set of attributes and the proportionality constant but, does not indicate how these parameters obtained by this procedure can be used to give a price estimate of an interest asset.

There are at least two ways to use the weights obtained when applying the model presented:

3.4.1- *Price that minimizes inconsistencies*.

This is a fairly obvious option, if we call r_0 the property whose price we wish to estimate (P_0), from equation (5) it follows that

$$\epsilon_{i0} = |\Delta P_{i0}| - \Phi DC_{i0} \quad (14)$$

There will be as many new error terms as there are properties in the sample. The P_0 estimate we are looking for will be the value that minimizes the sum of the terms ϵ_{i0} that is

$$P_0 = \text{argmin} \sum |\epsilon_{i0}| \quad (15)$$

It should be noted that at this point the vector $\boldsymbol{\alpha}$ is determined, so that the distances between the property in question and the comparables are also known, with P_0 being the only variable to be estimated.

3.4.2 - *Fitting a constant in a linear model*

This option consists of assuming a linear model

$$P = a_1 X_1 + a_2 X_2 + \cdots . + a_n X_n + C \quad (16)$$

In this case, the components of the vector $\boldsymbol{a}$ do not have to add one, but a condition is added, they must respect proportionality (relative importance) according to the vector $\boldsymbol{\alpha}$. That is, if for example α_1 is $1.5\alpha_2$, then in the same way a_1 is $1{,}5a_2$ and so on. Then for (16) we simply search for *C* so that it minimizes the error within the sample.

In the next section, the results of both approaches will be presented on an example case. The procedure described is what we have called *Method of the Corrected Distance* (hereinafter MCD).

4. Numerical example

4.1 – *Calculation of the weights or relative importance of each variable*

In what follows we are going to work on a case with only two variables or attributes (X_1 and X_2), and five referents (R_1 to R_5). To make practical sense of the variables, we will assume that we are talking about apartments, and X_1 is the floor number (height within the building) and X_2 is the common expenses in the building (expressed in the corresponding monetary unit).

Table 1. *Data from the example*

	X_1	X_2	Unit Price ($/m²)
R 1	14	250	77
R 2	5	120	77
R 3	6	330	64
R 4	3	110	79
R 5	10	150	83

We will also assume a market willing to pay more for high floors and less if it is a low floor (due to apprehensions about noise, insecurity, etc.). Similarly, high common expenses have a negative influence on the price. As indicated in section 3, the model requires that the relationship of the variables to the price be direct, so we convert the common expenses (*ce*) and the variable X_2 will actually express $(1/ce) \cdot 10^3$.

Table 2. *Example with modified X_2 variable*

	X_1	X_2	Unit Price ($/m^2)
R 1	14	4,0000	77
R 2	5	8,3333	77
R 3	6	3,0303	64
R 4	3	9,0909	79
R 5	10	6,6667	83

The application of the MCD generates as a result $\alpha_1 = 0{,}228$; $\alpha_2 = 0{,}772$ and $\Phi = 6{,}245$.

The model assigns greater weight to common expenses, although without discarding the height of the apartment. Of course, as in a regression model, these weights will depend on the units assigned to the different variables, they are factors that guarantee internal coherence in the model, they do not have an absolute interpretation.

4.2 – *Comparing the results with the Goal Programming method*

Just to have a comparison with another method that objectively estimates the relative weights of the variables, we are going to calculate the results using Goal Programming (GP) - see [1, 17] for details of the model in this context - . GP works in a similar way to Linear Regression, but it is an optimization model where the hyperplane is obtained by minimizing a different loss function, without being subject to the same statistical assumptions, and being much less sensitive to the presence of outliers.

4.2.1 – Estimation using "price that minimizes inconsistencies".

As an indicator of the degree of fit within the sample, we will use the well known *Mean Absolute Percentage Error* (MAPE).

$$MAPE = \frac{1}{n} \sum \left| \frac{P_A - P_F}{P_A} \right| \tag{17}$$

The MAPE for MCD is 2 %.

The application of the GP algorithm produces the result

$$p = 47{,}125 + 1{,}250X_1 + 3{,}093X_2 \tag{18}$$

And the result is a MAPE of 1.22%.

This is a performance indicator, but only within the sample that is used to calculate the parameters of each model. To appreciate how similar the results are outside the sample, we can compare the estimates with some contrast scenarios and that cover a wide range of possibilities: E_1 [X_1 = 1, X_2 = 1]; E_2 [X_1 = 4, X_2 = 5]; E_3 [X_1 = 20, X_2 = 1]; E_4 [X_1 = 15, X_2 = 5]; E_5 [X_1 = 5, X_2 = 15]; E_6 [X_1 = 1, X_2 = 20] ; E_7 [X_1 = 20, X_2 = 10] ; E_8 [X_1 = 10, X_2 = 20] ; E_9 [X_1 = 20, X_2 = 20].

Table 3 shows the estimates of unit prices for nine apartments that have differences with respect to the two attributes considered in the example.

Table 3. *Comparison of methods*

Scenario	MCD	GP
E_1	46,67	51,46
E_2	66,77	67,59
E_3	75,49	75,21
E_4	81,81	81,34
E_5	95,29	99,78
E_6	114,87	103,06
E_7	103,21	110,25
E_8	118,05	121,50
E_9	139,18	134,00

The same results are presented in Figure 2.

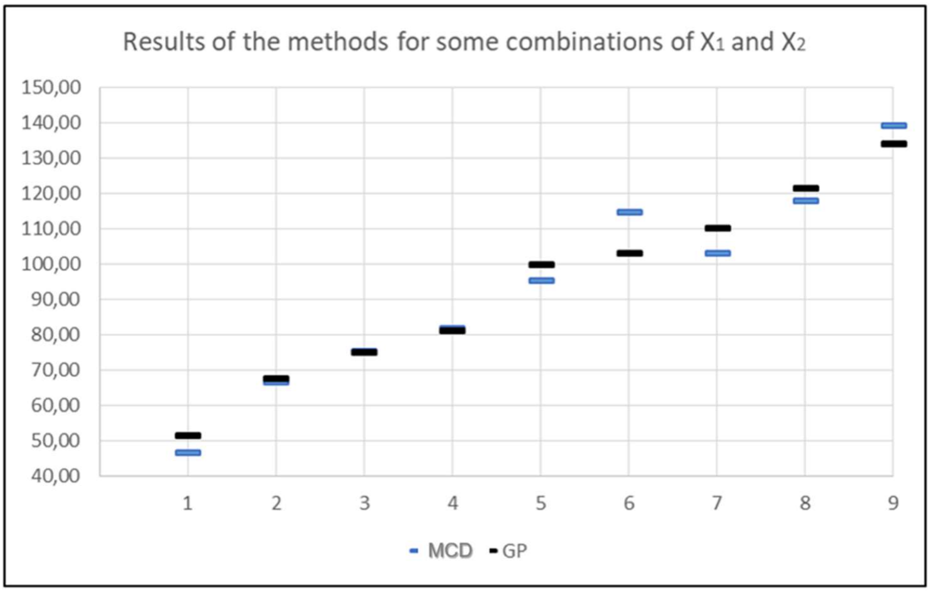

Figure 2. *Method comparison chart*

The results are similar in all cases, regardless of the level of the estimated price. Nor is there a bias in the sense that the estimate of one of the models is always above or below the estimate achieved by the other.

4.2.2 – Estimation using fitting a constant in a linear model.

This option yields a MAPE of 2.53%, and the results of the nine scenarios are as follows

Table 4. *Comparison of methods (for the second estimation option in MCD).*

Scenario	MCD	GP
E_1	53,50	51,46
E_2	68,12	67,59
E_3	70,29	75,21
E_4	77,85	81,34
E_5	98,94	99,78
E_6	97,23	103,06
E_7	110,37	110,25
E_8	118,32	121,50
E_9	127,16	134,00

Again, the data is presented graphically in figure 3.

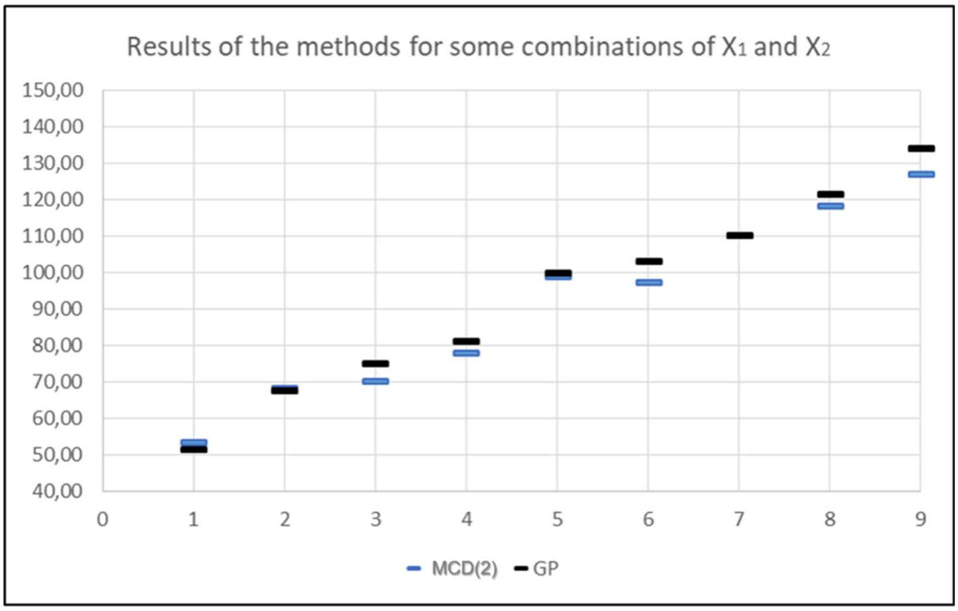

Figure 3. *Method comparison chart (for the second estimation option in MCD).*

The results are similar to the previous case, however, in this instance the difference with respect to GP is a little smaller than when the *price that minimizes inconsistencies* is used. However, although similar, the results are essentially different, as if they had been generated by completely different methods. Determining which of the two estimation approaches (*price that minimizes inconsistencies* or using *fitting a constant in a linear model*) could give better results in practice, will require future tests with robust databases, where both details of the properties and the price they achieved in the market are known.

Despite this, considering that the approach of fitting a constant actually requires determining several new parameters (all those present in equation 16), and that a linear structure is imposed, at least by a principle of parsimony, the option 3.4.1 which minimizes inconsistencies is preferable.

4.3– *Performance of the method with respect to outliers*

The method follows a similar behavior to GP also in terms of sensitivity to extreme values in some comparable, in general the results are not altered to a great extent when a data is varied.

As an example, the results of the original problem are compared below, when the value of a property is estimated with X1 = 7 and X2 = 7, with respect to a modified scenario where the comparable 5 takes a price of 98 \$ / m^2.

Table 5. *Performance with an outlier.*

Before (original problem)		
MCD	MCD (2)	GP
77,87	77,74	77,58
New results (price of 98 \$/m^2 for comparable 5)		
MCD	MCD (2)	GP
78,25	76,95	77,53

Conclusion

The article presents a novel valuation model, which operates under the premise of making the weights assigned to the factors that influence the value, manage to harmonize the price differences and the dissimilarities among the different properties that make up the sample of comparables.

The procedure is objective as far as possible, because once the variables of interest have been selected (process not covered in the methodology, but for which there are other techniques), the estimation is independent of the appraiser, and is based only on a nonlinear optimization program applied to the available data. This clearly implies a disadvantage, as it is not as simple as other comparison methods (the traditional grid adjustment for example). However, it may have applications beyond appraisals, and there is still work to be done to adapt it to estimates or forecasts in other areas. Likewise, since the method is based on the internal coherence of the data in the sample, it could be modified to help in the detection of outliers.

Conflicts of interest: This research has not been funded by any company or institution. The author declares that there are no conflicts of interest.

Conflictos de interés: Esta investigación no ha sido financiada por ninguna empresa o institución. El autor declara que no hay conflictos de interés.

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