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Adaptive Tracking Control of Robotic Manipulator Subjected to Actuator Saturation and Partial Loss of Effectiveness

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This paper introduces an adaptive control design tailored for robotic systems described by Euler–Lagrange equations under actuator saturation and partial loss of effectiveness. The adaptive law put forth not only retains conventional control properties but also extends its scope to effectively address challenges posed by actuator saturation and partial loss of effectiveness. The framework's primary focus is on bolstering system robustness, thereby ensuring the achievement of uniformly ultimate bounded tracking errors. The stability and convergence of the system's behavior are rigorously established through the application of the Lyapunov analysis technique. Moreover, the effectiveness and superiority of the introduced framework are compellingly demonstrated through a series of practical simulations and experimental instances. [DOI: 10.1115/1.4064653]

Keywords: actuator saturation, actuator partial loss-of-effectiveness, adaptive control, anti-windup, control applications, motion controls, nonlinear control, robotics

1 Introduction

In recent times, robots have found widespread applications across a vast array of fields, including manufacturing, human–robot collaboration, warehousing, and nursing. Actuators, playing a pivotal role in robots, serve as the driving force behind their motion. It's important to recognize that actuators possess physical limitations, which restrict their ability to generate torque or speed beyond certain thresholds [1,2]. In simpler terms, all actuators inherently exhibit saturation, stemming from their inherent physical properties. Moreover, alongside this natural saturation, actuators also exhibit artificial saturation, intentionally set by human operators to ensure the safety of both robots and the objects they interact with. Actuators can also be susceptible to malfunctions, including gradual loss of effectiveness over time. This makes it imperative to develop a comprehensive control framework for robotic systems that effectively addresses challenges arising from actuator saturation and loss of effectiveness, thereby averting potential instability [3].

While adaptive laws [4,5] offer the advantage of online parameter compensation to enhance system performance, the presence of saturation can introduce inaccuracies in updating the adaptive terms. This can result in windup behavior that not only compromises performance but also potentially triggers unstable states [1,6]. Extensive research efforts have been directed toward mitigating the detrimental effects of input saturation, leading to the classification of two primary approaches. The first method is exemplified by works such as Refs. [7,8]. Here, input saturation is eliminated using a smooth nonlinear function like the hyperbolic tangent function, i.e., $\tanh(\cdot)$. The second method adopts auxiliary functions, such as neural network functions [9,10] or stable linear functions [11,12], to estimate the discrepancy between the controller output and the saturation threshold. This estimation is subsequently harnessed to compensate for the deviation during the controller's design phase. Besides the challenges posed by saturation, the partial loss of effectiveness in actuators can similarly contribute to a decline in system performance. Consequently, numerous control algorithms have been devised to confront this concern, as evidenced by Refs. [13–15].

In existing literature, a range of studies have tackled issues pertaining to saturation and partial loss of effectiveness in both linear and nonlinear systems, as evidenced by Refs. [13,16–19]. These works encompass diverse control algorithm proposals, including adaptive control, model predictive control, backstepping control, and robust output-regulation control. However, despite the existence of these approaches, the exploration of these challenges within the context of Euler–Lagrange systems remains limited, even though Euler–Lagrange equations are typically the preferred modeling approach for practical mechatronics systems [20,21]. Consequently, there arises a need to develop dedicated control algorithms tailored to Euler–Lagrange systems, capable of effectively managing issues arising from actuator saturation and loss of effectiveness.

In this paper, we present a pioneering control framework to address this combined challenge. The proposed approach leverages the Lyapunov theory to analyze its effectiveness. Moreover, we provide experimental results featuring two-link manipulators, showcasing the practical viability of the framework. The novel controller demonstrates remarkable resilience against both actuator saturation and partial loss of effectiveness. It attains several pivotal objectives, including the following: First, the adaptive

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laws proposed within the framework effectively mitigate the undesirable impacts of nonlinearity within control inputs, encompassing the saturation and loss of effectiveness of actuators, thereby enhancing the performance of the closed-loop system. Second, the stability of the closed-loop system is ensured, with the filtered tracking error achieving uniform ultimate boundedness. Third, in the absence of saturation, the filtered tracking error exhibits asymptotic stability at the origin. This comprehensive approach addresses the intricate interplay between saturation and partial loss of effectiveness, contributing to the robustness and reliability of control in robotic systems modeled by Euler–Lagrange equations.

2 Problem Formulation

Let's consider a robotic system modeled by Euler–Lagrange equation

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where $q, \dot{q}, \ddot{q} \in \mathcal{R}^n$, respectively, denote the generalized position, velocity, acceleration vectors in the generalized coordinates, $M(q) \in \mathcal{R}^{n\times n}$ stands for the inertia matrix and is symmetric positive-definite, $C(q, \dot{q}) \in \mathcal{R}^{n\times n}$ represents the Coriolis and centripetal forces matrix, $G(q) \in \mathcal{R}^n$ denotes the gradient of the potential function, and $\tau \in \mathcal{R}^n$ is the output of the controller which assumes equal to the output of the actuators in the ideal case. The dynamic model, Eq. (1), possesses several useful properties, which are recalled here [22].

Property 1. $\dot{M}(q) - 2C(q, \dot{q})$ is a skew symmetric matrix, then, $\xi^T[\dot{M}(q) - 2C(q, \dot{q})]\xi = 0$, where $\xi \in \mathcal{R}^n$ is any vector.

Property 2. The dynamics (1) linearly depends on a constant vector $\Theta \in R^w$ of the system parameters such that $M(q)\dot{\xi} + C(q, \dot{q})\xi + G(q) = Y(q, \dot{q}, \xi, \dot{\xi})\Theta$, where $Y(q, \dot{q}, \xi, \dot{\xi}) \in R^{n\times w}$ is a known regressor matrix and $\xi \in R^n$ is any differential vector.

Let $X(t) = h(q) \in \mathcal{R}^p$ represent a trajectory of the system in the task space, where $h(q): \mathcal{R}^n \to \mathcal{R}^p$ is a mapping from the generalized coordinates to the task-space coordinates. We have the kinematic relationship between $X(t)$ and $q(t)$ as $\dot{X} = \frac{\partial h(q)}{\partial q}\dot{q} = J(q)\dot{q}$, where $J(q) = \frac{\partial h(q)}{\partial q} \in \mathcal{R}^{p\times n}$ is the Jacobian matrix. Assuming that the robot works in the singularity-free regions, the inverse of $J(q)$ always exists.

Given a desired trajectory $X^d(t) \in \mathcal{R}^p$ in the task space with bounded first and second derivatives such as $\dot{X}^d(t) \in \mathcal{L}_\infty$ and $\ddot{X}^d(t) \in \mathcal{L}_\infty$. $e(t) = X(t) - X^d(t)$ denotes the tracking error. It is well-known that the adaptive control algorithm, which can drive $X(t)$ to asymptotically track the desired value $X^d(t)$, e.g., $\lim_{t\to\infty} e(t) = 0$, is given as below [23]

$$\tau = Y\hat{\Theta} - K_s s - J^T K_J J s, \quad \dot{\hat{\Theta}} = -\Gamma Y^T s \quad (2)$$

where $Y\hat{\Theta} = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta}$, $\dot{q}_r = J^+(\dot{X}^d - \lambda e)$, $\ddot{q}_r = J^+(\ddot{X}^d - \lambda\dot{e}) + \dot{J}^+(\dot{X}^d - \lambda e)$, and $s = \dot{q} - \dot{q}_r$, respectively, denote the linearization dynamics, reference velocity, acceleration, and filtered tracking error, $J^+ := J^T(JJ^T)^{-1}$ stands for a pseudo-inverse of the Jacobian matrix J if $n > p$, and $J^+ := J^{-1}$ if $n = p$, K_s, K_J, Γ, λ represent control gains, which are positive-definite diagonal matrices with appropriate dimension, and $\dot{\hat{\Theta}}$ is the adaptive law to tune the parameters of the robot.

The fact is always existing saturation in actuators since all the actuators have physical torque limits. In addition, working in hostile environments or for long-time operations, robots may encounter unanticipated problems such as loss-of-effectiveness in actuators. To cover both the saturation and partial-loss-of-effectiveness in actuators, the robot dynamics is represented as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \mathrm{sat}\big((I_{n\times 1} - \delta) \odot \tau^p\big) \quad (3)$$

where $\odot$ denotes the Hadamard (element-wise) product, $\delta = [\delta_1, \ldots, \delta_i, \ldots, \delta_n]^T$ is the vector characterizing actuator fault severity with $\delta_i \in [0, 1)$, $(I_{n\times 1} - \delta) \odot \tau^p \in \mathcal{R}^n$ presents partial-loss-of-effectiveness of the actuators [24], $\tau^p \in \mathcal{R}^n$ denotes the output of the controller, $\mathrm{sat}\big((I_{n\times 1} - \delta) \odot \tau^p\big): \mathcal{R}^n \to \mathcal{R}^n$ represents the output of the actuators, and $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \ldots, \mathrm{sat}(u_i), \ldots, \mathrm{sat}(u_n)]^T \in \mathcal{R}^n$ is a saturation function whose elements are defined as [15]

$$\mathrm{sat}(u_i) = \begin{cases} \bar{u}_i, & \text{if } u_i > \bar{u}_i \\ u_i, & \text{if } -\bar{u}_i \le u_i \le \bar{u}_i \\ -\bar{u}_i, & \text{if } u_i < -\bar{u}_i \end{cases} \quad (4)$$

where $\bar{u}_i > 0$ presents the magnitude constrain of each actuator, $\bar{u} = [\bar{u}_1, \ldots, \bar{u}_i, \ldots, \bar{u}_n]^T$. To assure the feasibility of solutions, the following assumptions are provided:

Assumption 1. $\delta_i \in [0, 1)$ *is quasi-static such that* $\frac{\mathrm{d}}{\mathrm{d}t}(\delta_i) = 0$.

Assumption 2. $0 \le |g_i(q)| < \bar{u}_i, \forall i \in \{1, 2, \ldots, n\}$.

Assumption 1 is required for adaptive laws and Assumption 2 is needed to guarantee the working condition of actuators at any desired equilibrium configuration.

It is observed that the aforementioned control law, refer to Eq. (2), does not work well in Eq. (3). Because the outputs of the actuators, $\mathrm{sat}\big((I_{n\times 1} - \delta) \odot \tau^p\big)$, are different from the output of the controller, τ^p, in the presence of saturation or loss of effectiveness. When these issues occur, the outputs of actuators do not match the controller outputs, thus, the actuators cannot drive the robot as expected. Consequently, the states of the controller are incorrectly updated. This behavior is named controller windup [1]. Furthermore, the control law (2) has an integrator inside. When saturation occurs, the outputs of the actuators do not change but the output of the controller might keep increasing caused by the low response of the filtered tracking error, s, to the adaptive law (2). If the duration of the saturation is long enough, the controller output, τ, and the estimated value, $\hat{\Theta}$, can become very large that leads to an unstable state.

It is worth mentioning that, to reduce the effects of input saturation in Euler–Lagrange systems, consider there is no partial-loss-of-effectiveness $\delta = 0$, many papers, e.g., Refs. [11,12,15,25], have used the auxiliary function

$$\dot{\xi} = \begin{cases} -K_\xi \xi - \frac{\|s^T \Delta_\tau\| + 0.5\|\Delta_\tau\|^2}{\|\xi\|^2}\xi + \Delta_\tau, & \text{if } \|\xi\| > \nu \\ 0, & \text{if } \|\xi\| \le \nu \end{cases} \quad (5)$$

and the controller

$$\tau^p = Y\hat{\Theta} - J^T K_J J s - K_s s + K_P \xi \quad (6)$$

where $\xi \in \mathcal{R}^n$ is the state of the function, $\Delta_\tau = \mathrm{sat}(\tau^p) - \tau^p$ denotes the different between the output of the actuator and the output of the controller, ν is a small positive value, $K_\xi, K_J, K_s, K_P \in \mathcal{R}^{n\times n}$ denote positive-definite diagonal matrices. Substituting Eq. (6) in Eq. (3) leads to the closed-loop dynamics

$$M\dot{s} = \Delta_\tau + Y\tilde{\Theta} - Cs - J^T K_J J s - K_s s + K_P \xi \quad (7)$$

where $\tilde{\Theta} = \hat{\Theta} - \Theta$ is the estimated error of the robot's parameters. Let us analyze the above control framework to show that the auxiliary function (5) is not effective in handling the saturation issue. Consider a positive function

$$V = \frac{1}{2}s^T M s + \frac{1}{2}\tilde{\Theta}^T\tilde{\Theta} + \frac{1}{2}\xi^T\xi \quad (8)$$

Differentiating V along the dynamics (7) yields

$$\begin{aligned} \dot{V} &= s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s + \tilde{\Theta}^T\dot{\hat{\Theta}} + \xi^T\dot{\xi} \\ &= s^T\big(\Delta_\tau + Y\tilde{\Theta} - J^T K_J J s - K_s s + K_P\xi - Cs\big) \\ &\quad + \frac{1}{2}s^T\dot{M}s + \tilde{\Theta}^T\dot{\hat{\Theta}} + \xi^T\dot{\xi} \end{aligned} \quad (9)$$

From the adaptive law (2) and the skew symmetric property (Property 1), the above equation is reduced such that

$$\dot{V} = s^T\left(\Delta_\tau - J^T K_J J s_i - K_s s + K_P \xi\right) + \xi^T \dot{\xi} \tag{10}$$

Since $\xi^T \Delta_\tau \le \frac{1}{2}\xi^T\xi + \frac{1}{2}\Delta_\tau^T \Delta_\tau$, $s^T K_P \xi \le \frac{1}{2}s^T s + \frac{1}{2}\xi^T K_P^T K_P \xi$, and from Eq. (5), we have

$$\begin{aligned}\xi^T \dot{\xi} &= -\xi^T K_\xi \xi - \|s^T \Delta_\tau\| - 0.5\|\Delta_\tau\|^2 + \xi^T \Delta_\tau \\ &\le -\xi^T\left(K_\xi - \frac{1}{2}I_n\right)\xi - \|s^T \Delta_\tau\|\end{aligned} \tag{11}$$

Thus, we obtain

$$\dot{V} \le -s^T\left(J^T K_J J + K_s - I_n\right)s - \xi^T\left(K_\xi - \frac{1}{2}(I_n + K_P^T K_P)\right)\xi \tag{12}$$

By selecting $J^T K_J J + K_s > I_n$ and $K_\xi - \frac{1}{2}(I_n + K_P^T K_P) \ge \frac{1}{2}I_n$, we have that

$$\dot{V} \le -s^T\left(J^T K_J J + K_s - I_n\right)s \le 0 \tag{13}$$

Thus, Refs. [11,12,15,25] claim such that the control framework, which is addressed in Eqs. (5) and (6), guarantees asymptotic stability at the origin, if the control gains are selected such that $J^T K_J J + K_s > I_n$ and $K_\xi - \frac{1}{2}(I_n + K_P^T K_P) \ge 0$. However, the condition $K_\xi - \frac{1}{2}(I_n + K_P^T K_P) \ge 0$ indicates that $\xi^T \dot{\xi} \le 0$. The condition $\xi^T \dot{\xi} \le 0$ indicates that ξ in Eq. (5) exponentially converges to the origin and stays there forever. Therefore, the auxiliary function (5) does not effect to help with the actuator saturation issues.

In the following section, we propose a control framework that handles the issue of Eq. (5) and the problems of actuators such that: it reduces the difference between the controller output and the output of the actuator, and it also diminishes the increase of the controller output when saturation occurs. Furthermore, the framework can simultaneously solve the issue of the loss of effectiveness.

3 Controller Design

The dynamic model (3) can be rewritten as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Delta u + (I_{n\times 1} - \delta) \odot \tau^p \tag{14}$$

where $\Delta u = \text{sat}\left((I_{n\times 1} - \delta) \odot \tau^p\right) - (I_{n\times 1} - \delta) \odot \tau^p$. The magnitude of $|\Delta u|$ degrades system performance since it affects the size of the stability region and the transient response. Consider that the real-value of $|\Delta u|$ is unknown but its bounded value is know such as $|\Delta u| \le \bar{U}_{\max}$. To mitigate the influence of the saturation, a bumpless transfer compensation function is formed such that

$$\dot{\xi} = -K_\xi \xi + \bar{U}, \quad \bar{U} = \begin{cases} 0, & \text{if } |\tau^p| \le \bar{u} \\ |\tau^p| - \bar{u}, & \text{if } |\tau^p| > \bar{u} \end{cases} \tag{15}$$

To mitigate the adverse effect of the actuator problems, we proposed an adaptive control scheme of the form

$$(I_{n\times 1} - \hat{\delta}) \odot \tau^p = \tau^* \tag{16}$$

$$\tau^* = Y\hat{\Theta}_e - K_s s - J^T K_J J s + K_P \xi \odot \text{sgn}(s) \tag{17}$$

$$\dot{\hat{\delta}} = -\varepsilon^{(-K_\delta \|\xi\|)} \Gamma_\delta (\tau^p \odot s) \tag{18}$$

$$\dot{\hat{\Theta}}_e = -\varepsilon^{(-K\|\xi\|)} \Gamma Y^T s \tag{19}$$

$$K_s > I_n \tag{20}$$

$$K_\xi > \frac{1}{2}(I_n + K_P^T K_P) \tag{21}$$

where K_s, K_J, K_P, Γ_δ, and Γ are positive-definite diagonal matrices, K_δ and K are positive scale values, $\varepsilon^{(\cdot)}: \mathcal{R} \to \mathcal{R}_{\ge 0}$ denotes the exponential function, $\hat{\delta}$ and $\hat{\Theta}_e$ respectively denote learned values of δ and Θ.

Remark 1. Different from related works, e.g., Refs. [11,12,25], the exponential functions, e.g., $\varepsilon^{(-K_\delta\|\xi\|)}$ and $\varepsilon^{(-K\|\xi\|)}$, are integrated into the adaptive laws to reduce the adverse effect of the saturation on the estimated values.

Equation (14) can be rewritten such that

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Delta u + \tau^p - \delta \odot \tau^p \tag{22}$$

and Eq. (16) can be rewritten such that $\tau^p = \hat{\delta} \odot \tau^p + \tau^*$. Thus, substituting Eq. (16) into Eq. (14) yields to $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Delta u + \tilde{\delta} \odot \tau^p + \tau^*$, where $\tilde{\delta} = \hat{\delta} - \delta$. Subsequently, by using Eq. (17) we obtain the closed-loop dynamics

$$M\dot{s} = \Delta u + \tilde{\delta} \odot \tau^p + Y\tilde{\Theta} - Cs - K_s s - J^T K_J J s + K_P \xi \odot \text{sgn}(s) \tag{23}$$

The main result of the paper is summarized as follows.

Theorem 1. Consider the uncertain Euler–Lagrange system (3) satisfying Assumptions 1 and 2. By applying the control framework, from Eq. (15) to Eq. (21),

(1) the filtered error s achieves UUB under actuator saturation and partial loss of effectiveness.
(2) Moreover, the filtered error s achieves asymptotically stable at the origin if the actuators are not saturated.

Proof. Let's consider a Lyapunov-like function candidate $V = \frac{1}{2}s^T M s + \frac{1}{2}\varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\tilde{\Theta} + \frac{1}{2}\varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\tilde{\delta} + \frac{1}{2}\xi^T\xi$. The derivative of V along the dynamics (23) is

$$\begin{aligned}\dot{V} &= s^T M\dot{s} + \frac{1}{2}s^T \dot{M} s + \xi^T \dot{\xi} \\ &\quad + \varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\dot{\hat{\Theta}}_e + \frac{K}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\tilde{\Theta} \\ &\quad + \varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\dot{\hat{\delta}} + \frac{K_\delta}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\tilde{\delta} \\ &= s^T\left(\Delta u + \tilde{\delta} \odot \tau^p + Y\tilde{\Theta} - Cs - J^T K_J J s \right. \\ &\quad \left. - K_s s + K_P \xi \odot \text{sgn}(s)\right) + \frac{1}{2}s^T \dot{M} s + \xi^T \dot{\xi} \\ &\quad + \varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\dot{\hat{\Theta}}_e + \frac{K}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\tilde{\Theta} \\ &\quad + \varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\dot{\hat{\delta}} + \frac{K_\delta}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\tilde{\delta}\end{aligned} \tag{24}$$

Applying skew symmetric property, Property 1, we have

$$\begin{aligned}\dot{V} &= s^T\left(\Delta u - J^T K_J J s - K_s s + K_P \xi \odot \text{sgn}(s)\right) \\ &\quad + \xi^T \dot{\xi} + s^T Y\tilde{\Theta} + s^T \tilde{\delta} \odot \tau^p \\ &\quad + \varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\dot{\hat{\Theta}}_e + \frac{K}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\tilde{\Theta} \\ &\quad + \varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\dot{\hat{\delta}} + \frac{K_\delta}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\tilde{\delta}\end{aligned} \tag{25}$$

and then applying adaptive laws (18) and (19) on the above equation leads that

$$\begin{aligned}\dot{V} &= s^T\left(\Delta u - J^T K_J J s - K_s s + K_P \xi \odot \text{sgn}(s)\right) \\ &\quad + \xi^T \dot{\xi} + \frac{K}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K\|\xi\|)}\tilde{\Theta}^T \Gamma^{-1}\tilde{\Theta} + \frac{K_\delta}{2}\frac{\xi^T\dot{\xi}}{\|\xi\|}\varepsilon^{(K_\delta\|\xi\|)}\tilde{\delta}^T \Gamma_\delta^{-1}\tilde{\delta}\end{aligned} \tag{26}$$

Note that $s^T \Delta u \le \frac{1}{2} s^T s + \frac{1}{2} \Delta u^T \Delta u \le \frac{1}{2} s^T s + \frac{1}{2} \bar{U}^T \bar{U}$, $s^T K_P \xi \odot \mathrm{sgn}(s) \le \dfrac{1}{2} s^T s + \dfrac{1}{2} \xi^T K_P^T K_P \xi$. Since the system (15) is input to state stable and satisfies $\|\xi\| \le \|\bar{U}/K_\xi\|$ and $\|\dot{\xi}\| \le \|\bar{U}\|$ we have $\frac{\xi^T \dot{\xi}}{\|\xi\|} \le \|\dot{\xi}\| \le \|\bar{U}\|$, and from Eq. (15), we also have $\xi^T \dot{\xi} = -\xi^T K_\xi \xi + \xi^T \bar{U} \le -\xi^T (K_\xi - \dfrac{1}{2} I_n) \xi + \dfrac{1}{2} \bar{U}^T \bar{U}$. Hence, we can obtain that

$$\begin{aligned} \dot{V} &\le -s^T \left(J^T K_J J + K_s - I_n\right) s \\ &\quad - \xi^T (K_\xi - \frac{1}{2}(I_n + K_P^T K_P)) \xi + \Delta_{\bar{U}} \end{aligned} \tag{27}$$

where $\Delta_{\bar{U}} = \bar{U}^T \bar{U} + \dfrac{\|\bar{U}\|}{2} (K \varepsilon^{(K\|\xi\|)} \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} + K_\delta \varepsilon^{(K_\delta \|\xi\|)} \tilde{\delta}^T \Gamma_\delta^{-1} \tilde{\delta})$. By choosing $K_s > I_n$ and $K_\xi > \frac{1}{2}(I_n + K_P^T K_P)$, $\dot{V}$ is further bounded such that

$$\dot{V} \le -\lambda_{\min} \left(J^T K_J J + K_s - I_n\right) \|s\|^2 + \Delta_{\bar{U}} \tag{28}$$

(1) It is obvious that $\Delta_{\bar{U}} > 0$ if the system has actuator saturations. In this case, $\dot{V}$ is negative as long as $\|s\| \ge \sqrt{\frac{\Delta_{\bar{U}}}{\lambda_{min}\left(J^T K_J J + K_s - I_n\right)}}$. Thus, we can conclude that s is uniformly ultimately bounded under actuator saturations. It is worth mentioning that the upper bounds of $\|s\|$ is for the worst case, where divergence and convergence of $\|s\|$ tend to stabilize at $\sqrt{\frac{\Delta_{\bar{U}}}{\lambda_{\min}(J^T K_J J + K_s - I_n)}}$.

(2) $\Delta_{\bar{U}} = 0$ if the system has no actuator saturations, thus, Eq. (28) can be written as $\dot{V} \le -\lambda_{\min}\left(J^T K_J J + K_s - I_n\right) \|s\|^2$. Therefore, we can conclude that $\|s\| \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\tilde{\Theta} \in \mathcal{L}_\infty$, and $\tilde{\delta} \in \mathcal{L}_\infty$. In addition, according to Barbalat's lemma, we can conclude that $\|s\|$ asymptotically converges to zero. ■

4 Numerical Verification

Consider a two-link manipulator with revolute joints, where the manipulator parameters are given as $m_1 = 1.2\,\mathrm{kg}$, $m_2 = 0.6\,\mathrm{kg}$, $I_1 = 0.24\,\mathrm{kg\,m^2}$, $I_2 = 0.12\,\mathrm{kg\,m^2}$, $l_1 = 1.5\,\mathrm{m}$, $l_2 = 1.4\,\mathrm{m}$, $l_{c1} = l_1/2$, $l_{c2} = l_2/2$, $g = 9.81\,\mathrm{m/s^2}$. The detail of dynamic model (1) can be found in Ref. [22]. In the simulations, the initial conditions are selected as $q(0) = [-0.15,\ 1.5]^T$ rad, $\hat{\Theta}_e(0) = [2,\ 1,\ 1,\ 1,\ 1]^T$. The design trajectory is given as $X^d = [1.2 + 0.5\sin(0.3t),\ 1 + 0.3\cos(0.3t)]^T$. The control parameters are selected as follows, $K_s = 5$, $K_J = 2$, $K_P = 1$, $K_\xi = 3$, $K = 100$, $K_\delta = 100$, and $\lambda = 20$.

First, we conduct a simulation using control law (6) with actuator saturations, $\bar{u} = [25,\ 25]^T$. The simulation results are shown in Figs. 1 and 2. Figure 1 demonstrates that the output of the robot cannot track the desired trajectory. Figure 2 illustrates that the actuator outputs are bumped between saturation values.

Second, we conduct a simulation using control law (16) with both saturation and partial loss of effectiveness of actuators, $\bar{u} = [25,\ 25]^T$ and $\delta = [0.2,\ 0.3]^T$. The simulation results are shown in Figs. 3 and 4. Figure 3 illustrates that the output of the robot can track the desired trajectory very well. It is almost the same as the case of the ideal actuators. Figure 4 demonstrates that the actuator outputs shortly chatter between ± 25 and then stable.

The norms of the tracking errors, $\|e(t)\|$, of the previous two simulations and the ideal case are shown in Fig. 5. It further verifies that the proposed control framework effectively deals with both saturation and partial loss of effectiveness of actuators since the tracking error under actuator issues is close to the tracking error under ideal actuators.

5 Experimental Verification

In this section, we use a PHANToM Omni haptic device (Touch) to verify the proposed control algorithm. The control algorithm is generated by using C++ on the desktop computer, where Open-Haptic interface API is employed to exchange signals between Touch and the computer. The Touch's kinematic model, dynamic model, regressor matrix, and Jacobian matrix can be obtained in Ref. [26]. The end-effector of Touch will be controlled to track a desired trajectory $X_0(t) = [0;\ -40;\ -110] + [0;\ 1t;\ 1t]\,\mathrm{mm}$ on the vertical plane Oyz starting from its initial position selected randomly round $[0;\ -40;\ -110]\,\mathrm{mm}$. Control parameters are selected such as $K_s = \mathrm{diag}\{100, 30, 30\}$, $K_J = \mathrm{diag}\{0.01, 0.01, 0.01\}$, $K_\xi = \mathrm{diag}\{3, 3, 3\}$, $K_P = \mathrm{diag}\{1, 1, 1\}$, $\Gamma = \mathrm{diag}\{1, 1, 1, 1, 1,\ 1, 1, 1\}$, $\bar{u} = [350; 150; 75]$, $\lambda = 1$, $K = K_\delta = 1$, $\delta = [0; 0.2; 0.2]$, $\hat{\Theta}(0) = [0.3; -0.05; 0.3; 0.4; 0.4; 0.2; 50; 100]$, $\hat{\delta}(0) = [0; 0; 0]$, and $\Gamma_\delta = \mathrm{diag}\{1, 1, 1\}$.

We present two sets of experimental results. The first set involves saturation and utilizes a control law discussed in Eq. (2). The second set incorporates both saturation and partial loss of effectiveness, employing the control law we propose in Eq. (16). These results are depicted in Figs. 6 and 7, respectively. Upon observing the outcomes in Fig. 6, we notice significant chattering behavior between the 15th and 25th seconds and the robot has a significant vibration. It's worth noting that this is the best outcome we've achieved,

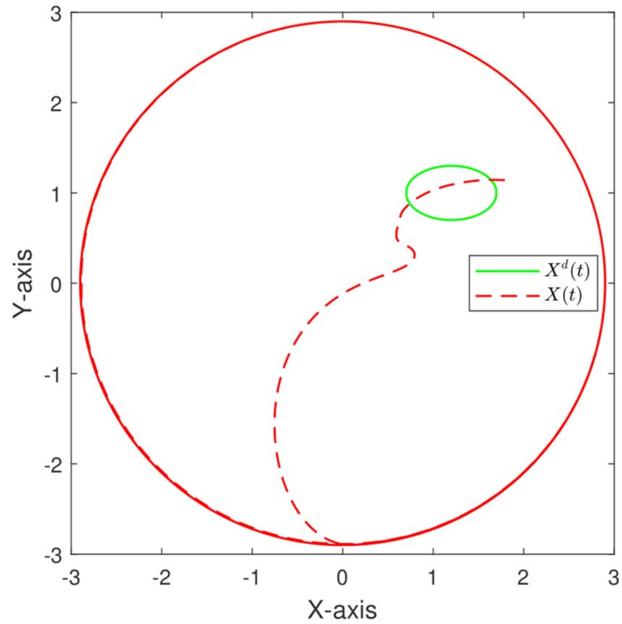

Fig. 1 Position tracking result under control law (6) with actuator saturations, $\bar{u} = [25, 25]^T$

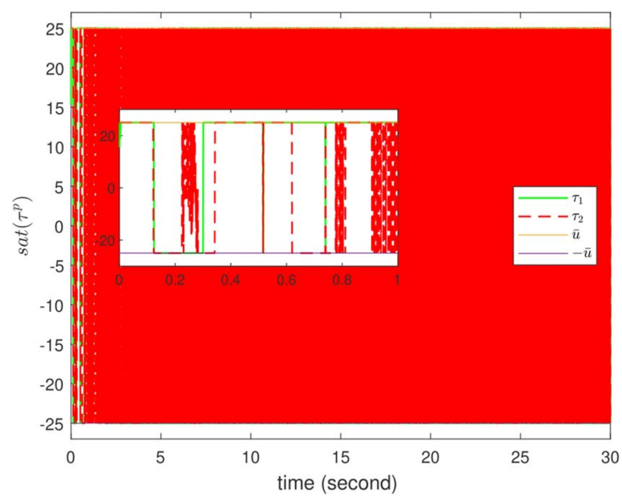

Fig. 2 Responses of actuators under control law (6) with actuator saturations, $\bar{u} = [25, 25]^T$

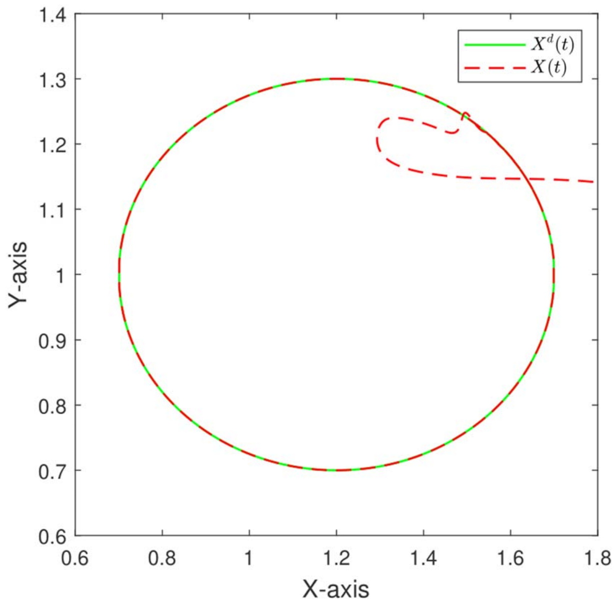

Fig. 3 Position tracking result under control law (16) with both saturation and partial loss of effectiveness of actuators, $\bar{u} = [25, 25]^T$ and $\delta = [0.2, 0.3]^T$

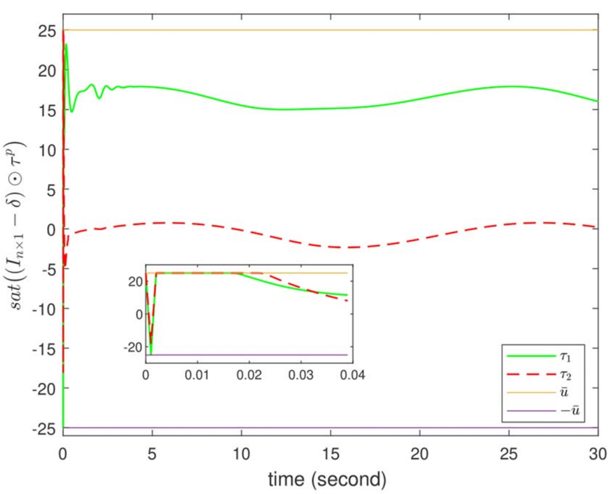

Fig. 4 Responses of actuators under control law (16) with both saturation and partial loss of effectiveness of actuators, $\bar{u} = [25, 25]^T$ and $\delta = [0.2, 0.3]^T$

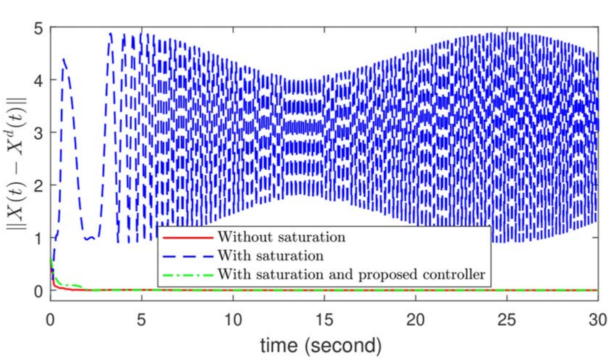

Fig. 5 Norms of the tracking errors

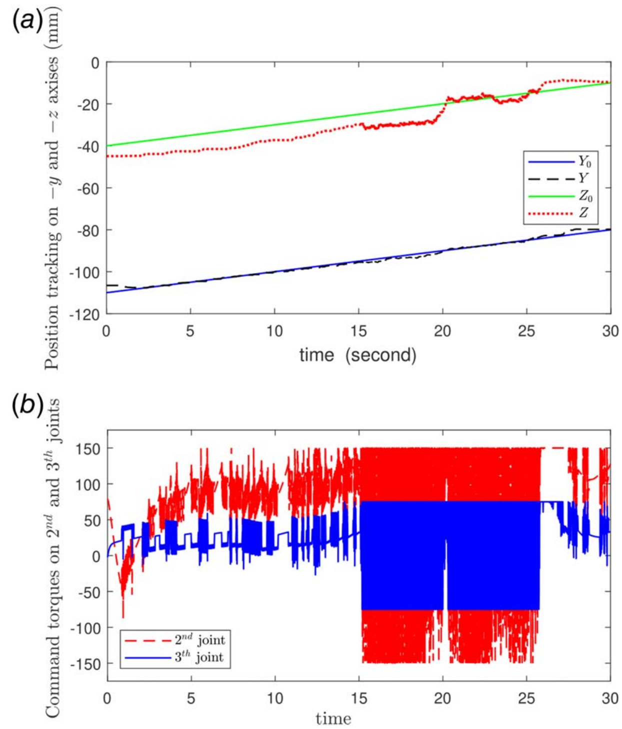

Fig. 6 Experimental results with saturation $\bar{u} = [350; 150; 75]$ and using control law (2): (*a*) tracking results on vertical plane and (*b*) command torques

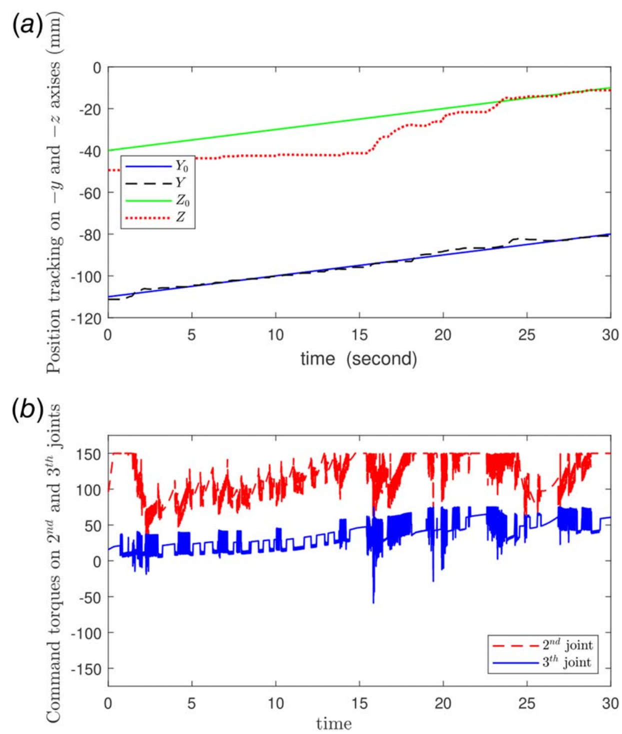

Fig. 7 Experimental results with saturation $\bar{u} = [350; 150; 75]$ and loss of effectiveness $\delta = [0; 0.2; 0.2]$ and using control law (16): (*a*) tracking results on vertical plane and (*b*) command torques

as in most cases, the system becomes unstable shortly after chattering. In contrast, Fig. 7 demonstrates minimal chattering behavior. This underscores the effectiveness of our proposed control algorithm in addressing issues related to actuators, including saturation and partial loss of effectiveness. It is worth mentioning that the proposed control framework can be directly applied to robotic systems under normal conditions. This enhancement significantly bolsters the safety and reliability of the control system, as it autonomously compensates for challenges associated with saturation and loss of effectiveness.

6 Conclusion

This paper presents an adaptive control framework tailored for robotic systems modeled by the Euler–Lagrange equation, while addressing significant actuator-related challenges. Specifically, we tackle the complex issues of saturation and partial loss of effectiveness. To counteract saturation, an auxiliary function is introduced, followed by the proposal of a modified adaptive law. This novel law facilitates the estimation of unknown loss of effectiveness values and system parameters. The application of Lyapunov stability theory substantiates that the filtered tracking error achieves uniform ultimate boundedness in the presence of saturation, and attains asymptotic stability at the origin when saturation is absent. In the forthcoming stages, our intent is to expand upon this research by delving into the synchronization of multi-robotic systems, where communication delays will be a crucial factor to consider. This extension would contribute to a broader application of the proposed framework and further advance the understanding and control of complex robotic systems.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

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