Humidification requirements in economizer-type HVAC systems

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Abstract

We develop a formulation to compute the maximum humidification load for economizer-type HVAC systems. The resulting formulae can be used, together with ASHRAE-provided climate data, to find the maximum humidification load in both temperature-based and enthalpy-based economizer systems.

1 Introduction

The humidification requirement of a heating, ventilation, and air-conditioning (HVAC) system is determined by the amount of outside air (OA) that the system uses and the difference between the psychrometric properties of outside air versus space design.

This requirement is computed relatively easily when the amount of outside air used by the HVAC system is fixed. The situation is more complicated when the amount of outside air varies as a function of its psychrometric properties: this is the case when an "economizer" type system is used [1], which is designed with the intent to minimize energy consumption. Two types of economizer systems are in widespread use: temperature-based and enthalpy-based economizers. In this paper, we derive explicit formulas for the humidification loads that occur in non-economizer, temperature-based economizer, and enthalpy-based economizer systems.

2 The properties of moist air

Moist air is a mixture of air (itself composed primarily of diatomic molecules, notably N_2 and O_2) and water vapor (H_2O). The specific heat c_V of a diatomic gas at constant volume is [2]

$$c_V^{\rm dg} = \frac{5R}{2M_n}, \tag{1}$$

where $R \simeq 8.31~{\rm JK^{-1}mol^{-1}} \simeq 3.40~{\rm ft{\cdot}lb{\cdot}R^{-1}mol^{-1}}$ is the ideal gas constant and M_n ($\simeq 0.029$ kg/mol $\simeq$ 0.064 lb/mol for air) is the molar mass of the gas. For air, we get

$$c_V^{\rm da} = 716~{\rm JK^{-1}kg^{-1}} = 133~{\rm lb\cdot ft_f\cdot R^{-1}lb_w^{-1}}, \tag{2}$$

which agrees well with the observed value ($718~{\rm JK^{-1}kg^{-1}} = 133~{\rm lb_f{\cdot}R^{-1}lb_w^{-1}}$).

The factor of 5 in the numerator of Eq. (1) is the number of kinetic degrees of freedom for a diatomic molecule at room temperature: in addition to the three translational degrees of freedom, diatomic molecules also have two rotational degrees of freedom. Water molecules are triatomic, with 6 kinetic degrees of freedom at room temperature (three translational, three rotational). Given a molar mass of $M_n = 0.018$ kg/mol for water vapor, we get

$$c_V^{\rm wv} = \frac{6R}{2M_n} \simeq 1.39~{\rm kJK^{-1}kg^{-1}} = 257~{\rm lb\cdot ft_f\cdot R^{-1}lb_w^{-1}}, \tag{3}$$

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which agrees well with the observed value of 1.38 $\text{kJK}^{-1}\text{kg}^{-1}$ (257 $\text{lb}\cdot\text{ft}_\text{f}\cdot\text{R}^{-1}\text{lb}_\text{w}^{-1}$) at room temperature.

The water vapor content of moist air is usually measured using a dimensionless number, the humidity ratio w, which is the ratio of the mass of water vapor and the mass of dry air. Thus, for a given quantity of moist air characterized by w, the specific heat can be calculated as

$$c_V^{\text{moist air}} = \frac{w}{1+w} c_V^{\text{wv}} + \frac{1}{1+w} c_V^{\text{da}}. \tag{4}$$

Given that $w \ll 1$, we have $1/(1+w) \simeq 1 - w$, and terms quadratic in w can be dropped, allowing us to obtain a further simplification:

$$c_V^{\text{moist air}} \simeq (c_V^{\text{wv}} - c_V^{\text{dryair}})w + c_V^{\text{da}}. \tag{5}$$

The specific heat at constant pressure, c_P, is connected with c_V by

$$c_P - c_V = \frac{R}{M_n}, \tag{6}$$

which is known as Mayer's relation and can be derived from axiomatic thermodynamics. Thus, for dry air we get

$$c_P^{\text{da}} = \frac{7}{5} c_V^{\text{da}}, \tag{7}$$

and for water vapor,

$$c_P^{\text{wv}} = \frac{4}{3} c_V^{\text{wv}}, \tag{8}$$

again in good agreement with the observed values of 1.01 and 1.84 $\text{kJK}^{-1}\text{kg}^{-1}$ (0.24 and 0.44 $\text{Btu}\cdot\text{R}^{-1}\text{lb}^{-1}$), respectively. For moist air, we get

$$c_P^{\text{moist air}} = \frac{w}{1+w} c_P^{\text{wv}} + \frac{1}{1+w} c_P^{\text{da}} \simeq c_P^{\text{da}} + (c_P^{\text{wv}} - c_P^{\text{da}})w. \tag{9}$$

The enthalpy of moist air, $h^{\text{moist air}}$ is the sum of the heat content of moist air plus the latent heat of vaporization h^{vap}:

$$h^{\text{moist air}} = c_P^{\text{moist air}} T + h^{\text{vap}} w. \tag{10}$$

By convention, ASHRAE chooses the heat content of moist air zero at the starting point T_0 of the temperature scale[1], while calculating the vaporization heat h_0^{vap} at T_0 [3]. Thus, the enthalpy is written as:

$$h_{\text{ASHRAE}}^{\text{moist air}} = c_P^{\text{moist air}}(T - T_0) + h_0^{\text{vap}} w. \tag{11}$$

The latent heat of vaporization for water vapor at T_0 is $h_0^{\text{vap}} \simeq 2501$ kJ/kg (1061 Btu/lb).

Per unit of *dry air* (1 unit of dry air is contained in $(1+w)$ units of moist air), therefore, we obtain, in SI units,

$$h_{\text{ASHRAE}}^{\text{moist air}} \simeq (1.85w + 1.00)(T - T_0) + 2501w \text{ kJ/kg}. \tag{12a}$$

[1] A note about a subtle trap that one can fall into at this point: we are no longer free to choose an arbitrary starting point for our temperature scale! Consider the partial derivative of $h^{\text{moist air}}$ with respect to w, given that neither T nor h^{vap} depend on w:

$$\frac{\partial h^{\text{moist air}}}{\partial w} = T \frac{\partial c_P^{\text{moist air}}}{\partial w} + h_{\text{vap}}.$$

Clearly, changing the starting point of the temperature scale (from 0 K to, say, 273.15 K) changes the right-hand side of this equation in a way that is not accounted for by a change of dimensions. In other words, at this point the choice of temperature scale is no longer a matter of convention; the temperature scale must be an absolute scale, otherwise the results are incorrect. Specifically, if we naively assume that we can set the enthalpy scale such that the enthalpy of moist air at 0°C is its vaporization heat only, we make the invalid assumption that the heat content of air and water vapor are the same at a temperature other than absolute zero; this, of course, is wrong, as (assuming both are ideal gases) the heat content of a quantity of air at 0°C matches the heat content of a similar quantity of water vapor at −125.41°C. This problem does not affect calculations in which the humidity content of air remains constant (i.e., heating and cooling) but it can impact calculations that involve humidification or dehumidification.

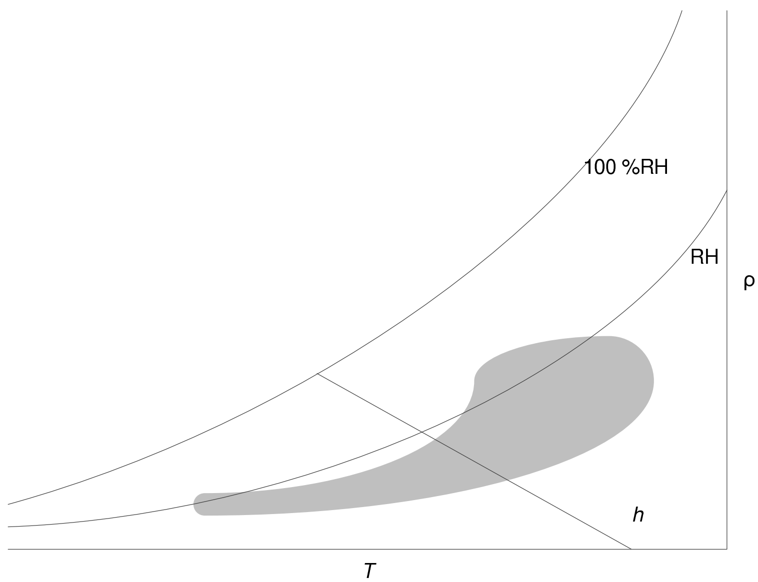

Figure 1: Schematic of a psychrometric chart, with one constant relative humidity (RH) and one constant enthalpy (h) line shown. Gray area schematically illustrates the recorded range of outside air conditions at a given location.

This agrees closely with the standard ASHRAE formulation, which reads

$$h^{\text{moist air}}_{\text{ASHRAE-standard}} = (1.86w + 1.006)(T - T_0) + 2501w \text{ kJ/kg}. \tag{13a}$$

Similarly, in conventional units, we obtain

$$h^{\text{moist air}}_{\text{ASHRAE}} \simeq (0.44w + 0.24)(T - T_0) + 1061w \text{ Btu/lb}, \tag{12b}$$

which is again in agreement (with differences being due to rounding) with the ASHRAE result:

$$h^{\text{moist air}}_{\text{ASHRAE-standard}} = (0.444w + 0.240)(T - T_0) + 1061w \text{ kJ/kg}. \tag{13b}$$

At a given atmospheric pressure, the temperature T and humidity ratio w completely determine the properties of moist air. These two properties are usually plotted in the form of a *state point* in a psychrometric chart, in which temperature is represented by the horizontal axis, the humidity ratio on the vertical axis, and additional plotlines indicate states of constant enthalpy, relative humidity, and wet bulb temperature (see Figure 1).

When quantities of air are mixed, the properties of the resulting mixed air can be computed by recognizing that the following quantities are additive and conserved: the amount of dry air, the amount of moisture, and the energy content (enthalpy). Therefore, given m_1, w_1, h_1 representing the dry air mass, humidity ratio, and enthalpy of a first quantity of air, with m_2, w_2 and h_2 the same for a second quantity of air, a mixture would be represented by

$$m = m_1 + m_2, \tag{14}$$

$$w = \frac{w_1 m_1 + w_2 m_2}{m_1 + m_2}, \tag{15}$$

$$h = \frac{h_1 m_1 + h_2 m_2}{m_1 + m_2}. \tag{16}$$

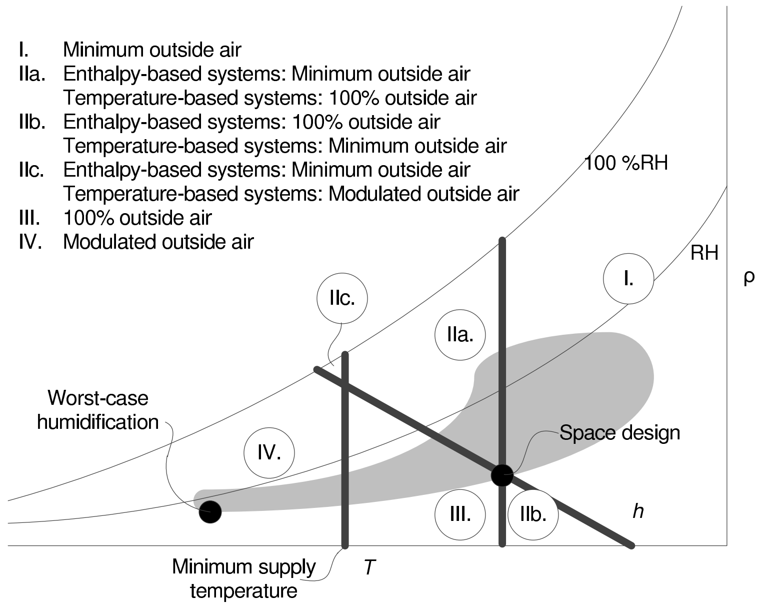

Figure 2: Economizer systems. Thick lines correspond to the design condition temperature and enthalpy, and the minimum supply air temperature. Temperature and enthalpy based economizers behave identically in regions I, III and IV; their behavior is different in regions IIa and IIb and IIc. Adapted from [1].

In a standard HVAC system, a fixed amount of outside air is taken in, either by itself ("Makeup Air" system) or mixed with a quantity of recirculated air ("Mixed Air" system.) Clearly, such a system is not always energy efficient. It may draw in more outside air when necessary (e.g., during summer months), requiring excessive cooling; in winter months, it may be drawing in less air than possible, making it necessary to cool interior parts of larger buildings actively.

3 Economizer systems

Issues of energy efficiency are meant to be addressed by "economizer" type HVAC systems, in which the ratio of outside air and recirculated air is varied in response to changing outside air conditions.

Generally speaking, an economizer type system has three operating modes:

1. When the outside air is hotter than the design conditions, the amount of outside air being drawn in is the minimum amount of fresh air required;

2. When the outside air is colder than the design conditions but warmer than a predetermined minimum supply air temperature, the system operates in a 100% outside air mode;

3. When the outside air is colder than the minimum supply air temperature, the ratio is modulated to ensure that the supply air temperature does not fall below the permitted minimum; however, minimum fresh air requirements are still observed and take precedence (requiring perhaps that the outside air be preheated in order to achieve the minimum supply air temperature.)

The difference between temperature and enthalpy based economizer systems boils down to the definition of "hotter": specifically, is outside air considered hotter if its temperature is higher than the design conditions, or when its enthalpy is higher than the design enthalpy?

The behavior of economizer systems is illustrated in Figure 2.

The difference between the two types of economizer systems can be striking. For instance, a temperature-based economizer may be drawing in 100% very moist air that is only slightly colder than the design conditions; after being heated, this air would be significantly more moist than what is acceptable, requiring excessive dehumidification.

Insofar as humidification requirements are concerned, the difference is effectively non-existent. For a non-economizer type system, the worst-case humidification condition occurs at the bottom edge of the shaded region, when the humidity ratio of the outside air is lowest, which is when the most water vapor needs to be added in order to achieve the desired design conditions. If this point falls within region III (in Figure 2), the location of the worst-case point does not change; humidification requirements may, however, increase because 100% outside air is being used. More likely, however, the worst-case condition falls within region IV, where the amount of outside air is being modulated. In this case, as a result of the modulation, less humidification is required than in a corresponding makeup air system; indeed, chances are that the worst-case humidification scenario coincides with the minimum supply air temperature, as left of this temperature, the amount of outside air being used rapidly decreases due to modulation.

If we assume that in region IV, the amount of outside air is modulated such that the temperature of mixed air remains constant at the minimum supply air temperature, we can compute the percentage of outside air as a function of its state. First, given (13), we can compute the temperature of a quantity of moist air if its enthalpy and humidity ratio are known:

$$T^\star = \frac{h - h_0^{\rm vap} w}{c_P^{\rm wv} w + c_P^{\rm da}}, \tag{17}$$

where we use the notation $T^\star = T - T_0$ to indicate temperatures measured in a non-absolute temperature scale.

Mixing a quantity of outside air (with dry air mass $m_{\rm OA}$, humidity ratio $w_{\rm OA}$, and enthalpy $h_{\rm OA}$) and recirculated air (with dry air mass $m_{\rm RA}$, humidity ratio $w_{\rm RA}$, and enthalpy $h_{\rm RA}$), we obtain mixed air, in accordance with Eqs. (14–16), with mixed air temperature

$$T^\star_{\rm MA} = \frac{h_{\rm OA} m_{\rm OA} + h_{\rm RA} m_{\rm RA} - h_0^{\rm vap}(w_{\rm OA} m_{\rm OA} + w_{\rm RA} m_{\rm RA})}{c_P^{\rm wv}(w_{\rm OA} m_{\rm OA} + w_{\rm RA} w_{\rm RA}) + c_P^{\rm da}(m_{\rm OA} + m_{\rm RA})}. \tag{18}$$

This equation can be solved for $m_{\rm OA}$, yielding

$$m_{\rm OA} = -\frac{(c_P^{\rm wv} T^\star_{\rm MA} + h_0^{\rm vap}) w_{\rm RA} + c_P^{\rm da} T^\star_{\rm MA} - h_{\rm RA}}{(c_P^{\rm wv} T^\star_{\rm MA} + h_0^{\rm vap}) w_{\rm OA} + c_P^{\rm da} T^\star_{\rm MA} - h_{\rm OA}} m_{\rm RA}. \tag{19}$$

Alternatively, we can express this result in terms of the outside air temperature $T^\star_{\rm OA}$ and recirculated air temperature $T^\star_{\rm RA}$ using (13):

$$m_{\rm OA} = -\frac{(c_P^{\rm wv} w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv} w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})} m_{\rm RA}. \tag{20}$$

The humidity ratio of mixed air, then, can be calculated as

$$\begin{aligned} w_{\rm MA} &= \frac{w_{\rm OA} m_{\rm OA} + w_{\rm RA} m_{\rm RA}}{m_{\rm OA} + m_{\rm RA}} \\ &= \frac{(c_P^{\rm wv} w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA}) w_{\rm OA} + (c_P^{\rm wv} w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA}) w_{\rm RA}}{(c_P^{\rm wv} w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA}) + (c_P^{\rm wv} w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})}. \end{aligned} \tag{21}$$

This formula can be used directly to assign mixed air humidity ratios to all state points in a psychrometric chart; in particular, to the area representing measured weather conditions at a particular location, as indicated schematically in Figs. 1 and 2. The actual humidification load is proportional to the difference between $w_{\rm MA}$ and the humidity ratio of the desired design conditions; the lower $w_{\rm MA}$ is, the higher the humidification load. Finding the maximum humidification load for an economizer system, therefore, amounts to finding the outside air state point for which $w_{\rm MA}$ is minimal.

4 Minimum outside air requirement

In a typical HVAC system, a minimum amount of outside air must always be used in order to satisfy fresh air requirements. Even when no such minimum exists, the amount of outside air can never be negative, something that our Eq. (21) does not guarantee.

Given a minimum fresh air ratio expressed in the form $\alpha = m_{\rm OA}/m_{\rm RA}$, the minimum fresh air requirement can be formulated as follows:

$$m_{\rm OA} = \begin{cases} -\dfrac{(c_P^{\rm wv}w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv}w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})}m_{\rm RA} & \text{if } -\dfrac{(c_P^{\rm wv}w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv}w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})} > \alpha, \\ \alpha m_{\rm RA} & \text{if } -\dfrac{(c_P^{\rm wv}w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv}w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})} \le \alpha. \end{cases} \tag{22}$$

Consequently, the humidity ratio of mixed air reads:

$$w_{\rm MA} = \begin{cases} \dfrac{w_{\rm OA}m_{\rm OA} + w_{\rm RA}m_{\rm RA}}{m_{\rm OA} + m_{\rm RA}} & \text{if } -\dfrac{(c_P^{\rm wv}w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv}w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})} > \alpha, \\ \dfrac{\alpha w_{\rm OA} + w_{\rm RA}}{1 + \alpha} & \text{if } -\dfrac{(c_P^{\rm wv}w_{\rm RA} + c_P^{\rm da})(T^\star_{\rm RA} - T^\star_{\rm MA})}{(c_P^{\rm wv}w_{\rm OA} + c_P^{\rm da})(T^\star_{\rm OA} - T^\star_{\rm MA})} \le \alpha, \end{cases} \tag{23}$$

with $m_{\rm OA}$ given by (20).

5 Conclusions

In most design scenarios, all of the variables in Eq. (23): notably, the range of outside air state points to be considered (characterized by $T^\star_{\rm OA}$ and $w_{\rm OA}$), the recirculated air state point ($T^\star_{\rm RA}$, $w_{\rm RA}$, often coinciding with design conditions), the (minimum) supply duct temperature ($T^\star_{\rm MA}$) and the minimum fresh air requirement (α), are known, and the expression can be easily and repeatedly evaluated, and the minimal value of $w_{\rm MA}$ can be found rapidly.

Although our present focus is humidification, we note that the same approach can be used to determine the maximum heating, cooling, and dehumidification load of a system as well. In a non-economizer system, these correspond to the maximum and minimum enthalpy, and maximum humidity ratio within the range of outside air state points that occur at the design location. For economizer systems, the load is varied by the modulation effected by the system.

Lastly, we note that although data sets published by ASHRAE provide per location state point frequency information (i.e., a frequency function in the form of $f(T, w)$ or equivalent, assigning frequencies of occurrence to each grid point in a psychrometric chart at some grid resolution), the data are not broken down by the time-of-day τ. If a designer has at his disposal a three-dimensional data set available that provides frequency of occurrence of specific state points at specific times of day (i.e., a frequency function $f(T, w, \tau)$), this could be used to determine system loads as a function of building occupancy. This information could also be used to improve the accuracy of analyzing operational costs.

References

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[3] *2005 ASHRAE Handbook: Fundamentals*. ASHRAE, 2005.