Development of an Engineering Reasoning Course to Improve the Persistence of Non-Calculus Ready First Year Engineering Students

Anika C. Pirkey[a], Melissa Morris[b], David J. Klinke II[a], D. Jake Follmer[c], and Lizzie Y. Santiago[d]

a. Department of Chemical and Biomedical Engineering

West Virginia University

b. Department of Mechanical Engineering

University of Nevada, Las Vegas

c. School of Education, College of Applied Human Sciences

West Virginia University

d. Fundamentals of Engineering Program

West Virginia University

RUNNING HEAD: PIRKEY, MORRIS, KLINKE, FOLLMER, AND SANTIAGO

Correspondence should be addressed to:

Lizzie Y. Santiago

lizzie.santiago@mail.wvu.edu

Phone: (304) 293-0998

Fundamentals of Engineering Program

West Virginia University

P.O. Box 6101

Morgantown, WV 26506-6102

ABSTRACT

Background

As college readiness continues to decline, the proportion of students entering engineering programs with low mathematics proficiency is increasing. These students have lower retention rates than their calculus-ready peers.

Purpose

This work describes the design and implementation of an Introduction to Engineering Problem Solving course targeted to first-year engineering students who place into College Algebra with the goal of increasing student retention in engineering.

Design/Method

Eighty-one (81) students were enrolled in the course over two years. The course implements the Paul-Elder Critical Thinking Theory to promote engineering problem solving and critical thinking skills through problem and project-based learning. Institutional data including cumulative GPA, mathematics course grades, and enrollment status within the institution were collected for four semesters. The experimental group took the Critical Thinking Assessment Test (CAT) at the start and end of the intervention. All statistical analysis was completed in R using appropriate Bayesian statistical methods.

Results

Students participating in the intervention course saw improved critical thinking skills compared to their baseline along with higher pass rates in their College Algebra course and improved progression through the mathematics curriculum compared to the control group. Cumulative GPA and retention within engineering and the university were also improved. Results were statistically significant.

Conclusion

The importance of supporting non-calculus ready engineering students will continue to increase in the coming years. Intervention coursework has the potential to significantly increase student success in engineering. Future work should explore the possibility of multi-semester interventions to support students until they reach calculus.

KEYWORDS: Engineering Reasoning, Critical Thinking, Non-calculus ready, Course Intervention, Retention, Persistence

INTRODUCTION

The American Society of Engineering Education reports that nearly 80% of all engineering students in the United States are retained between their first and second years of college, yet the four-year and six-year graduation rates remain below 35% and 60% respectively (American Society for Engineering Education, 2017). These numbers drop drastically when analyzing students who are placed in College Algebra or Pre-Calculus. In a study monitoring the persistence and graduation of engineering students who began their education in Pre-Calculus, a mid-sized private university reported a second-year retention of only 66% and four- and five-year graduation rates of 34% and 43% (Jones et al., 2021). As the number of students from underrepresented minorities and low-income backgrounds who often don't have access to higher level STEM coursework increases (American Society for Engineering Education, 2022; Fry & Cilluffo, 2019) and the aftereffects of the COVID-19 Pandemic on students' education continue to come to light, college readiness will continue to decline (ACT Inc., 2022) and the number of students arriving to campus with low math proficiency will likely to increase. As such, it is important for institutions to improve the persistence and graduation rates of this population. Successful engineers must, along with strong skills in STEM disciplines, possess strong problem solving and critical thinking skills. While developing critical thinking skills is desired and even deemed important by many educators at the K-12 and university levels, they often feel unable to effectively teach these skills in the classroom (Innabi & El Sheikh, 2007; Sezer, 2008). The goal of this study was to develop and implement an intervention course for students admitted to an engineering program and placed in College Algebra. The course introduces students to the engineering design and problem-solving processes along with critical thinking skills and concepts. These skills are then applied to mathematics problems through an algebra-based math recitation in order to improve student success in College Algebra and better prepare students for advanced math coursework. It is expected that by developing students' mathematics, engineering problem solving, and critical thinking skills, non-calculus ready students' retention rates will improve.

LITERATURE REVIEW

Engineering Program Enrollment Trends. The number of domestic student applicants to bachelor's level engineering programs at public institutions decreased by a median of 1.4% between 2020 and 2021 while international applicants decreased by a median of 6.0% in the same period. Total freshmen enrollment decreased by a median percent of 4.1% across all institutions between 2019 and 2020. While some of this decline can be attributed to the COVID-19 pandemic, a decrease in total freshmen enrollment of a median percent of 3.6% was reported between 2018 and 2019, suggesting that there was a pre-pandemic enrollment issue that was merely exacerbated by the pandemic. In fact, the data suggests that the beginning of the decline began as early as 2016 (American Society for Engineering Education, 2021).

Although overall freshman enrollment has declined in recent years, enrollment of some underrepresented groups has increased. Female engineering students made up 23.8% of those enrolled in 2019 and saw a modest increase to 24.6% in 2021. In that same timeframe, Hispanic engineering students increased from 14.5% to 15.8%. Enrollment of Black/African American students has stayed relatively steady, increasing from 5.2% to 5.4% between 2019 and 2021 (American Society for Engineering Education, 2020, 2022).

Trends in College Readiness. ACT Inc reports the percentage of students each year who met the college readiness benchmarks. In 2018, 27% of test takers were considered college ready. That number has declined to only 22% in 2022. The average composite ACT score has dropped one point in that time, from 20.8 to 19.8 (ACT Inc., 2022). Similarly, the average SAT score has dropped from 1068 to 1050 between 2018 and 2022 and the percentage of students who met both the reading and math benchmarks set by College Board decreased from 47% to 43% (College Board, 2018, 2022).

College Board reports that the average SAT Math score for students interested in pursuing engineering in 2022 was 599. At many universities across the country, this score places students into College Algebra (Temple University, 2023; University of Arizona, 2023; University of Wisconsin Milwaukee, 2022; West Virginia University, 2023). With the majority of universities ill prepared to handle engineering students below a Calculus I math proficiency, the average student is unable to be accepted to an engineering program at their current math level and must either enter a general studies program or choose another major, potentially leading to decreased enrollment in engineering program.

Existing Interventions for Non-Calculus Ready Students. Institutions have taken different approaches to addressing the needs of pre-calculus first year engineering students. Texas A&M modified their curriculum to integrate pre-calculus students in 1996 by rearranging course requirements so that these students could make progress towards their degrees while improving their mathematics skills (Whiteacre & Malave, 1998). Other universities created engineering courses for pre-calculus students to introduce them to engineering principles and how pre-calculus is used in engineering to improve student interest and enthusiasm, and thus retention, in engineering (Monte & Hein, 2003; Standridge et al., 2003). Still, other universities focused on building a learning community for pre-calculus students and worked together with their math departments to create special sections of pre-calculus that showcased engineering applications of various mathematics principles (Bennett et al., 2013). All these interventions saw improvements in metrics such as cumulative GPA (cGPA), course grades in core STEM curriculum, and engineering retention. Few universities have introduced these necessary interventions for first-year engineering students placed in College Algebra.

THEORETICAL FRAMEWORK

Paul-Elder Critical Thinking Theory. According to Paul and Elder, “critical thinking is the art of analyzing and evaluating thinking with a view to improve it” and is self-directed, self-disciplined, self-monitored, and self-corrective in nature (Paul & Elder, 2006). The theory is built upon eight elements of thought (Purpose, Question at Issue, Information, Interpretation and inference, Concept, Assumptions, Implications and Consequences, and Point of View) and ten universal intellectual standards (Clarity, Accuracy, Relevance, Logicalness, Breadth, Precision, Significance, Completeness, Fairness, and Depth). Application of the intellectual standards to the elements of thought allow for the development of eight key intellectual traits (fairmindedness, intellectual humility, intellectual courage, intellectual autonomy, intellectual empathy, intellectual perseverance, intellectual integrity, and confidence in reason), which is considered the ultimate goal of developing critical thinking skills under the model. This framework is summarized in Figure S1 (Paul & Elder, 2005, 2006).

METHODOLOGY

Participants. Eighty-one (81) first year engineering students that were not calculus-ready at the time of enrollment participated in this study. Non-calculus ready students are defined as any student who has entered the engineering program and was placed in a math course below the level of Calculus 1, Part A such as College Algebra or Trigonometry based on SAT/ACT scores, previous college credit, or a math placement exam. All participants were first enrolled in a first-year engineering program at a Land Grant Institution in the Mid-Atlantic Region in 2016 or 2017. Participants were all placed in College Algebra at the start of the study.

The characteristics of the student participants in the control and experimental groups are summarized in Table 1. Institutional data from students enrolled in the same first-year engineering program and placed in College Algebra, but who did not enroll in the intervention course was used to establish the control group for the study. The majority of students involved in the study were white males, which is consistent with the demographics of the program overall. Average high school GPA (hGPA), and Math SAT/ACT scores were not statistically different between groups (hGPA average difference = 0.049 GPA points, 95% Highest Density Interval (95%HDI) = [-0.06-0.16 GPA points], 18.3% probability of a difference < 0 ; Math SAT average difference = 1.17 points, 95%HDI = [-13.6-16.4], 44.0% probability of a difference < 0; Math ACT average difference = -0.07 points, 95%HDI = [-0.48-0.34 points], 62.5% probability of a difference < 0, Figures S2 A-C).

First Year Engineering Program. At the time of the study, all students accepted to the engineering school were initially enrolled in a first-year engineering program in one of 3 tracks. Track 1 was designed for calculus ready students placed in Calculus 1, Part A or above and required a 3.0 high school GPA with a Math SAT or ACT score of 660 or 28 respectively for admittance. Track 2 was designed for pre-calculus ready students and required a 2.75 high school GPA with a Math SAT or ACT score of 610 or 26 respectively for admittance. Students admitted to the program and enrolled in College Algebra were considered Track 3 students and accounted for approximately 21% of students accepted to the program. Students in Track 3 were required to have a 2.5 high school GPA with a Math SAT or ACT score of 540 or 22 respectively for admittance.

To move from the first-year program into their chosen department, students were required to maintain a cumulative GPA (cGPA) of at least 2.25 and complete a set of six core courses ((Fundamentals of Chemistry 1, Calculus 1, Introduction to Composition and Rhetoric, First Year Seminar, Engineering Problem Solving 1, and Engineering Problem Solving 2) with a grade of C or better. While completion of Calculus 2 was not required, enrollment in the course was a corequisite of Engineering Problem Solving 2. At the time of this study, students who did not successfully complete all requirements to move into a department by the end of four semesters (excluding Summers) were required to leave the engineering college until prerequisites were met and then apply for reentry.

To earn an engineering degree, students in Computer science must complete 3 semesters of college calculus while those in any other engineering discipline must complete four semesters. Track 3 students were required to complete both College Algebra and Trigonometry before being allowed to take their first calculus course. Some students who earned As or Bs in College Algebra had the possibility to advance to Pre-Calculus instead of Trigonometry, and students had the opportunity to take a math placement test to advance to Calculus 1 at the end of their first semester.

Course Description. The outline of the course was first described by Santiago et. al (Santiago et al., 2016). A list of topics addressed during the lecture portion of the course is provided in Table 2. In brief, a course was designed to foster critical thinking skills in first-year engineering students. The course was developed using Paul and Elder's critical thinking theory and introduced students to problem solving and critical thinking in research, experimentation, and engineering design through two hours of lecture per week (Paul & Elder, 2005, 2006). The course focused on the development of twelve different critical thinking skills, which are listed in Table 3.

A mathematics recitation was added to the original course to assist students in improving their College Algebra competency and demonstrate real world engineering applications for various topics. The recitation met once per week for two hours and was operated as described in Santiago et al. (Santiago et al., 2017). A list of topics reinforced through the math recitation is provided in Table 4.

Metrics Used. Baseline data for all students enrolled in the first-year engineering program and placed in College Algebra were collected at the start of the study. This included gender, high school GPA, and Math SAT and ACT scores. Institutional data was collected at the end of each of students' first four semesters for the control and experimental groups. This data included cumulative GPA, mathematics course grades, and enrollment status within the institution. In addition, students enrolled in the intervention course took the Critical thinking Assessment Test (CAT) developed by Tennessee Tech at the start and end of the semester to measure changes in critical thinking skills (Haynes et al., 2015).

Statistical Analysis. All statistical analysis was performed in R v4.3.1.

Bayesian estimation using the BEST (Bayesian Estimation Supersedes the t Test) method developed by John K. Kruschke was completed using The BEST package (v0.5.4) to estimate the size of the effect the intervention course had on summary statistics such as the mean cumulative GPA between treatment groups. The package uses minimally informative priors including normal priors with large standard deviations for the mean, broad uniform priors for standard deviation, and a shifted-exponential prior for the normality parameter and equal variance is assumed between treatment groups (Kruschke, 2013; Meredith & Kruschke, 2021). The effect size is reported as a 95% Highest Density Interval (HDI) along with a mean of the credible values for the difference between groups (average difference) and a probability that the difference is less than 0. A probability of 5% or less was considered statistically significant. Comparison of proportions between groups was completed using the brms package (v2.19.0) in R and methods described by Andrew Heiss (Heiss, 2023). Data is reported as the mean credible value of the difference between proportions as well as 95% HDI and the probability that the difference in proportions is less than or greater than 0. All R code and deidentified data is available at https://github.com/arcoolbaugh/EngineeringReasoningCourse.

Study Approval. This study was approved by the West Virginia University Institutional Review Board (WVU-IRB).

RESULTS

Identification of Deficiencies in College Algebra Skills. Throughout the math recitation, student work such as homework and quizzes were collected and analyzed after grading to manually identify deficiencies in student knowledge. Common errors that were identified can be broadly categorized into two areas: Misconceptions associated with factoring procedures and a lack of knowledge needed to simplify complex expressions such as rational or radical expressions. While factoring, common mistakes included incorrectly identifying common factors and factoring coefficients along with incorrect procedural practices for simplifying basic mathematical expressions. When attempting to simplify rational equations, most students were unable to correctly identify a common denominator and rewrite each term using that denominator. Errors when simplifying rational expressions were varied, but often included being unable to identify all factors to reach the simplest possible form of the equation. Specific examples of provided student work are summarized in Table 5.

Improvement in Mathematics Performance. In order to track both short- and long-term effects of the intervention course on students' performance in mathematics coursework, data was collected for each student indicating the math course they enrolled in each semester and their final performance (grade) in that class. During their first semester, students in both the control and experimental groups were enrolled in College Algebra. Figure 1 shows the percentages of each group that received a certain grade in their College Algebra course. In the control group, 57/242 students (19.4%) received a D, F, or W grade in the course compared with only 5/81 (6.2%) of the experimental group (average difference = -12.6%, 95% HDI = [-21.2% - -2.83%], 1.0% probability of a difference > 0, Figure S3 A). In addition, only 74/242 (30.6%) of the control group received an A in the course while 39/81 (48.1%) of the experimental group received an A (average difference = 17.3%, 95% HDI = [2.34% - 31.2%], 1.40% probability of a difference < 0, Figure S3 B). These results indicate a statistically significant improvement in student performance.

Students' progression through the engineering mathematics curriculum was also monitored over the course of four semesters (Figure 2). Students are expected to progress to Trigonometry in Semester 2 (or higher). In the control group, 72.7% progressed as expected while 86.4% of students in the experimental group progressed (average difference = 13.1%, 95% HDI = [1.04% - 24.2%], 1.93% probability of a difference < 0, Figure S4 A). Depending on summer course participation (data not collected), students should be in one of the available Calculus 1 courses (1 semester full course version or Part A or B of a two-part version of the course) or in Calculus 2 in Semester 3. The fraction of students that were enrolled in an appropriate math course in Semester 3 was 48.3% in the control group and 64.2% in the experimental group (average difference = 15.4%, 95% HDI = [0.10% - 30.1%], 2.53% probability of a difference < 0, Figure S4 B).

By Semester 4, students who are considered “on track” could be anywhere between Calculus 1 Part B and Calculus 3. By the end of Semester 4, 48.1% of students who participated in the study were considered “on track” in their math course progression while only 38.0% remained “on track” in the control group (average difference = 9.97%, 95% HDI = [5.63% - 24.2%], 8.95% probability of a difference < 0, Figure S4 C). This is not considered statistically significant, however, the differences in the proportions of on track students is at least in part due to a statistically significant difference in the number of students who were not enrolled in an engineering required math course in the control group compared to the experimental by the end of Semester 4 (50.4% vs 37.0% (average difference = -13.0%, 95% HDI = [-27.5% - -2.09%], 4.3% probability of a difference > 0, Figure S4 D)). This difference in math course participation is a preliminary indicator for persistence in the program, as the most common reason for students to not enroll in a math course or to enroll in a course outside of the engineering progression such as Business Calculus is a change of major or attrition from the university.

Improvements in Critical Thinking Skills from the Intervention. The CAT was administered at the beginning and end of the semester in the intervention course to measure the impact of the intervention on students’ development of critical thinking skills. Of the 81 students in the experimental group, 73 of them had pre and post CAT scores available for comparison. Shown in Figure 3, the average CAT score at the start of the semester was 11±5 points and rose to 18±5 points by the end of the course (average difference = 6.82 points, 95% HDI = [5.64 – 8.04 points], 0% probability of a difference < 0, Figure S2 D), indicating a significant improvement in critical thinking skills over the course of the intervention.

Difference in cGPA Between Groups. Figure 4 shows the average cumulative GPA (cGPA) for students in the control and experimental groups. The average cGPA in the control group at the end of the first semester was 2.82 ±0.93 while the experimental group had an average cGPA of 3.15 ±0.64 (average difference = 0.30 GPA points, 95% HDI = [0.121- 0.48 GPA points], 0.1% probability of a difference < 0, Figure S2 E).

Changes in Persistence. Engineering and institutional persistence were monitored for each student for four semesters and the trend is shown in Figure 5. Engineering persistence is defined as remaining enrolled in a department within the College of Engineering, while institutional persistence includes both those who remained enrolled in engineering and those who remained enrolled at the institution, but within a different college. Significant drops in persistence occurred at the end of the second semester, particularly in engineering persistence of the control group. By the end of four semesters, engineering persistence for the experimental group was 72%, compared to 49% in the control group (average difference = 22.3%, 95% HDI = [6.97% - 35.2%], 0.62% probability of a difference < 0, Figure S3 C). The difference in institutional persistence was smaller, but still significant, at 84% for the experimental group compared to 68% for the control group (average difference = 15.2%, 95% HDI = [2.05% - 27.1%], 1.50% probability of a difference < 0, Figure S3 D).

DISCUSSION

Freshman enrollment in engineering programs began declining as early as 2016, but has seen a significantly steeper decline since the COVID-19 pandemic (American Society for Engineering Education, 2022). This is not a unique problem to engineering programs. University enrollment overall took a steep decline during the pandemic, and while it appears to have stabilized, there has been no recovery in enrollment numbers nationally (Berg et al., 2023). In addition to the enrollment decline, students are arriving at universities significantly underprepared for college in comparison to previous cohorts (ACT Inc., 2022). As these trends continue, the number of students interested in engineering who do not have a calculus-level math proficiency is likely to increase, leading to a decrease in the number of students who would meet admission requirements for many engineering programs. In addition, the significant decline in U.S. birth rates since the Great Recession will lead to an Enrollment Cliff in higher education beginning in 2025 that could result in a decrease in the college-going population by as much as 15% between 2025 and 2029, further decreasing the number of engineering students on campus (Schuette, 2023). With these factors in mind, it would be beneficial to engineering programs to consider enrolling students who are non-calculus ready to help boost enrollment. However, if institutions are to accept students who are non-calculus ready, appropriate course work and support systems such as this intervention course are necessary in order to increase their likelihood of success in their programs.

Data collected from the 2016-2017 cohort of students enrolled in this course was used to make improvements to the intervention. Misconceptions identified in students' College Algebra skills during this study informed modifications to the mathematics recitation portion of the course to focus more on those skills and correct the deficiencies. In addition, students' difficulty using these mathematics skills to solve real-world inspired word problems led to further review of students' problem-solving skills and further modifications to improve student outcomes (Santiago & Pirkey, 2020).

When reviewing the difference in proportions of students who remain on track in their engineering mathematics curriculum, it is noted that the proportions begin to converge and the likelihood that the difference is greater than zero increases in Semester 4 to 8.95% from 2.53% in

Semester 3. While the intervention appears to have a strong short-term effect on persistence and math progression, students in the experimental cohort are leaving the engineering math track at similar rates to those of the control group between Semesters 3 and 4 (8.7% increase in No Math/Left Track in the control group, 8.6% in the experimental group, Figure 2) and those in the experimental group remaining in the engineering math curriculum begin struggling as they reach higher level math courses (1.7% increase in Behind Track students in the control group, 7.4% increase in the experimental group, Figure 2). The implementation of an additional intervention to support students in Trigonometry before moving through the Calculus curriculum may have beneficial effects to further improve students' long-term performance and retention within engineering.

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AUTHOR BIOGRAPHIES:

Anika C. Pirkey is a Teaching Assistant Professor in the Department of Chemical and Biomedical Engineering at West Virginia University, P.O. Box 6102, Morgantown, WV, USA, 26506; anika.pirkey@mail.wvu.edu

Melissa Morris is an Associate Professor-In-Residence and the Undergraduate Program Coordinator in the Department of Mechanical Engineering at University of Nevada, Las Vegas, Box 454027, 4505 S. Maryland Parkway, Las Vegas, NV, 89154; melissa.morris@unlv.edu

David J. Klinke is a Professor in the Department of Chemical and Biomedical Engineering and an Adjunct Associate Professor in the Department of Microbiology, Immunology, and Cell Biology at West Virginia University, P.O. Box 6102, Morgantown, WV, USA, 26506; David.klinke@mail.wvu.edu

D. Jake Follmer is an Assistant Professor in the School of Education in the College of Applied Human Sciences at West Virginia University, 375 Birch Street, P.O. Box 6115, Morgantown, WV, USA, 26506; djf00001@mail.wvu.edu

Lizzie Y. Santiago is a Teaching Professor and the Director of the Fundamentals of Engineering Program at West Virginia University, P.O. Box 6101, Morgantown, WV, USA, 26506; lizzie.santiago@mail.wvu.edu

BIBLIOGRAPHY:

ACT Inc. (2022). *The ACT Profile Report - National Graduating Class of 2022*.

American Society for Engineering Education. (2017). *Engineering by the Numbers: ASEE Retention and Time-to-Graduation Benchmarks for Undergraduate Engineering Schools, Departments and Programs*.

American Society for Engineering Education. (2020). *ASEE 2019 Edition Engineering & Engineering Technology By the Numbers*.

American Society for Engineering Education. (2021). *ASEE 2020 Edition Engineering & Engineering Technology By the Numbers*. www.asee.org

American Society for Engineering Education. (2022). *ASEE 2021 Edition Engineering & Engineering Technology By the Numbers*. American Society for Engineering Education. www.asee.org

Bennett, R. M., Russell, M. H., & Rawn, C. J. (2013, June). Engineering Introduction in Pre-Calculus Courses. *120th ASEE Annual Conference & Exposition*. http://ef.e

Berg, B., Lee, S., Randolph, B., Ryu, M., & Shapiro, D. (2023). *Current Term Enrollment Estimates: Spring 2023*.

College Board. (2018). *2018 Total Group SAT Suite of Assessments Annual Report*.

College Board. (2022). *2022 Total Group SAT Suite of Assessments Annual Report*. www.collegeboard.org.

Fry, R., & Cilluffo, A. (2019). *A Rising Share of Undergraduates Are from Poor Families, Especially at Less Selective Colleges* (Vol. 22). www.pewresearch.org.

Haynes, A., Lisic, E., Harris, K., Leming, K., Shanks, K., & Stein, B. (2015). Using the Critical Thinking Assessment Test (CAT) as a Model for Designing Within-Course Assessments. *Inquiry: Critical Thinking Across the Disciplines*, *30*(3), 38–48. https://doi.org/10.5840/inquiryct201530316

Heiss, A. (2023, May 15). *A guide to Bayesian proportion tests with R and {brms}*. Andrewheiss.Com.

Innabi, H., & El Sheikh, O. (2007). The change in mathematics teachers' perceptions of critical thinking after 15 years of educational reform in Jordan. *Educational Studies in Mathematics*, *64*(1), 45–68. https://doi.org/10.1007/s10649-005-9017-x

Jones, S. A., Cairncross, C., Vandergrift, T., & Kalnin, J. (2021). Persistence of Students who Begin Engineering Programs in Precalculus. *Advances in Engineering Education*, *9*(4).

Kruschke, J. K. (2013). Bayesian estimation supersedes the t test. *Journal of Experimental Psychology: General*, *142*(2), 573–603. https://doi.org/10.1037/a0029146

Meredith, M., & Kruschke, J. (2021). *Bayesian Estimation Supersedes the t-Test*. http://sourceforge.

Monte, A. E., & Hein, G. L. (2003). Using Engineering Courses to Improve Pre-Calculus Students' Success Background-First Year Engineering at MTU. *Proceedings of the 2003 American Society for Engineering Educations Annual Conference & Exposition*.

Paul, R., & Elder, L. (2005). *A Guide For Educators to Critical Thinking Competency Standards Standards, Principles, Performance Indicators, and Outcomes With a Critical Thinking Master Rubric*. Foundation for Critical Thinking. www.criticalthinking.org

Paul, R., & Elder, L. (2006). *The Miniature Guide to Critical Thinking Concepts and Tools* (Fourth). Foundation for Critical Thinking. www.criticalthinking.org

Santiago, L. Y., Coolbaugh, A. R., Markle, H. A., Hensel, R. A. M., & Morris, M. L. (2018, June). Algebra-Related Misconceptions Identified in a First-Year Engineering Reasoning Course Algebra-Related Misconceptions Identified in a First Year Engineering Reasoning Course. © 2018, American Society for Engineering Education. *2018 ASEE Annual Conference & Exposition*.

Santiago, L. Y., Coolbaugh, A. R., & Veeramachaneni, S. S. (2016, June). Critical Thinking Skills in First-Year Engineering Students Critical Thinking Course for First Year Engineering Students. © 2016, American Society for Engineering Education. *2016 ASEE Annual Conference & Exposition*.

Santiago, L. Y., Coolbaugh, A. R., Veeramachaneni, S. S., & Morris, M. L. (2017, June). Introducing First Year Engineering Students to Engineering Reasoning. © 2017, American Society for Engineering Education. *2017 ASEE Annual Conference & Exposition*.

Santiago, L. Y., & Pirkey, A. C. (2020, June). Identifying Deficiencies in Engineering Problem Solving Skills. © 2020, American Society for Engineering Education. *2020 ASEE Virtual Annual Conference Content Access*.

Schuette, A. (2023). Navigating the Enrollment Cliff in Higher Education. In *Trellis Company*.

Sezer, R. (2008). Integration of Critical Thinking Skills into Elementary School Teacher Education Courses in Mathematics. *Education*, *128*(3).

Standridge, C. R., Fleischmann, S. T., Larson, H. T., & Johnson, P. D. (2003). An Engineering Experiences Course for Non-Calculus Freshman. *Proceedings of the 2003 American Society for Engineering Education Annual Conference & Exposition*.

Temple University. (2023). *Math Placement Assessment*. https://ira.temple.edu/placement-assessments/math-placement-assessment

University of Arizona. (2023). *Placement Chart and MPLF Scores*. https://www.math.arizona.edu/support/kb/placement/index.php?pg=kb.page&id=567

University of Wisconsin Milwaukee. (2022). *Math Placement Levels and Scores*. https://uwm.edu/math/undergraduate/resources/math-placement/math-course-placement-information/#placement_heading2

West Virginia University. (2023). *Admission Criteria*. https://www.statler.wvu.edu/students/admissions-criteria

Whiteacre, M. M., & Malave, C. O. (1998). An Integrated Freshman Engineering Curriculum for Pre-calculus Students. *1998 FIE Conference*.

FIGURE LEGENDS:

Figure 1: Breakdown of College Algebra final course grades by group (Control n = 242, Experimental n = 81).

Figure 2: Student progression through the mathematics curriculum over four semesters. All students in both groups started in College Algebra in semester 1. Values in the data table are given in percent (%) (Control n = 242, Experimental n = 81). On Track students are those who are enrolled in Trigonometry or higher in Semester 2, Calculus 1 Part A in Semester 3, and Calculus 1 Part B in Semester 3. Behind Track students are those who are still enrolled in a course within the engineering math curriculum, but are behind on their expected progression through the curriculum. No Math/Left Track indicates that students were either not enrolled in a math course or took a math course outside of the required engineering math curriculum.

Figure 3: Average Critical Thinking Assessment Test (CAT) scores for students participating in the intervention (n = 73 students with pre and post scores available). The CAT was administered at the beginning and end of the course to measure improvement in critical thinking skills. Data is presented as the average ± standard deviation.

Figure 4: Average cumulative GPA (cGPA) at the end of the first semester for the control and experimental groups (Control n = 242, Experimental n = 81). Data is presented as the average ± standard deviation.

Figure 5: Engineering and institutional persistence trends over four semesters stratified by group (Control n = 242 shown using dashed lines, Experimental n = 81 shown using solid lines). Data is represented as the % of the cohort retained at the end of each semester. Engineering persistence is shown in black and institutional persistence is shown in grey.

Table 1: Summary characteristics of the control and experimental groups for the study (Control n = 242, Experimental n = 81)

Table 2: Summary of topics taught during the lecture portion of the intervention course.

Table 3: A list of the critical thinking skills promoted through lectures and activities throughout the intervention course.

Table 4: A list of the College Algebra topics reinforced through the math recitation portion of the intervention

Table 5: Examples of student errors in simplifying mathematical expressions that correlate with common deficiencies in College Algebra skills identified during the intervention (Santiago et al., 2018)

APPENDIX A: SUPPLEMENTARY INFORMATION ON PAUL-ELDERS CRITICAL THINKING THEORY

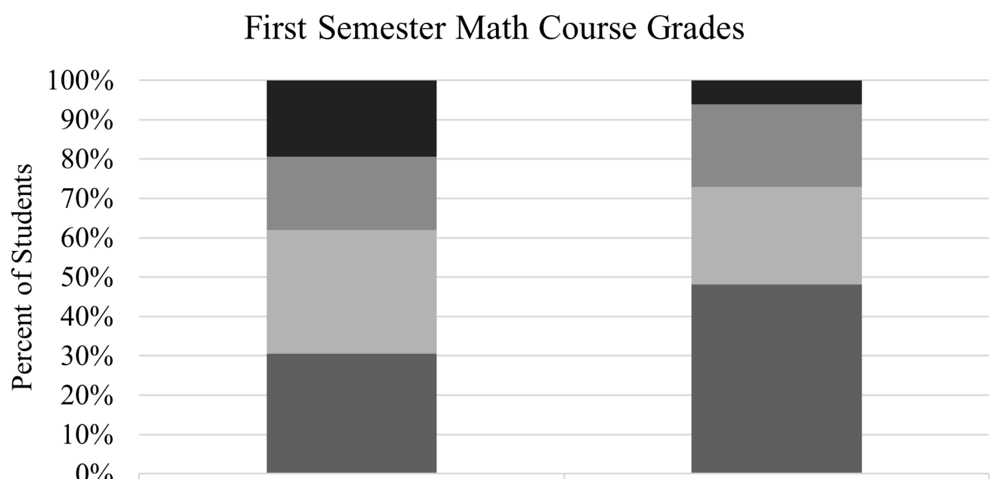
Figure S1: Graphical Depiction of the structure of the Paul-Elders Critical Thinking Theory (Paul & Elder, 2005, 2006).

APPENDIX B: POSTERIOR DISTRIBUTIONS FROM BAYESIAN ANALYSIS

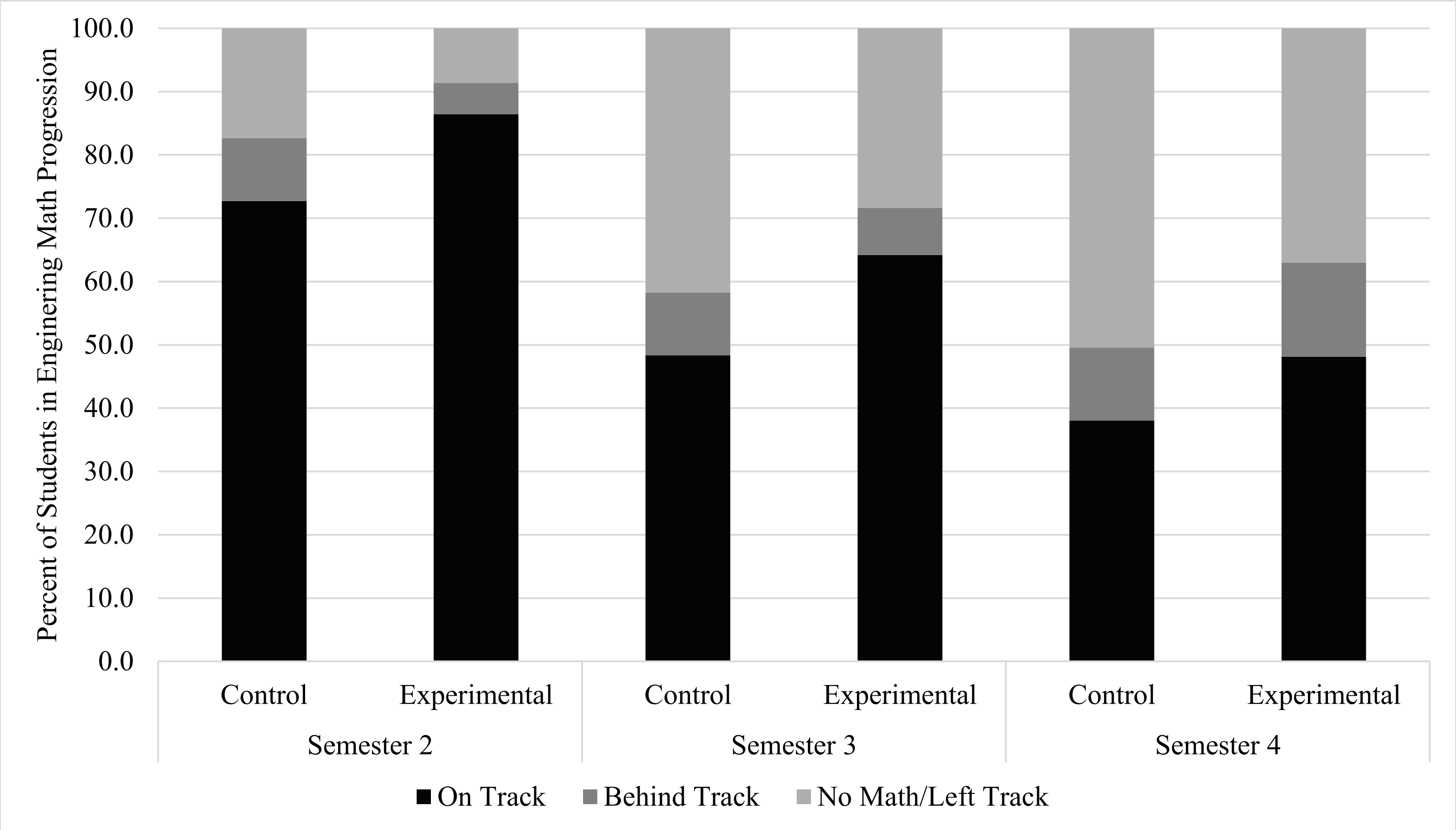
Figure S2: Results from BEST statistical testing comparing the difference of means between the experimental and control groups for the following parameters: (A) High School GPA, (B) Math SAT score, (C) Math ACT score, (D) CAT score, and (E) First-semester cumulative GPA.

Figure S3: Posterior distributions of the proportion of students in each group to receive and A (A) or a DFW (B) in their College Algebra course in Semester 1 along with the proportion of students who persisted in Engineering (C) and at the Institution (D) after Semester 4. Each plot also includes a posterior distribution of the percentage point difference in proportions between the experimental and control groups.

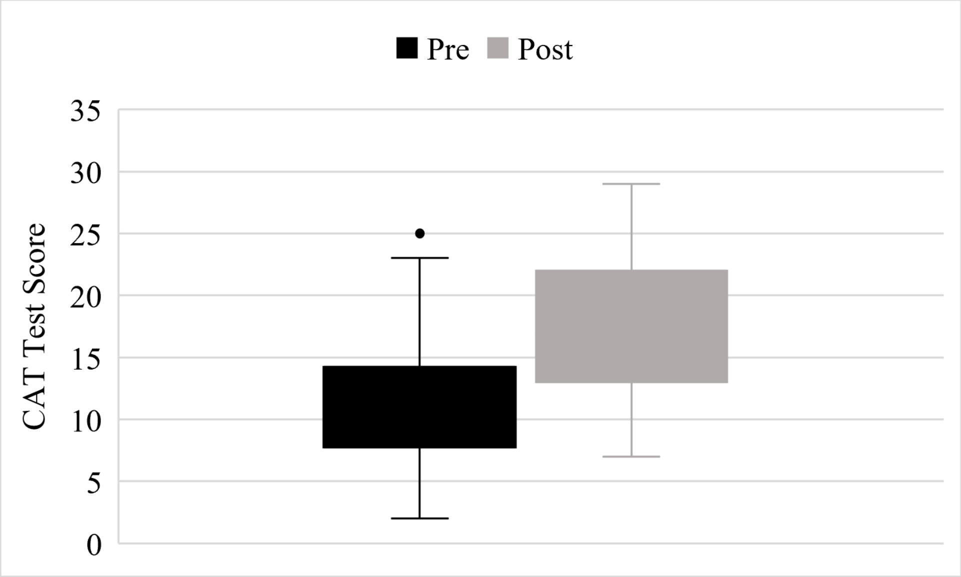
Figure S4: Posterior distributions of the proportion of students in each group to progress at the expected rate through the engineering math curriculum and remain on track in (A) Semester 2, (B) Semester 3, and (C) Semester 4 along with the proportion of students who have left the engineering math curriculum by Semester 4 (D). In Semester 2, on track students should be in Trigonometry or higher. In Semester 3, on track students should be in Calculus 1 or Part A of the two-part Calculus series (or higher). In Semester 4, on track students should be in Calculus 1B or higher. Students who are considered to leave the engineering math curriculum have either stopped enrolling in mathematics courses or are enrolled in courses outside of the engineering progression. Each plot also includes a posterior distribution of the percentage point difference in proportions between the experimental and control groups.

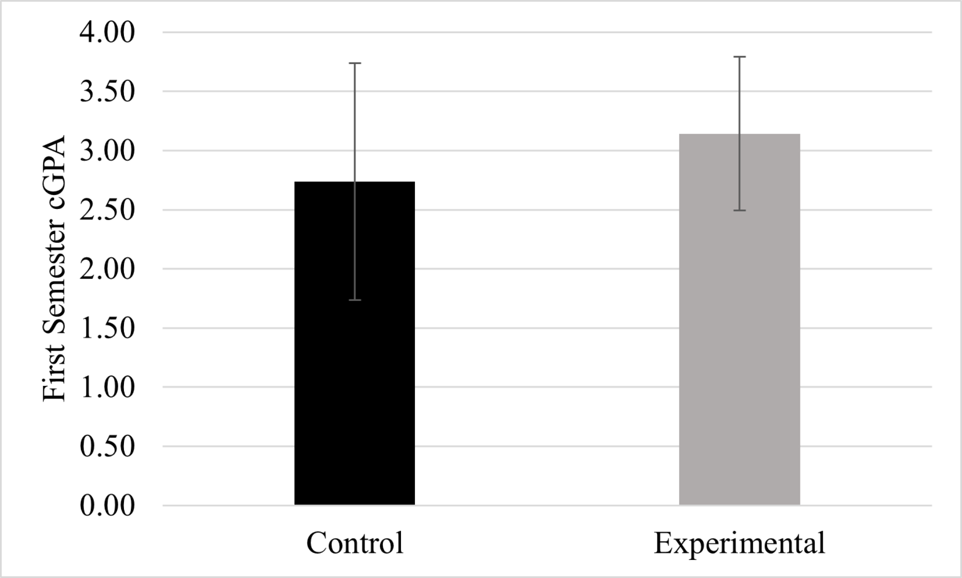


	Control	Experimental
D/F/W	19.4%	6.2%
C	18.6%	21.0%
B	31.4%	24.7%
A	30.6%	48.1%

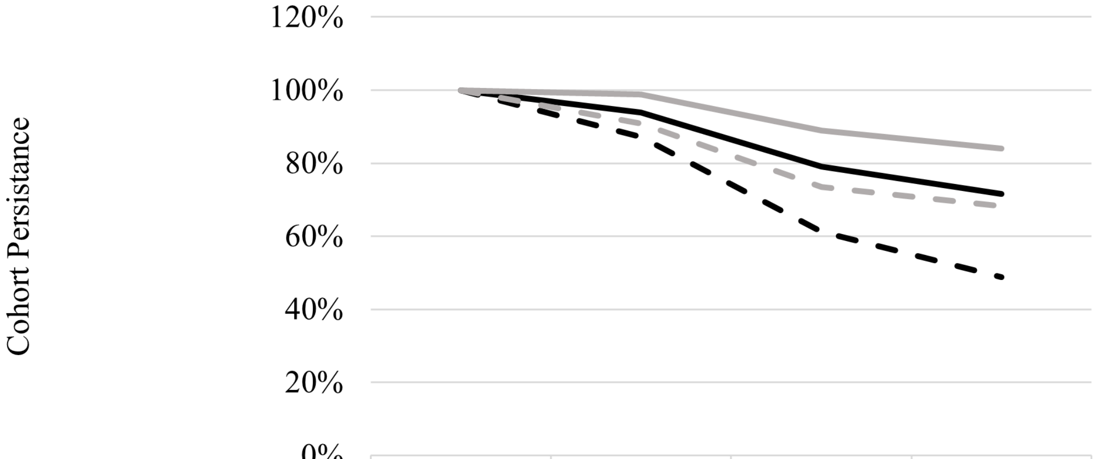


	Semester 2		Semester 3		Semester 4	
	Control	Experimental	Control	Experimental	Control	Experimental
On Track	72.7	86.4	48.3	64.2	38.0	48.1
Behind Track	9.9	4.9	9.9	7.4	11.6	14.8
No Math/Left Track	17.4	8.6	41.7	28.4	50.4	37.0


Pre
Post
CAT Test Score
35
30
25
20
15
10
5
0



First Semester cGPA
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
Control
Experimental



	1	2	3	4
ENGR Control	100%	87%	61%	49%
ENGR Experimental	100%	94%	79%	72%
INST Control	100%	91%	74%	68%
INST Experimental	100%	99%	89%	84%

Number of Semesters Enrolled

Characteristics of Participants						
Parameter	Control			Experimental		
Gender	M	197	81.4%	M	69	85.2%
	F	45	18.6%	F	12	14.8%
	Total	242		Total	81	
	Mean	**Stdev**		**Mean**	**Stdev**	
High School GPA	3.51	0.46		3.56	0.40	
Math SAT	531	40		533	35	
Math ACT	24	1		24	1	

Lecture Topics
What is Engineering: Engineering Disciplines and History of Engineering
The Human Brain
Critical Thinking
Emotional Intelligence
Experimentation and Knowledge Discovery
Engineering Reasoning and Problem Solving
Engineering Design
Entrepreneurship and Innovation
Sustainability
Oral and Written Communication

#	Description of Critical Thinking Skill Promoted
1	Summarize a pattern of information without making inappropriate inferences
2	Evaluate how strongly correlational-type data supports a hypothesis
3	Provide alternative explanations to observations
4	Identify additional information needed to evaluate a hypothesis or particular explanation of an observation
5	Determine whether an invited inference in an advertisement is supported by the information
6	Provide relevant alternative interpretations of information
7	Separate relevant from irrelevant information when solving a real-world problem
8	Analyze and integrate information from separate sources to solve a real-world problem
9	Use basic mathematical skills to help solve a real-world problem
10	Identify suitable solutions to real-world problems using relevant information
11	Identify and explain the best solution for a real-world problem using relevant information
12	Explain how changes in a real-world problem situation might affect the solution

Math Recitation Topics
Factoring Quadratic Equations
Algebraic Manipulation
Developing and Solving Linear Equations from Word Problems
Developing and Solving Polynomial Equations from Word Problems
Graphing Quadratic Equations
Solving Logarithmic Equations

Original Problem Students Must Simplify	Example of Work Provided by Students	Description of Student Error
$\frac{(-15)-(-6)}{(-3)+6}$	$\frac{-5+1}{(-1)+1}$	Attempted to factor out 3 from the first terms and 6 from the second terms in the numerator and denominator – **incorrect factoring**
$5x^4+3x^2-8$	$(5x^2+4)(5x^2-2)$	Incorrect identification of factoring coefficients – **incorrect factoring**
x^4+2x^2-8	$x^2(x^2+2)-8$ $(x^2-8)(x^2+2)$	Treated (x^2+2) as a common factor to attempt to further simplify the equation – **incorrect identification of common factors**
$\frac{3}{p-4}+\frac{5p}{p+1}$	$\frac{3+5p}{p-4+p}$ OR $\frac{3+5p}{p^2-3p-4}$	Incorrectly adding rational expressions by either simple addition or improper creation of a like denominator – **lack of knowledge on simplifying rational expressions**
$-5\sqrt{80x^5}$	$-5(2)\sqrt{20x^5}$ $-20\sqrt{5x^5}$ $-4\sqrt{25x^5}$	Incorrect simplification of radical expressions and failure to identify factors that can be simplified – **lack of knowledge on simplifying radical expressions**

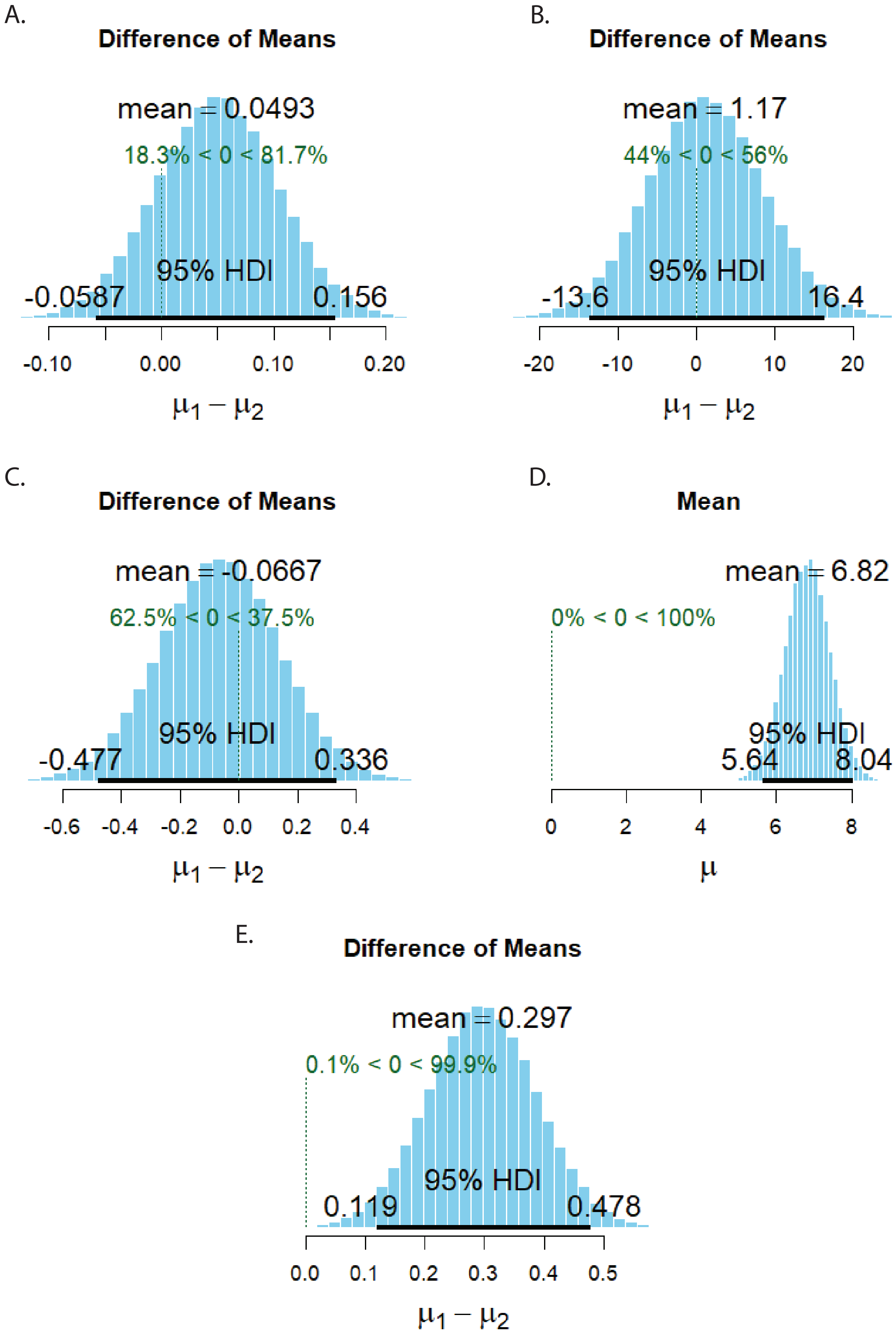
Universal Intellectual Standards

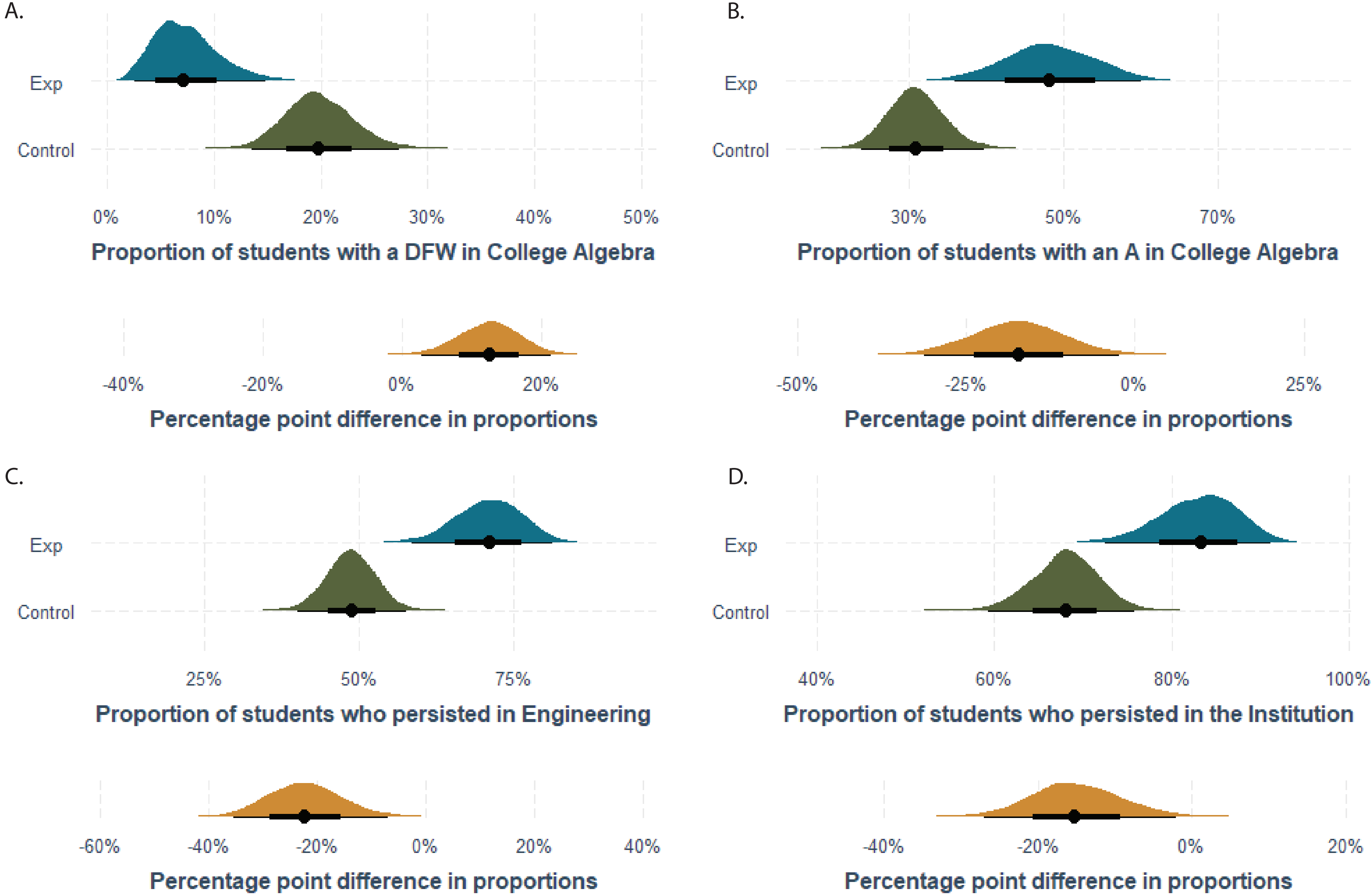
- Clarity
- Accuracy
- Relevance
- Logicalness
- Breadth
- Precision
- Significance
- Completeness
- Fairness
- Depth

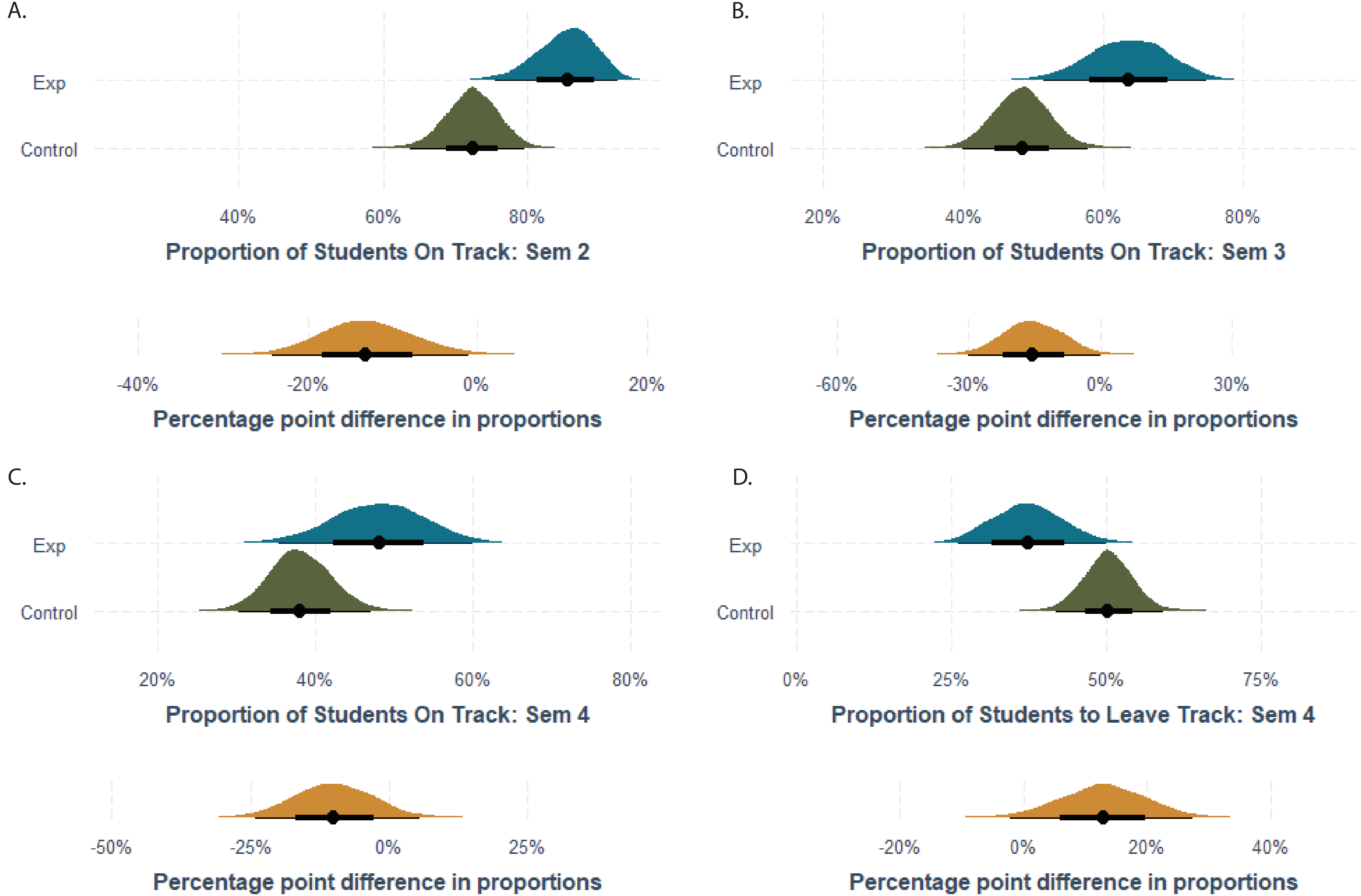
Elements of Thought

- Purpose
- Question at Issue
- Information
- Interpretation and inference
- Concept
- Assumptions
- Implications and Consequences
- Point of View

Intellectual Traits
Intellectual Humility
Intellectual Autonomy
Intellectual Integrity
Intellectual Courage
Intellectual Perseverance
Confidence in Reason
Intellectual Empathy
Fairmindedness


A.
Difference of Means
mean = 0.0493
18.3% < 0 < 81.7%
95% HDI
-0.0587
0.156
-0.10
0.00
0.10
0.20
$\mu_1 - \mu_2$
B.
Difference of Means
mean = 1.17
44% < 0 < 56%
95% HDI
-13.6
16.4
-20
-10
0
10
20
$\mu_1 - \mu_2$
C.
Difference of Means
mean = -0.0667
62.5% < 0 < 37.5%
95% HDI
-0.477
0.336
-0.6
-0.4
-0.2
0.0
0.2
0.4
$\mu_1 - \mu_2$
D.
Mean
mean = 6.82
0% < 0 < 100%
95% HDI
5.64
8.04
0
2
4
6
8
μ
E.
Difference of Means
mean = 0.297
0.1% < 0 < 99.9%
95% HDI
0.119
0.478
0.0
0.1
0.2
0.3
0.4
0.5
$\mu_1 - \mu_2$


A.
Exp
Control
0%
10%
20%
30%
40%
50%
Proportion of students with a DFW in College Algebra
-40%
-20%
0%
20%
Percentage point difference in proportions
B.
Exp
Control
30%
50%
70%
Proportion of students with an A in College Algebra
-50%
-25%
0%
25%
Percentage point difference in proportions
C.
Exp
Control
25%
50%
75%
Proportion of students who persisted in Engineering
-60%
-40%
-20%
0%
20%
40%
Percentage point difference in proportions
D.
Exp
Control
40%
60%
80%
100%
Proportion of students who persisted in the Institution
-40%
-20%
0%
20%
Percentage point difference in proportions


A.
Exp
Control
40%
60%
80%
Proportion of Students On Track: Sem 2
-40%
-20%
0%
20%
Percentage point difference in proportions
B.
Exp
Control
20%
40%
60%
80%
Proportion of Students On Track: Sem 3
-60%
-30%
0%
30%
Percentage point difference in proportions
C.
Exp
Control
20%
40%
60%
80%
Proportion of Students On Track: Sem 4
-50%
-25%
0%
25%
Percentage point difference in proportions
D.
Exp
Control
0%
25%
50%
75%
Proportion of Students to Leave Track: Sem 4
-20%
0%
20%
40%
Percentage point difference in proportions