Multi-objective optimisation of VCR systems by applying TOPSIS to the single-objective solutions obtained with Excel Solver

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Abstract

This paper describes two methods for using the single-objective solver that comes with Microsoft Excel and the TOPSIS decision-making technique for multi-objective optimisation of a two-stage vapour-compression refrigeration (VCR) system. The Excel-aided model developed for analysing the exergetic and economic performance of the system was first used to obtain six optimised solutions by using the two solution methods provided by Solver that separately maximised the system's exergetic efficiency (ε) and minimise its total cost-rate (C_{total}) and equipment cost rate (C_{equip}). A dual-objective optimised solution that simultaneously maximises ε and minimises C_{total} was also obtained by using the MIDACO solver. The first method applies TOPSIS to rank the seven optimised solutions by using five weighting schemes. As should be expected, the results of this method show that MIDACO's dual-objective solution achieved the first rank, while the two Solver solutions that minimised the equipment cost rate occupied the lowest two ranks. According to this method, the two Solver solutions that maximised the exergetic efficiency closely satisfy the dual objective of the 2E optimisation. The second method applies TOPSIS in an active manner by using Solver to adjust the relevant design variables so that TOPSIS parameter that measures the closeness of the base design to the ideal dual-objective target is maximised. The results of this method show that it can produce a solution that is closer to the dual-objective target than MIDACO's 2E solution.

Keywords: multi-objective optimisation, single-objective optimisation, multi-stage vapour compression refrigeration, Microsoft Excel Solver, MIDACO solver

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1. Introduction

The significant share of vapour-compression refrigeration (VCR) system in the energy consumption of residential, commercial, and industrial sectors necessitates improving the efficiency of these systems. One method for increasing the systems' coefficient of performance (COP) is to use multi-stage compression [1,2]. Since the improved systems cost more than the simple systems, their feasibility depends on careful compromises between their electrical energy consumption and cost. The increasing concern about the environmental change due to global warming and the ozone-layer depletion added the need for replacing the conventional synthetic refrigerants with more environment-friendly fluids as a third factor [3]. This inspired many researchers to be involved in developing suitable multi-objective optimisation (MOO) methods for optimising the energetic, economic, and environmental (3E) performance of VCR systems using natural or environment-friendly refrigerants [4].

Ahmed et al. [1] analysed a two-stage VCR system by using a novel hybrid multi-objective grey wolf optimizer (HMOGWO) algorithm. The system was modelled using response surface methods (RSM) to investigate the impacts of design variables on the set responses. Three conflicting scenarios in bi-objective optimisation were built focusing on the overall system following the TOPSIS and LINMAP decision-making methods. The optimal solutions indicate that for the first to third scenarios, the exergetic efficiency (ε) and capital expenditure are optimised by 33.4% and 7.5%, and the ε and operational expenditure are improved by 27.4% and 19.0%. The ε and global warming potential are also optimised by 27.2% and 19.1%, where the proposed HMOGWO outperforms the MOGWO and NSGA-II. Based on the research outcomes, they concluded that the combined RSM and HMOGWO technique is an excellent solution to simulate and optimise two-stage VCR systems.

Singh et al. [4] analysed an ammonia-based multi-stage VCR system incorporated with a flash intercooler which also works as a sub-cooler. They carried out a thermo-economic optimisation of the system in order to maximise its exergetic efficiency and minimise its total capital cost rate. The evaporator temperature, condenser temperature, subcooling parameter, and de-superheating parameter were considered as design variables for their MOO analysis. They also employed the multi-objective genetic algorithm tool provided with MATLAB to carry out the optimisation analysis and used EES to determine the thermodynamic properties of the refrigerants. TOPSIS [5] was used to select unique solutions for five different weighting factors of exergetic efficiency and total cost. Their results revealed that the exergetic efficiency and total capital cost of the system at the thermo-economic optimal operating conditions were 41.76% and $223,717.6, respectively.

Being widely-available general-purpose software with powerful analytical tools, Microsoft Excel allows more researchers and engineering students to join the search for alternative environment-friendly refrigerants and contribute to the development of innovative VCR systems. Excel is supported by a versatile solver for single-objective optimisation (SOO) analyses and the VBA programming language that comes with MS applications can be used for developing property functions for various conventional and alternative refrigerants [6]. However, currently Excel doesn’t have its own solver for MOO analyses. Although a free version of the MIDACO solver [7] is available for Excel users, it allows only four design variables to be considered in the analysis; which is not adequate for analysing multi-stage compression systems with various design parameters such as compressor efficiency and sub-cooling and superheating degrees.

This paper shows how Excel’s Solver can be used with TOPSIS for MOO analyses of two-stage compression VCR systems. The idea of using a SOO solver for a MOO analysis is not new. Balabanov [8] demonstrated the method by solving a mathematical multi-objective problem by using the NLP Solver submodule in LibreOffice Calc. However, there is no published work on the use of similar methods for MOO analyses of VCR systems on any platform. The paper presents two methods for using TOPSIS with Solver to conduct MOO analyses the first of which applies TOPSIS simply to identify a Solver solution(s) that is close to satisfy the multi-objective requirement. The second method uses Solver and TOPSIS to improve a base design so as to achieve the multi-objective requirement. The results of the two methods are compared with a real MOO solution obtained by using the limited-version of the MIDACO solver.

2. The analytical model for the two-stage compression VCR system

Figures 1 and 2 show schematic and *T-s* diagrams of the two-stage compression VCR system which is to be analysed with the assumed input parameters shown on Table 1. The liquid refrigerant expands in the first expansion valve to the flash chamber pressure (p_{FC}). Part of the liquid vaporises during this process and the resulting saturated vapour (state 3) is mixed with the superheated vapour from the low-pressure compressor (state 2) before entering the high-pressure compressor (state 9). Although the refrigerant's flow to the evaporator is reduced, the total compression work is also reduced and the net effect is an increase in the system's COP [9]. Since the COP depends on the flash-chamber pressure, this pressure has to be optimised.

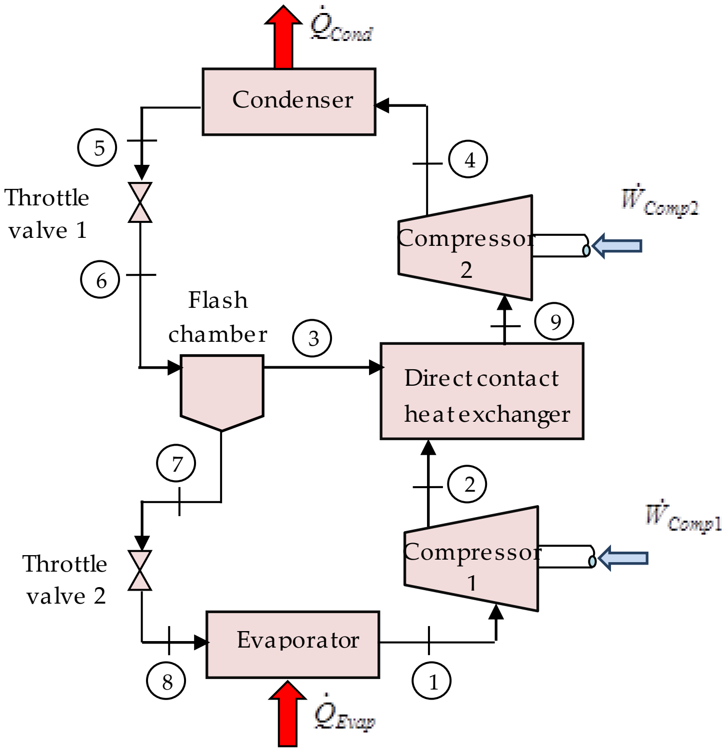


Figure 1. Schematic of the two-stage VCR system

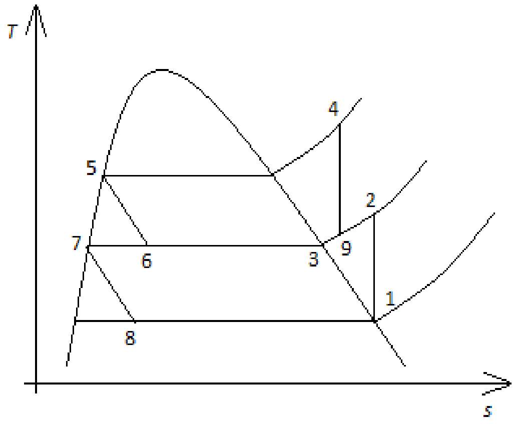


Figure 2. *T-s* diagram of the two-stage VCR system

Table 1. Assumed input parameters for thermodynamic analysis of the system [10]

Parameter	Value
Cooling capacity of the system, *CC*	10 kW
Overall heat transfer coefficient for evaporator, U_{Evap}	0.03 kW/m^2.K
Overall heat transfer coefficient for condenser, U_{Cond}	0.04 kW/m^2.K
Ambient temperature, T_0	25^{o}C
Temperature change for air in evaporator and condenser	± 5^{o}C
Temperature of the inlet air to evaporator	0^{o}C

2.1. The thermodynamic model

The system will be analysed for a constant condenser temperature of 40^{o}C, but the evaporator temperature is allowed to vary in the range $-25^oC \le T_{Evap} \le -15^oC$. The inter-stage pressure, p_{FC}, is initially determined as:

$$p_{FC} = \sqrt{p_{Evap} \times p_{Cond}} \tag{1}$$

The isentropic efficiencies of the two compressors are assumed to vary with the corresponding suction and discharge pressures as follows [4]:

$$\eta_{Comp} = 0.85 - 0.04667 (P_{Out} / P_{In}) \tag{2}$$

Table 2 shows the mass, energy and exergy balance equations for the different system components.

Table 2. Equations for mass and energy balances and rates of exergy destruction in the individual system components

	Mass balance	Energy balance	Exergy destruction
Evaporator	$\dot{m}_1 = \dot{m}_8 = \dot{m}_r$	$\dot{m}_1 h_1 = \dot{m}_8 h_8 + CC$	$T_0 [\dot{m}_1 (s_1 - s_8) - CC / T_{Evap}]$
Compressor 1	$\dot{m}_2 = \dot{m}_1$	$\dot{W}_{Comp1} = \dot{m}_r (h_2 - h_1)$	$T_0 \dot{m}_1 (s_2 - s_1)$
Compressor 2	$\dot{m}_4 = \dot{m}_9 = \dot{m}_r / (1 - x_6)$	$\dot{W}_{Comp2} = \dot{m}_3 (h_4 - h_3)$	$T_0 \dot{m}_4 (s_4 - s_9)$
Condenser	$\dot{m}_5 = \dot{m}_4 = \dot{m}_r / (1 - x_6)$	$\dot{m}_5 h_5 = \dot{m}_4 h_4 - \dot{Q}_{Cond}$	$T_0 [\dot{m}_4 (s_5 - s_4) + \dot{Q}_{cond} / T_{Cond}]$
Throttle valve 1	$\dot{m}_6 = \dot{m}_5 = \dot{m}_r / (1 - x_6)$	$h_6 = h_5$	$\dot{m}_5 T_0 (s_6 - s_5)$
Throttle valve 2	$\dot{m}_8 = \dot{m}_7 = \dot{m}_r$	$h_8 = h_7$	$\dot{m}_7 T_0 (s_8 - s_7)$
Flash chamber	$\dot{m}_3 = x_6 \dot{m}_5 = x_6 \dot{m}_r$ $\dot{m}_7 = (1 - x_6) \dot{m}_r$	$\dot{m}_6 h_6 = \dot{m}_3 h_8 + \dot{m}_7 h_7$	$\frac{T_0 \dot{m}_1}{1 - x_6} \times [s_6 - x_6 s_3 - (1 - x_6) s_7]$
Direct contact heat exchanger	$\dot{m}_9 = \dot{m}_2 + \dot{m}_3 = \dot{m}_r$	$\dot{m}_2 h_2 + \dot{m} h_3 = \dot{m}_9 h_9$	$\dot{m}_1 T_0 [s_9 / (1 - x_6) - s_2 - x_6 s_3 / (1 - x_6)]$

The system's total compression work, *COP*, and exergetic efficiency (ε), are given by:

$$\dot{W}_{Total} = \dot{W}_{Comp1} + \dot{W}_{Comp2} = \dot{m}_r (h_2 - h_1) + \dot{m}_r (h_4 - h_9)/(1 - x_6) \tag{3}$$

$$COP = CC / \dot{W}_{Total} \tag{4}$$

$$\varepsilon = \left(\dot{W}_{Total} - \dot{E}^D_{Total} \right) / \dot{W}_{Total} \tag{5}$$

Where CC is the system's cooling capacity and $\dot{E}^D_{Total}$ is its total exergy destruction rate which is the summation of the exergy destruction rates in the eight components:

$$\dot{E}^D_{Total} = \dot{E}^D_{Evap} + \dot{E}^D_{Comp1} + \dot{E}^D_{Comp2} + \dot{E}^D_{Cond} + \dot{E}^D_{TV1} + \dot{E}^D_{TV2} + \dot{E}^D_{FC} + \dot{E}^D_{DCHX} \tag{6}$$

Table 2 also shows the equations used to determine the exergy destruction rates.

2.1. The economic model

The total annualised cost rate of the system is given by [10]:

$$\dot{C}_{total} = \sum_{1}^{4} \dot{C}_k + \dot{C}_{op} + \dot{C}_{env} \tag{7}$$

where, $\dot{C}_k$ is the capital and maintenance cost rate of individual components, $\dot{C}_{op}$ is the operational cost rate of the system, and $\dot{C}_{env}$ is the CO2 penalty cost rate of the system. The total capital and maintenance cost rate is calculated by adding the capital and maintenance cost rates of the individual component given by [11]:

$$\dot{C}_k = C_k . \phi . CRF \tag{8}$$

where, ϕ is the maintenance factor and CRF is the capital recovery factor defined as:

$$CRF = i(1+i)^n / \left[(1+i)^n - 1 \right] \tag{9}$$

Where i is the interest rate and n is the system's expected lifetime. The capital costs of the various system components are estimated using the relations shown on Table 3.

Table 3. Capital cost functions of the various system components [10]

Component	Capital cost function
Evaporator	$C_{eva} = 1397 \times A_{eva}^{0.89}$
Low-temperature compressor	$C_{Comp1} = 10167.5 \times \dot{W}_{Comp1}^{0.46}$
High-temperature compressor	$C_{comp2} = 9624.2 \times \dot{W}_{Comp2}^{0.46}$
Throttle valve 1	$C_{TV1} = 114.5 \times \dot{m}_5$
Condenser	$C_{con} = 1397 \times A_{con}^{0.89}$
Throttle valve 2	$C_{TV2} = 114.5 \times \dot{m}_7$
Flash chamber and direct contact heat exchanger	$C_{FC} = 280.3 \dot{m}_r^{0.67}$ [12]

The heat-transfer areas of the evaporator and condenser given in Table 3 are obtained by using the log-mean temperature difference (LMTD) method.

The operational cost rate of the system is mainly the cost of electricity as given by:

$$\dot{C}_{op} = \dot{W}.N.c_{elec} \tag{10}$$

Where N is the annual operational hours and c_{elec} is the cost of electricity in \$/kWh.

Following Wang et al. [13], the CO_2 penalty cost rate of the systems is calculated from:

$$\dot{C}_{env} = m_{CO_2e}.c_{CO_2} \tag{11}$$

Where, c_{CO_2} is the penalty cost of the avoided CO_2 emission and m_{CO_2e} is the amount of annual CO_2 emission from the system that can be estimated by:

$$m_{CO_2e} = \mu_{CO_2e}.E_{annual} \tag{12}$$

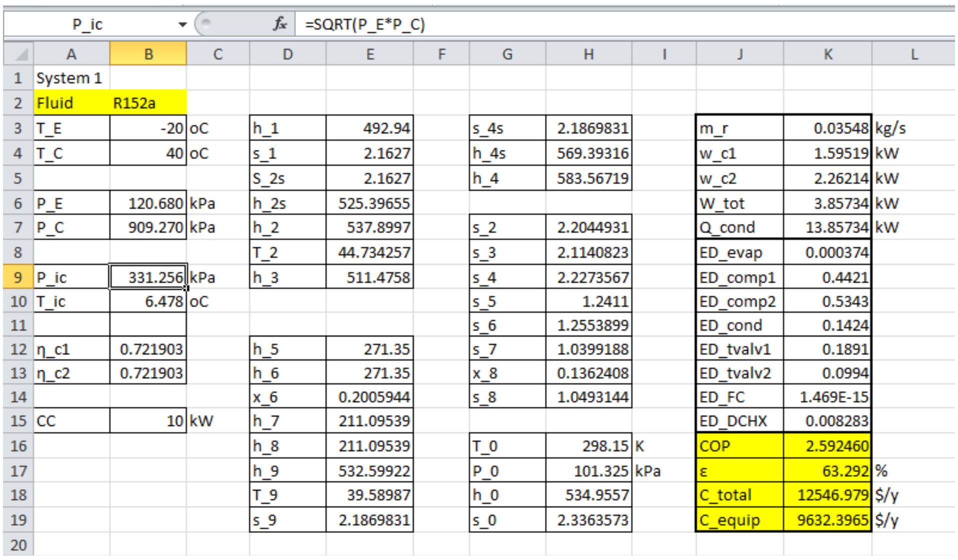
Where μ_{CO_2e} is the regional (country) electricity conversion factor and E_{annual} is the annual amount of energy consumed by the system. The values of N, μ_{CO_2e}, c_{elec}, and c_{CO_2} used in the present analysis are shown on Table 4.

Table 4. Assumed input parameters for economic analysis of the system [10]

Parameter	Value
Maintenance factor, ϕ	1.06
Interest rate, i	14%
Plant life time, n	15 Years
Annual operation hours, N	4266 hours
Electrical power cost, c_{elec}	0.09 \$/kWh
Emission factor, μ_{CO_2e}	0.968 kg/kWh
Cost of CO_2 avoided, c_{CO_2}	0.09 \$/kg of CO_2 emission

2.2. Development of the Excel-aided model

Figure 3 shows the front sheet (Sheet 1) of the Excel-aided model developed for the VCR system. The data part on the left side of the sheet shows the specified values of the evaporator and condenser temperatures (T_E and T_C), the values of the flash-chamber pressure and temperature (P_fc and T_fc), the isentropic efficiencies of the two compressors (η_c1 and η_c2), and the system's cooling capacity (CC). The calculations part in the central part of the sheet determines the enthalpy and entropy of the refrigerant at all the 8 states by using VBA functions [6]. Note that the refrigerant name is stored as a variable (Fluid) so that the same model can be used for other refrigerants without modification. The results part on the right-hand side of the sheet determines the refrigerant's mass flow rate, the compressors' work, the rates of exergy destruction in the eight system components, the COP, and the overall exergetic efficiency of the system. Finally, the sheet displays the total cost rate and the total equipment cost rate as calculated by the back sheet (Sheet 2) that applies the economic model as shown on Figure 4.Sheet 2 determines the areas of the evaporator and condenser by using the LMTD method to calculate their costs from the relations shown on Table 3. Values of the temperatures involved are imported from Sheet 1.

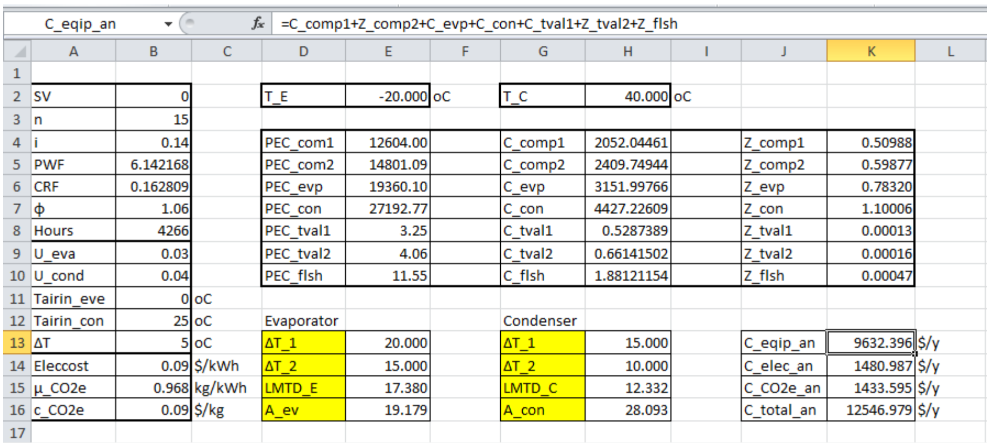


P_ic

=SQRT(P_E*P_C)

	A	B	C	D	E	F	G	H	I	J	K	L
1	System 1											
2	Fluid	R152a										
3	T_E	-20	oC	h_1	492.94		s_4s	2.1869831		m_r	0.03548	kg/s
4	T_C	40	oC	s_1	2.1627		h_4s	569.39316		w_c1	1.59519	kW
5				S_2s	2.1627		h_4	583.56719		w_c2	2.26214	kW
6	P_E	120.680	kPa	h_2s	525.39655					W_tot	3.85734	kW
7	P_C	909.270	kPa	h_2	537.8997		s_2	2.2044931		Q_cond	13.85734	kW
8				T_2	44.734257		s_3	2.1140823		ED_evap	0.000374	
9	P_ic	331.256	kPa	h_3	511.4758		s_4	2.2273567		ED_comp1	0.4421	
10	T_ic	6.478	oC				s_5	1.2411		ED_comp2	0.5343	
11							s_6	1.2553899		ED_cond	0.1424	
12	η_c1	0.721903		h_5	271.35		s_7	1.0399188		ED_tvalv1	0.1891	
13	η_c2	0.721903		h_6	271.35		x_8	0.1362408		ED_tvalv2	0.0994	
14				x_6	0.2005944		s_8	1.0493144		ED_FC	1.469E-15	
15	CC	10	kW	h_7	211.09539					ED_DCHX	0.008283	
16				h_8	211.09539		T_0	298.15	K	COP	2.592460	
17				h_9	532.59922		P_0	101.325	kPa	ε	63.292	%
18				T_9	39.58987		h_0	534.9557		C_total	12546.979	$/y
19				s_9	2.1869831		s_0	2.3363573		C_equip	9632.3965	$/y
20												

Figure 3. The front sheet of the model for the thermodynamic calculations



C_eqip_an

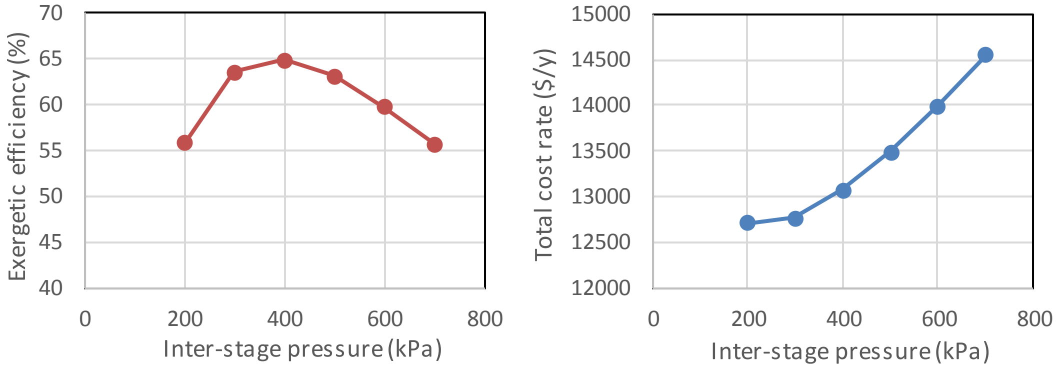
=C_comp1+Z_comp2+C_evp+C_con+C_tval1+Z_tval2+Z_flsh

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	SV	0		T_E	-20.000	oC	T_C	40.000	oC			
3	n	15										
4	i	0.14		PEC_com1	12604.00		C_comp1	2052.04461		Z_comp1	0.50988	
5	PWF	6.142168		PEC_com2	14801.09		C_comp2	2409.74944		Z_comp2	0.59877	
6	CRF	0.162809		PEC_evp	19360.10		C_evp	3151.99766		Z_evp	0.78320	
7	ϕ	1.06		PEC_con	27192.77		C_con	4427.22609		Z_con	1.10006	
8	Hours	4266		PEC_tval1	3.25		C_tval1	0.5287389		Z_tval1	0.00013	
9	U_eva	0.03		PEC_tval2	4.06		C_tval2	0.66141502		Z_tval2	0.00016	
10	U_cond	0.04		PEC_flsh	11.55		C_flsh	1.88121154		Z_flsh	0.00047	
11	Tairin_eve	0	oC									
12	Tairin_con	25	oC	Evaporator			Condenser					
13	ΔT	5	oC	ΔT_1	20.000		ΔT_1	15.000		C_eqip_an	9632.396	$/y
14	Eleccost	0.09	$/kWh	ΔT_2	15.000		ΔT_2	10.000		C_elec_an	1480.987	$/y
15	µ_CO2e	0.968	kg/kWh	LMTD_E	17.380		LMTD_C	12.332		C_CO2e_an	1433.595	$/y
16	c_CO2e	0.09	$/kg	A_ev	19.179		A_con	28.093		C_total_an	12546.979	$/y
17												

Figure 4. The back sheet of the model for the economic calculations

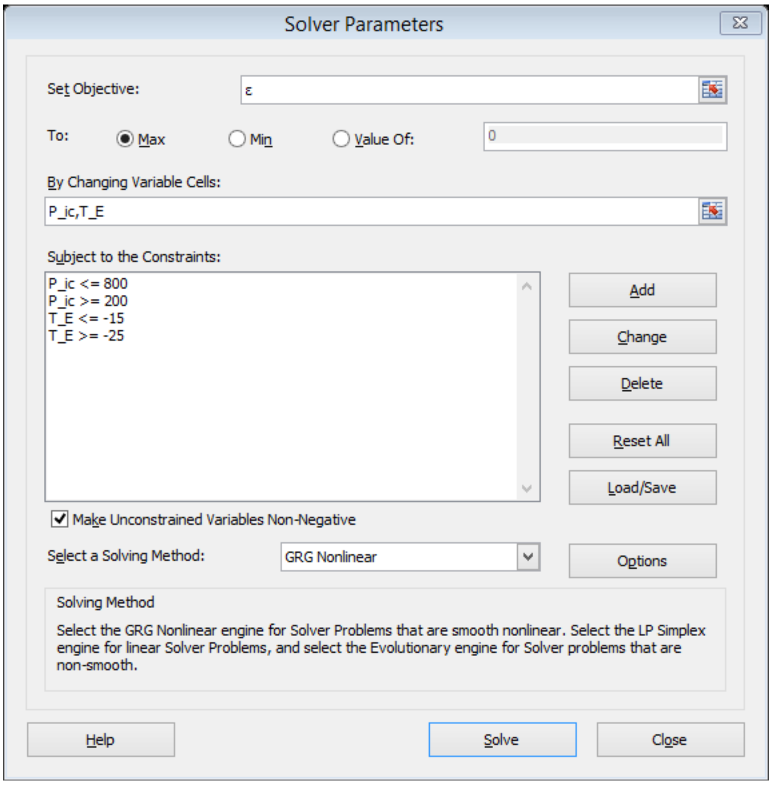
3. Single-objective optimisation by using Solver

Two design variables are considered in the present analysis for optimising the VCR system; which are the inter-stage pressure, p_{FC}, and evaporator temperature, T_{Evap}. The thermodynamic and economic performance of the VCR system is measured by two performance indicators; which are the exergetic efficiency, ε, and total cost rate C_{total}. Figure 5 shows the variation of the two indicators with p_{FC}. Figure 5.a shows that the exergetic efficiency reaches its maximum value at an inter-stage pressure of about 400 kPa, while Figure 5.b shows that the total cost rate increases gradually with the pressure and does not reach a maximum within the specified range of p_{FC}. Determining the values of p_{FC} and T_{Evap} that optimise each of the two performance indicators requires the use of a single-objective solver such as Excel's Solver. Figure 6 shows Solver's set-up for determining the values of p_{FC} and T_{Evap} that maximise the system's exergetic efficiency. Four constraints are imposed on the solution to keep the values of p_{FC} and T_{Evap} within the ranges $200 \leq p_{FC} \leq 800$ kPa and $-25^{\circ}C \leq T_{Evap} \leq -15^{\circ}C$, respectively.



(a) (b)

Figure 5. Variation of the exergetic efficiency and total cost rate with the inter-stage pressure



Set Objective: ε

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells: P_ic,T_E

Subject to the Constraints:
P_ic <= 800
P_ic >= 200
T_E <= -15
T_E >= -25

☑ Make Unconstrained Variables Non-Negative

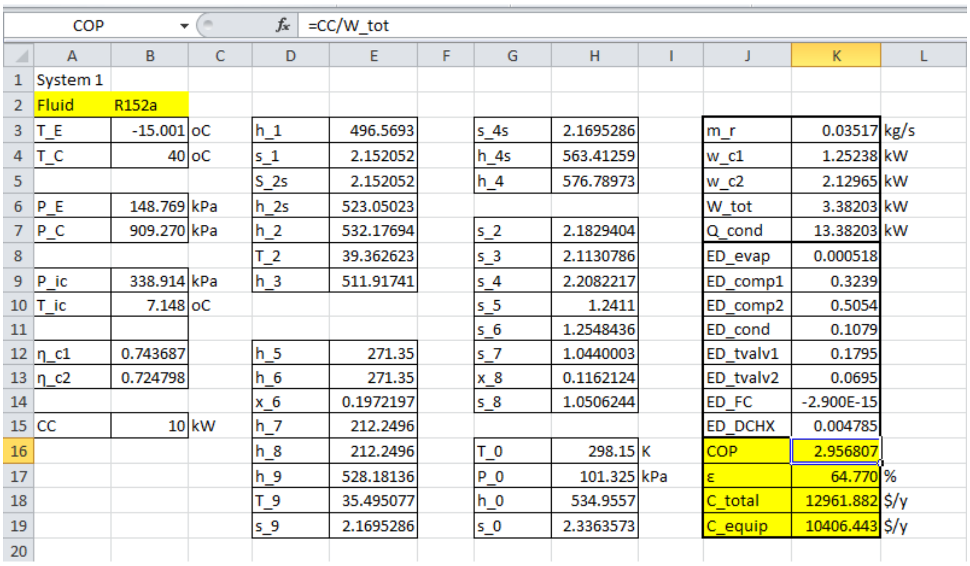
Select a Solving Method: GRG Nonlinear

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure6. Solver set-up for maximising the exergetic efficiency

Figure 7 shows the solution obtained by the GRG Nonlinear method of Solver. According to this solution, the maximum exergetic efficiency is 64.77% which is achieved at T_{Evap} of -15°C and p_{FC} of 338.914kPa. Another solution was obtained for maximising ε with the Evolutionary method and four other solutions were obtained for minimising the total cost rate or the equipment cost rate by using Solver's two solution methods. Table 5 shows the six values of the exergetic efficiency and total cost rate as determined by the six solutions that will be used for obtaining a dual-objective solution by using the proposed Solver-TOPSIS technique to be discussed in Section 5.Areal MOO solution is needed as a reference for verifying and also clarifying the proposed method. This is dealt with in the following section.



COP

=CC/W_tot

	A	B	C	D	E	F	G	H	I	J	K	L
1	System 1											
2	Fluid	R152a										
3	T_E	-15.001	oC	h_1	496.5693		s_4s	2.1695286		m_r	0.03517	kg/s
4	T_C	40	oC	s_1	2.152052		h_4s	563.41259		w_c1	1.25238	kW
5				S_2s	2.152052		h_4	576.78973		w_c2	2.12965	kW
6	P_E	148.769	kPa	h_2s	523.05023					W_tot	3.38203	kW
7	P_C	909.270	kPa	h_2	532.17694		s_2	2.1829404		Q_cond	13.38203	kW
8				T_2	39.362623		s_3	2.1130786		ED_evap	0.000518	
9	P_ic	338.914	kPa	h_3	511.91741		s_4	2.2082217		ED_comp1	0.3239	
10	T_ic	7.148	oC				s_5	1.2411		ED_comp2	0.5054	
11							s_6	1.2548436		ED_cond	0.1079	
12	η_c1	0.743687		h_5	271.35		s_7	1.0440003		ED_tvalv1	0.1795	
13	η_c2	0.724798		h_6	271.35		x_8	0.1162124		ED_tvalv2	0.0695	
14				x_6	0.1972197		s_8	1.0506244		ED_FC	-2.900E-15	
15	CC	10	kW	h_7	212.2496					ED_DCHX	0.004785	
16				h_8	212.2496		T_0	298.15	K	COP	2.956807	
17				h_9	528.18136		P_0	101.325	kPa	ε	64.770	%
18				T_9	35.495077		h_0	534.9557		C_total	12961.882	$/y
19				s_9	2.1695286		s_0	2.3363573		C_equip	10406.443	$/y
20												

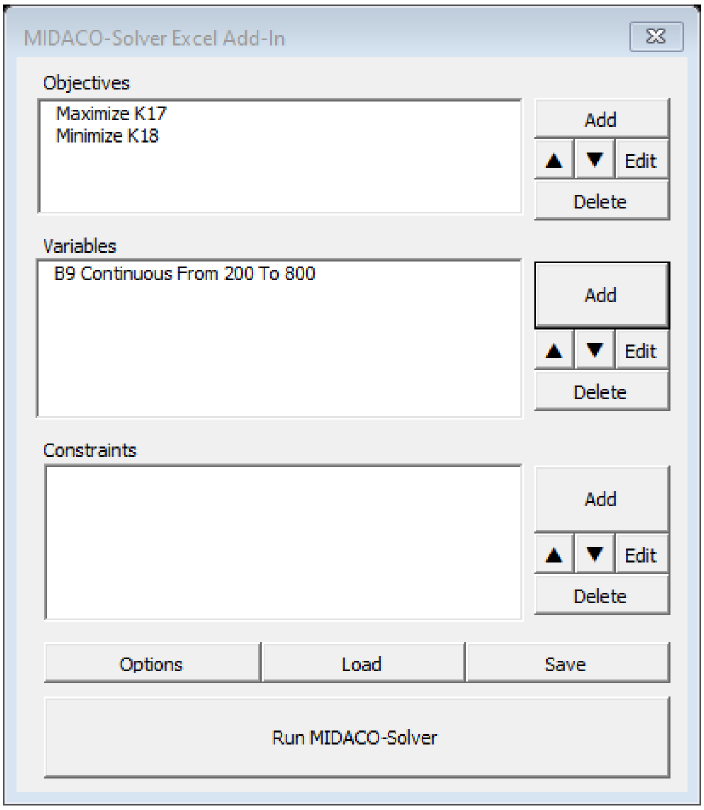
Figure 7. Solver solution for maximising the exergetic efficiency with the GRG Nonlinear method

Table 5. Results of the single-objective optimisation analyses using Solver

Solution method	Objective	ε	C_{total}
GRG Nonlinear	Maximise ε	64.770	12961.882
	Minimise C_{total}	63.292	12546.965
	Minimise C_{equip}	56.688	12493.246
Evolutionary	Maximise ε	65.172	13114.228
	Minimise C_{total}	59.722	12342.795
	Minimise C_{equip}	56.688	12493.246

4. Dual-objective optimisation by using MIDACO

MOO analyses, in which two or more conflicting performance indicators are to be simultaneously optimised, require a multi-objective solver to be used. This is illustrated by using the MIDACO solver [7] for a dual-objective (2E) optimisation analysis of the two-stage compression system. The conflict between the two objectives, which are maximising the system's total exergy efficiency and minimising its total cost rate, is clearly shown on Figure 5.The MIDACO solver can be used with Excel as an add-in just like Solver. Figure 8shows the set-up for the MIDACO solver for this analysis. As Figure 8 shows, there are two changing variables in the analysis which are the evaporator temperature T_{Evap} stored in cell B3 and the inter-stage pressure p_{FC} stored in cell B9. Since a MOO analysis does not lead to a single solution but a Pareto front, a decision-making method has to be applied to select the most desirable solution. Figure 9 shows the Pareto front obtained by MIDACO and Figure 10 shows the selected solution. According to this solution, the optimum evaporator temperature and inter-stage pressure are -17.7°C and 302.95kPa, respectively, at which the values of the exergetic efficiency and total cost rate are 63.447% and \$12544.7/y, respectively.



Maximize K17
Minimize K18

B9 Continuous From 200 To 800

Figure 8. MIDACO set-up for the dual-objective optimisation analysis

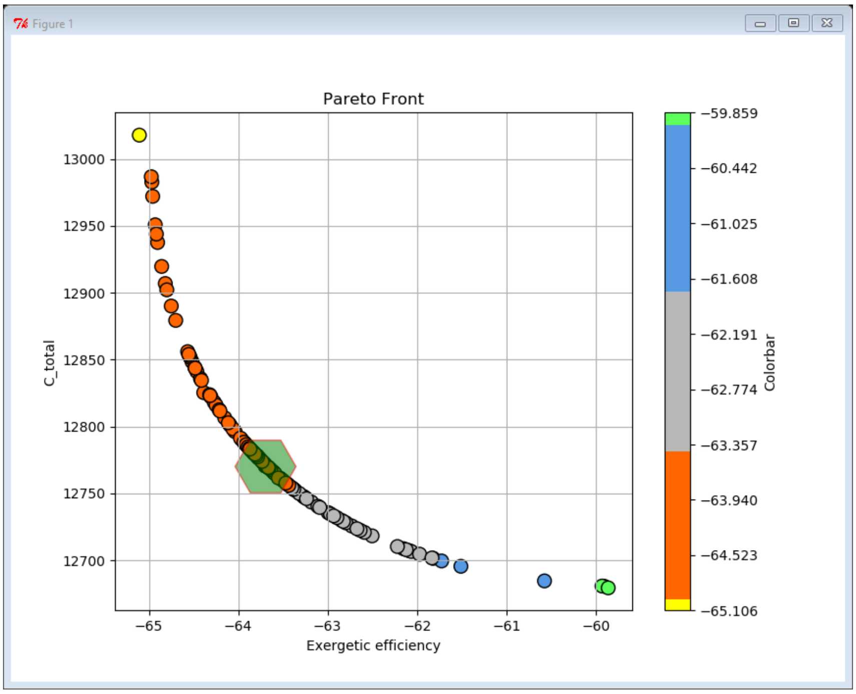
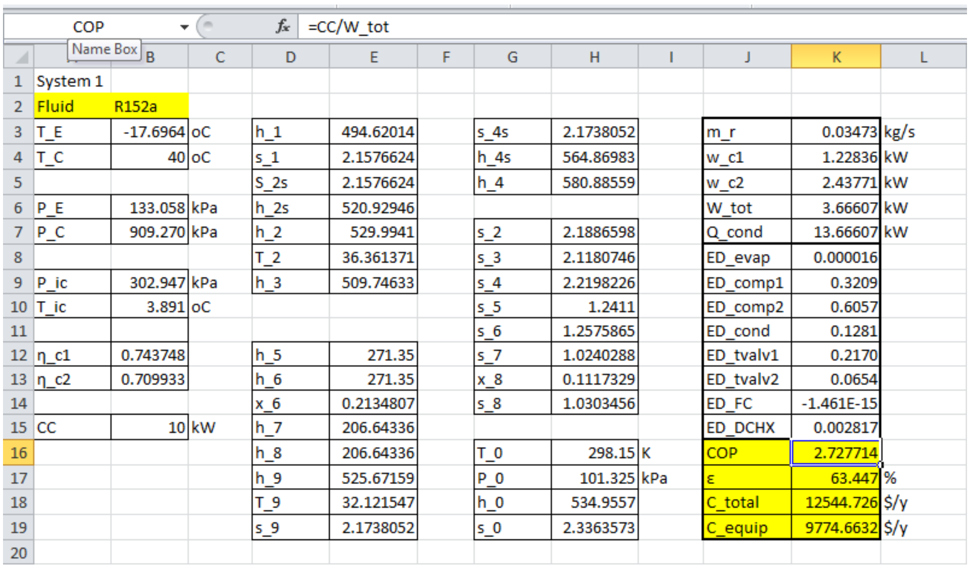


Figure 9. Pareto front of the 2E solution obtained by MIDACO



COP =CC/W_tot

		B	C	D	E	F	G	H	I	J	K	L
1	System 1											
2	Fluid	R152a										
3	T_E	-17.6964	oC	h_1	494.62014		s_4s	2.1738052		m_r	0.03473	kg/s
4	T_C	40	oC	s_1	2.1576624		h_4s	564.86983		w_c1	1.22836	kW
5				S_2s	2.1576624		h_4	580.88559		w_c2	2.43771	kW
6	P_E	133.058	kPa	h_2s	520.92946					W_tot	3.66607	kW
7	P_C	909.270	kPa	h_2	529.9941		s_2	2.1886598		Q_cond	13.66607	kW
8				T_2	36.361371		s_3	2.1180746		ED_evap	0.000016	
9	P_ic	302.947	kPa	h_3	509.74633		s_4	2.2198226		ED_comp1	0.3209	
10	T_ic	3.891	oC				s_5	1.2411		ED_comp2	0.6057	
11							s_6	1.2575865		ED_cond	0.1281	
12	η_c1	0.743748		h_5	271.35		s_7	1.0240288		ED_tvalv1	0.2170	
13	η_c2	0.709933		h_6	271.35		x_8	0.1117329		ED_tvalv2	0.0654	
14				x_6	0.2134807		s_8	1.0303456		ED_FC	-1.461E-15	
15	CC	10	kW	h_7	206.64336					ED_DCHX	0.002817	
16				h_8	206.64336		T_0	298.15	K	COP	2.727714	
17				h_9	525.67159		P_0	101.325	kPa	ε	63.447	%
18				T_9	32.121547		h_0	534.9557		C_total	12544.726	$/y
19				s_9	2.1738052		s_0	2.3363573		C_equip	9774.6632	$/y
20												

Figure 10. The dual-objective optimised solution obtained by MIDACO

Figure 11 shows the values of the two performance indicators as obtained by the dual-objective solution of MIDACO together with the six single-objective solutions of Solver. The figure shows that Solver's solution that minimised C_{total} with the GRG Nonlinear method (the blue colour) is practically the same as MIDACO's entry (the red colour). The following section shows how Solver and the TOPSIS decision-making method [5] can be used to obtain a solution that satisfies the dual-objective requirement by applying a systematic procedure.

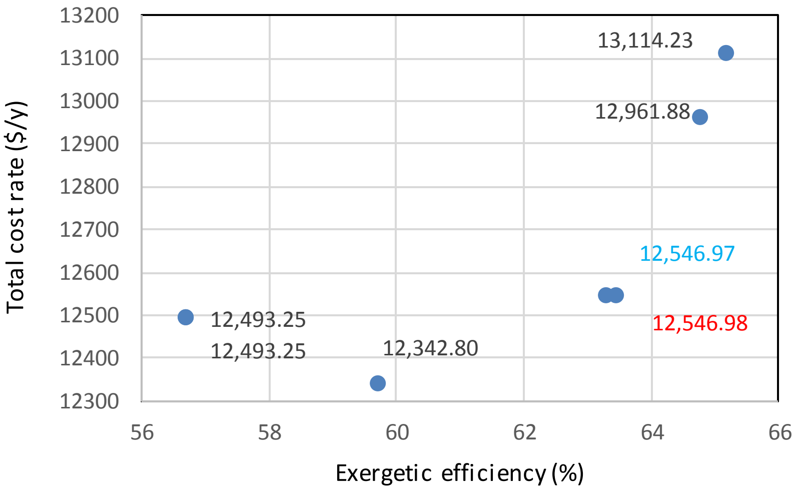


Figure 11. The seven optimised solutions obtained by Solver and MIDACO

5. Dual-objective optimisation by using Solver solutions and TOPSIS

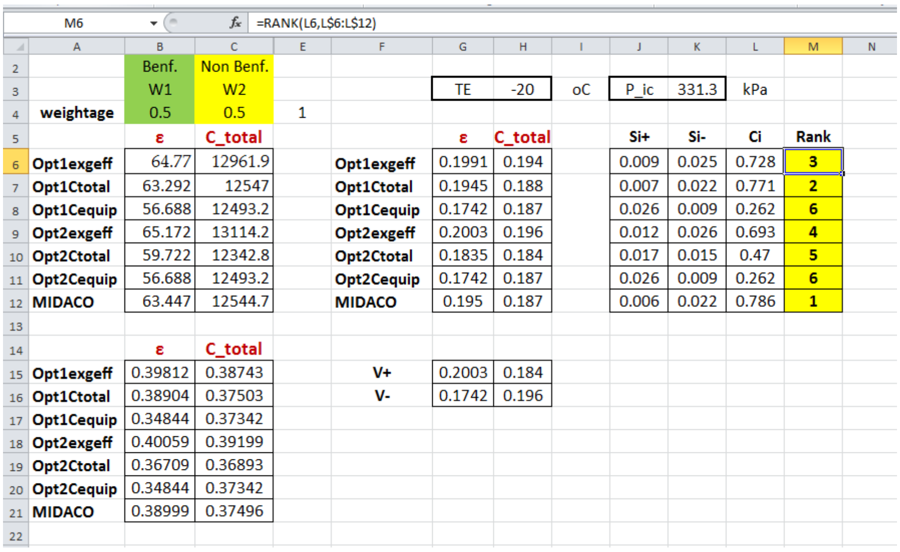
Being developed as a tool for SOO analyses, Solver does not produce a Pareto front but its two solution methods, the GRG Nonlinear method and the Evolutionary method, provide multiple optimised solutions for the various performance indicators. The TOPSIS technique [5, 14] allows the solution that is closest to satisfying the dual-objective requirement to be identified. Although, the number of optimised solutions that can be available for the selection process will be far less than a MOO solver can generate, the decision-making procedure that follows allows various weighting factors to be tested; something that may not be allowable by a proper MOO solver.

TOPSIS follows six well-defined steps explained in the appendix to rank the different solutions depending on the weighting factors provided to it (refer to Equation A.3). Five schemes of weighting factors are applied as shown on Table 6 to compare the seven optimised solutions shown on Figure 11.Schemes 1 and 5 are strongly unbalanced schemes that give a large weight factor of 0.75 to one performance indicator and only 0.25 to the other indicator. Schemes 2 and 4 are weakly unbalanced schemes (0.6/0.4 or 0.4/0.6),while Scheme 3 (0.5/0.5) is a balanced scheme that gives the same weight to both performance indicators.

Table 6. The five weighting schemes for applying TOPSIS method

Weighting scheme	Maximise exergetic efficiency (benefit)	Minimise total cost rate (non-benefit)
1	0.75	0.25
2	0.6	0.4
3	0.5	0.5
4	0.4	0.6
5	0.25	0.75

Figure 12 shows the Excel sheet developed for applying the TOPSIS method by modifying an example sheet available at [15]. Note that the 'benefit' objective for this analysis is maximising the exergetic efficiency (ε), while the 'non-benefit' objective is minimising the total cost rate (C_total). The sheet shown on Figure12 applies Scheme 3 that gives the same weight to the benefit and non-benefit objectives. As the formula bar shows, ranking of the seven solutions is done by using Excel's function "Rank" so that the different schemes can be applied by simply adjusting the values of the weight factors W1 and W2 stored in cells B4 and C4.



M6 =RANK(L6,L$6:L$12)

	A	B	C	E	F	G	H	I	J	K	L	M	N
2		Benf.	Non Benf.										
3		W1	W2			TE	-20	oC	P_ic	331.3	kPa		
4	weightage	0.5	0.5	1									
5		ε	C_total			ε	C_total		Si+	Si-	Ci	Rank	
6	Opt1exgeff	64.77	12961.9		Opt1exgeff	0.1991	0.194		0.009	0.025	0.728	3	
7	Opt1Ctotal	63.292	12547		Opt1Ctotal	0.1945	0.188		0.007	0.022	0.771	2	
8	Opt1Cequip	56.688	12493.2		Opt1Cequip	0.1742	0.187		0.026	0.009	0.262	6	
9	Opt2exgeff	65.172	13114.2		Opt2exgeff	0.2003	0.196		0.012	0.026	0.693	4	
10	Opt2Ctotal	59.722	12342.8		Opt2Ctotal	0.1835	0.184		0.017	0.015	0.47	5	
11	Opt2Cequip	56.688	12493.2		Opt2Cequip	0.1742	0.187		0.026	0.009	0.262	6	
12	MIDACO	63.447	12544.7		MIDACO	0.195	0.187		0.006	0.022	0.786	1	
13													
14		ε	C_total										
15	Opt1exgeff	0.39812	0.38743		V+	0.2003	0.184						
16	Opt1Ctotal	0.38904	0.37503		V-	0.1742	0.196						
17	Opt1Cequip	0.34844	0.37342										
18	Opt2exgeff	0.40059	0.39199										
19	Opt2Ctotal	0.36709	0.36893										
20	Opt2Cequip	0.34844	0.37342										
21	MIDACO	0.38999	0.37496										
22													

Figure 12. TOPSIS sheet using Scheme 3 to rank the seven solutions

According to Scheme 3, the solution closest to the ideal configuration (rank 1) is that obtained with the MIDACO solver. The two solutions obtained by the GRG Nonlinear method and the Evolutionary method of Solver for maximising the exergetic efficiency occupied the second and third ranks, respectively, while the two solutions that minimised the total cots rate occupied the fourth and fifth ranks. The two solutions that minimised the equipment cost occupied the two lowest ranks.

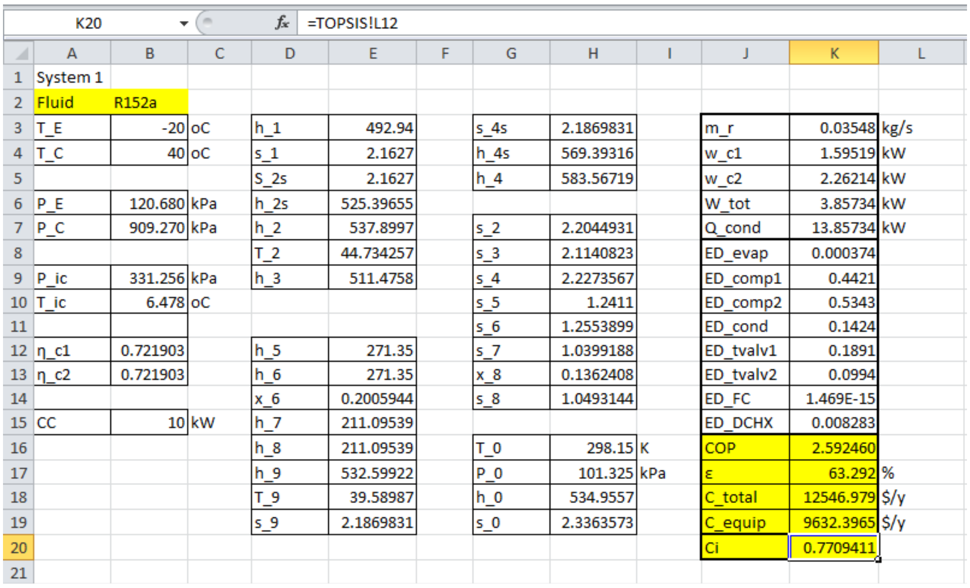
Table 7 summarises TOPSIS results with the five schemes applied to the seven solutions with the two Solver solutions obtained by the GRG Nonlinear method listed first followed by the two solutions obtained by the Evolutionary method and then the 2E solution obtained by MIDACO.The figures on Table 7 show that the five weighting schemes give different ranks to each solution, but MIDACO's 2E solution achieved the highest average rank. Also, note the high ranks reached by the two solutions obtained with GRG Nonlinear method that minimised the total cost rate and maximised the exergetic efficiency with the corresponding solutions obtained with the Evolutionary method occupying the fourth and fifth ranks. In agreement with Figure 11, the method gives the lowest two ranks to Solver's solutions that minimised the equipment cost.

Table 7. Results of applying the TOPSIS method to Solver and MIDACO optimised solutions with the five weighting schemes

	Schemes					
	1	*2*	*3*	*4*	*5*	Average
Solver1_ε	6	3	3	1	1	2.8
Solver1_Ctotal	2	2	2	3	4	2.6
Solver1_Cequip	4	6	6	6	6	5.6
Solver2_ε	7	4	4	4	2	4.2
Solver2_Ctotal	3	5	5	5	5	4.6
Solver2_Cequip	4	6	6	6	6	5.6
MIDACO	1	1	1	2	3	1.6

A more effective application of the Solver-TOPSIS technique is to use TOPSIS, not only in a passive manner to rank the different Solver solutions, but in an active manner to improve the base design variables so as to achieve a multi-objective target. For that purpose, the sheet that applies TOPSIS is integrated with the analytical model as the third sheet and Solver is used to maximise the value of C_i in Equation (A.8) by adjusting the base-design variables. Figure 13 shows the first sheet of the integrated model that copies the value of C_i from sheet 3as shown on the formula bar. The third sheet (not shown) copies the values of the exergetic efficiency and total cost rate from sheet 1 as the seventh option (replacing the MIDACO's2E solution). Note that the value of $C_i$for the base design (0.771) is lower than that of the 2E solution obtained by MIDACO (0.786). Solver can now be used to maximise the value of C_i by adjusting the values of T_{evap} and p_{FC}in Sheet 1 using the same set-up shown on Figure 6. Figure 14 shows the third sheet of the solution obtained in which the value of C_i

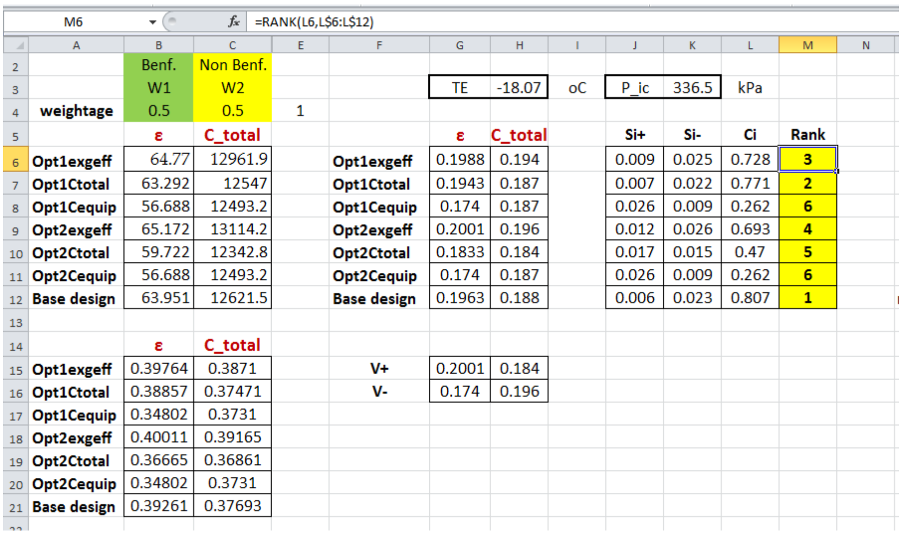
increased to 0.807 which is higher than that of MIDACO's 2E solution. According to this solution, T_{evap} increased to -18.07°C and p_{FC} increased to 336.5kPa.



K20 =TOPSIS!L12

	A	B	C	D	E	F	G	H	I	J	K	L
1	System 1											
2	Fluid	R152a										
3	T_E	-20	oC	h_1	492.94		s_4s	2.1869831		m_r	0.03548	kg/s
4	T_C	40	oC	s_1	2.1627		h_4s	569.39316		w_c1	1.59519	kW
5				S_2s	2.1627		h_4	583.56719		w_c2	2.26214	kW
6	P_E	120.680	kPa	h_2s	525.39655					W_tot	3.85734	kW
7	P_C	909.270	kPa	h_2	537.8997		s_2	2.2044931		Q_cond	13.85734	kW
8				T_2	44.734257		s_3	2.1140823		ED_evap	0.000374	
9	P_ic	331.256	kPa	h_3	511.4758		s_4	2.2273567		ED_comp1	0.4421	
10	T_ic	6.478	oC				s_5	1.2411		ED_comp2	0.5343	
11							s_6	1.2553899		ED_cond	0.1424	
12	η_c1	0.721903		h_5	271.35		s_7	1.0399188		ED_tvalv1	0.1891	
13	η_c2	0.721903		h_6	271.35		x_8	0.1362408		ED_tvalv2	0.0994	
14				x_6	0.2005944		s_8	1.0493144		ED_FC	1.469E-15	
15	CC	10	kW	h_7	211.09539					ED_DCHX	0.008283	
16				h_8	211.09539		T_0	298.15	K	COP	2.592460	
17				h_9	532.59922		P_0	101.325	kPa	ε	63.292	%
18				T_9	39.58987		h_0	534.9557		C_total	12546.979	$/y
19				s_9	2.1869831		s_0	2.3363573		C_equip	9632.3965	$/y
20										Ci	0.7709411	
21												

Figure 13. Sheet 1 of the TOPSIS-integrated analytical model (before maximising C_i)



M6 =RANK(L6,L$6:L$12)

	A	B	C	E	F	G	H	I	J	K	L	M	N
2		Benf.	Non Benf.										
3		W1	W2			TE	-18.07	oC	P_ic	336.5	kPa		
4	weightage	0.5	0.5	1									
5		ε	C_total			ε	C_total		Si+	Si-	Ci	Rank	
6	Opt1exgeff	64.77	12961.9		Opt1exgeff	0.1988	0.194		0.009	0.025	0.728	3	
7	Opt1Ctotal	63.292	12547		Opt1Ctotal	0.1943	0.187		0.007	0.022	0.771	2	
8	Opt1Cequip	56.688	12493.2		Opt1Cequip	0.174	0.187		0.026	0.009	0.262	6	
9	Opt2exgeff	65.172	13114.2		Opt2exgeff	0.2001	0.196		0.012	0.026	0.693	4	
10	Opt2Ctotal	59.722	12342.8		Opt2Ctotal	0.1833	0.184		0.017	0.015	0.47	5	
11	Opt2Cequip	56.688	12493.2		Opt2Cequip	0.174	0.187		0.026	0.009	0.262	6	
12	Base design	63.951	12621.5		Base design	0.1963	0.188		0.006	0.023	0.807	1	
13													
14		ε	C_total										
15	Opt1exgeff	0.39764	0.3871		V+	0.2001	0.184						
16	Opt1Ctotal	0.38857	0.37471		V-	0.174	0.196						
17	Opt1Cequip	0.34802	0.3731										
18	Opt2exgeff	0.40011	0.39165										
19	Opt2Ctotal	0.36665	0.36861										
20	Opt2Cequip	0.34802	0.3731										
21	Base design	0.39261	0.37693										

Figure 14. Sheet 3 of the TOPSIS-integrated analytical model (after maximising C_i)

6. Conclusions

Single-objective optimisation of VCR systems can easily be performed by using Excel's Solver with any practical number of changing variables, but a solver for multi-objective optimisation is currently unavailable for the general Excel users. Although the free version of the MIDACO solver can be used, its capability is limited to four changing variables. This paper presents two methods for utilising the TOPSIS decision-making method and Solver so as to achieve multi-objective optimisation of VCR systems. In the first method, Solver's two solution methods are used to generate multiple optimised solutions for the key performance indicators and TOPSIS is then used to rank the various solutions according to the designer's preference. In the second method, Solver is used to maximise the value of TOPSIS parameter that measures the closeness of the base design to the ideal dual-objective target by adjusting the relevant design variables.

The two methods are applied for a dual-objective optimisation of a two-stage compression VCR system that simultaneously maximises its exergetic efficiency and minimises its total cost rate by changing the values of the evaporator temperature and inter-stage pressure. The reliability of the first method is tested by including a true 2E solution for the optimisation problem obtained by using the MIDACO solver. The results of this method show that MIDACO's 2E solution obtained the first rank closely followed by Solver's solutions with the two solution methods for maximising the exergetic efficiency. As should be expected, the method shows that Solver's solutions for minimising the equipment cost rate are the farthest from satisfying the dual-objective requirement of the optimisation. The result of the second method shows that it is more effective than the first one since it could produce a solution that is closer to the dual-objective target than MIDACO's 2E solution.

7. Nomenclature

Latin Letters

A	Heat-transfer area (m^2)
$\dot{C}$	Annual cost rate (\$/year)
$\dot{E}^D$	Rate of exergy destruction (kW)
$\dot{m}$	Mass flow (kg/s)
$\dot{Q}$	Heat-transfer rate (kW)
$\dot{W}$	Compressor power (kW)
CC	Cooling capacity (kW)
c	Unit cost (e.g. electricity price) (\$/kWh)
COP	Coefficient of performance (-)
CRF	Capital recovery factor (-)
E	Electrical energy consumption (kWh)
h	Specific enthalpy (kJ/kg)
i	Interest rate (%)
N	Number of operation hours per year (hours)
n	Plant life time(years)
p	Pressure (kPa)
s	Specific entropy (kJ/kg)
T	Temperature (K or ^{o}C)
U	Heat transfer coefficient (kW/m^2.K)

x	Vapour quality (-)

Greek Letters

η	Compressor's isentropic efficiency (%)
μ_{CO_2e}	Regional (country) electricity conversion factor
ϕ	Maintenance factor

Indices

k	System component number

Abbreviations

MOO	Multi-objective optimisation
SOO	Single-objective optimisation
TEWI	Total equivalent warming impact
VBA	Visual Basic for Applications language
VCR	Vapour-compression refrigeration

Subscripts

0	Ambient
$Casc$	Cascade
CO_2	Avoided carbon dioxide emission
$Comp$	Compressor
$Cond$	Condenser
$DCHX$	Direct contact heat exchanger
$elec$	Electrical
env	Environmental
$evap$	Evaporator
FC	Flash chamber
op	Operation
r	Refrigerant
$Total$	Total work or exergy destruction
TV	Throttle valve

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Appendix: TOPSIS decision-making method

TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) is a decision making method that has been utilised for similar optimisation studies in the recent years [1, 4, 14]. According to this method, ideal and non-ideal points should be first obtained. The ideal point is the point at which optimum value of each single objective is achieved regardless of satisfaction of other objectives. While, the non-ideal point is defined as the point at which the worst value for each objective is obtained. The fundamental principle of this approach is that the chosen final optimal point must be in the shortest possible distance from the ideal point and the furthest distance from the non-ideal one [14]. Therefore, both the distance from the ideal point ($d+$), and non-ideal point ($d-$) are evaluated for all of achieved solution points and the solution with maximum value of the closeness coefficient [$d-/(d- + d+)$] is selected as the final optimal point. The method follows the following six steps [14]:

Step 1: Creation of an mxn evaluation matrix [A] with m alternatives and n criteria. For the present analyses there are 7 alternative solutions and 2 performance criteria. Therefore, the evaluation matrix is:

$$[\mathrm{A}] = \begin{bmatrix} a_{11} & a_{12} \\ - & - \\ a_{71} & a_{72} \end{bmatrix} \tag{A.1}$$

Where, the a_{ij} are the values of ε and C_{total} determined by the two methods of Solver and the MIDACO solver as shown on Table 5.

Step 2: Normalisation of the evaluation matrix:

Since the dimensions of the two objectives are different (i.e., the total cost rate is expressed in terms of US dollar per unit of time, while the exergetic efficiency has no dimension), the values of the objective functions are then non-dimensionalised by using the following equation:

$$\bar{a}_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^{2}} \tag{A.2}$$

Step 3: Determination of the weighted matrix by multiplying the mass factor with the normalized matrix as follows:

$$V_{ij} = w_i \times a_{ij} \tag{A.3}$$

Step 4: Determination of positive and negative ideal solutions.

$$A_j^{+} = \left\{ Max\, V_{ij} \middle| j \in K \right\}, \left\{ Min\, V_{ij} \middle| j \in K^{'} \right\} \tag{A.4}$$

$$A_j^{-} = \left\{ Min\, V_{ij} \middle| j \in K \right\}, \left\{ Max\, V_{ij} \middle| j \in K^{'} \right\} \tag{A.5}$$

where K is the benefit parameters and K' is the non-benefit parameters or cost parameters

Step 5: Calculation of distances from positive and negative ideal solutions from the following equations

$$S_i^{+} = \left[\sum_{j=1}^{n} \left(V_{ij} - V_j^{+} \right)^2 \right]^{0.5} \tag{A.6}$$

$$S_i^{-} = \left[\sum_{j=1}^{n} \left(V_{ij} - V_j^{-} \right)^2 \right]^{0.5} \tag{A.7}$$

Step 6: Determination of relative closeness from the ideal solution can be expressed as:

$$C_i = \frac{S_j^{-}}{S_j^{+} + S_j^{-}} \tag{A.8}$$

Rank C_i in descending order where highest value gives the solution that is closer to the ideal.