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A Solver-TOPSIS technique for multi-objective optimisation of innovative multi-stage VCR systems by using Microsoft Excel

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الملخص

تصف هذه الورقة طريقة لتحسين الأداء متعدد الاهداف لأنظمة التبريد بضغط البخار متعدد المراحل باستخدام برنامج حل الهدف الواحد لبرنامج ميكروسوفت أكسل بمساعدة طريقة اتخاذ القرار توبسس. توضح الورقة كيفية تطبيق الطريقة المقترحة على نظام مبتكر ذى مرحلتين يضيف الى النظام التقليدى مبردًا داخليًا لاستعادة الحرارة بعد ضاغط المرحلة الأولى لإنتاج ماء ساخن. يقوم النموذج الحاسوبى للنظام الذى تم تطويره باستخدام اكسل بحساب معامل الأداء, الكفاءة الأكسيرجية, معدل التكلفة الإجمالي, و إجمالي تأثير الاحترار المكافئ بأعتبار أربعة خصائص للتصميم كمتغيرات و هى درجتا حرارة المبخر و المكثف, كفاءة الضاغط الحرارية, و معدل سريان الماء فى المبرد . بمقارنة مؤشرات الأداء الأربعة للنظام التى تم تحسينها بالطريقة المقترحة مع تلك المحسنة باستخدام برنامج "ميداكو" متعدد الأهداف يتضح أن الطريقة المقترحة تنتج قيمًا أعلى لـمعامل الأداء والكفاءة الأكسيرجية مع خفض فى إجمالي تأثير الاحترار المكافئ على حساب زيادة طفيفة في معدل التكلفة الإجمالي للنظام.

ABSTRACT

This paper describes a method for conducting multi-objective optimisation of multi-stage vapour-compression refrigeration (VCR) systems by utilising the single-objective solver of Microsoft Excel and TOPSIS decision-making method. The paper illustrates the Solver-TOPSIS method by analysing an innovative two-stage VCR system that adds a heat-recovery intercooler to the conventional system after the first-stage compressor. The Excel-aided model developed for the system calculates its coefficient of performance (COP), exergetic efficiency, total cost rate, and total equivalent warming impact (TEWI) with four design parameters as variables which are the evaporator and condenser temperatures, the isentropic efficiency of the compressor, and the cooling water flow rate. Comparison of the four performance indicators of the system as optimised by the proposed method and by the MIDACO solver shows that the method yields higher values of the system's COP and exergetic efficiency and lower TEWI at the expense of increasing the total cost rate. The total rate of exergy destruction in the system which is optimised by method is also lower than the one optimised by MIDACO.

KEYWORDS: Multi-stage VCR systems, multi-objective optimisation, TOPSIS, Excel

INTRODUCTION

The significant share of vapour-compression refrigeration (VCR) systems in the energy consumption of residential, commercial, and industrial sectors emphasises the importance of improving the efficiency of these systems [1,2]. In this respect, multi-stage compression allows various innovative methods to be used for reducing the systems' energy consumption. Since the improved systems cost more than the simple ones, their economic feasibility requires careful trade-offs between their electrical energy consumption and capital costs. A third factor has now become equally important due to the increasing concern about the effects of global warming and ozone-layer depletion, which is the need to replace the harmful synthetic refrigerants with more environment-friendly fluids [3]. The quest to design innovative VCR systems using environment-friendly refrigerants and to develop suitable methods for their energetic, economic, and environmental optimisation has inspired many researchers to be involved.

Roy and Mandal [4] conducted a thermo-economic analysis of a simple VCR system using three refrigerants with low GWP namely, R152a, R600a and R1234ze. Developing their model with Engineering Equation Solver (EES), they evaluated the effect of evaporator and condenser temperatures on the system's coefficient of performance (COP), exergetic efficiency, and annual plant cost rate. They carried out their multi-objective optimisation (MOO) analysis by using MATLAB toolbox and used the TOPSIS method [5] to select the best optimised solution. Their results showed that R152a gave the best performance among the three investigated refrigerants. Aminyavari et al. [6] analysed a CO_2/NH_3 cascade refrigeration system by considering its exergetic, economic and environmental performance. They also developed their model in MATLAB but employed a specific MATLAB function for calling the REFPROP data base to obtain the refrigerants' thermodynamic data. Their MOO analyses used a genetic algorithm method to achieve the optimal design parameters of the system and used TOPSIS to select the final optimum point from the set of optimal solutions achieved. Their results showed that, for the considered plant with cooling capacity of 50 kW, the optimum design results in exergetic efficiency of 45.89% and a total cost rate of 0.01099 US$/s

Singh et al. [7] analysed an ammonia-based multi-stage compression VCR system incorporated with a flash intercooler which also works as a sub-cooler. They carried out a thermo-economic optimisation of the system in order to maximise its exergetic efficiency and minimise its total capital cost rate. The evaporator temperature, condenser temperature, subcooling parameter, and de-superheating parameter were considered as design variables for their MOO analysis. They also employed the multi-objective genetic algorithm tool provided with MATLAB to carry out the optimisation analysis and used EES to determine the thermodynamic properties of the refrigerants. TOPSIS was used to select unique solutions for five different weighting factors of exergetic efficiency and total cost. Their results revealed that the exergetic efficiency and total capital cost of the system at the thermo-economic optimal operating conditions were 41.76% and 223,717.6 USD, respectively.

The above example studies show that most researchers used commercial software for their MOO analyses like MATLAB, EES, and REFPROP. However, the use of general-purpose applications can encourage independent researchers and engineering students to contribute to the development of innovative VCR systems using refrigerants that are less harmful to the environment. Microsoft Excel, which is an easy-to-learn general-purpose spreadsheet application, has powerful analytical tools that include a versatile solver for single-objective optimisation (SOO) analyses. Regarding MOO analyses, a free version of the MIDACO solver [8] is available for Excel users, but it allows only four design variables to be considered in the analysis; which is not adequate for analysing multi-stage compression and cascade VCR systems with multiple design parameters. Since the development of MOO solvers is too complex and takes a long time even for top software professionals [9, 10], the present paper describes a method for using the same SOO Solver provided by Excel with the TOPSIS method for conducting MOO analyses with practically any number of design variables. The paper applies the method for energetic, exergetic, economic, and environmental (4E) optimisation of an innovative two-stage compression VCR system.

DESCRIPTION OF THE TWO-STAGE VCR SYSTEM

The two-stage compression VCR system shown on Figure (1) adds a water intercooler to the conventional system for cooling the superheated refrigerant leaving the first-stage compressor. Apart from reducing the compression work in the second-stage compressor, the hot water exiting the intercooler can be utilised for various needs. This system is a modified version of that described by Anjum et al. [11] in which the cooled refrigerant goes to the flash chamber which it exits as dry saturated vapour. The system shown on Figure (1) adds a direct-contact heat-exchanger (DCHX) for mixing the refrigerant leaving the intercooler with the dry saturated vapour leaving the flash chamber so that the refrigerant enters the second-stage compressor as slightly-superheated vapour to prevent liquid carry-over. Figure (2) shows the *T*-*s* diagram of the modified system.

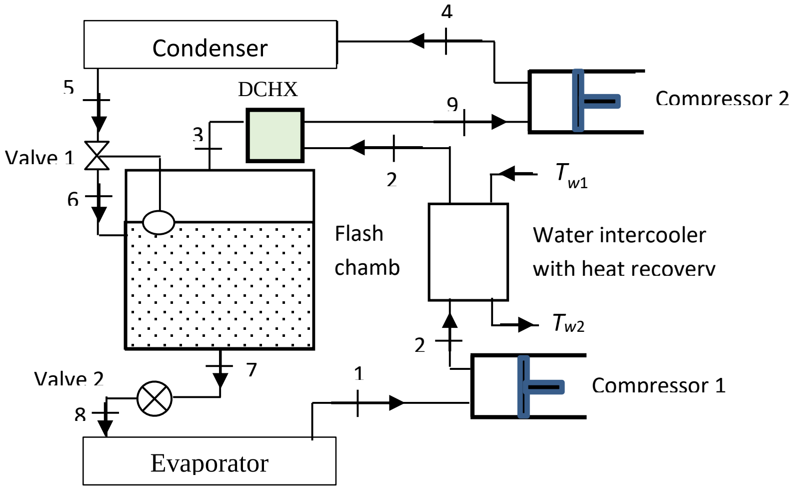


Figure 1: Schematic of the two-stage compression system with heat-recovery.

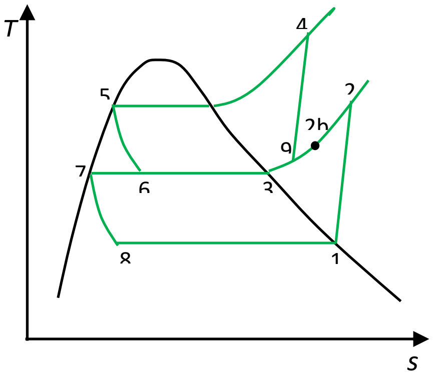


Figure 2: *T*-*s* diagram of the two-stage compression system with heat-recovery.

Anjum et al. [11] conducted a thermodynamic analysis of the original two-stage system by using ammonia as the refrigerant. The present multi-objective assumes the same input parameters shown on Table 1, but using another environment-friendly refrigerant, which is R152a.

Table 1: Assumed values of the input parameters for analysing the VCR system [11]

Parameter	Value
Cooling capacity of the system, *CC*	10 kW
Isentropic efficiency of compressor, η_c	80%
Overall heat transfer coefficient for evaporator	0.03 kW/m^2.K
Overall heat transfer coefficient for condenser	0.04 kW/m^2.K
Overall heat transfer coefficient for intercooler	0.1 kW/m^2.K
Ambient temperature, T_0	25°C
Temperature change for air in evaporator and condenser	± 5°C
Temperature of the inlet air to evaporator	0°C
Inlet temperature of cooling water	17°C
Maintenance factor, ϕ	1.06
Interest rate, *i*	14%
Plant life time, *n*	15 Years
Annual operation hours, *N*	4266 hour
Electrical power cost, c_{elec}	0.09 $/kWh
Emission factor, μ_{CO_2e}	0.968 kg/kWh [12]
Cost of CO$_2$ avoided, c_{CO_2}	0.09 $/kg of emitted CO$_2$

THE THERMODYNAMIC MODEL

Table 2 shows the mass and energy balance equations for the different system components. The mass flow rate of the refrigerant in the evaporator is given by:

$$\dot{m}_r = CC/(h_1 - h_8) \tag{1}$$

Where *CC* is the cooling capacity of the system. The exit temperature of the cooling water T_{w2} is determined from the specified value of the heat-exchanger effectiveness, ε, as follows:

$$T_{w2} = T_{w1} + \varepsilon(T_2 - T_{w1}) \tag{2}$$

The enthalpy of the cooled refrigerant is then determined from energy balance across the intercooler as shown on Table 2. The total compression work is given by:

$$\dot{W}_{Total} = \dot{W}_{comp1} + \dot{W}_{comp2} = \dot{m}_r(h_2 - h_1) + \dot{m}_r(h_4 - h_9)/(1 - x_6) \tag{3}$$

The COP and total exergetic efficiency, ε, of the system are given by:

$$COP = \frac{CC}{\dot{W}_{Total}} \tag{4}$$

$$\varepsilon = \frac{\dot{W}_{Total} - E_D^{Total}}{\dot{W}_{Total}} \tag{5}$$

Where $\dot{E}^D_{Total}$ is the total exergy destruction in the system given by:

Table 2: Mass and energy balance equations and exergy destruction rates in the system components

	Mass balance	Energy balance	Exergy destruction rate
Evaporator	$\dot{m}_1 = \dot{m}_8 = \dot{m}_r$	$\dot{m}_1 h_1 = \dot{m}_8 h_8 + CC$	$T_0\left[\dot{m}_1\left(s_1 - s_8\right) - \frac{CC}{T_E}\right]$
Compressor 1	$\dot{m}_2 = \dot{m}_1$	$\dot{W}_{comp1} = \dot{m}_r\left(h_2 - h_1\right)$	$T_0\dot{m}_1\left(s_2 - s_1\right)$
Compressor 2	$\dot{m}_4 = \dot{m}_9 = \dot{m}_r / \left(1 - x_6\right)$	$\dot{W}_{comp2} = \dot{m}_3\left(h_4 - h_9\right)$	$T_0\dot{m}_3\left(s_4 - s_9\right)$
Condenser	$\dot{m}_5 = \dot{m}_4 = \dot{m}_r / \left(1 - x_6\right)$	$\dot{m}_5 h_5 = \dot{m}_4 h_4 - \dot{Q}_{cond}$	$T_0\left[\dot{m}_4\left(s_5 - s_4\right) + \frac{\dot{Q}_{cond}}{T_C}\right]$
Throttle valve 1	$\dot{m}_6 = \dot{m}_5 = \dot{m}_r / \left(1 - x_6\right)$	$h_6 = h_5$	$\dot{m}_5 T_0\left(s_6 - s_5\right)$
Throttle valve 2	$\dot{m}_8 = \dot{m}_7 = \dot{m}_r$	$h_8 = h_7$	$\dot{m}_7 T_0\left(s_8 - s_7\right)$
Flash chamber	$\dot{m}_3 = x_6\dot{m}_5 = x_6\dot{m}_r$ $\dot{m}_7 = \left(1 - x_6\right)\dot{m}_r$	$h_3 = h_{g\,@\,P_{fc}}$ $h_7 = h_{f\,@\,P_{fc}}$ $\dot{m}_6 h_6 = \dot{m}_3 h_8 + \dot{m}_7 h_7$	$\frac{T_0\dot{m}_1}{1 - x_6} \times \left[s_6 - x_6 s_3 - \left(1 - x_6\right)s_7\right]$
Intercooler	$\dot{m}_3 = \dot{m}_r$ $\dot{m}_9 = \dot{m}_2$	$h_{2b} = h_2 - \dot{m}_w c_{pw}\left(T_{w2} - T_{w1}\right) / \dot{m}_2$	$T_0\left[\dot{m}_w s_{w2} + \dot{m}_r s_{2b} - \dot{m}_w s_{w1} - \dot{m}_r s_2\right]$
Direct heat exchanger	$\dot{m}_9 = \dot{m}_2 + \dot{m}_3 = \dot{m}_r$	$\dot{m}_2 h_{2b} + \dot{m}_3 h_3 = \dot{m}_9 h_9$	$T_0\left[\dot{m}_9 s_9 - \dot{m}_r s_{2b} - \dot{m}_3 s_3\right]$

$$\dot{E}^{D}_{Total} = \dot{E}^{D}_{Evap} + \dot{E}^{D}_{Comp1} + \dot{E}^{D}_{TV1} + \dot{E}^{D}_{IC} + \dot{E}^{D}_{FC} + \dot{E}^{D}_{DCHX} + \dot{E}^{D}_{Comp2} + \dot{E}^{D}_{TV2} + \dot{E}^{D}_{Cond} \tag{6}$$

Table 2 also shows the exergy destruction rates in the different system components.

THE ECONOMIC MODEL

The total annualised cost rate of the system is given by [11]:

$$\dot{C}_{total} = \sum_{k=1}^{9} \dot{C}_k + \dot{C}_{op} + \dot{C}_{env} \tag{7}$$

where, $\dot{C}_k$ is the capital and maintenance cost rate of individual components, $\dot{C}_{opt}$ is the maintenance cost rate of the system, and $\dot{C}_{env}$ is the CO_2 penalty cost rate of the system. The total capital and maintenance cost rate of the system is calculated by adding up the capital and maintenance cost rate of the individual components which is given by:

$$\dot{C}_k = C_k.\phi.CRF \tag{8}$$

where, ϕ is the maintenance factor and *CRF* is the capital recovery factor obtained from:

$$CRF = \frac{i\left(1 + i\right)^n}{\left(1 + i\right)^n - 1} \tag{9}$$

where *i* is the interest rate and *n* is the system's expected lifetime. The values of *i* and *n* used in the present analysis are given in Table 1. The capital costs of individual components are estimated using the relations shown on Table 3 [7,8].

Table 3: Capital cost functions of the different components [7,8]

Component	Capital cost function
Evaporator	$C_{eva} = 1397 \times A_{eva}^{0.89}$
Compressor	$C_{comp} = 10167.5 \times \dot{W}^{0.46}$
Condenser	$C_{con} = 1397 \times A_{con}^{0.89}$
Throttle valve	$C_{TV} = 114.5 \times \dot{m}$
Flash chamber	$C_{FC} = 280.3 \dot{m}_{ref}^{0.67}$
Intercooler	$C_{intr} = 2382.9 \times \left(A_{intr}\right)^{0.68}$

Since the costs of the two throttle valves, flash chamber, and direct contact heat exchanger are minor compared to those of the two compressors, the evaporator, and the condenser, they have been ignored in some analyses. In the present analysis, the cost of the DCHX is taken as equal to that of the flash chamber. The heat-transfer areas of the evaporator, condenser, and intercooler in the relations shown on Table 3 are determined from the respective temperature differences by using the log-mean temperature difference method. The operational cost rate of the system is the cost of electricity given by:

$$\dot{C}_{op} = \dot{W}.N.c_{elec} \tag{10}$$

where *N* is the annual operational hours and c_{elec} is the cost of electricity in \$/kWh. Following Wang et al. [13], the penalty for the system's CO_2 emission is calculated from:

$$\dot{C}_{env} = m_{CO_2e}.c_{CO_2} \tag{11}$$

where, c_{CO_2} is the penalty cost of the avoided CO_2 emission and m_{CO_2e} is the amount of annual CO_2 emission from the system that can be estimated from:

$$m_{CO_2e} = \mu_{CO_2e}.E_{annual} \tag{12}$$

where μ_{CO_2e} is the emission factor and E_{annual} is the annual amount of energy consumed by the system. The values of *N*, μ_{CO_2e} c_{elec}, and c_{CO_2} used in the present analysis are given in Table 1.

THE TOTAL EQUIVALENT WARMING IMPACT (TEWI)

TEWI is a non-monetary measure that evaluates the direct and indirect global warming effects of the refrigeration systems. The direct effect results from the refrigerants being directly released or leaked into the atmosphere and the indirect effect is caused by the CO_2 emissions in thermal power plants that use fossil fuels to produce the electrical energy. The refrigerant TEWI is calculated by using the following correlation [14].

$$TEWI = GWP_{ref}\left[m_{ref} \times L_{annual} \times n + m_{ref} \times (1-\alpha)\right] + \underline{\left(E_{annual} \times \beta \times n\right)} \tag{13}$$

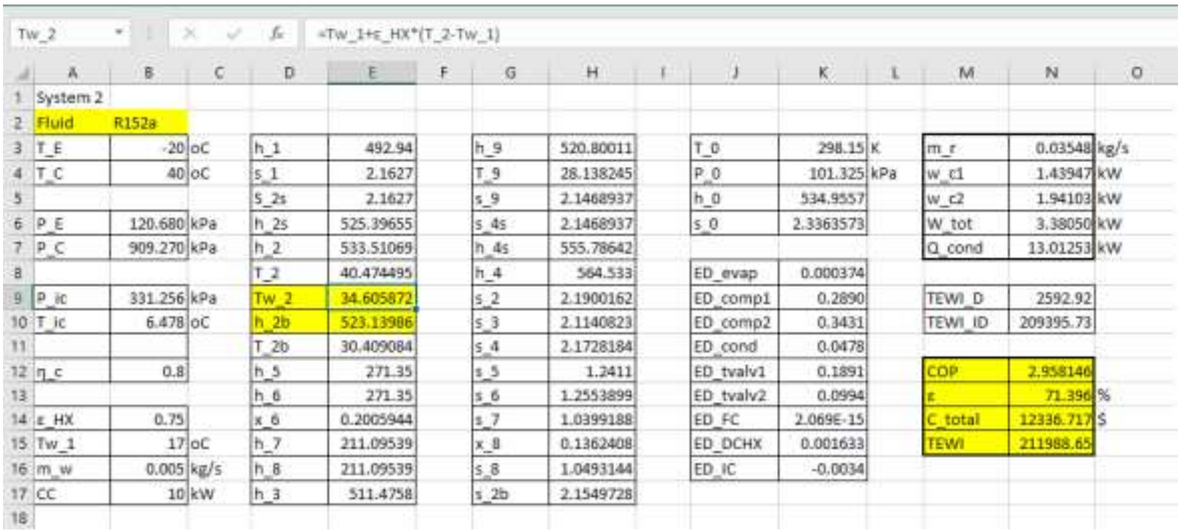
where GWP_{Ref} is the GWP of the refrigerant, m_{Ref} is the total refrigerant charge, L_{annual} is the refrigerant leakage rate, α is the recycling factor, E_{annual} is the energy consumed per year, and β is the electricity regional conversion factor. Table 4 show how m_{ref} and L_{annual} are calculated and gives the values of α, β, and GWP_{ref} for R152a [14]. The underlined on the right side of Equation (13) is the indirect part of the TEWI.

Table 4: TEWI analysis assumptions [14]

Parameter	m_{ref} [kg]	L_{annual} [%]	α	β [kg.CO$_2$/kWh]	GWP_{ref}
Assumed value	$\dot{m}_{ref}\left(240\,s\right)$	12.5	0.7	0.65	140

THE EXCEL-AIDED MODEL AND SINGLE-OBJECTIVE OPTIMISATION BY USING SOLVER

Figure (3) shows the first sheet of the Excel-aided model for the two-stage system. The first column on the left side of the sheet stores the assumed data for the thermodynamic model such as the evaporator and condenser temperatures, the mass flow rate and inlet temperature of the cooling water, the intercoolers effectiveness, etc. The following two columns determine the temperature, enthalpy, and entropy values of the refrigerant and water at the various states by using the relevant Thermax property functions [15]. The formula bar reveals the formula in cell E9 that calculates T_{w2} according to Equation (2). The fourth column from the left calculates the rates of exergy destruction in all the system components. The last column on the right determines the mass flow rate of the refrigerant, the compression work and four performance indicators; COP, ε, C_{Total}, and TEWI.

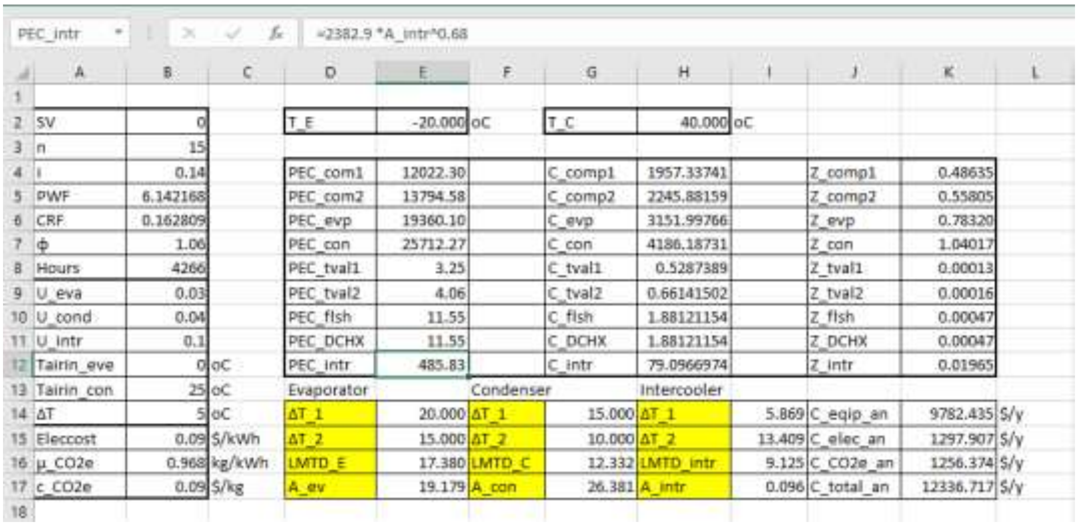


Tw_2 =Tw_1+ε_HX*(T_2-Tw_1)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-20	oC	h_1	492.94		h_9	520.80011		T_0	298.15	K	m_r	0.03548	kg/s
4	T_C	40	oC	s_1	2.1627		T_9	28.138245		P_0	101.325	kPa	w_c1	1.43947	kW
5				S_2s	2.1627		s_9	2.1468937		h_0	534.9557		w_c2	1.94103	kW
6	P_E	120.680	kPa	h_2s	525.39655		s_4s	2.1468937		s_0	2.3363573		W_tot	3.38050	kW
7	P_C	909.270	kPa	h_2	533.51069		h_4s	555.78642					Q_cond	13.01253	kW
8				T_2	40.474495		h_4	564.533		ED_evap	0.000374				
9	P_ic	331.256	kPa	Tw_2	34.605872		s_2	2.1900162		ED_comp1	0.2890		TEWI_D	2592.92	
10	T_ic	6.478	oC	h_2b	523.13986		s_3	2.1140823		ED_comp2	0.3431		TEWI_ID	209395.73	
11				T_2b	30.409084		s_4	2.1728184		ED_cond	0.0478				
12	η_c	0.8		h_5	271.35		s_5	1.2411		ED_tvalv1	0.1891		COP	2.958146	
13				h_6	271.35		s_6	1.2553899		ED_tvalv2	0.0994		ε	71.396	%
14	ε_HX	0.75		x_6	0.2005944		s_7	1.0399188		ED_FC	2.069E-15		C_total	12336.717	$
15	Tw_1	17	oC	h_7	211.09539		x_8	0.1362408		ED_DCHX	0.001633		TEWI	211988.65	
16	m_w	0.005	kg/s	h_8	211.09539		s_8	1.0493144		ED_IC	-0.0034				
17	CC	10	kW	h_3	511.4758		s_2b	2.1549728							
18															

Figure 3: Sheet 1 of the model for the modified two-stage compression system

Figure (4) shows the second sheet of the model that calculates the cost rates of the nine system components using the relevant cost relations shown on Table 3. The formula bar shows the formula in cell E12 that calculates the capital cost of the intercooler. Figure (4) show that the total cost of the system is dominated by the costs of the evaporator, the condenser, and the two compressors.

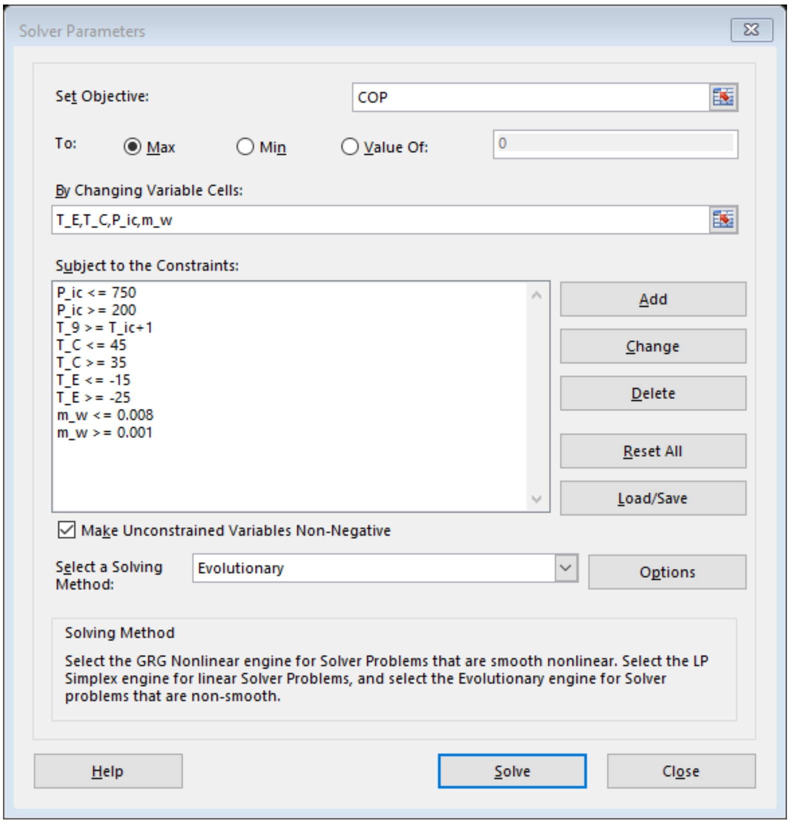


PEC_intr =2382.9 *A_intr^0.68

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	SV	0		T_E	-20.000	oC	T_C	40.000	oC			
3	n	15										
4	i	0.14		PEC_com1	12022.30		C_comp1	1957.33741		Z_comp1	0.48635	
5	PWF	6.142168		PEC_com2	13794.58		C_comp2	2245.88159		Z_comp2	0.55805	
6	CRF	0.162809		PEC_evp	19360.10		C_evp	3151.99766		Z_evp	0.78320	
7	ϕ	1.06		PEC_con	25712.27		C_con	4186.18731		Z_con	1.04017	
8	Hours	4266		PEC_tval1	3.25		C_tval1	0.5287389		Z_tval1	0.00013	
9	U_eva	0.03		PEC_tval2	4.06		C_tval2	0.66141502		Z_tval2	0.00016	
10	U_cond	0.04		PEC_flsh	11.55		C_flsh	1.88121154		Z_flsh	0.00047	
11	U_intr	0.1		PEC_DCHX	11.55		C_DCHX	1.88121154		Z_DCHX	0.00047	
12	Tairin_eve	0	oC	PEC_intr	485.83		C_intr	79.0966974		Z_intr	0.01965	
13	Tairin_con	25	oC	Evaporator		Condenser		Intercooler				
14	ΔT	5	oC	ΔT_1	20.000	ΔT_1	15.000	ΔT_1	5.869	C_eqip_an	9782.435	$/y
15	Eleccost	0.09	$/kWh	ΔT_2	15.000	ΔT_2	10.000	ΔT_2	13.409	C_elec_an	1297.907	$/y
16	μ_CO2e	0.968	kg/kWh	LMTD_E	17.380	LMTD_C	12.332	LMTD_intr	9.125	C_CO2e_an	1256.374	$/y
17	c_CO2e	0.09	$/kg	A_ev	19.179	A_con	26.381	A_intr	0.096	C_total_an	12336.717	$/y
18												

Figure 4: Sheet 2 of the model for the modified two-stage compression system

Solver [16], the single-objective solver that comes with Excel, can be used to optimise each of the four key performance indicators shown on Figure 3 by using either the GRG Nonlinear method, which is a deterministic gradient-based method, or the Evolutionary method. Figure (5) shows the set-up for maximising the COP of the two-stage system by using four changing variables which are the evaporator temperatures, the condenser temperature, the inter-stage pressure, and the flow rate of the cooling water.



Solver Parameters

Set Objective: COP

To: ● Max ○ Min ○ Value Of: 0

By Changing Variable Cells:

T_E,T_C,P_ic,m_w

Subject to the Constraints:

P_ic <= 750
P_ic >= 200
T_9 >= T_ic+1
T_C <= 45
T_C >= 35
T_E <= -15
T_E >= -25
m_w <= 0.008
m_w >= 0.001

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: Evolutionary

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Figure 5: Solver set-up for maximising the COP of the modified system

The four design variables are allowed to vary within specified ranges shown on the Figure 5. The other three performance indicators were similarly optimised by selecting the respective objective cell and minimising or maximising it with Solver. Table 5 shows the optimised key-performance indicators for the four Solver solutions obtained with the Evolutionary method.

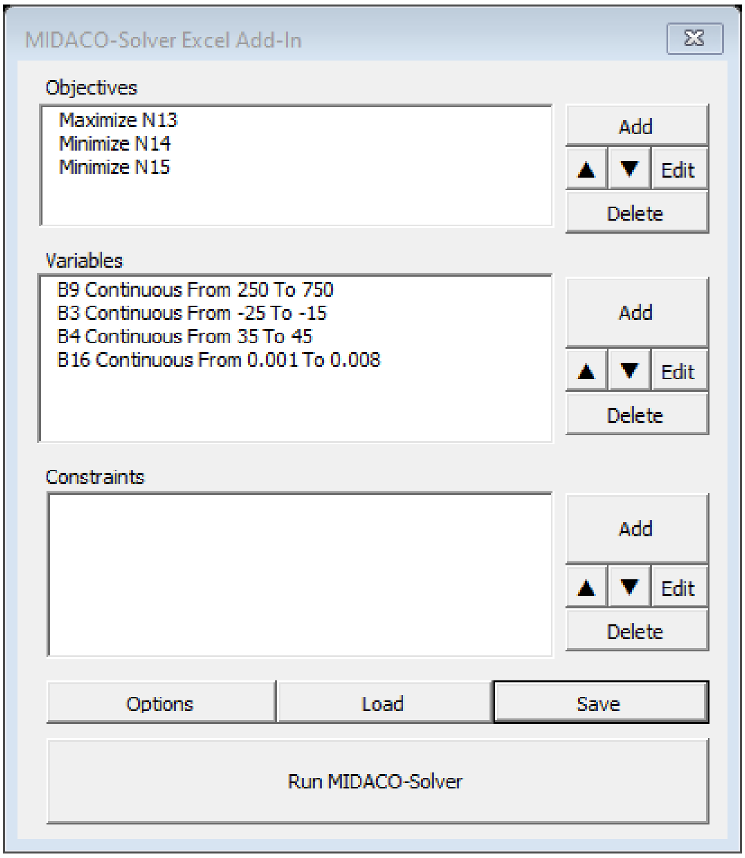
Table 5: Single-objective optimised solutions obtained by Solver for the system

Optimisation objective	COP	ε [%]	C_{total} [$/y]	TEWI [kg CO_2/y]
Maximise COP	3.474	72.189	14456.96	181066.2
Maximise ε	3.445	72.655	14200.60	182500.8
Minimise total cost rate	2.296	67.810	11166.29	272165.0
Minimise TEWI	3.574	72.460	14214.03	176028.0

MULTI-OBJECTIVE OPTIMISATION OF THE SYSTEM BY USING MIDACO

Although four performance indicators were considered in the previous single-objective optimisation analyses of the system, the dominance of the indirect global-warming effect on the TEWI enables the multi-objective optimisation analysis to be conducted with only three performance indicators which are the exergetic efficiency, the total cost rate, and either the COP or TEWI. The best trade-off between the three conflicting objectives can be found by using a MOO solver such as the MIDACO solver [8]. The limited version of MIDACO allows up-to four design variables which is adequate for the present analysis. Figure (6) shows the set-up for MIDACO to optimise the system by maximising its exrgetic efficiency, minimising its total cost rate, and minimising its TEWI. The changing variables are the same as those used for Solver as shown on Figure (5). As a multi-objective solver, MIDACO produces a Pareto front containing a set of un-dominated optimum solutions from which the best optimum solution can then selected. Figure (7) shows the Pareto front obtained by MIDACO and Figure (8) shows the selected 3E optimal solution.

Figure (9) compares the 3E solution obtained by MIDACO with the four single-objective solutions obtained by Solver as shown on Table 5. Comparisons with Solver’s three solution that minimised the system’s TEWI, maximised the exergetic efficiency, or maximised COP shows that the 3E solution reduced both the COP and exergetic efficiency of the system. The trade-off for degrading these two performance indicators is that the 3E solution reduced the system’s total cost rate and increased the TEWI. However, compared to the solution for minimising the total cost rate, the 3E solution considerably increased both the COP and exergetic efficiency and reduced the TEWI but increased the total cost rate. It can be judged from the scale of deviations shown on the figure that the closest single-objective solution to the 3E solution is the one that maximised the system’s exergetic efficiency followed by the one that maximised its COP.



MIDACO-Solver Excel Add-In

Objectives

Maximize N13
Minimize N14
Minimize N15

Variables

B9 Continuous From 250 To 750
B3 Continuous From -25 To -15
B4 Continuous From 35 To 45
B16 Continuous From 0.001 To 0.008

Constraints

Figure 6: MIDACO set-up for the 3E optimisation of the modified system

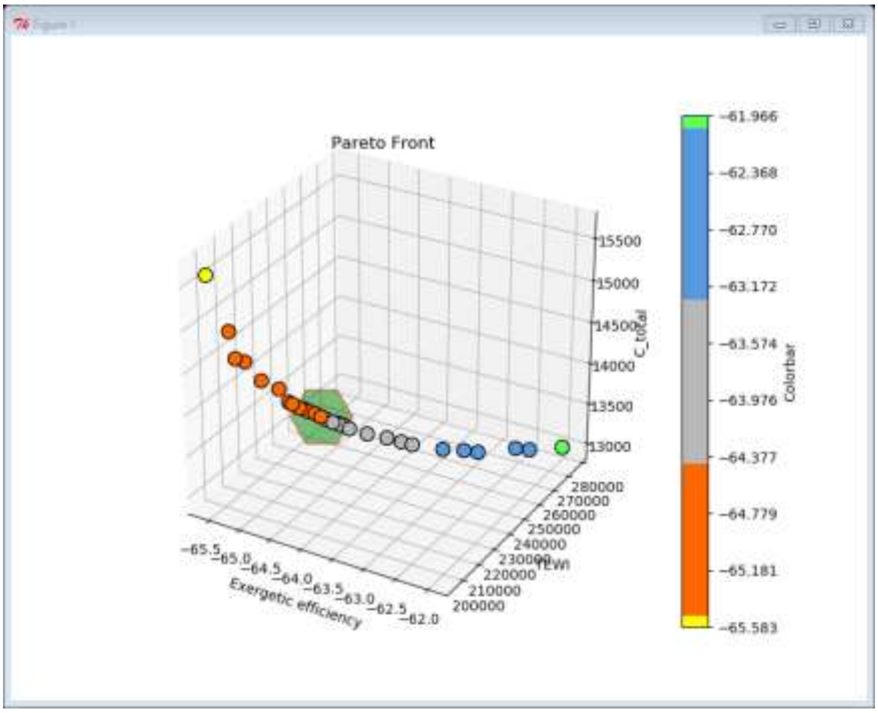
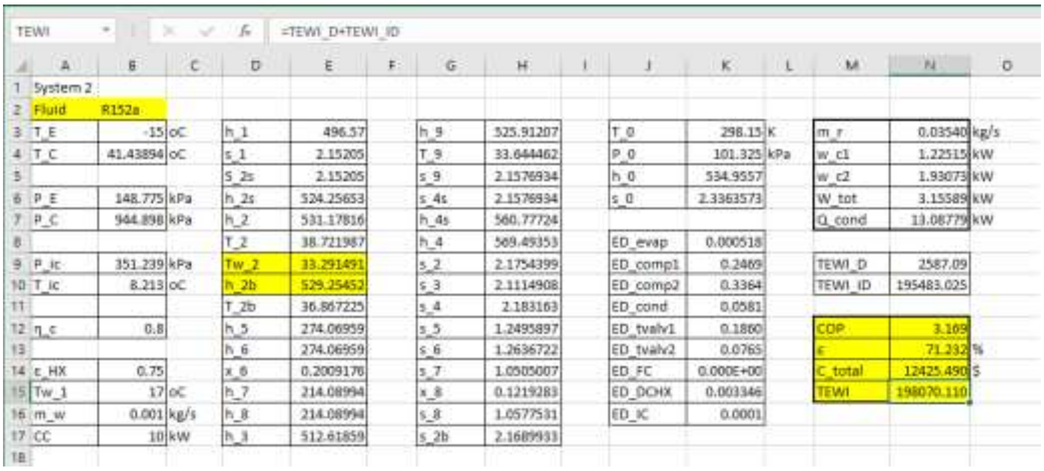


Figure 7: Pareto front of the 3E optimised solutions for the modified system



TEWI =TEWI_D+TEWI_ID

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-15	oC	h_1	496.57		h_9	525.91207		T_0	298.15	K	m_r	0.03540	kg/s
4	T_C	41.43894	oC	s_1	2.15205		T_9	33.644462		P_0	101.325	kPa	w_c1	1.22515	kW
5				S_2s	2.15205		s_9	2.1576934		h_0	534.9557		w_c2	1.93073	kW
6	P_E	148.775	kPa	h_2s	524.25653		s_4s	2.1576934		s_0	2.3363573		W_tot	3.15589	kW
7	P_C	944.898	kPa	h_2	531.17816		h_4s	560.77724					Q_cond	13.08779	kW
8				T_2	38.721987		h_4	569.49353		ED_evap	0.000518				
9	P_ic	351.239	kPa	Tw_2	33.291491		s_2	2.1754399		ED_comp1	0.2469		TEWI_D	2587.09	
10	T_ic	8.213	oC	h_2b	529.25452		s_3	2.1114908		ED_comp2	0.3364		TEWI_ID	195483.025	
11				T_2b	36.867225		s_4	2.183163		ED_cond	0.0581				
12	η_c	0.8		h_5	274.06959		s_5	1.2495897		ED_tvalv1	0.1860		COP	3.169	
13				h_6	274.06959		s_6	1.2636722		ED_tvalv2	0.0765		ε	71.232	%
14	ε_HX	0.75		x_6	0.2009176		s_7	1.0505007		ED_FC	0.000E+00		C_total	12425.490	$
15	Tw_1	17	oC	h_7	214.08994		x_8	0.1219283		ED_DCHX	0.003346		TEWI	198070.110	
16	m_w	0.001	kg/s	h_8	214.08994		s_8	1.0577531		ED_IC	0.0001				
17	CC	10	kW	h_3	512.61859		s_2b	2.1689933							
18															

Figure 8: The selected 3E optimised solution by MIDACO

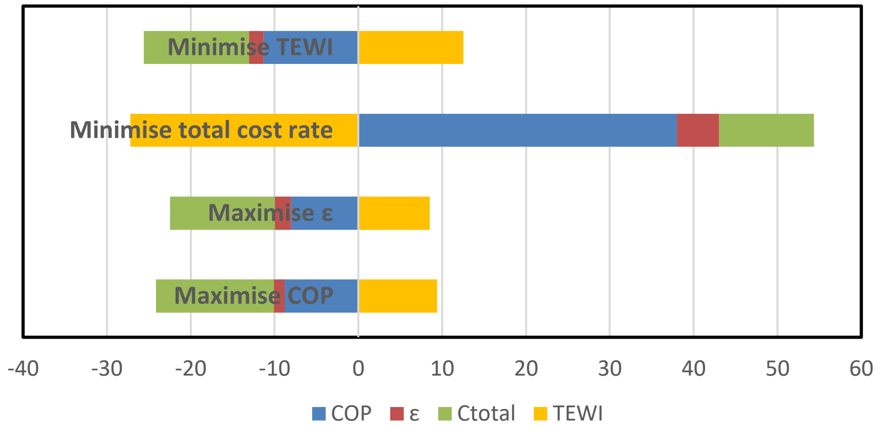


Figure 9: Percentage deviations of the 3E optimised solution by MIDACO from Solver solutions

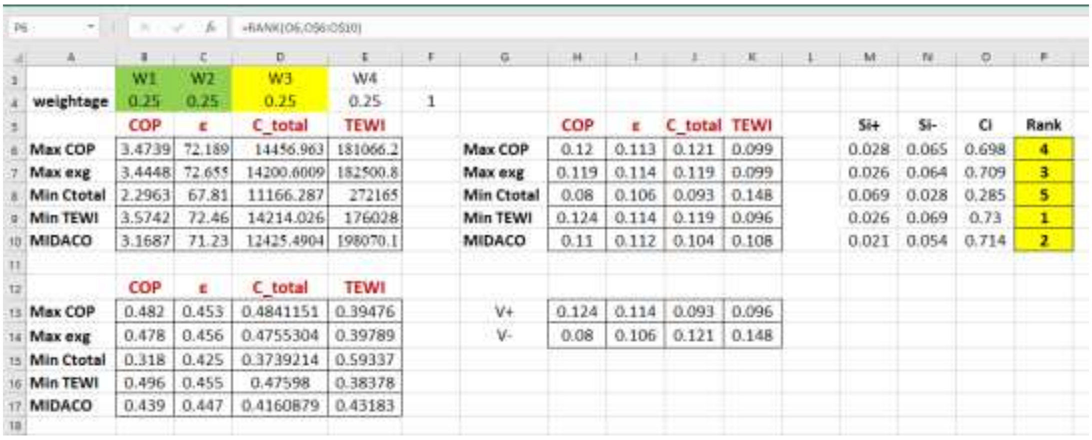
MULTI-OBJECTIVE OPTIMISATION OF THE SYSTEM USING SOLVER AND TOPSIS

The TOPSIS decision-making technique ranks different choices by evaluating an overall index, C_i, that measures their relative distance from the ideal choice according to the following relationship [6, 17]:

$$C_i = \frac{S_j^-}{S_j^+ + S_j^-} \tag{14}$$

Where S_j^+ and S_j^- are the distances from “benefit” and “non-benefit” ideal choices depending on the weighting factors provided to it. Therefore, the method requires the benefit and non-benefit objectives to be identified. This section shows how the single-objective solution obtained by Solver can be used to achieve multi-objective optimisation of the system by using the TOPSIS technique. The advantage of this method is that it can be used to obtain optimised solutions with any number of design variables and various weighting factors.

The Excel sheet shown on Figure (10), which is a modified version of an example sheet available at [18], applies the TOPSIS method to the four single-objective optimised solutions obtained by Solver together with the 3E optimised solution obtained by MIDACO. The values of the four performance indicators obtained by the five solutions are stored as a matrix in cells B6:E10. Note that there are two "benefit" objectives for this analysis, which are maximising the system's COP and exergetic efficiency (ε), and two "non-benefit" objectives, which are minimising the system's total cost rate (C_total) and TEWI. The sheet shown on Figure (10) applies a balanced weighting scheme that gives equal weights to these four objectives by assigning the value 0.25 to each of the four weighting factors W1 to W4 stored in cells B4 to E4.



P6 =RANK(O6,O$6:O$10)

	W1	W2	W3	W4	
weightage	0.25	0.25	0.25	0.25	1

	COP	ε	C_total	TEWI
Max COP	3.4739	72.189	14456.963	181066.2
Max exg	3.4448	72.655	14200.6009	182500.8
Min Ctotal	2.2963	67.81	11166.287	272165
Min TEWI	3.5742	72.46	14214.026	176028
MIDACO	3.1687	71.23	12425.4904	198070.1

	COP	ε	C_total	TEWI
Max COP	0.482	0.453	0.4841151	0.39476
Max exg	0.478	0.456	0.4755304	0.39789
Min Ctotal	0.318	0.425	0.3739214	0.59337
Min TEWI	0.496	0.455	0.47598	0.38378
MIDACO	0.439	0.447	0.4160879	0.43183

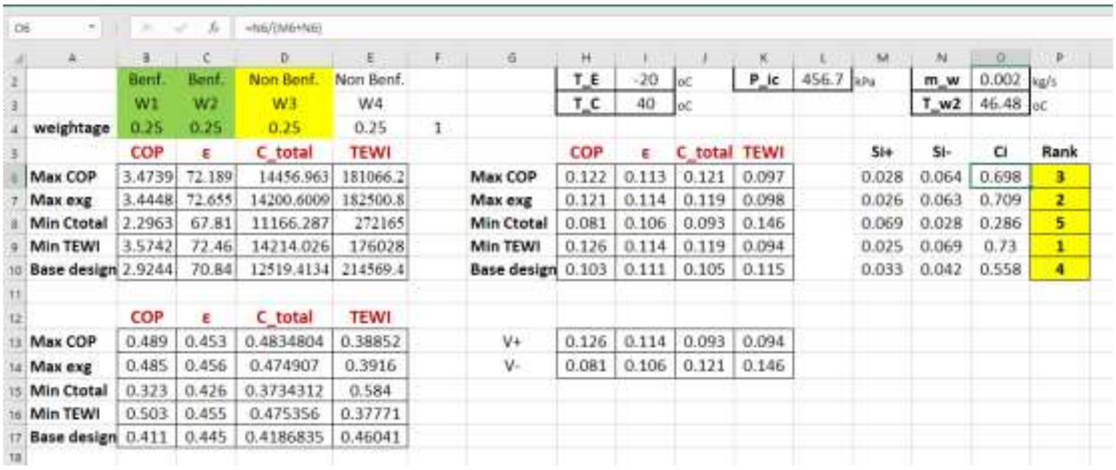
	COP	ε	C_total	TEWI		Si+	Si-	Ci	Rank
Max COP	0.12	0.113	0.121	0.099		0.028	0.065	0.698	4
Max exg	0.119	0.114	0.119	0.099		0.026	0.064	0.709	3
Min Ctotal	0.08	0.106	0.093	0.148		0.069	0.028	0.285	5
Min TEWI	0.124	0.114	0.119	0.096		0.026	0.069	0.73	1
MIDACO	0.11	0.112	0.104	0.108		0.021	0.054	0.714	2

V+	0.124	0.114	0.093	0.096
V-	0.08	0.106	0.121	0.148

Figure 10: TOPSIS sheet for ranking the different optimised solutions obtained by Solver and MIDACO

As the formula bar shows, ranking of the five solutions is done by using Excel's function "Rank"; which makes it easy to judge the different optimised solutions with different values of the four weight factors by giving more weight to any of these factors. With the balanced weighting scheme, Figure (10) shows that the solution that is the nearest solution to satisfying the multi-objective requirement, i.e. the one with the largest value of C_i, is not that obtained by MIDACO but that obtained by Solver for minimising the TEWI. The figure also shows that the solution with the smallest value of C_i is that for minimising the total cost rate.

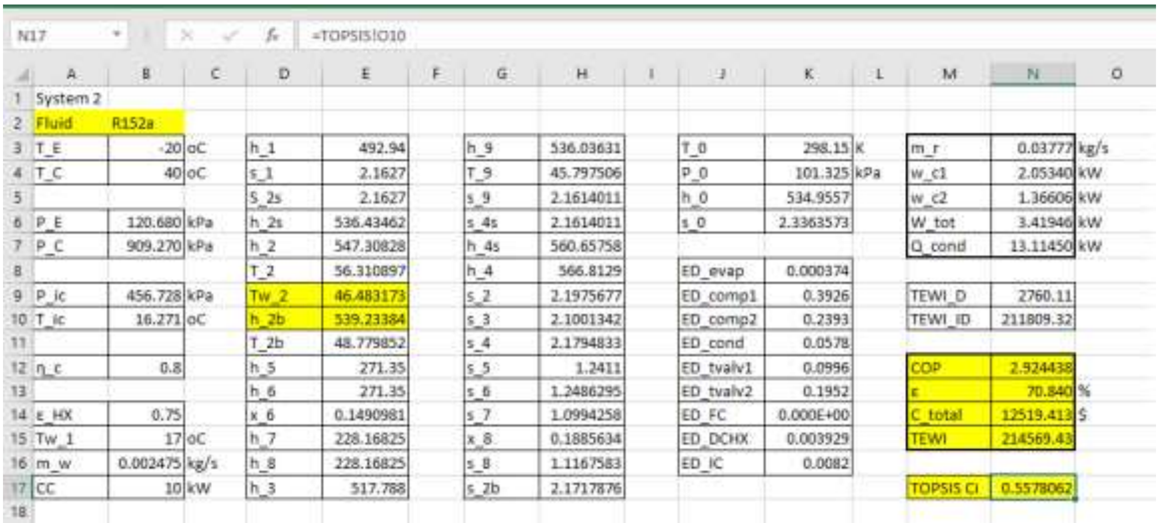
By incorporating the TOPSIS sheet with the Excel-aided model of the VCR system, the scheme can be used not only to rank the four Solver solutions, but to maximise the value of C_i for a selected solution by using Solver also. Figure (10) shows the added third sheet of the extended Excel-aided model. This sheet copies the values of the four performance indicators from Sheet 1 into its cells B9 to E9 as shown on Figure (11), while Sheet 1 copies the corresponding value of C_i from Sheet 3 into its cell N17 as shown on Figure (12). Note that the values of the four changing variables (the evaporator and condenser temperatures, the inter-stage pressure, and the water mass flow rate) are those of the base design that replaced the MIDACO solution in Sheet 3. Also note that Sheet 3 now shows the values of the four design variables, together with the value of T_{w2}, before Solver is used to maximise the value of C_i by adjusting the four changing variables in Sheet 1. Figures (13) and (14) show Sheet 1 and Sheet 3 with the solution obtained by Solver with the Evolutionary method and same constraints shown on Figure (5).



O6 =N6/(M6+N6)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
2		Benf.	Benf.	Non Benf.	Non Benf.			T_E	-20	oC	P_ic	456.7	kPa	m_w	0.002	kg/s
3		W1	W2	W3	W4			T_C	40	oC				T_w2	46.48	oC
4	weightage	0.25	0.25	0.25	0.25	1										
5		COP	ε	C_total	TEWI			COP	ε	C_total	TEWI		Si+	Si-	Ci	Rank
6	Max COP	3.4739	72.189	14456.963	181066.2		Max COP	0.122	0.113	0.121	0.097		0.028	0.064	0.698	3
7	Max exg	3.4448	72.655	14200.6009	182500.8		Max exg	0.121	0.114	0.119	0.098		0.026	0.063	0.709	2
8	Min Ctotal	2.2963	67.81	11166.287	272165		Min Ctotal	0.081	0.106	0.093	0.146		0.069	0.028	0.286	5
9	Min TEWI	3.5742	72.46	14214.026	176028		Min TEWI	0.126	0.114	0.119	0.094		0.025	0.069	0.73	1
10	Base design	2.9244	70.84	12519.4134	214569.4		Base design	0.103	0.111	0.105	0.115		0.033	0.042	0.558	4
11																
12		COP	ε	C_total	TEWI											
13	Max COP	0.489	0.453	0.4834804	0.38852		V+	0.126	0.114	0.093	0.094					
14	Max exg	0.485	0.456	0.474907	0.3916		V-	0.081	0.106	0.121	0.146					
15	Min Ctotal	0.323	0.426	0.3734312	0.584											
16	Min TEWI	0.503	0.455	0.475356	0.37771											
17	Base design	0.411	0.445	0.4186835	0.46041											
18																

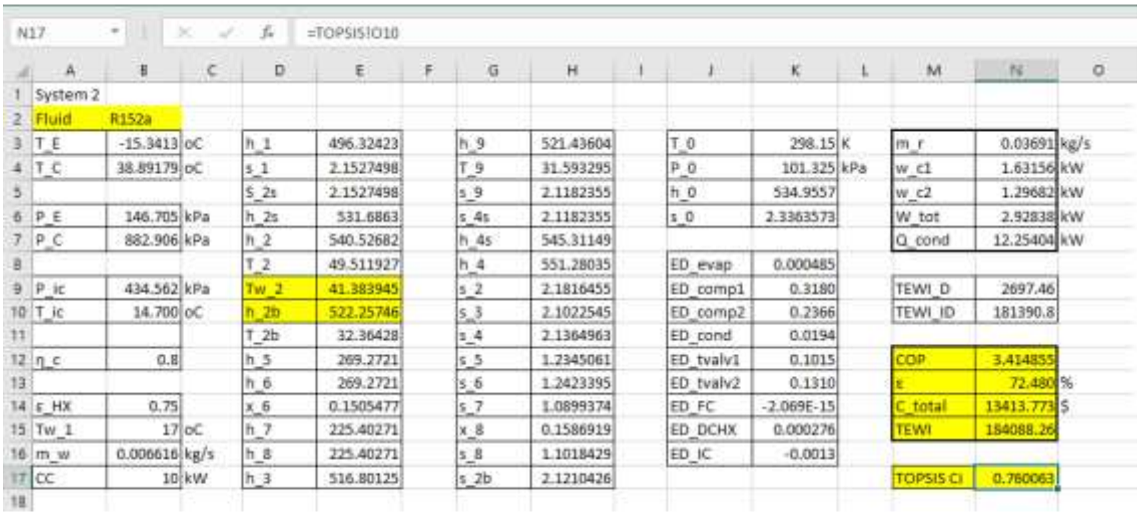
Figure 11: Sheet 3 of the extended Excel-aided model for the system (base design)



N17 =TOPSIS!O10

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-20	oC	h_1	492.94		h_9	536.03631		T_0	298.15	K	m_r	0.03777	kg/s
4	T_C	40	oC	s_1	2.1627		T_9	45.797506		P_0	101.325	kPa	w_c1	2.05340	kW
5				S_2s	2.1627		s_9	2.1614011		h_0	534.9557		w_c2	1.36606	kW
6	P_E	120.680	kPa	h_2s	536.43462		s_4s	2.1614011		s_0	2.3363573		W_tot	3.41946	kW
7	P_C	909.270	kPa	h_2	547.30828		h_4s	560.65758					Q_cond	13.11450	kW
8				T_2	56.310897		h_4	566.8129		ED_evap	0.000374				
9	P_ic	456.728	kPa	Tw_2	46.483173		s_2	2.1975677		ED_comp1	0.3926		TEWI_D	2760.11	
10	T_ic	16.271	oC	h_2b	539.23384		s_3	2.1001342		ED_comp2	0.2393		TEWI_ID	211809.32	
11				T_2b	48.779852		s_4	2.1794833		ED_cond	0.0578				
12	η_c	0.8		h_5	271.35		s_5	1.2411		ED_tvalv1	0.0996		COP	2.924438	
13				h_6	271.35		s_6	1.2486295		ED_tvalv2	0.1952		ε	70.840	%
14	ε_HX	0.75		x_6	0.1490981		s_7	1.0994258		ED_FC	0.000E+00		C_total	12519.413	$
15	Tw_1	17	oC	h_7	228.16825		x_8	0.1885634		ED_DCHX	0.003929		TEWI	214569.43	
16	m_w	0.002475	kg/s	h_8	228.16825		s_8	1.1167583		ED_IC	0.0082				
17	CC	10	kW	h_3	517.788		s_2b	2.1717876					TOPSIS CI	0.5578062	
18															

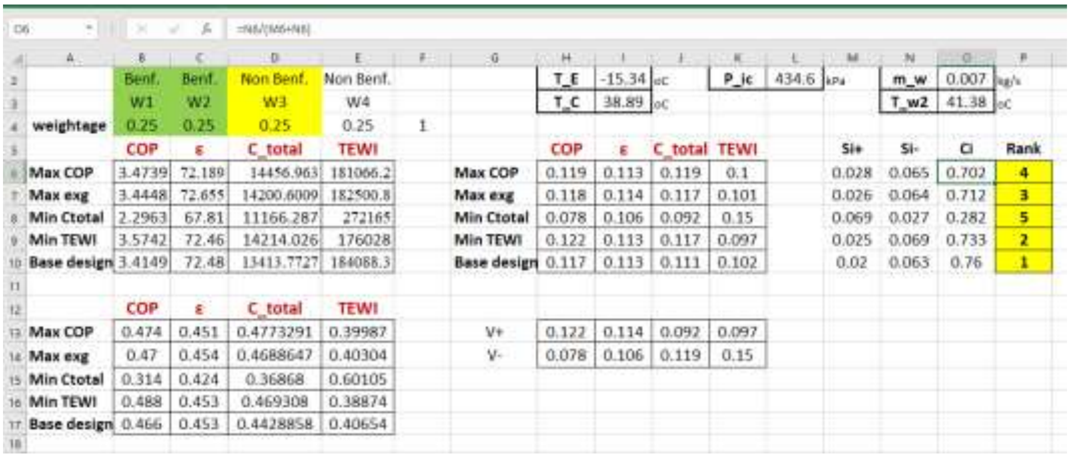
Figure 12: Sheet 1 of the extended Excel-aided model for the system (base design)



N17 =TOPSIS!O10

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-15.3413	oC	h_1	496.32423		h_9	521.43604		T_0	298.15	K	m_r	0.03691	kg/s
4	T_C	38.89179	oC	s_1	2.1527498		T_9	31.593295		P_0	101.325	kPa	w_c1	1.63156	kW
5				S_2s	2.1527498		s_9	2.1182355		h_0	534.9557		w_c2	1.29682	kW
6	P_E	146.705	kPa	h_2s	531.6863		s_4s	2.1182355		s_0	2.3363573		W_tot	2.92838	kW
7	P_C	882.906	kPa	h_2	540.52682		h_4s	545.31149					Q_cond	12.25404	kW
8				T_2	49.511927		h_4	551.28035		ED_evap	0.000485				
9	P_ic	434.562	kPa	Tw_2	41.383945		s_2	2.1816455		ED_comp1	0.3180		TEWI_D	2697.46	
10	T_ic	14.700	oC	h_2b	522.25746		s_3	2.1022545		ED_comp2	0.2366		TEWI_ID	181390.8	
11				T_2b	32.36428		s_4	2.1364963		ED_cond	0.0194				
12	η_c	0.8		h_5	269.2721		s_5	1.2345061		ED_tvalv1	0.1015		COP	3.414855	
13				h_6	269.2721		s_6	1.2423395		ED_tvalv2	0.1310		ε	72.480	%
14	ε_HX	0.75		x_6	0.1505477		s_7	1.0899374		ED_FC	-2.069E-15		C_total	13413.773	$
15	Tw_1	17	oC	h_7	225.40271		x_8	0.1586919		ED_DCHX	0.000276		TEWI	184088.26	
16	m_w	0.006616	kg/s	h_8	225.40271		s_8	1.1018429		ED_IC	-0.0013				
17	CC	10	kW	h_3	516.80125		s_2b	2.1210426					TOPSIS CI	0.760063	
18															

Figure 13: Sheet 1 of the extended Excel-aided model for the optimised system



T_E -15.34 oC
T_C 38.89 oC
P_ic 434.6 kPa
m_w 0.007 kg/s
T_w2 41.38 oC

	Benf.	Benf.	Non Benf.	Non Benf.	
	W1	W2	W3	W4	
weightage	0.25	0.25	0.25	0.25	1
	COP	ε	C_total	TEWI	
Max COP	3.4739	72.189	14456.963	181066.2	
Max exg	3.4448	72.655	14200.6009	182500.8	
Min Ctotal	2.2963	67.81	11166.287	272165	
Min TEWI	3.5742	72.46	14214.026	176028	
Base design	3.4149	72.48	13413.7727	184088.3	

	COP	ε	C_total	TEWI
Max COP	0.474	0.451	0.4773291	0.39987
Max exg	0.47	0.454	0.4688647	0.40304
Min Ctotal	0.314	0.424	0.36868	0.60105
Min TEWI	0.488	0.453	0.469308	0.38874
Base design	0.466	0.453	0.4428858	0.40654

	COP	ε	C_total	TEWI	Si+	Si-	Ci	Rank
Max COP	0.119	0.113	0.119	0.1	0.028	0.065	0.702	4
Max exg	0.118	0.114	0.117	0.101	0.026	0.064	0.712	3
Min Ctotal	0.078	0.106	0.092	0.15	0.069	0.027	0.282	5
Min TEWI	0.122	0.113	0.117	0.097	0.025	0.069	0.733	2
Base design	0.117	0.113	0.111	0.102	0.02	0.063	0.76	1

V+	0.122	0.114	0.092	0.097
V-	0.078	0.106	0.119	0.15

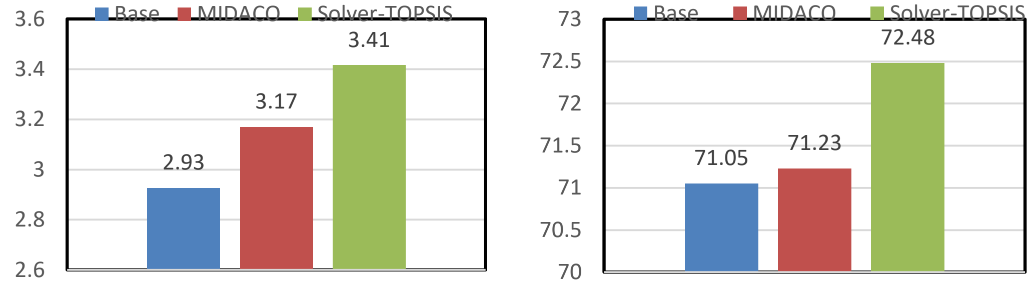
Figure 14: Sheet 3 of the extended Excel-aided model for the optimised system

Comparison of Figure (14) with Figure (11) shows that Solver increased the value of C_i from 0.56 to 0.76 by adjusting the values of the four design variables as shown on Table 6. Figure (15) compares the modified values of the four optimised performance indicators obtained by Solver with those of the base design and those obtained by the MOO solution of MIDACO. The figure shows that the increments in the COP and exergetic efficiency obtained by Solver-TOPSIS solution are significantly higher than those of the solution obtained by MIDACO while the TEWI is significantly lower. However, these improvements are achieved by increasing the total cost rate which increased from \$12,519.413/y to \$13,413.773/y. (Actually, the cost of electricity decreased from \$1,311.773/y to \$1,124.323/y and the penalty for CO_2 emissions decreased from \$1,269.796/y to \$1,088.345/y, but the equipment cost rate increased from \$9,625.328/y to \$11,201.105/y). With respect to the hot water, Table 6 shows that the method increased the flow rate from 0.0025 kg/s to 0.0066 kg/s, and increased the exit temperature from 34.6°C to 41.4°C. For eight hour of operation per day, the total hot water produced by the 3E optimised system is about 190 litres.

Table 6: Particulars of the base design, the MIDACO solution, and the Solver-TOPSIS solution

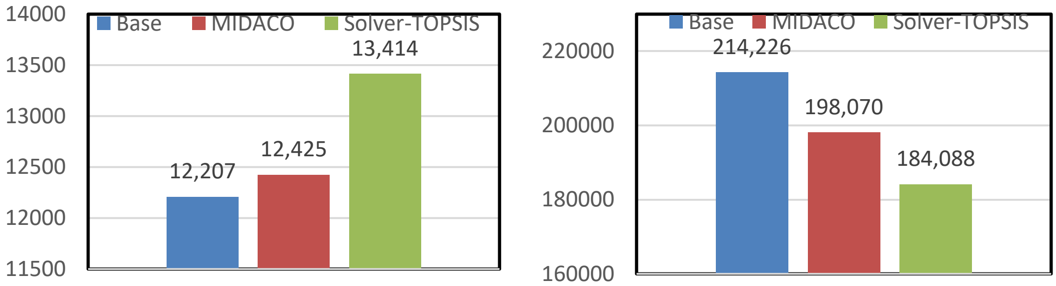
	Base design	MIDACO	Solver -TOPSIS
T_E [°C]	-20	-15	-15.341
T_C [°C]	40	41.439	38.892
P_{ic} [kPa]	331.256	351.239	434.562
$\dot{m}_{water}$ [kg/s]	0.002475	0.001	0.006616
T_{w2} [°C)	34.61	33.29	41.38

Figure (16) compares the rates of exergy destruction in the different system components of the base design to those adjusted by the MIDACO solver and the Solver-TOPSIS technique. As the figure shows, both solutions rank the rates of components exergy destructions in the same order according to which the highest rates of exergy destruction occur in compressor 2 followed by compressor 1, throttle valve1, throttle valve 2 and then the condenser. By comparison, the rates of exergy destruction in the remaining three components are negligible. The figure also shows that the total rate of exergy destruction in the system optimised by the Solver-TOPSIS technique is less than the corresponding values of the base design and that optimised by the MIDACO solver.



(a) COP [-]

(b) ε [%]



(c) Total cost rate, [$/y]

(d) TEWI [kg/y]

Figure 15: Comparison of the four key performance indicators for the base design with those obtained by MIDACO and the Solver-TOPSIS technique

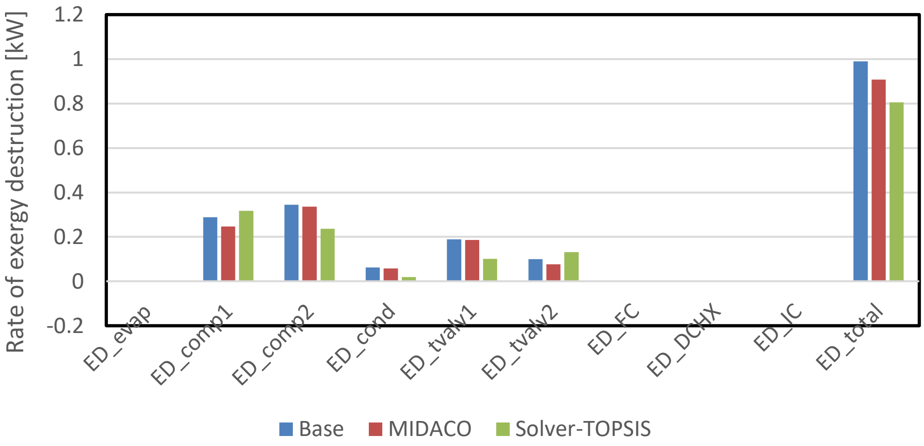


Figure 16: Exergy destruction rates in the base design and at the optimal solutions obtained by MIDACO and the Solver-TOPSIS technique

CONCLUDING REMARKS

This paper describes a method for utilising the TOPSIS decision-making technique with the single-objective Solver of Microsoft Excel for multi-objective optimisation analyses of multi-stage compression VCR systems. The method is illustrated by analysing an innovative two-stage system that incorporates an intercooler after the first compression stage and recovers the waste energy for producing hot water. Four single-objective solutions are first obtained by using Solver to maximise the COP and exergetic efficiency and minimise the total cost and TEWI using four design variables, which are the evaporator temperature, the condenser temperature, the isentropic efficiency of the compressor, and the water flow rate. TOPSIS is then used to improve base design to simultaneously satisfy the four objectives. Comparison of the optimised system obtained by this method with that obtained by using the MIDACO multi-objective solver shows that the proposed method leads to higher values of the system's COP and exergetic efficiency and a lower value of its TEWI at the expense of increasing the total cost rate.

List of Symbols and Abbreviations

c - Unit cost
$\dot{C}$ - Cost rate
CC - Cooling capacity of the system
COP - Coefficient of performance
CRF - Capital-recovery factor
$\dot{E}^{D}$ - Rate of exergy destruction
GWP - Global-warming potential
h - Enthalpy
i - Interest rate
MOO - Multi-objective optimisation
m - Mass of refrigerant
$\dot{m}$ - Mass flow rate
N - Annual operation hours
n - Plant life time
$\dot{Q}$ - Rate of heat transfer
s - Entropy
SOO - Single-objective optimisation
T - Temperature
$TEWI$ - Total equivalent warming index
$\dot{W}$ - Work
x - Quality or dryness fraction of refrigerant

Greek letters:

α - Refrigerant recycling factor
β - Electricity regional conversion factor
ε - Exergetic efficiency
η - Isentropic efficiency of compressor
μ - Emission factor
ϕ - Maintenance factor,

Subscripts:

CO_2	-	Carbon-dioxide
$elec$	-	Electricity
r	-	Refrigerant
ref	-	Refrigerant
w	-	Cooling water

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