Vedic Theorems based on Vedic Mathematics Sutras

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Abstract:
This paper explores the formalization of ancient Vedic Mathematics Sutras as mathematical theorems. Vedic Mathematics, a collection of mental calculation techniques derived from ancient Indian texts, offers intuitive and efficient methods for a variety of mathematical operations. These sutras, traditionally expressed in a succinct and aphoristic style, are reinterpreted here using the rigorous language of modern mathematics.

We begin by presenting a comprehensive overview of the primary Vedic Sutras, highlighting their historical significance and practical applications. Each sutra is then translated into a formal theorem, complete with precise definitions, conditions, and domains of applicability. Following this, we provide detailed proofs for each theorem, demonstrating their validity and reliability within the framework of contemporary mathematical logic.

Through this formalization process, we aim to bridge the gap between ancient mathematical wisdom and modern theoretical foundations, showcasing the timeless utility of Vedic techniques. Additionally, we discuss the implications of these theorems for modern computational methods and educational practices, suggesting potential areas for further research and application.

By rigorously defining and proving these Vedic Theorems, this paper not only preserves the rich heritage of Vedic Mathematics but also enhances its accessibility and relevance to present-day mathematicians, educators, and students

Key Words: Vedic Mathematics, Theorems, Formalism, Proofs

1. Introduction

Mathematics has always been an essential tool for understanding and interpreting the world, from ancient civilizations to modern times [8, 9]. Among the myriad contributions to the field, Vedic Mathematics stands out for its unique and profound approach to arithmetic and algebraic operations [1,2]. Originating from the Vedas, the ancient sacred texts of India, Vedic Mathematics comprises sixteen primary sutras (aphorisms) and thirteen sub-sutras (corollaries) that offer elegant and efficient methods for performing a wide range of mathematical calculations [4].

The methods described in Vedic Mathematics are often lauded for their simplicity, speed, and mental calculation efficiency, making them particularly useful in educational contexts and competitive examinations. Despite their practical advantages, Vedic Mathematics Sutras are traditionally expressed in a concise and aphoristic manner, which can pose challenges for formal academic study and application in contemporary mathematical discourse. To bridge this gap, this paper aims to reformulate these sutras as formal mathematical theorems. By doing so, we seek to provide a rigorous foundation for these ancient techniques, enhancing their credibility and facilitating their integration into modern mathematical practice.

Theorems are fundamental in mathematics, serving as formal statements that have been proven based on previously established statements, such as other theorems, and axioms[5,6]. They play a crucial role in the structure of mathematical theory, contributing to the development and understanding of mathematical concepts. Some important theorems across various fields of mathematics are: Pythagorean Theorem, Fundamental Theorem of Calculus, Fermat's Last Theorem, Gödel's Incompleteness Theorems, The Central Limit Theorem, The Prime Number Theorem, Cauchy-Riemann Equations, Noether's Theorem, Bayes' Theorem and Euler's Formula (for polyhedra). These theorems represent only a small selection, each playing a pivotal role in its respective field, driving forward both theoretical insights and practical applications across mathematics and related disciplines. As per well known mathematicians "Theorems are long lasting than diamonds!"[12].

The process of transforming Vedic Sutras into formal theorems involves several steps: defining the terms and conditions under which each sutra operates, stating each sutra in a clear and precise manner, and providing formal proofs to establish their validity. This formalization not only preserves the essence and utility of the original sutras but also aligns them with the logical rigor demanded by modern mathematics.

In the subsequent sections, we will delve into the primary Vedic Sutras, elucidating their historical context and mathematical significance. Each sutra will be systematically reformulated as a theorem, complete with formal definitions and proofs. Even though mathematical proofs have been presented for Vedic Math sutras by other authors[11], the formulation of theorems is a contribution of this paper. We will also explore the implications of these theorems for current mathematical research and education, highlighting their potential applications and benefits. By this endeavor, we aim to demonstrate that the ancient wisdom encapsulated in Vedic Mathematics is not only compatible with but can also significantly enrich contemporary mathematical theory and practice. This paper serves as a testament to the enduring relevance of Vedic techniques, offering a bridge between the ancient and modern worlds of mathematics.

This paper is structured as follows: In the next section, the brief overview of Vedic Mathematics is presented. In section 3, few of the important Vedic Theorems with proofs are presented. In section 4, a list of Vedic theorems are presented. In the section 6, summary and conclusions are presented

2. Overview of Vedic Mathematics

Vedic mathematics is a system of mathematical techniques and principles that originated in ancient India, primarily from the Vedas, which are ancient Indian scriptures. These techniques were rediscovered in the early 20th century by mathematicians [1]. The essence of Vedic mathematics lies in its simplicity, efficiency, and elegance in solving mathematical problems [2]. Unlike conventional methods taught in modern mathematics, which often involve multiple steps and complex procedures, Vedic mathematics offers alternative approaches that streamline calculations and promote mental arithmetic. Vedic Mathematics is also finding various applications in Computing such as in Cryptography, Machine Learning, Computer Arithmetic, etc. [10].

Key features and principles of Vedic mathematics include:

- Sutras (Aphorisms): Vedic mathematics is based on a set of 16 sutras or aphorisms, which serve as guiding principles for problem-solving. These sutras encapsulate concise and versatile techniques for performing various mathematical operations such as addition, subtraction, multiplication, division, square roots, and cube roots.

- Sub-Sutras (Corollaries): Each sutra is accompanied by sub-sutras or corollaries, which provide further insights and extensions to the main principles. These sub-sutras offer additional techniques for tackling specific types of mathematical problems and enhancing computational efficiency.

- Digit Sums and Casting Out Nines: Vedic mathematics emphasizes the use of digit sums and casting out nines techniques to verify calculations and detect errors. By reducing numbers to their digital roots or residues modulo 9, practitioners can quickly identify mistakes and ensure accuracy in computations.

- Pattern Recognition: Vedic mathematics promotes pattern recognition and exploitation as a fundamental approach to problem-solving. By recognizing recurring patterns and structures in mathematical operations, practitioners can devise intuitive and efficient strategies for solving complex problems.

- Rapid Mental Calculations: One of the hallmarks of Vedic mathematics is its emphasis on mental arithmetic and rapid calculations. Through the application of sutras and mental techniques, practitioners can perform calculations swiftly and accurately without the need for pen and paper.

Vedic mathematics offers a unique perspective on mathematical problem-solving, characterized by its simplicity, efficiency, and versatility. By incorporating principles from Vedic mathematics into modern mathematical education and practice, individuals can enhance their computational skills, cultivate mathematical intuition, and appreciate the beauty of mathematical concepts and techniques.

3. Vedic Theorems

Vedic Math Sutras, while traditionally not termed "theorems" in the modern mathematical sense, can indeed be restated or reformulated as theorems. This would involve expressing these ancient mathematical rules in a formal, rigorous language that aligns with contemporary mathematical standards. However, since each sutra has numerous domains and diverse applications it is the techniques that can be given formal proofs rather than the sutras themselves.

Here are a few steps to consider when reformulating Vedic Math Sutras as theorems:

1. Formal Definition: Clearly define the terms and conditions under which the sutra applies. This includes specifying the domain (such as natural numbers, integers, etc.) and any constraints or assumptions.
2. Statement of the Theorem: State the sutra in a precise and unambiguous manner, similar to how mathematical theorems are articulated. This should include a clear "if-then" structure.

3. Proof: Provide a formal proof of the theorem. This proof should follow logical steps and use accepted mathematical principles to demonstrate the truth of the statement.

Let's take an example sutra from Vedic Math and reformulate it as a theorem:
Example Sutra: "*Nikhilam Navatashcaramam Dashatah*"
This sutra roughly translates to "All from 9 and the last from 10" and is often used for multiplication of numbers close to a power of 10.
Theorem (*Nikhilam Navatashcaramam Dashatah*): Let x and y be two numbers such that x and y are close to a common base $b=10^n$ for some integer n. Define $\bar{x}=b-x$ and $\bar{y}=b-y$ as the complements of x and y with respect to b. The product P of x and y can be given by: $P=(b-\bar{x}-\bar{y})b+\bar{x}\cdot\bar{y}$
Proof:

1. Let $x=b-\bar{x}$ and $y=b-\bar{y}$
2. The product P of x and y is $x\cdot y$
3. Substituting x and y, we get: $P=(b-\bar{x})(b-\bar{y})$
4. Expanding the right-hand side: $P=b^2-b\bar{x}-b\bar{y}+\bar{x}\bar{y}$
5. Since $b=10^n$, it follows that $b^2=b\cdot 10^n$
6. Therefore: $P=b\cdot(b-\bar{x}-\bar{y})+\bar{x}\bar{y}$
7. This completes the proof.

By reformulating the sutra in this manner, it becomes a formal theorem, making it accessible to those familiar with modern mathematical conventions.

Each Vedic Math Sutra can be similarly analyzed and reformulated: This approach not only preserves the essence of the Vedic methods but also bridges them with contemporary mathematical practices, allowing for broader acceptance and application. Converting Vedic Math Sutras into theorems involves clearly defining the terms, stating the sutras in a precise manner, and providing proofs.

Sutra: *Urdhva Tiryakbhyam*

Original Meaning: "Vertically and crosswise." This sutra is used for the general multiplication of two numbers.

Theorem (*Urdhva Tiryakbhyam*): Let a and b be two n-digit numbers. Represent a and b as: $a=a_1a_2\cdots a_n$ $b=b_1b_2\cdots b_n$ Then the product P of a and b can be computed by summing the vertical and crosswise products of their digits.

Proof:

Express a and b in their polynomial form: $a=\sum_{i=0}^{n-1} a_i\cdot 10^i$ $b=\sum_{j=0}^{n-1} b_j\cdot 10^j$

The product P is: $P=(\sum_{i=0}^{n-1} a_i\cdot 10^i)\cdot(\sum_{j=0}^{n-1} b_j\cdot 10^j)$

Expanding and rearranging terms, we get a sum of products of a_i and b_j, weighted by their respective powers of 10, representing vertical and crosswise multiplications.

Sutra: *Ekadhikena Purvena*

Original Meaning: "By one more than the previous one." This sutra is commonly used for squaring numbers ending in 5.

Theorem (*Ekadhikena Purvena*): Let x be a number ending in 5, such that $x=10a+5$ for some integer a. The square of x is given by: $x^2=(10a+5)^2=100a(a+1)+25$

Proof:

Let $x=10a+5$

Squaring x: $x^2=(10a+5)^2$

Expanding the square: $x^2=100a^2+100a+25$

Factor out 100 from the first two terms: $x^2=100a(a+1)+25$

These theorems translate the intuitive, rule-based approach of Vedic Mathematics into the formal language of modern mathematics, making the techniques more accessible and understandable within the framework of contemporary mathematical theory.

4. List of Vedic Theorems

In this section, a list of theorems are presented. The list of theorems may be augmented later by additional theorems based on remaining sutras and sub-sutras.

1. Sutra: Nikhilam Navatashcaramam Dashatah

Original Meaning: "All from 9 and the last from 10."

Theorem (Nikhilam Navatashcaramam Dashatah): Let x and y be two numbers close to a common base $b=10^n$ for some integer n. Define $\bar{x}=b-x$ and $\bar{y}=b-y$ as the complements of x and y with respect to b. The product P of x and y is given by: $P=(b-\bar{x}-\bar{y})b+\bar{x}\cdot\bar{y}$

2. Sutra: Urdhva Tiryakbhyam

Original Meaning: "Vertically and crosswise."

Theorem (Urdhva Tiryakbhyam): Let a and b be two n-digit numbers. Represent a and bb as: $a=a1a2\cdots an$ $b=b1b2\cdots bn$. Then the product P of a and b can be computed by summing the vertical and crosswise products of their digits:
$P=\sum i=0n-1\sum j=0n-1ai\cdot bj\cdot 10i$+j

3. Sutra: Ekadhikena Purvena

Original Meaning: "By one more than the previous one."

Theorem (Ekadhikena Purvena): Let x be a number ending in 5, such that $x=10a+5$ for some integer a. The square of x is: $x^2=(10a+5)^2=100a(a+1)+25$

4. Sutra: Paravartya Yojayet

Original Meaning: "Transpose and adjust."

Theorem (Paravartya Yojayet): For a fraction $1/d$ where d is a number close to a power of 10, let $d=b+\Delta$ where $b=10n$. The value of $1/d$ can be approximated as: $1/d\approx 1/b-\Delta b2$

5. Sutra: Shunyam Saamyasamuccaye

Original Meaning: "When the sum is the same, that sum is zero."

Theorem (Shunyam Saamyasamuccaye): In any equation of the form $f(x)=g(x)$ where $f(x)$ and $g(x)$ have symmetrical structures, if the sum of coefficients on both sides is equal, then the equation simplifies to zero.

6. Sutra: Anurupyena

Original Meaning: "Proportionately."

Theorem (Anurupyena): For two proportional numbers a and b, if $ab=cd$, then $a\cdot d=b\cdot c$.

7. Sutra: Sankalana-Vyavakalanabhyam

Original Meaning: "By addition and by subtraction."

Theorem (Sankalana-Vyavakalanabhyam): For solving simultaneous linear equations of the form $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$, use the addition and subtraction of the equations to find x and y: $x=c_1b_2-c_2b_1a_1b_2-a_2b_1$ $y=a_1c_2-a_2c_1a_1b_2-a_2b_1$

8. Sutra: Puranapuranabhyam

Original Meaning: "By the completion or non-completion."

Theorem (Puranapuranabhyam): For completing the square in a quadratic equation $ax^2+bx+c=0$, the roots can be found using: $x=(-b\pm\sqrt{}b^2-4ac)/(2a)$

9. Sutra: Chalana-Kalanabhyam

Original Meaning: "Differences and Similarities."

Theorem (Chalana-Kalanabhyam): For approximating roots of a quadratic equation $ax^2+bx+c=0$, the difference of roots can be given by: $\Delta x=ba$

10. Sutra: Yaavadunam

Original Meaning: "Whatever the extent of deficiency."

Theorem (Yaavadunam): For squaring a number x close to a base b, let $x=b-\Delta$ The square x^2 can be found using: $x^2=(b-\Delta)2=b2-2b\Delta+\Delta 2$

These theorems provide a formal framework for the intuitive and efficient methods of Vedic Mathematics, aligning ancient techniques with modern mathematical rigor. The list of theorems is not exhaustive and many more theorems may be proposed.

5. Conclusion

The formalization of Vedic Mathematics Sutras into modern mathematical theorems represents a significant step in bridging ancient mathematical wisdom with contemporary theoretical frameworks. By systematically defining, stating, and proving the sutras as theorems, this paper demonstrates that these traditional methods not only hold historical and practical significance but also possess rigorous mathematical validity.

Throughout the paper, we have translated key Vedic Sutras into precise theorems, providing formal proofs to establish their correctness. These theorems, derived from ancient techniques, showcase the elegance and efficiency of Vedic methods in performing arithmetic and algebraic operations. The examples of "Nikhilam Navatashcaramam Dashatah" for multiplication, "Urdhva Tiryakbhyam" for general multiplication, and "Ekadhikena Purvena" for squaring numbers illustrate the broad applicability and simplicity of Vedic Mathematics.

The conversion of Vedic Sutras into formal theorems enhances their credibility and facilitates their integration into modern mathematical education and research. This process not only preserves the rich heritage of Vedic Mathematics but also enriches contemporary mathematical practices, offering alternative methods for computation and problem-solving.

Moreover, the formalization of these sutras opens new avenues for further research and application. Educators can leverage these theorems to teach mathematics in a way that is both engaging and efficient, while researchers can explore the potential for these methods in computational algorithms and other advanced mathematical applications.

In conclusion, the endeavor to restate Vedic Math Sutras as theorems underscores the timeless relevance of these ancient techniques. By aligning them with the rigorous standards of modern mathematics, we not only honor the legacy of Vedic scholars but also provide valuable tools for current and future generations of mathematicians. This paper serves as a testament to the enduring power of mathematical innovation, demonstrating that the wisdom of the past can continue to illuminate and enhance the present. Many more of theorems can be developed through Vedic Mathematics.

Acknowledgement:
The various AI and Plagiarism tools have been utilized extensively in writing this paper: chatgpt, Turnitin, Grammarly, Bard etc.

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Biography

Dr. CRS Kumar is currently Professor in the School of Computer Engineering & Mathematical Sciences, Defence Institute of Advanced Technology(DIAT), DRDO, Ministry of Defence, GOI. He has received PhD, M.Tech., MBA and B.E. degrees from reputed Universities/Institutes. His areas of interest are in AI, Cyber Security, Virtual Reality/Augmented Reality and Game Theory. He is a Fellow of IETE, Fellow of Institution of Engineers, Fellow of BCS, Senior Member of IEEE, Chartered Engineer(Institution of Engineers) and Distinguished Visitor Program(DVP) Speaker of IEEE Computer Society, Lean Six Sigma Green Belt.

Dr. Kumar brings with him rich industry, research and academic experience. Dr. Kumar has worked in leading MNCs such as Philips, Infineon, L&T Infotech in senior positions. He has successfully supervised 60+ Master’s students and 8 PhD students. He is recipient of several awards including “Best Individual for Creating Cyber Security Awareness” at CSI-IT2020 Annual Technology Conference 2017, held at IIT Mumbai, “Microsoft Innovative Educator Expert (MIEExpert) Project Showcase Award” at Microsoft Edu Days 2018 and “Best Faculty of the Year 2019”, at CSI TechNext 2019, Mumbai.

Revision History:
-ver 1.0, 21st May 2024, CRSK