Framework for the fabrication of scalable, flat foldable, thick origami Kresling structures via non-rigid methods.

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Abstract: *The fabrication of thick origami structures often faces two challenges. Thick origami is usually rigid and not flat foldable. This work demonstrates a technique for creating non-rigid, fully flat foldable, and thick origami. By harnessing the material properties of 3D-printed Polylactic Acid (PLA), thick origami panels that selectively exhibit non-rigid behaviors can be fabricated using living hinges. Constraining the deformation of these living hinges within the elastic regime of the material, robust flat foldable thick origami structures can be created. These methods are demonstrated in a parametric model of a thick origami flat foldable Kresling pattern. The kinematics and bistable mechanical properties of these Kresling structures will be characterized to demonstrate the viability and scalability of this technique.*

1 Introduction

This paper will present a novel design technique for creating flat foldable derivatives of the Kresling pattern using thick, nonrigid origami methods. The Kresling origami pattern arises naturally when a thin sheet of paper is stretched between the ends of two cylinders and a twisting moment is applied to one of the two cylinders. This twist results in the spontaneous formation of the Kresling pattern on the piece of paper between the two cylinders, allowing the paper to fold flat between the two cylinders as illustrated in Figure 1 (a-e). The Kresling pattern is also found in nature within the air sac of the Hawk moth which uses it to expand and contract [Dalaq and Daqaq 22]. The simple design and flat foldability of the Kresling pattern make it ideal for use in deployable structures as it can be easily extended and collapsed just by twisting one face [Kresling 08]. This twist however is not always desired and can be eliminated by adding together multiple Kresling structures with opposing chirality. As the Kresling transitions from the collapsed state to the expanded state, one can feel the presence of an energy barrier resulting in the bistability of the pattern. The presence of this energy barrier indicates that some deformation occurs outside of the predefined creases. Thus the pattern is inherently non-rigid. This non-rigid deformation poses a challenge for current thick origami methods that rely on the use of thick rigid materials [Lang et al. 18]. Another challenge

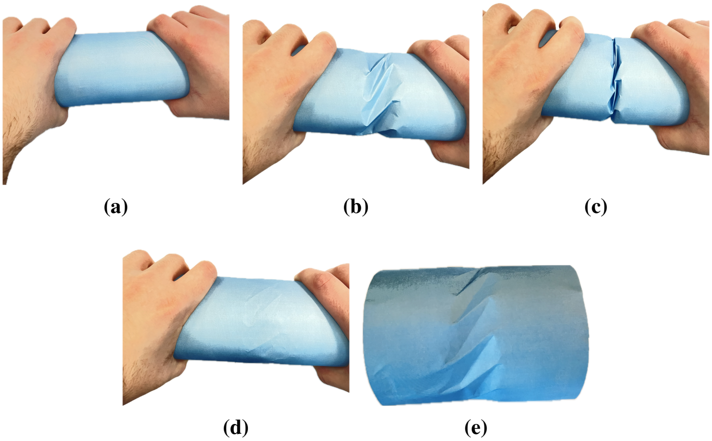


Figure 1: *The figures show the process of spontaneously generating a Kresling pattern using two cylindrical tubes and a piece of paper. (a) the paper is wrapped around the tube. (b) shows the paper collapsing as the cylinders are twisted and pushed together. (c) shows the fully collapsed Kresling. (d) shows the paper after actuation. (e) shows the final crease pattern generated by the movement.*

posed by the Kresling pattern regarding thick origami is the number of overlapping layers of material present within the flat folded state of the structure. The spontaneously generated Kresling contains many overlapping layers of thin paper that fold nicely into each other, however, if the paper has non-negligible thickness it results in the structure jamming before being able to fully flat fold. This paper seeks to overcome these limitations and create a framework for generating fully flat foldable, non-rigid, thick origami Kresling structures.

2 Thick Origami Methods

In this section, we will discuss the simple methods developed to design fully flat foldable thick origami structures that enabled the creation of the thick Kresling pattern.

2.1 Hinge Design

The most important, and simplest of these is the design of the thick origami crease. The simplest method of creating a crease in thick origami is to simply mimic the behavior of a thin origami fold and create a hinge that pivots around a single point Figure 2 (a). This primitive approach works for creating simple thick origami structures but inherently does not allow for flat foldability as the two adjacent panels will always intersect. A slight modification of the design allows for full 180°rotation of the hinge Figure 2 (b). Moving on from the simplest design, one can create a hinge with two pivot points, one on each panel, with a small third panel between

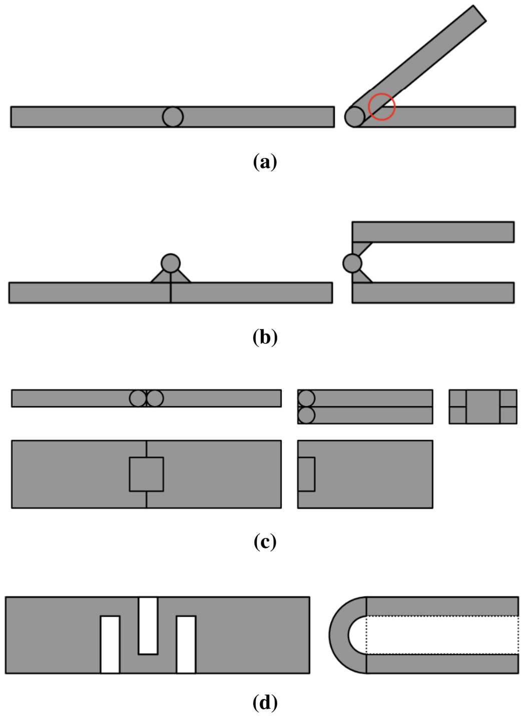


Figure 2: *The figure illustrates several hinge mechanisms that can be used in thick origami (a) A simple single pivot hinges have a very limited range of motion due to the intersection highlighted in red. (b) An offset single pivot hinge that can go a full 180°. (c) A dual pivot hinge that allows for a full 360°rotation and flat foldability. (d) A living hinge is made by removing material from the hinge and relying on the elastic properties of the hinge material to enable bending.*

them. This third panel will have a width equal to at least two times the thickness of the material, allowing the two panels to fold flat against each other, linked together through the smaller panel. This very simple double-pivot hinge allows for the full 360-degree rotation of both panels around each other Figure 2 (c). A third method is that of cutting small slats into the location of the crease, creating thinner pieces of the rigid material and taking advantage of elastic deformations to create a living hinge Figure 2 (d). This method is slightly more complicated and depends greatly on the properties of the thick material being used. Living hinges offer a distinct advantage over the other types of folds discussed in Figure 2. Their flexible nature, arising from their reliance on elastic deformation of the material, enables them to flex along multiple axes and not only around a pivot point. This flexibility allows for out-of-plane deformations and even facilitates compression and stretching of the panel. This flexibility the creation of non-rigid origami as discussed in section 2.3.

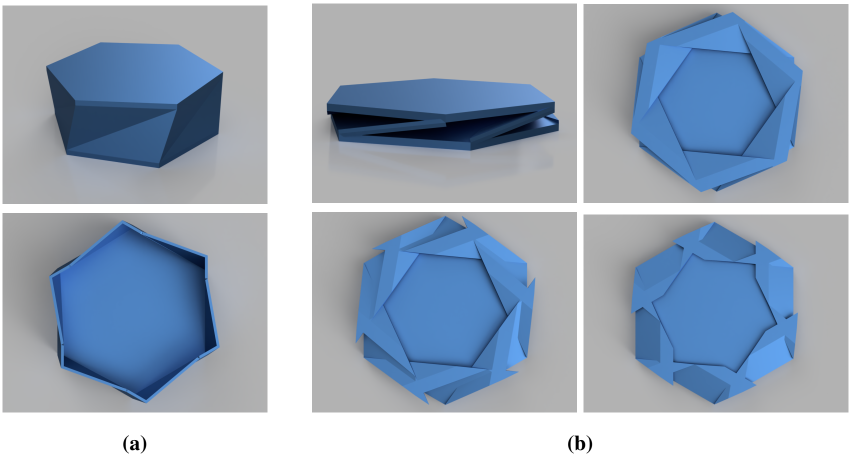


Figure 3: *(a) Illustrates a Kresling with thick walls in an extended state, where collisions are not present. (b) shows three cross-sections of the collapsed state where the intersections of the panels become evident.*

2.2 Pattern Intersection Reduction

An origami pattern is considered flat foldable when there exists some folded state where all the panels of the structure are parallel to each other. In thin origami, it is assumed the material is sufficiently thin, such that when folding its thickness does not need to be accounted for. Thus the materials do not consume physical space, allowing one to create patterns where multiple layers of material occupy the same volume in the folded state. In reality, however, this is never the case. In an example like paper origami, the thickness of the material is so thin, that in most cases this approximation is a helpful one. Even though the faces in the pattern are not truly parallel, the angles between them are so small that to an outside observer, the structure appears flat. Even with thin materials like paper, this approximation begins to break down as more and more layers of paper are folded into the same space. This can be easily observed when trying to fold a piece of paper in half seven times. To achieve truly flat foldable structures it is necessary to modify the geometry of the origami to resolve any conflicts where materials are folded into the same space.

When looking at the cross-section of the Kresling pattern with thick walls, we see that the intersections between panels are minimal in the extended state as shown in Figure 3 (a). As the structure is flat folded, these intersections become more and more severe as shown in Figure 3 (b). To create a thick, flat foldable Kresling pattern, it was necessary to reduce the number of material overlaps (N_i) at any given location to be less than or equal to the number of creases (N_f) along the vertical axis of the structure plus one.

$$N_i \leq N_f + 1 \tag{1}$$

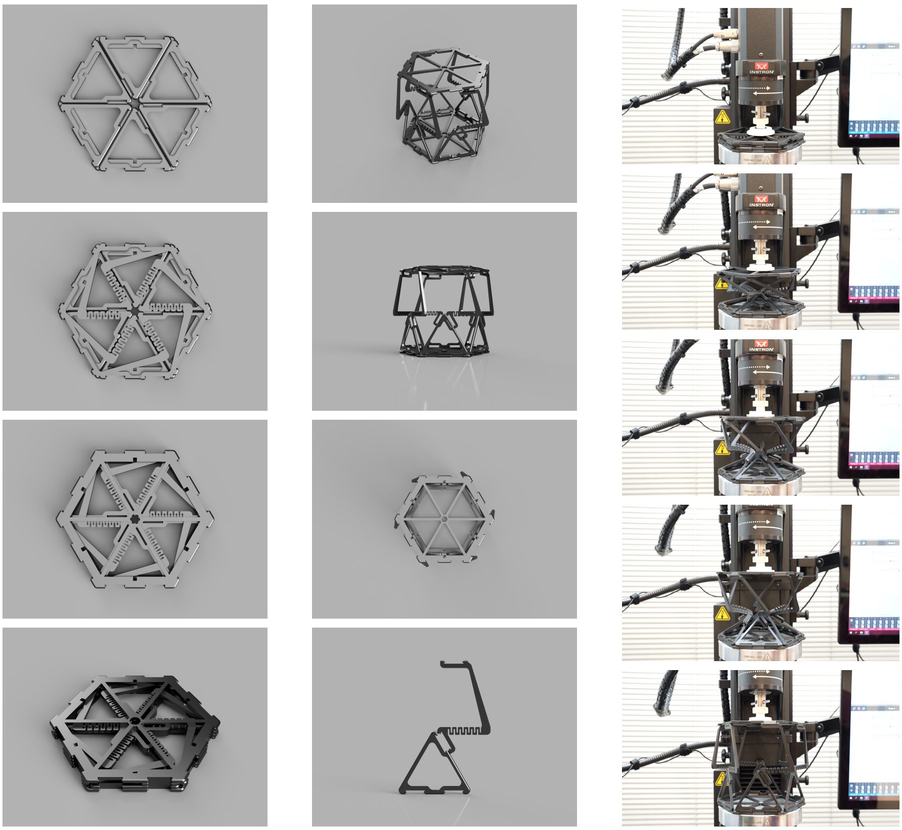
This equation arises from the fact that the maximum number of layers that can be stacked on top of each other is directly related to the number of folds along the vertical direction of the structure. Each fold along the vertical direction allows another layer of material to be staked on the structure in the folded state. Thus the maximum overlaps at any point is $N_f + 1$. For the Kresling pattern, the maximum allowed number of overlapping layers of material in the flat folded state at any point is four, as the structure contains three folds along the vertical structure.

To achieve this, the material within the faces of the Kresling pattern was reduced to enable panels within the same layer to snake around each other while still preserving the location of the creases. This can be seen in Figure 4 (a). This optimization also enables the fabrication of the Kresling pattern a single print-in-place assembly if 3D printed in the flat-folded state reducing time and labor required to assemble the completed structures.

2.3 Non-Rigid Methods

The energy barrier that is felt in the deployment of the Kresling structure is a result of non-rigid deformation in the panels. The presence of a non-rigid deformation, by Gauss's Theorema Egregium[1], implies a change in the Gaussian curvature of the structure during the transition. In thick origami, however, the panels are often made from thick rigid materials that do not permit stretching. It is possible to create a composite structure using several materials, such as the inclusion of rubber sections in the panels, the inclusion of a spring, or even a telescopic structure, thus enabling the panel to change shape and size [Tachi 11]. In this paper, however, we explore the inclusion of living hinges within the panel faces to enable this non-rigid deformation as shown in the last panel of Figure 4 (b). As described above in the Hinge Design section, the addition of living hinges is done by creating thin sections of material that can deform due to the natural elasticity of the material Figure 2 (d). The living hinges enable the panel material to be compressed and stretched and even allow for out-of-plane deformation. In this work, the living hinge is placed in the middle of the Kresling side panel, just above the middle fold. This provides sufficient flexibility to transition between the open and closed states while also ensuring a stiff structure. Altering the size, placement, and orientation of the living hinges can be used to tune the actuation force of the Kresling pattern. Increasing the length of the living hinge section reduces the required bending force and the magnitude of elastic deformation on each segment is reduced. In our case the living hinge near the center of the Kresling structure and oriented orthogonally to the top and bottom of the Kresling, as shown in our hexagonal example Figure 4, creates a significant energy barrier. This is because the orthogonal living hinge experiences significant outward bowing and twisting during actuation as seen in Figure 4 (c). Altering the location and orientation of the living hinge to be in line with the structure and located near the top and bottom plates would presumably result in far less bowing but more twisting of the flexible hinge. The locations and orientations of the living hinges in our design were largely dictated by our efforts to

[1] Any Change in Gaussian curvature must be accompanied by stretching of the material [Pressley 10].



(a) (b) (c)

Figure 4: *(a) Shows the layers of the 6-sided thick Kresling in its collapsed state. (b) Shows the 6-sided Kresling in its extended state. The last panel of (b) shows the alignment of the two arms in the fully extended state (c) Depicts a six-sided thick Kresling being actuated using our Instron tensile tester while collecting force and torque data*

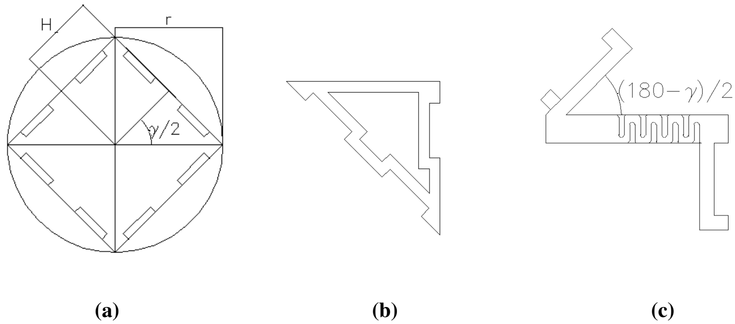


Figure 5: *Sketches for designing a 4-sided Kresling. (a) The Sketch of the bottom polygon with hinge locations marked, (b) Sketch of the lower arm with hinge locations. (c) Sketch of the upper arm with hinge locations.*

reduce material collisions in the flat folded state as discussed previously in section 2.2.

3 Designing the Kresling

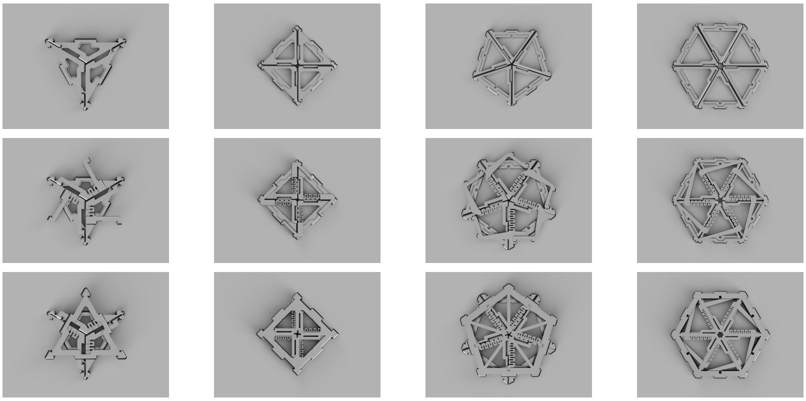
The key parameters of a fully flat foldable Kresling structure for any regular N-gon are thickness (T), side length (L), and number of sides (N). If non-rigid deformation is achieved through a living hinge, one also needs to define the thickness, location, and length of the living hinge portion. From the geometric parameters, we can define the rest of the intrinsic parameters of the structure: angle γ and radius r.

$$\gamma = 360/N \tag{2}$$

$$r = \frac{L}{2 * \sin\gamma/2} \tag{3}$$

One starts by drawing a regular polygon of side length L and then radially subdividing it into equal triangles Figure 5 (a). The regular polygon will act as both the top and bottom face of the completed structure. Hinges are placed along each of the polygon edges, allowing one to affix another layer to the structure. The only constraints on the inner layers of the Kresling structure are that any two arms corresponding to any edge of the structure do not intersect with any of the other arms, the axis through the hinges located at the ends of the arms are parallel to the axis of the hinges on the polygon faces. This normally contains the extended arms to a plane orthogonal to the polygon face, however, this constraint can be relaxed for cases where the side lengths of the two regular polygons are different, resulting in a slanted Kresling structure such as in [Lu et al. 22], and that some non-rigid deformation is permitted within the inner layers.

In the examples shown here the hinges connecting the two inner layers are located along the radial edges of the subdividing triangles, thus being rotated by an angle of γ from the X-axis Figure 5 (b). This hinge connects the two middle



(a) **(b)** **(c)** **(d)**

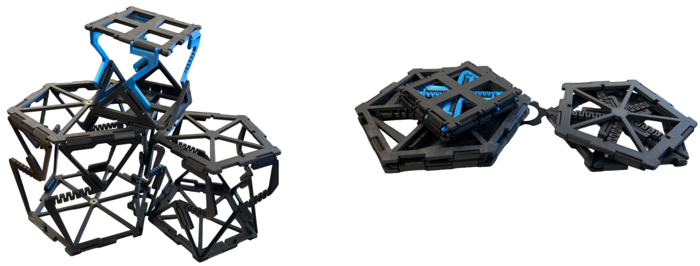
Figure 6: *Various layers of the: (a) 3-sided Kresling (b) 4-sided Kresling (c) 5-sided Kresling (d) 6-sided Kresling shows*

layers. To connect the upper of the two middle layers to the top of the structure, the location of the hinge must be selected such that when rotated around the middle hinge it becomes parallel to the hinge linking the middle arms to the base structure. This is done by placing it at an angle of $180-\gamma$ from the location of the middle hinge Figure 4 (b), Figure 5 (c). Non-rigid deformation is enabled by the inclusion of a living hinge portion in at least one of the two middle arms, enabling the arms to stretch, compress, and otherwise deform as needed to account for the change in Gaussian curvature experienced by the Kresling pattern. In doing this, one ensures that in the extended state of the Kresling, the hinges connecting to the top face and bottom face to the middle sections are parallel, allowing the structure to actuate.

Following this method results in N-gon Kreslings where in the flat folded state the top face is rotated relative to the bottom face by an angle of γ. This means that for any even N the two faces will be aligned and for any odd N they will be offset. This, of course, can be modified by changing the angles and locations of the hinges as long as they still satisfy the required constraints outlined above. The creation of Kresling structures with irregular polygonal faces is also possible.

4 Fabrication

Using the technique described above, 3, 4, 5, and 6-sided Kresling Structures were generated as depicted in Figure 6 (a-d). The individual components were then 3D printed out of polylactic acid (PLA) on a consumer FMD 3D printer using a 0.4mm nozzle and 0.2mm layer height. The serpentine living hinge pattern of the top arm section of each Kresling structure enables a non-rigid deformation by harnessing the elastic deformation of thin-walled PLA extrusions. To improve the longevity and flexibility, of these 3D printed springs the printer was set to use 3 continuous



(a)

Figure 7: *3D printed flat foldable Kresling structures*

perimeters, ensuring that no infill structure would be printed in the spring area. Doing this greatly increases the spring lifespan because it aligns the extrusion axis with the axis of deformation in the spring. This in turn allows us to take advantage of the material's full strength in tension and not the adhesion strength of the various extrusions. The removal of infill also avoids the presence of filament strands that oppose the deformation of the spring. A subset of the fabricated samples are depicted in Figure 7. In this work, each component was printed separately and then assembled, however, due to the removal of intersections in the flat folded state, as discussed in section 2.2, these structures can be printed as a single print in place assembly in the flat folded state, assuming that sufficient airgaps are incorporated into the structure to prevent unwanted boding of the stacked components during the printing process.

5 Results

The generated Kresling structures, pictured in Figure 6 and Figure 7 were placed into a modified Instron tensile tester that collects tensile force from the freely rotating top surface as well as torque data from the constrained bottom surface as depicted in Figure 4 (c). Each sample was then tested to its full extension length of approximately 2 times its radius. All 4 structures exhibited a strong bi-stability at an actuation range of approximately $85-98\%$ extension Figure 8 (a). The critical point is a function of many parameters including: N (number of sides), spring length, spring orientation, radius, side length, and the elasticity of the 3D-printed material. As a result of these numerous contributing factors, our samples are not directly comparable as many of these parameters vary between them. We can however gain a strong intuition about the general properties of these structures. Looking at the torque exerted on the constrained bottom face Figure 8 (b) we can see the axial forces required to actuate the structure. At the critical points, we see a rapid change in the torque as the structure switches to being in compression rather than tension. This change often results in rapid oscillations, characteristic of a damped harmonic oscillator, of torque exerted on the torque sensor. These Kresling struc-

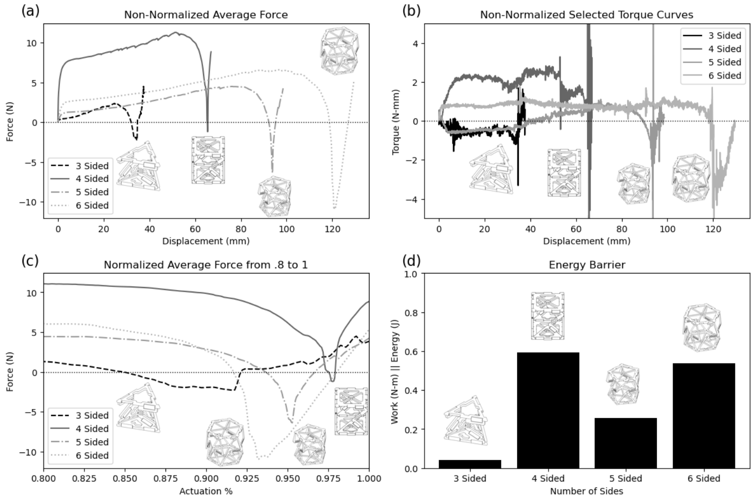


Figure 8: *(a) Tensile force vs displacement plots for 3, 4, 5, and 6-sided thick Kresling structures. (b) Torque on the constrained face of the Kresling structure as a function of displacement. (c) A closer look at the forces around the bi-stability point (the X axis has been rescaled to percent actuation). (d) The magnitude of the energy barrier of each Kresling structure. The integral of the force from 0 displacement to the point when the force crosses zero.*

tures exhibit strong bi-stability, easily snapping between stable points once a critical strain is reached. The energy barrier between these states depends on the elastic modulus of the material and various spring parameters. Across our samples, we generally see that samples with larger N have a greater energy barrier. In Figure 6 (a), the triangular structure has quite a low energy barrier as the three arms are short and the required deformation is small. However, as the arms grow longer and the required deformation to actuate increases, so does the required energy as seen with the haxagonal structure Figure 8 (d). In Figure 6 (b) the spring is in a straight line between the two hinges, in Figure 6 (c, d) the spring is oriented at an angle and then linked with a rigid bar. This kink in the arm increases the required deformation of the structure, the height of the energy barrier between the two states, and therefore the magnitude of the reversal of force observed in Figure 8 (a,c).

6 Discussion and Applications

This technique for designing fully flat foldable, thick Kresling structures, scales to any size size structure while also preserving the intrinsic properties of the Kresling pattern. The structures exhibit a bistable behavior with a clear energy barrier between the collapsed and extended states. By varying the chosen material and geometry on the nonrigid section of the structure, one can programmably tune the energy barrier between the bistable states for various energy absorption and controlled failure applications such as vehicle crash boxes. The twisting motion of the Kresling pattern is preserved and remains additive, as a result of this it can be removed by staking several Kresling structures with opposing chirality. Common truss-based structures like construction cranes can be manufactured as a chain of flat foldable Kreslings and be deployed and collapsed in the field as needed. Other applications include flat foldable deployable shelters, prefabricated assemblies construction of larger buildings, and deployable spacecraft [Navarro et al.]. This design framework enables the creation of larger and more complicated deployable origami structures by enabling a transition from thin sheet materials to thicker structural materials. Using hinge design optimizations and techniques for the incorporation of non-rigid segments in thick origami structures we can translate many classical non-rigid thin origami patterns to their thick counterparts. This framework allows us to create complex origami-inspired structures while harnessing the structural properties of thick materials such as various plastics, woods, and metals.

7 Conclusions

In this paper, we demonstrated and characterized a framework for the creation of fully flat foldable, non-rigid, thick origami through a generalized implementation of the Kresling pattern. Implementations of thick rigid origami structures have already existed, however, the demonstration of non-rigid techniques with thick origami enables the translation of thin non-rigid origami patterns to their thick origami equivalents. This in turn enables the implementation of many deployable origami structures currently only manufacturable out of thin sheet materials to be made using thick origami methods, increasing strength and reliability.

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