Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 1629–1634

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

On dynamic analysis and control of an elevator system using polynomial chaos and Karhunen-Loève approaches

Diego Colón[a,], Americo Cunha Jr[b], Stefan Kaczmarczyk[c], José M. Balthazar[d]

[a] University of São Paulo - USP, Escola Politécnica da USP, Laboratório de Automação e Controle (LAC -PTC), Brazil
[b] Rio de Janeito State University - UERJ, NUMERICO - Nucleus of Modeling and Experimentation with Computers, Brazil
[c] The University of Northampton, Faculty of Arts, Science and Technology, UK
[d] Technological Institute of Aeronautics - ITA, Graduate Program in Aeronautics and Mechanical Engineering, Brazil

Abstract

The paper deals with the quantification of uncertainties in an elevator system, subjected to lateral vibration induced by the car guide rail irregularities, with and without a feedback control system. A mathematical model for the elevator system is derived, and a stochastic model for the rail profile irregularities is constructed. This model of uncertainties describes the rail profile as a random field, which is represented by means of Karhunen-Loève expansion. Polynomial chaos is employed to compute the propagation of uncertainties through the stochastic model, in order to evaluate the disturbance rejection properties and robustness of the closed-loop system.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: elevator systems; nonlinear dynamics; uncertainty quantification; polynomial chaos.

1. Introduction

Elevator systems are very important devices for the modern society, being essential in many places, from office buildings to industrial transportation of material and goods. A typical elevator consists of vertical cables and a car, that slides on lateral rails, which induce lateral vibrations due to the irregular shape of its profile. The understanding of the impact of this vibration on a elevator dynamics, at its control, are essential for a safe operation [1].

In this sense, this paper deals with the quantification of uncertainties in an elevator system, subjected to lateral vibrations induced by rail's irregularities. In particular, it is interested in study the effect of the uncertainties with and without a feedback control system for the horizontal vibration.

The rest of this paper is organized as follows. Section 2 presents the modeling and a control strategy for the system dynamics. Numerical results are presented and discussed in section 3. Finally, the main conclusions and paths for future works are indicated in section 4.

E-mail address: diego@lac.usp.br

1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.083

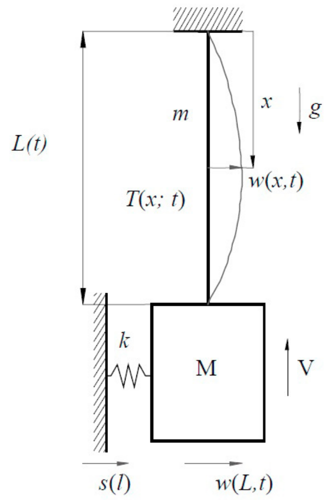

Fig. 1. Sketch of an idealized mechanical system which represents the elevator system.

2. Modeling elevator system dynamics

2.1. Deterministic mechanical-mathematical model

A simplified schematics of the decoupled elevator system, which is the object of study of this work, is shown in Figure 1. This idealized mechanical system, subjected to lateral and vertical motions, is composed by a car attached to a vertically moving rope (cable), and is excited by an external load due to the irregularities in rail's profile.

The rope's linear specific mass is m, and it has the same linear velocity V as the car. The rail's profile $s(l)$ is a function of l, the vertical distance travelled by mass M, given by $l = Vt$. The coupling with the car is done through a linear spring, with constant k. The rope is under tension $T(x,t)$ and its horizontal motion is described by the field variable $w(x,t)$, where x is the vertical coordinate. The rope length (from the traction sheave to the load) is represented by $L(t)$. If we define the slow time factor $\epsilon = V/\omega_0 L_0$, in the practical cases one has $\epsilon << 1$ and the *slow time* is defined as $\tau = \epsilon t$. It is possible then separate the slow dynamic related to the motion of the car (and variation of the rope's length) and the fast dynamic of the horizontal vibration. The time derivative of L is related to the slow time derivative by $\dot{L} = \epsilon\, \mathrm{d}L/\,\mathrm{d}\tau = \epsilon L^{'}$.

The dynamical behavior of this mechanical system evolves according to

$$m(w_{tt} + 2\epsilon L^{'} w_{xt} + \epsilon^2 (L')^2 w_{xx} + \epsilon^2 L^{''} w_x) + \gamma(w_t + \epsilon L^{'} w_x) - (T w_x)_x = 0, \tag{1}$$

which is supplemented by the boundary conditions

$$w(0,t) = 0 \qquad \text{and} \qquad M(w_{tt} + 2\epsilon L^{'} w_{xt} + \epsilon^2 (L')^2 w_{xx} + \epsilon^2 L^{''} w_x)_{x=L} + k(w - s)_{x=L} + (T w_x)_{x=L} = 0. \tag{2}$$

This mathematical model emulates the behavior of a distributed parameter system with a discrete mass (car) in one end, but with the length slowly variable. For details the reader is addressed to [1].

We search for solutions of the form

$$w(x,t) = \sum_{n=1}^{N} \Psi_n(x,\tau) q_n(t), \tag{3}$$

where $\Psi_n(x,\tau) = \sin(\beta_n(\tau)x)$, and the *wave-number* $\beta_n(\tau)$, is a slowly variable in time, that satisfy the equation

$$\left(k - \frac{M}{m} T(\tau)\beta_n^2\right) \sin(\beta_n L(\tau)) + T(\tau)\beta_n(\cos \beta_n L(\tau)) = 0 \tag{4}$$

for $T(\tau) = [M + (1/2)mL(\tau)]g$, a time-varying mean tension.

After a substitution, we obtain a set of ordinary differential equation for the time-varying coefficients of the form

$$\ddot{q}_r(t) + 2\,\varsigma_r\,\omega_r(\tau)\,\dot{q}_r(t) + \omega_r^2(\tau)\,q_r(t) = \frac{1}{m_r(\tau)}\,\Psi_r\,(L(\tau))\;k\,s(t), \qquad r = 1, 2\cdots, N, \tag{5}$$

where ς_r is the damping coefficient for the r natural mode, and

$$m_r(\tau) = m\int_0^L \Psi_r^2(x,\tau)\;\mathrm{d}x + M\,\Psi_r^2(L). \tag{6}$$

Note that wave-numbers β_n are related to the natural frequencies of the several modes by $\omega_n(\tau) = \bar{c}\beta_n(\tau)$, where $\bar{c} = \sqrt{T(\tau)/m}$ represents the time-dependent velocity of the lateral wave.

The set of equations in Eq.(5) are decoupled in the sense that the vibration modes are independent of one another. The equations are linear, but (slowly) time-varying.

2.2. Strategy of vibration control

In order to damp the car's horizontal vibration, a position sensor is used in order to measure the horizontal position $y(t)$. Also, an actuator that applies an horizontal force $f(y(t))$ in the car must be used. This force is calculated in real time, according to $y(t)$, as prescribed by a feedback control law. The mathematical model of the system, that is presented Eq.(5), is them modified by the inclusion of the force term $f(y(t))$ in the second equation, which turns it in a closed-loop system. We choose to implement a state feedback linear control law, in which $f(y(t))$ is calculated as a linear combination of the states in real time. Those states, that are not measured, are estimated by a state observer, based in the measurements of the position sensor $y(t)$ [2].

2.3. Karhunen-Loève representation of random field

The function $s(l)$ has a random nature, in reason of the rail's profile irregularities. In this way, it is modeled as a second-order, mean-square continuous, stationary random process $\{S(l), l \in [l_{min}, l_{max}] \subset \mathbb{R}\}$, with zero-mean and autocovariance function given by

$$K_s(l_2 - l_1) = \sigma_s^2\;\exp\left(-|l_2 - l_1|/l_{corr}\right), \tag{7}$$

where σ_s^2 and l_{corr} denotes, respectively, the variance and the correlation length for the process $S(l)$.

In order to represent $S(l)$, a truncated Karhunen-Loèeve expansion is employed, so that

$$S(l) \approx \sum_{n=1}^{N_{kl}} \sqrt{\lambda_n}\,\varphi_n(l)\,Y_n, \tag{8}$$

where the integer N_{kl} is the number of terms used in the expansion, the pairs (λ_n, φ_n) are solution of the Fredholm integral equation associated to K_s, and $\{Y_n\}_{n=1}^N$ is a family of, zero-mean, mutually uncorrelated random variables. In this case, there is an analytical solution to the eigenvalue problem, that can be found in [3].

2.4. Polynomial chaos expansion

Plugging $S(l)$, defined in section 2.3, into (5) we obtain a stochastic model of uncertainties for the mechanical system, defined by a random system of differential equations of the form

$$\dot{X}(t, Y) = F(X(t, Y), Y, t), \tag{9}$$

where t is the time variable, $X(t, Y)$ is a stochastic vector of state, and $Y = (Y_1, \cdots, Y_{N_{kl}})^T$ is a vector which collects the finite variance random variables which parametrize the random field $S(l)$.

The polynomial chaos expansion of this stochastic vector of state is given by

$$X(t, Y) = \sum_{i=0}^{\infty} x_i(t)\,\phi_i(Y), \tag{10}$$

where $\{\phi_i(Y)\}_{i=0}^{\infty}$ is an orthogonal basis for the Hilbert space of the finite variance random variables. The coefficients $\{x_i(t)\}_{i=0}^{\infty}$ are deterministic functions of time that must be found [4,5]. If the random variables are Gaussian, this basis is formed by Hermite polynomials, but it is also possible to do this expansion for other probability distributions and corresponding orthogonal polynomials, known as *Wiener-Askey scheme* [5]. There are two methods to find the temporal coefficients: 1) intrusive method; 2) non-intrusive method. The intrusive method is useful in some situations, as can be found in [5–9], on the other hand, it can represent a very heavy burden to the computational system. In the non-intrusive method, the expansion coefficients are calculated by the following formula

$$x_i(t_k) = \frac{E[X(t_k, Y)\,\phi_i(Y)]}{E[\phi_i(Y)\,\phi_i(Y)]}, \tag{11}$$

where the denominator is calculated by an explicit formula (the basis polynomials in the Hilbert space are known), and the numerator is calculated by solving an integral in the space $\mathbb{R}^{N_{kl}}$ of the N_{kl} random variables. A grid in this space is established and the original stochastic system is simulated for each point in this grid to obtain $X(t_k, Y)$. This must be done to each time value t in which we are interested [10]. If a grid is chosen such that there are n sample points, the total number of samples is $n^{N_{kl}}$, which grows very rapidly with N_{kl}. In order to avoid this problem, we use the Sparse Grid Integration, which consists in selecting convenient grids to $\mathbb{R}^{N_{kl}}$ in which the number of point does not grow exponentially with the dimension, but in a much slower rate (depending on the smoothness of the integrand) [11]. The non-intrusive polynomial chaos approach has other advantages over the intrusive, as presented in [10,12,13].

3. Numerical results

In this section, the results of numerical simulation tests carried out to predict the dynamic behaviour of the elevator system with, and without control system are presented. The results demonstrate the efficacy of the proposed control strategy. For random field representation, it is adopted $N_{kl} = 2$, which means that there will be two random parameters Y_1 and Y_2, that will be Gaussian and independent. The polynomial chaos expansion will use bi-dimensional Hermite polynomials, with maximum degree five. Only the first vibration mode will be considered for control, and the plant state variables will be $x_1 = q_1$ and $y_1 = \dot{q}_1$. The damping ratio is will be considered $\varsigma_1 = 0.4$. The stiffness coefficient for the car-rail interface is $k = 2083$ N/m, and $\sigma_s = 7.07 \times 10^{-4}$ mm and $l_{corr} = 100$ m. We suppose that that rope's specific mass is $m = 0.65$ kg/m, the total car's weight (car + load) is $M = 3250$ kg, the well height is 70 m, the car height is 3.2 m and the total travel height is 60 m, which is also the maximum length of the rope L_0. The vertical velocity of the elevator is $V = 6$ m/s and it is ascending in the simulations. This means that the rope has the slowly variable length $L(\tau) = (V/\epsilon)\tau + L_0$, and $\epsilon = (V/\omega_0 L_0)$ is the slow time constant factor.

The open loop equation is then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & -2\varsigma\omega_1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\Psi_1(L(\tau))}{m_1(\tau)} k \end{bmatrix} S(l; Y_1, Y_2) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f \tag{12}$$

Figure 2 shows important parameters for the problem when the elevator is ascending with vertical velocity $V = 6$ m/s. We can see that the velocity of the wave propagation does not change too much, but the wake number β_1 approach infinity, as well as the fundamental frequency, when $L(\tau)$ approaches zero. The amplitude of the forcing term in Eq.(12), that is $\frac{\Psi_r(L(\tau))}{m_r(\tau)} k$, also goes to infinity, which means that the forcing term has growing influence as the elevator approaches the uppermost part.

In Figure 3 it is presented the covariance surface for the case without the vibration control, and in Figure 4 it is presented the covariance surface for the case with vibration control. One can see that the magnitudes are reduced by more than one order of magnitude, which means that the vibration control designed is effective.

The designed control law imposed as closed-loop poles $-3, -4$, which resulted in the state feedback gains $k_1 = 11.1978$ and $k_2 = 6.2835$. The state observer poles was imposed to be $-6, -8$, which is the double, in order to guarantee that the observer is more rapid than the plant dynamics. The resulting observer gains are $h_1 = 13.2835$ and $h_2 = 37.6800$.

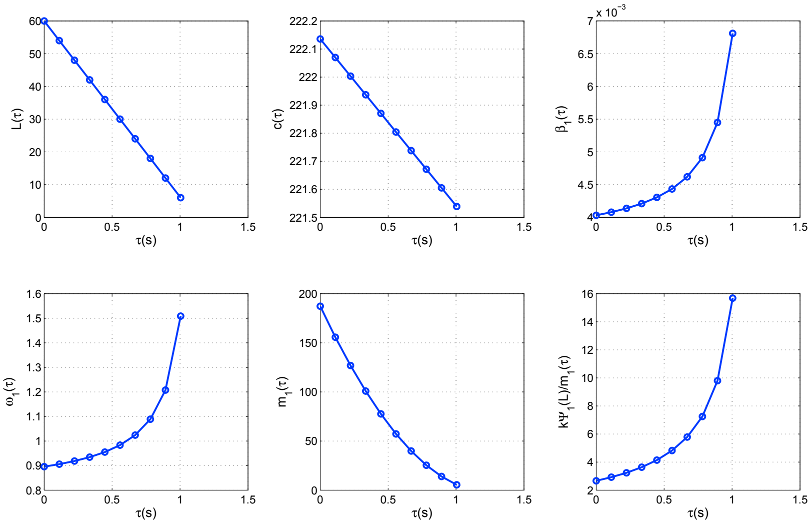

Fig. 2. Time-varying parameters for velocity $V = 6$ m/s.

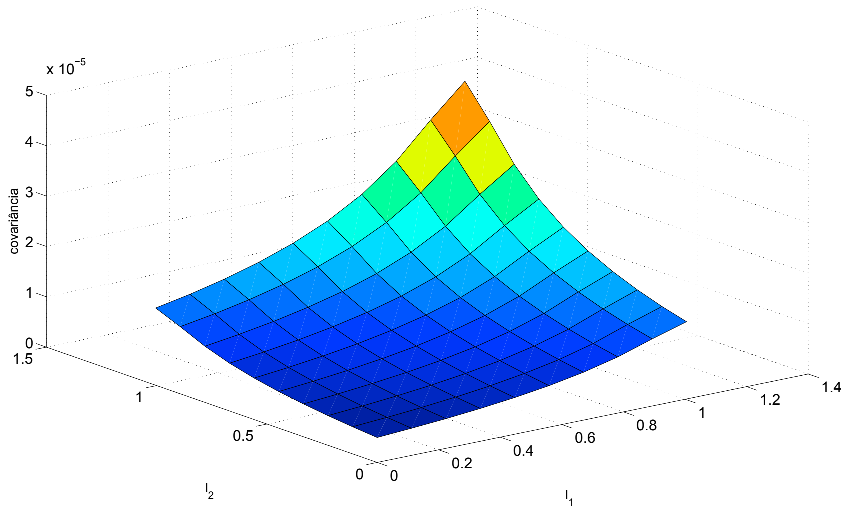

Fig. 3. Covariance without vibration control.

4. Concluding remarks

In this paper a model of an elevator system with a stochastic excitation originating from the guide rail profile is presented. The excitation is represented as a random field and with a truncated Karhunen-Loève expansion being employed. Stochastic numerical simulations were conducted, with and without vibration control, by using polynomial chaos expansion, in its intrusive form, to compute the propagation of uncertainties. The covariance surfaces were obtained for both situations, and it was confirmed that the control law designed is effective in reducing the stochastic vibrations. For future works, it is intended to include an improved model for the vibration source and use different control techniques to compare its efficacy.

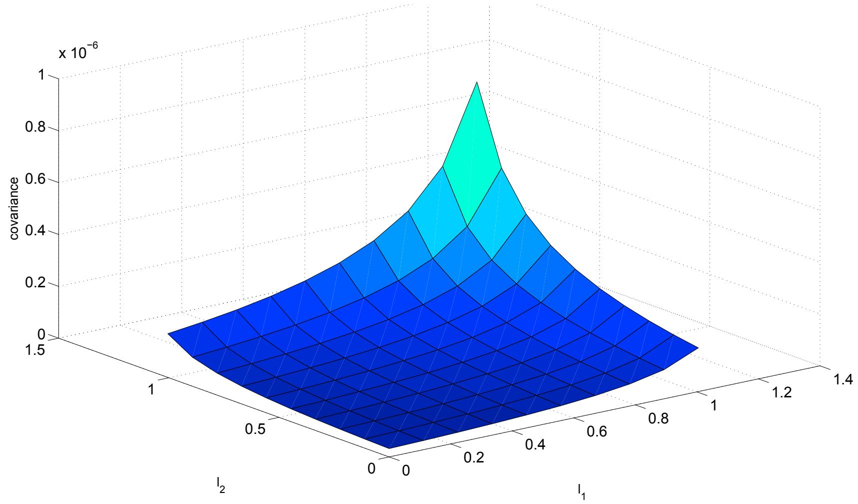

Fig. 4. Covariance with vibration control.

Acknowledgments

The authors are indebted to Brazilian agencies CNPq, CAPES, FAPESP, and FAPERJ for the financial support given to this research.

References

[1] S. Kaczmarczyk, R. Iwankiewicz, Dynamic response of an elevator car due to stochastic rail excitation, in: Proceedings of the Estonian Academy of Sciences, volume 55, 2006.
[2] J. Hespanha, Linear Systems Theory, Princeton University Press, 2009. URL: `https://books.google.com.br/books?id=uiRymAEACAAJ`.
[3] D. Xiu, Numerical Methods for Stochastic Computations: A Spectral Method Approach, Princeton University Press, 2010.
[4] R. G. Ghanem, P. D. Spanos, Stochastic Finite Element Method, Dover Publications, 2002.
[5] D. Xiu, G. E. Karniadakis, The Wiener-Askey polynomial chaos for stochastic differential equations, SIAM Journal of Scientific Computing 24 (2002) 619–644.
[6] D. Colón, J. M. Balthazar, C. A. dos Reis, A. M. Bueno, I. S. Diniz, S. de S. R. F. Rosa, Control design and robustness analysis of a ball and plate system by using polynomial chaos, AIP Conference Proceedings 1637 (2014) 226–234.
[7] D. Colón, M. A. S. Ferreira, J. M. Balthazar, A. M. Bueno, S. de S. R. F. Rosa, Robustness analysis of an air heating plant and control law by using polynomial chaos, AIP Conference Proceedings 1637 (2014) 235–244.
[8] J. Fisher, R. Bhattacharya, Stability analysis of stochastic systems using polynomial chaos, in: Proceedings of the 2008 American Control Conference, 2008, pp. 4250–4255.
[9] A. Smith, A. Monti, F. Ponci, Robust controller using polynomial chaos theory, in: Industry Applications Conference, 2006. 41st IAS Annual Meeting. Conference Record of the 2006 IEEE, volume 5, 2006, pp. 2511–2517.
[10] F. Xiong, S. Chen, Y. Xiong, Dynamic system uncertainty propagation using polynomial chaos, Chinese Journal of Aeronautics 27 (2014) 1156 – 1170.
[11] T. Gerstner, M. Griebel, Numerical integration using sparse grids, Numerical Algorithms 18 (1998) 209–232.
[12] B. Sudret, Global sensitivity analysis using polynomial chaos expansions, Reliability Engineering & System Safety 93 (2008) 964 – 979. Bayesian Networks in Dependability.
[13] M. Eldred, Recent advances in non-intrusive polynomial chaos and stochastic collocation methods for uncertainty analysis and design, in: Structures, Structural Dynamics, and Materials and Co-located Conferences, American Institute of Aeronautics and Astronautics, 2009, pp. –.