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Damage detection in an uncertain nonlinear beam

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Abstract

The present paper proposes the use of discrete time Volterra series expanded with Kautz filters for predicting the experimental response of a nonlinear beam, taking into account uncertainties that are inherent to the system. Nonlinear behavior is simulated considering large amplitude of vibration in a clamped-free beam, with a magnet near the free end to cause cubic stiffness effect. The Volterra kernels and Kautz functions are estimated in a stochastic way, where the limits of confidence are established to the system response and used as a reference model to detect damage. The detection approach is based on a nonlinear stochastic index and hypothesis test, considering different levels of simulated damage, associated with loss of mass. The results have shown that considering the uncertainties when identifying the model is very important in the process of damage detection and the nonlinear metric proved to be more appropriated to describe the system response with better results mainly in the nonlinear regime of motion.

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Keywords: Nonlinear dynamics; Volterra series; Uncertainties quantification; SHM.

1. Introduction

The structural health monitoring (SHM) problem is more complicated in systems with intrinsic nonlinear behavior, because the nonlinearities can be confused with damage [1]. In addition, many mechanical systems can operate with strong nonlinear behavior that make them exhibit complex responses containing harmonic distortion, jumps, modal interactions, bifurcation, and possible chaos [2]. In this situation, it is necessary to predict the nonlinear behavior in the healthy condition and there is not a general approach to be applied in all nonlinear systems, once the nonlinearities have a lot of particularities. Furthermore, another source of errors in SHM is the presence of uncertainties, since any real system is uncertain with respect to the project nominal values, due to material imperfections, irregularities on the manufacturing process, noise in the measurements, etc. and the natural variability of systems can be confused with damage [3,4]. To overcome these problems, the Volterra series expanded in the Kautz orthonormal basis have been used to predict the dynamic responses of systems with nonlinear behavior [5,6]. However, in the previous papers,

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the nonlinear model considered was deterministic and here this approach is extended considering the uncertainties in data acquisition and the variabilities of the system over different days. The mainly advantage of this nonlinear robust model is the capability to differ the presence of damage for the nonlinear behavior and uncertainties. The system in study is a clamped-free beam with nonlinear behavior induced by the presence of a magnet near the free end, excited in two different levels, and the Volterra kernels are identified considering the data variation as stochastic reference model. The damage detection approach is based on the model limits and hypothesis test, considering different levels of damage simulated with loss of mass. The linear and nonlinear analysis are compared since the Volterra series allow the separation of linear and nonlinear contributions in the response. The nonlinear approach proved to be more adequate with a higher rate of damage detection.

2. The Stochastic Volterra Series

In this paper, a stochastic version of the Volterra series is used to identify the nonlinear system of interest. The details to the deterministic version of the Volterra series theory can be found in [6–9]. Since only the data uncertainties are considered in this work, a parametric probabilistic approach is employed [10,11]. So, the model parameters subjected to uncertainties, the experimental output signal and consequently the Volterra kernels, are described as random processes, defined on the probability space $(\Theta, \Sigma, \mathbb{P})$, where Θ is a sample space, Σ is a σ-algebra over Θ, and $\mathbb{P}$ is a probability measure. It is assumed that any random variable $\theta \in \Theta \mapsto \mathbb{Y}(\theta) \in \mathbb{R}$ in this probabilistic setting, with probability distribution $P_{\mathbb{Y}}(dy)$ on $\mathbb{R}$, admits a probability density function (PDF) $y \mapsto p_{\mathbb{Y}}(y)$ with respect to dy. The discrete time Volterra series can be used to represent different types of nonlinear systems using the convolution concept [7]. Considering the presence of uncertainties, the random output signal $\mathbb{y}(\theta, k)$ of a nonlinear system can be described by

$$\mathbb{y}(\theta, k) = \sum_{\eta=1}^{\infty} \sum_{n_1=0}^{N_1-1} \dots \sum_{n_\eta=0}^{N_\eta-1} \mathbb{H}_\eta(\theta, n_1, \dots, n_\eta) \prod_{i=1}^{\eta} u(k-n_i) = \mathbb{y}_1(\theta, k) + \mathbb{y}_2(\theta, k) + \mathbb{y}_3(\theta, k) + \cdots , \tag{1}$$

where $u(k)$ is the deterministic input signal, the random process $\mathbb{y}(\theta, k)$ is the uncertain response of the system, $(\theta, n_1, .., n_\eta) \in \Theta \times \mathbb{Z}^\eta \mapsto \mathbb{H}_\eta(\theta, n_1, \dots, n_\eta)$ represents the random version of the η-order Volterra kernel, $\mathbb{y}_1(\theta, k)$ is the linear contribution and $\mathbb{y}_2(\theta, k)$ e $\mathbb{y}_3(\theta, k)$ are the quadratic and cubic contributions of $\mathbb{y}(\theta, k)$.

The Volterra series' inconvenience is the difficulty in its convergence when a high number of terms is used $N_1, ..., N_\eta$. So, the kernels can be expanded in a base of orthonormal functions to minimize the problem, in this case, with the use of Kautz functions [12–14]. Now, the Volterra kernels can be defined by

$$\mathbb{H}_\eta(\theta, n_1, \dots, n_\eta) \approx \sum_{i_1=1}^{J_1} \dots \sum_{i_\eta=1}^{J_\eta} \mathbb{B}_\eta\left(\theta, i_1, \dots, i_\eta\right) \prod_{j=1}^{\eta} \psi_{i_j}(\theta, n_j) , \tag{2}$$

where $J_1, \dots, J_\eta$ are the number of samples in each orthonormal projections of the Volterra kernels, the random processes $(\theta, i_1, \dots, i_\eta) \in \Theta \times \mathbb{Z}^\eta \mapsto \mathbb{B}_\eta(\theta, i_1, \dots, i_\eta)$ represent the random Volterra kernels expanded in the orthonormal basis. The definition of Kautz functions is related to the dynamic response of the system $\mathbb{y}(\theta, k)$ and it depends on the physical parameters of the system (damping ratio and natural frequency). Then, the Kautz functions are assumed as random processes $(\theta, n) \in \Theta \times \mathbb{Z} \mapsto \psi(\theta, n)$. In this context, the Volterra series can be rewritten as the multidimensional convolution between the random orthonormal kernels $\mathbb{B}_\eta(\theta, i_1, \dots, i_\eta)$ and the deterministic input signal filtered by the random Kautz functions

$$\mathbb{y}(\theta, k) \approx \sum_{\eta=1}^{\infty} \sum_{i_1=1}^{J_1} \dots \sum_{i_\eta=1}^{J_\eta} \mathbb{B}_\eta\left(\theta, i_1, \dots, i_\eta\right) \prod_{j=1}^{\eta} \mathbb{l}_{i_j}(\theta, k) , \tag{3}$$

where the random process $\mathbb{l}_{i_j}(\theta, k)$ is a simple filtering of the deterministic input signal $u(k)$ by the random Kautz function $\psi_{i_j}(\theta, n_j)$. Then, for each realization θ, the terms of the orthonormal kernels $\mathbb{B}_\eta\left(\theta, i_1, \dots, i_\eta\right)$ can be grouped into a vector $\mathbf{\Phi}$ and estimated by the least squares method

$$\mathbf{\Phi} = (\mathbf{\Gamma}^T\mathbf{\Gamma})^{-1}\mathbf{\Gamma}^T\mathbf{Y} , \tag{4}$$

where the matrix $\mathbf{\Gamma}$ has the filtered input signal $\mathbb{l}_{i_j}(\theta, k)$ and $\mathbf{Y} = [\mathbb{y}(\theta, 1) \cdots \mathbb{y}(\theta, k)]$ for each realization θ, where k is the number of samples used. More information about the method can be found in [5,6,14].

2.1. Damage detection approach

The identified Volterra kernels can be used as the reference model for the system. The stochastic response of the system can be obtained through convolution between the input signal and the Volterra kernels given by (Eq. 3) and the envelope of reliability, with probability $P_c = 99\%$, can be calculated based on the percentiles. The reliability interval at discrete time k is defined by the set

$$\mathcal{I}_{99}(k) = y(k)|r^{-}(k) <= y(k) <= r^{+}(k)\,, \tag{5}$$

where $y(k)$ is the model response in each discrete time k, $r^{-}(k)$ and $r^{+}(k)$ are the inferior and superior limits of the interval, respectively. If only the first kernel is used $y(k) = y_{lin}(k) = y_1(k)$, it means linear analysis; and if the first three kernels are used $y(k) = y_{nlin}(k) = y_1(k) + y_2(k) + y_3(k)$, nonlinear analysis. For each discrete time k the distance between the experimental signal $y_{exp}(k)$ and the set $\mathcal{I}_{99}(k)$ can be calculated by

$$d_{lin}(k) = \inf\left\{|y_{exp}(k) - y_{lin}(k)| \;\; for\ all \;\; y_{lin}(k) \in \mathcal{I}_{99}(k)\right\}\,, \tag{6}$$

$$d_{nlin}(k) = \inf\left\{|y_{exp}(k) - y_{nlin}(k)| \;\; for\ all \;\; y_{nlin}(k) \in \mathcal{I}_{99}(k)\right\}\,. \tag{7}$$

So, by considering the calculated distances, a damage index can be computed, assuming the linear and nonlinear analysis

$$\lambda_{lin} = \frac{||d_{lin}||}{A}\,, \tag{8}$$

$$\lambda_{nlin} = \frac{||d_{nlin}||}{A}\,, \tag{9}$$

where $||.||$ is the Euclidean norm operator and the excitation amplitude A is used to normalize the index considering high and low levels of amplitude, making the comparison between the indexes in different conditions easier. If the structure is in reference condition it is expected that the index has low value because the distance between the experimental signal and the response confidence bands is zero or close to zero. The damage detection is done considering the hypothesis test

$$\begin{cases} H_0 : \lambda \leq \Lambda\,, \\ H_1 : \lambda > \Lambda\,, \end{cases} \tag{10}$$

where Λ is the threshold value that is determined based on the reference data. The reference data has unknown distribution, thus the threshold is determined considering the interquartile range

$$\Lambda = Q_3 + 1.5(Q_3 - Q_1)\,, \tag{11}$$

where Q_1 and Q_3 are the first and third quartiles and Λ is the upper quartile of the Box plot theory [15]. By applying this equation, every quantity above the Λ value is considered an outlier. In order to estimate the threshold value the index is applied to the reference data considering linear and nonlinear behavior. The next section presents an experimental example of this approach.

3. Application in a Nonlinear Beam

The experimental setup is the same used in [6]. It is composed by an aluminum beam with dimensions of 300×18×3 [mm] with a steel mass connected to the free end to cause a magnetic interaction. To simulate structural variations and study the proposed approach, a bolted connection is placed 150 [mm] from the free end with four nuts with 1 [g] each one. The healthy state considers the four nuts in the system and the damage is simulated with the loss of mass. Figures 1 and 2 show the experimental setup used. A MODAL SHOP shaker (Model Number: K2004E01) is attached 50 mm from the clamped and used to excite the structure considering different levels of voltage amplitude 0.01 V (low level), 0.10 V (medium level) and 0.15 V (high level). A vibrometer *laser Polytec* (Model: OFV-525/-5000-S) and a *Dytran* load cell (Model: 1022V) are used to measure the velocity at the free end of the beam and the force excitation, respectively. All signals are measured considering a sampling rate of 1024 Hz and 4096 samples saved using a m+p

Vib Pilot data acquisition system. The magnet interaction with the beam generates a nonlinear hardening behavior. This effect is illustrated on the beam velocity curves obtained during the stepped sine test shown in Fig. 3. When the input signal is high, the beam presents a jump phenomenon that confirms the nonlinear behavior of the system in this condition of excitation. More information about the hardening behavior and jump phenomenon can be found in [16].

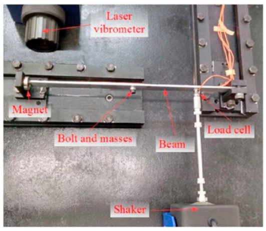


Fig. 1. Experimental setup (photo) [6].

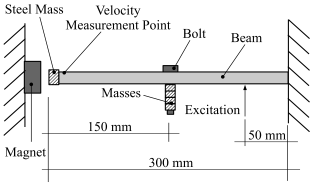


Fig. 2. Experimental setup (scheme) [6].

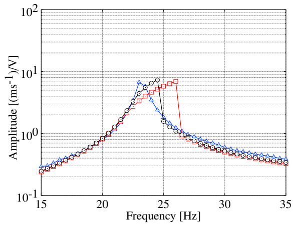
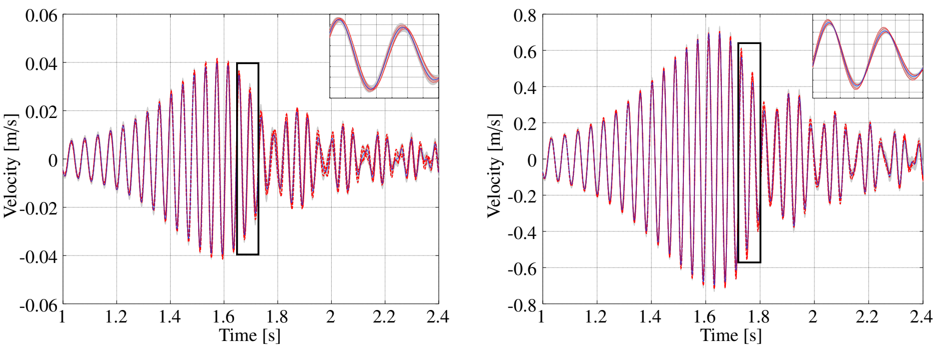


Fig. 3. Beam velocity during a stepped sine test, where △ represents the low level of input signal (0.01 V), ∘ the medium level (0.10 V) and □ the high level (0.15 V).

3.1. Volterra kernels identification

This section describes the model identification/validation through the Volterra kernels extraction. All tests used a sampling frequency of 1024 Hz and 4096 samples. The tests were repeated 156 times in different days during 2 weeks to obtain large variation of data. The estimation of Volterra kernels was done in two steps. First, a chirp signal varying the frequency from 10 to 50 Hz with rate of 10 Hz/s, considering low level of amplitude (0.01 V), was applied to estimate the first kernel (linear contribution). Then, the same chirp signal considering high level of amplitude (0.15 V) was applied to estimate the second and third kernels (nonlinear contribution).

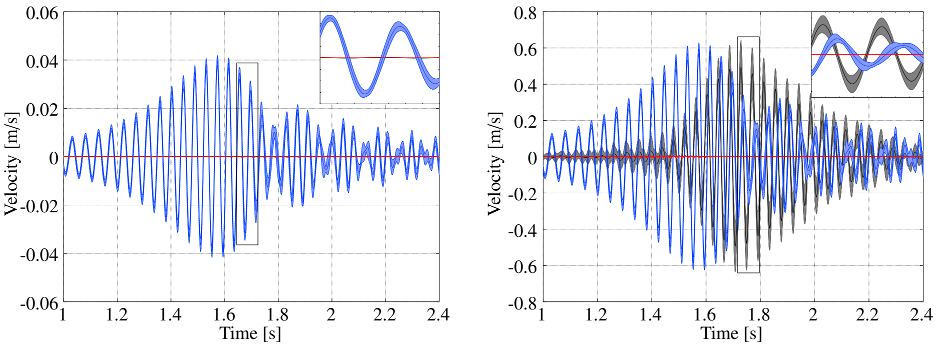
With the Volterra kernels estimated the system response can be obtained considering the convolution of the input signal with the kernels identified. Figure 4 shows the 99% confidence bands of the response in comparison with new experimental data to a chirp input varying the frequency from 10 to 50 Hz with rate of 10 Hz/s, considering two levels of amplitude (low - 0.01 V and high - 0.15 V). In both cases, linear and nonlinear behavior, the model is able to describe the responses, considering the data variations. The dispersion of the responses is probably caused by noise in the measurements, sensor position, actuator position, etc. since the tests were carried out in different days, precisely to consider a great dispersion of data, differently from what was done in [6], where only the statistical analysis of the damage index was enough to describe the dispersion of the responses without the uncertainties analysis in Volterra series.



(a) Low input. (b) High input.

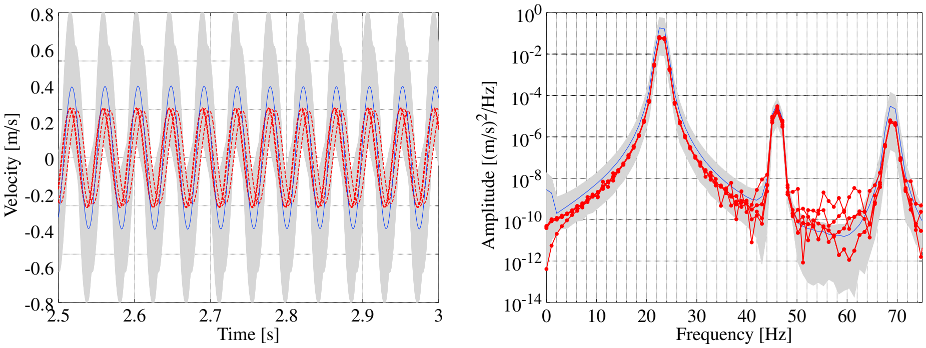
Fig. 4. Confidence bands for the system responses obtained through the Volterra model with 99% of confidence. The mean is represented by the blue line and the experimental data by the red dots.

As a great advantage, Volterra series allow the separation of linear and nonlinear contributions of the response through filtering by the kernels. Figure 5 shows the contributions of the first, second and third kernels with 99% of confidence, considering low and high input. When the system has linear behavior (low input - 0.01 V) only the first kernel contributes to the response. Now, with nonlinear behavior (high input - 0.15 V) the response is a sum of the first and third kernel contributions. The second kernel has no contribution because the response is approximately symmetric. The model validation was done considering a simple sine input in the region of the first mode shape ($\approx$ 23 Hz) with high level of input (0.15 V), considering new experimental data. Figure 6 shows the response in time and frequency domains with 99% of confidence bands. Multiples harmonics in the system response can be seen (nonlinear behavior) and the model is able to describe the multiple components in frequency domain. The frequency band in which the model is not able to describe the system behavior ($\approx$ 50 to $\approx$ 68 Hz) has low amplitude and little contribution in the response predicted.



(a) Low input. (b) High input.

Fig. 5. Confidence bands for the kernels contribution to the output signals obtained through the Volterra model with 99% of confidence. ■ represents the linear contribution, ■ represents the quadratic contribution and ■ represents the cubic contribution.



(a) Time domain. (b) Frequency domain.

Fig. 6. Confidence bands for the system responses obtained through the Volterra model with 99% of confidence. The mean is represented by the blue line and the experimental data are represented by the red line.

3.2. Damage detection

In order to include structural changes and to apply the stochastic Volterra series model in the damage detection, some mass reductions were analyzed by removing some nuts of the structure (Figs. 1 and 2). The conditions used are shown in the Tab. 1. To establish the threshold value, the method described in section 2.1 was applied considering 312 samples in reference condition with different excitation amplitudes. The threshold value was determined as $\Lambda = 1.14$. In each damage and repair condition, 156 samples were measured in different days to study the sensitivity of the method in presence of uncertainties.

The results in the Tab. 2 represent the percentages of damage detection through the hypothesis test applied in the two damage indexes (λ_{lin} and λ_{nlin}) considering two different levels of input applied (0.0 1 V - linear and 0.15 V - nonlinear behavior). The false alarm refers to the situation where the hypothesis test indicates damage and the system is in healthy/repair condition (states 0 and R). It can be seen that when it was used the linear index to detect damage and the structure was operating in nonlinear regime of motion the hypothesis test has shown that in every state the structure was damaged, i. e., the model can not differ the damage from the nonlinear behavior. Additionally, when the structure operated in linear regime of motion the effectiveness of the linear and nonlinear indexes has been the same, with no false alarms and 50% of true detection in the situation where just one mass was removed. The nonlinear index has presented better results when the structure operated with nonlinear behavior with 100% of true detection and no false alarms showing that the nonlinear index is more sensitive to detect damage.

Table 1. Structure variations.

State	Condition	State	Condition
0	4 masses (reference)	III	1 mass
I	3 masses	R	4 masses (repair)
II	2 masses	–	–

Table 2. Damage detection obtained through the hypothesis test applied to damage indexes considering linear and nonlinear regime of motion.

Condition	Linear regime (input - 0.01 V)		Nonlinear regime (input - 0.15 V)	
	Linear index (λ_{lin}) [%]	Nonlinear index (λ_{nlin}) [%]	Linear index (λ_{lin}) [%]	Nonlinear index (λ_{nlin}) [%]
0	0 ⇢ **false alarm**	0 ⇢ **false alarm**	100 ⇢ **false alarm**	0 ⇢ **false alarm**
I	50 ⇢ true detection	50 ⇢ true detection	100 ⇢ true detection	100 ⇢ true detection
II	100 ⇢ true detection	100 ⇢ true detection	100 ⇢ true detection	100 ⇢ true detection
III	100 ⇢ true detection	100 ⇢ true detection	100 ⇢ true detection	100 ⇢ true detection
R	0 ⇢ **false alarm**	0 ⇢ **false alarm**	100 ⇢ **false alarm**	0 ⇢ **false alarm**

4. Final remarks

The proposed methodology based on stochastic Volterra series had a satisfactory result considering the large data variation along the samples, mainly in the nonlinear regime of motion. The nonlinear approach has proved to be more efficient with better results principally in nonlinear regime of motion, when it was able to tell the difference between the nonlinear behavior compared to damage. Other indexes and hypothesis tests have to be studied to improve the approach whereas the estimation of the threshold based on reference samples is always subject to errors.

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References

[1] L. Bornn, C. R. Farrar, G. Park, Damage detection in initially nonlinear systems, International Journal of Engineering Science 48 (2010) 909 – 920. Structural Health Monitoring in the Light of Inverse Problems of Mechanics.
[2] J.-P. Noël, G. Kerschen, Nonlinear system identification in structural dynamics: 10 more years of progress, Mechanical Systems and Signal Processing 83 (2016) 2 – 35.
[3] C. Soize, A comprehensive overview of a non-parametric probabilistic approach of model uncertainties for predictive models in structural dynamics, Journal of Sound and Vibration 288 (2005) 623 – 652. Uncertainty in structural dynamics.
[4] G. Iaccarino, Quantification of uncertainty in flow simulations using probabilistic methods, Technical Report, DTIC Document, 2009.
[5] S. da Silva, Non-linear model updating of a three-dimensional portal frame based on Wiener series, International Journal of Non-Linear Mechanics 46 (2011) 312–320.
[6] S. B. Shiki, S. da Silva, M. D. Todd, On the application of discrete-time volterra series for the damage detection problem in initially nonlinear systems, Structural Health Monitoring 16 (2017) 62–78.
[7] M. Schetzen, The Volterra and Wiener Theories of Nonlinear Systems, Wiley, 1980.
[8] W. J. Rugh, Nonlinear system theory - The Volterra/Wiener Approach, Johns Hopkins University Press Baltimore, 1981.
[9] S. da Silva, S. Cogan, E. Foltête, Nonlinear identification in structural dynamics based on Wiener series and Kautz filters, Mechanical Systems and Signal Processing 24 (2010) 52–58.
[10] C. Soize, Stochastic models of uncertainties in computational mechanics, American Society of Civil Engineers Reston, 2012.
[11] C. Soize, Stochastic modeling of uncertainties in computational structural dynamics - recent theoretical advances, Journal of Sound and Vibration 332 (2013) 2379 – 2395.
[12] W. H. Kautz, Transient synthesis in the time domain, Circuit Theory, Transactions of the IRE Professional Group on CT-1 (1954) 29–39.
[13] P. S. Heuberger, P. M. Van den Hof, B. Wahlberg, Modelling and identification with rational orthogonal basis functions, Springer, 2005.
[14] S. da Silva, Non-parametric identification of mechanical systems by Kautz filter with multiple poles, Mechanical Systems and Signal Processing 25 (2011) 1103 – 1111.
[15] D. F. Williams, R. A. Parker, J. S. Kendrick, The box plot: A simple visual method to interpret data, Annals of Internal Medicine 110 (1989) 916–921.
[16] M. J. Brennan, I. Kovacic, The Duffing Equation - Nonlinear Oscillators and their Behaviour, 1 ed., John Wiley and Sons, Chichester, West Sussex, United Kingdom, 2011, pp. 25–43.