A Perturbation Approach to Modeling Eddy Current Signals of Compound Defects

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Abstract

Multiple defects located close to each other can significantly complicate eddy current testing. This is particularly problematic in applications like railway track inspection, where detecting and evaluating head checks (surface cracks in close proximity) is crucial. This paper proposes a novel approach using perturbation theory to model the interaction between multiple defects. By simulating single defects and applying perturbation theory, we can generate synthetic data for compound defects, enabling the training of machine learning algorithms for automated defect detection. This approach offers several advantages, including reduced computational cost, improved accuracy, and enhanced flexibility. In the case of rail testing it makes the systematic generation of synthetic head check data feasible in the first place. We present the theoretical framework of the perturbation approach, and validate for surface cracks in the context of rail testing using laboratory experiments and numerical simulations.

Keywords: Eddy current testing, non-destructive testing, synthetic data, perturbation theory

1 Introduction

Compound defects, that is defects in close spacial proximity, influence each others signals in eddy current testing. Possibly to a degree that individual signals are no longer distinguishable. This effect is of course highly dependent on the testing setup, in particular the eddy current probe and there is active research in the field of probe design to improve the resolution of compound defects, see for instance [6] or [4]. However, when working with established testing procedures and equipment the resolution can hardly be improved. One such area is the testing of railway tracks. Simultaneously, in rail testing one of the top priorities is the detection of Head Checks. These are cracks in the rail surface with a distance of 0.5 mm to 2 mm, often extending over several meter along the rail. At the same time the HC10 probe employed for rail testing in Germany has an interaction range of 10 mm. Thus we need to understand and model the interaction of up to 20 cracks in order to provide synthetic data. Due to this high complexity, 20 cracks each with several parameters sampled in a wide range, classical FEM simulation is prohibitively expansive.

Thus we propose to simulate single cracks only and use perturbation theory to generate compound defects. This approach is highly flexible and can be applied to various defect configurations and test scenarios, it reduces the computational cost compared to a full FEM simulation, and increases the model accuracy compared to simple signal addition. Thereby it enables the systematic generation of synthetic testing data for compound defects, which are used in large quantities for training machine learning based evaluation algorithms.

Furthermore, we carry out our approach for surface cracks in the context of railway rail testing. We use laboratory testing data as well as simulation data to fit free parameters and validate the method. The data used in this article is available here [3].

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2 Methodology

2.1 Perturbation Theory

The goal is to construct the signal of two or more cracks in close proximity from the signals of the individual cracks alone. This is done by adding the signals and then correcting with a convolution operator a follows. Let $s_1,\, s_2 \in C^0_c(\mathbb{R},\mathbb{C})$ be two compactly supported signals stemming from two separate crack defects. For

$$\begin{aligned} \phi &:= \begin{cases} \exp\left(\frac{-1}{1-x^2}+1\right) & \text{for } |x|>1 \\ 0 & \text{for } |x|\le 1 \end{cases}, \\ k_a(x) &:= a_0\left(\phi(a_2(x-a_1))+\phi(a_2(x+a_1))\right), \end{aligned}$$

and a perturbation parameter ϵ, we define the compound signal

$$s_{\text{total}} := s_1 \tilde{+} s_2 := s_1 + s_2 + \epsilon K(s_1,s_2), \text{ where} \tag{2.1}$$

$$K(s_1,s_2)(x) := \int_{\mathbb{R}} k_a(x-y)\,\mathrm{Re}(s_1(y))\,\mathrm{Re}(s_2(y)) + ik_b(x-y)\,\mathrm{Im}(s_1(y))\,\mathrm{Im}(s_2(y))\,dy. \tag{2.2}$$

Here the parameters $a=(a_1,a_2,a_3)$ and $b=(b_1,b_2,b_3)$ need to be determined from experimental data. To fit these parameters and validate the approach, we manufactured a reference specimen and carried out simulations of compound defects. Note also that in practice the signals and the integrals are of course discrete.

This approach ensures the key property that $s_1 \tilde{+} s_2 = s_1 + s_2$ if the support s_1 and s_2 is disjoint, i.e. if there is no interaction between the defects related to the signals. In particular, we have $s_1 \tilde{+} 0 = s_1$. Additionally, we have that $\tilde{+}$ is associative up to first order of ϵ.

$$\begin{aligned} \left(s_1 \tilde{+} s_2\right) \tilde{+} s_3 &= \left(s_1+s_2+\epsilon K(s_1,s_2)\right) \tilde{+} s_3 \\ &= s_1+s_2+s_3+\epsilon K(s_1,s_2)+\epsilon K(s_1+s_2,s_3)+O(\epsilon^2) \\ &= s_1 \tilde{+} \left(s_2+s_3+\epsilon K(s_2,s_3)\right)+O(\epsilon^2) \\ &= s_1 \tilde{+} \left(s_2 \tilde{+} s_3\right)+O(\epsilon^2) \end{aligned}$$

However, it is not distributive, i.e. scaling invariant, as K is bilinear. The (approximate) associativity is the other key property of this approach, as it allows us to successively add signals of single cracks, according to equation 2.1, to generate an extended compound signal.

2.2 Model Fit and Validation

Reference Specimen The reference specimen is manufactured from the underside of a cut off rail head. It has groups of defects consisting of straight, rectangular notches with a depth of 2 mm or 5 mm. There is one 2 mm and one 5 mm notch as reference. There are six groups where the notches are separated by 0.36 mm, 0.86 mm or 1.87 mm, and have a depth of either 2 mm and 5 mm or 5 mm and 5 mm. These groups are used to fit the parameters a and b in equation (2.1).

Further, there are two groups of three notches, separated by 0.86 mm. One with depths 2 mm, 5 mm, 2 mm and one with depths 5 mm, 5 mm, 5 mm. These are used to validate the approach. We focus here on the parameter ranges relevant for head checks, that is notches with a depth from 1 mm to 10 mm and distances between notches of 0.5 mm to 5 mm, see for instance [1] and [5]. We also note, that for a distance of more then 4 mm the naive signal addition and the perpetuation approach largely agree. Hence, we use lower distances for our reference Specimen.

Additionally, we carry out a larger associativity test by comparing $\left(s_1 \tilde{+} s_2\right) \tilde{+} s_3$ to $s_1 \tilde{+} \left(s_2 \tilde{+} s_3\right)$ directly. We choose the depth of the signals to be either 2 mm or 5 mm taking the single signals from the reference specimen. However, we vary their distance from 0.25 mm to 5.25 mm in steps of 0.5 mm, which results in 968 different combinations.

Simulation Experiment We perform two further tests to validate the perturbation approach with double defects with smaller depth. For one, we simulate two straight rectangular notches of width 0.1 mm, separated by 1 mm; one with fixed depth 2 mm and the other varying from 1 mm over 2 mm to 5 mm. Secondly, we simulate groups of two defects angled at 65° with respect to the z-axis. One with depth 1 mm and the other with depth 0.5 mm, 1 mm, or 2 mm. This angle of 65° is typical for cracks in railway rails.

The simulation was carried out as detailed in [2], using a mesh with a characteristic size of the skin depth of the reference specimen.

Error Measures We quantify the errors ϵ with respect to the discrete C^0 and L^2 norms as we did in [2]. We regard the eddy current signals s as continuous, compactly supported, complex valued functions and for a reference signal s_0 define

$$\begin{aligned}\epsilon_{L^2}(s) &:= \frac{\|s - s_0\|_{L^2}}{|\operatorname{supp}(s - s_0)|\operatorname{var}(s_0)},\\ \epsilon_{C^0}(s) &:= \frac{\|s - s_0\|_{C^0}}{\operatorname{var}(s_0)}.\end{aligned}$$

Here $\operatorname{var}(f) := \sup_{x,\,y \in I}(|f(x) - f(y)|)$ and supp is the support of a function, i.e. the set where it is non-zero.

Whenever possible we choose the laboratory data as reference signal s_0. To illustrate the effectiveness of the perturbation approach, we also report the errors of naive signal addition, that is for n signals $s_{\text{total, naive}} = \sum_{i=0}^{n} s_i$. The corresponding error is denoted by the "naive" subscript.

3 Results

3.1 Model Fit

We fit the free parameters of the perturbation approach on the double defects of reference specimen and find the following. Note that the perturbation parameter ϵ has been conflated with the parameters a_0 and b_0.

$$\begin{aligned}a &= (-0.30, 0.10, 0.22)\\ b &= (-0.21, 0.89, 0.53)\end{aligned}$$

The signal accuracy of the fits can be seen in table 1. For double defects d_1 and d_2 refer to the depth of the first and second defect, and x refers to the distance between them; always measured in mm. They are combined to the parameter tuple (d_1, d_2, x).

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(2, 5, 0.36)	8.8	27.7	4.4	8.8
(2, 5, 0.86)	10.4	26.9	6.0	7.7
(2, 5, 1.87)	22.7	35.4	14.2	9.5
(5, 5, 0.36)	4.2	28.0	2.3	8.7
(5, 5, 0.86)	7.8	21.3	4.2	6.1
(5, 5, 1.87)	7.3	19.1	3.7	5.4

Table 1: Error estimates for the fit on double defects

Table 1 clearly shows that the perturbation approach significantly reduces the C^0 of the signal compared to a naive addition. In one case the error is improved by a factor of 7 and by a factor of 3 on average. The L^2 error is also improved, except for one case.

3.2 Validation Error Estimates

We validate the perturbation approach with the triple defects of the reference specimen as well as with simulated double defects. The results for the simulated notches, table 2, and the simulated angled defects, table 3, are in very good agreement with each other and with the fit errors of table 1.

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(1.0, 2.0, 1.0)	11.2	17.4	7.0	4.9
(2.0, 2.0, 1.0)	3.4	11.6	1.6	3.9
(5.0, 2.0, 1.0)	11.9	19.1	6.4	5.3

Table 2: Validation error estimates for simulated double notches

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(0.5, 1.0, 1.0)	3.5	12.6	1.9	4.1
(1.0, 1.0, 1.0)	11.8	26.4	6.2	8.3
(2.0, 1.0, 1.0)	14.4	30.1	7.5	9.4

Table 3: Validation error estimates for simulated double angled defect

For triple defects the parameter tuple (d_1, d_2, d_3) refers to the depth of the first, second third defect. The distance between them is 0.86 mm and they are added in the order $\left(s_1 \tilde{+} s_2\right) \tilde{+} s_3$. The results in table 4 show similar errors to the other validation cases.

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(2, 5, 2)	5.9	32.1	2.9	10.5
(5, 5, 5)	9.2	39.4	4.6	12.1

Table 4: Validation error estimates on the triple notches, variant $\left(s_1 \tilde{+} s_2\right) \tilde{+} s_3$

Validation Error Estimates for the Associativity Since the associativity is of utmost importance for the generation of compounds of three or more defects we validate it separately. In table 4 we have already seen errors for triple defects. In tables 5 and 6 we present the errors for two different summation orders. The errors for the three different ways to add the signals are virtually the same, differing by at most 0.4 %pt.

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(2, 5, 2)	5.9	32.1	2.8	10.5
(5, 5, 5)	9.2	39.4	4.3	12.1

Table 5: Validation error estimates on the triple defects, variant $s_1 \tilde{+} \left(s_2 \tilde{+} s_3\right)$

Parameter	ϵ_{C^0} in %	$\epsilon_{C^0,\text{naive}}$ in %	ϵ_{L^2} in %	$\epsilon_{L^2,\text{naive}}$ in %
(2, 5, 2)	6.0	32.1	2.8	10.5
(5, 5, 5)	9.5	39.4	4.7	12.1

Table 6: Validation error estimates on the triple defects, variant $s_2 \tilde{+} \left(s_1 \tilde{+} s_3\right)$

The results for the systematic associativity test are presented in table 7. They show comparable, although slightly higher, errors compared to the differences between tables 4, 5, and 6.

error	min	max	mean
ϵ_{C^0}	0.00 %	2.32 %	0.71 %
ϵ_{L^2}	0.00 %	0.81 %	0.26 %

Table 7: Validation error estimates on the triple defects

4 Conclusion

The fit and validation errors for the perturbation approach are similar or lower then the expected simulation errors for the present eddy current setup, see [2]. Additionally, they are low with respect to other influences on the rail defect model accuracy, see [2] and [5]. Note also that the L^2 error is always significantly lower then C^0 meaning the the overall signal shape is well approximated and the error largely stems from the amplitude. This leads us to conclude that the first order perturbation approach for compound defects is well justified.
In tables 4, 5, and 6 we present the error estimates for the validation cases with three defects. We present the different combinations three signals can be added; always comparing the result data gathered in our laboratory. In every case the errors are comparable to the fit errors of table 1 and the differences between the three different ways to add the signals are negligible. This, together with the systematic associativity test (table 7), shows that we indeed have associativity up to second order and thus the successive addition of single defects in any order to model compound defects is valid.

In case higher model accuracy is required, we conjecture that higher order perturbation would yield improved error estimates.

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