# A Theoretical Proposal: Utilizing Quantum Walks for Feature Extraction in PSG-Based Sleep Stage Classification

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## I. Abstract

This work proposes a novel theoretical framework utilizing quantum walk principles for feature extraction from polysomnographic (PSG) signals, focusing on the classification of sleep stages. By mapping EEG signals to a quantum graph and leveraging the probabilistic and path-dependent properties of quantum walks, this approach offers a potentially transformative methodology for analyzing brain activity. Also discussing the potential solution to class imbalance in sleep datasets using the innate qualities of quantum mechanics like superposition, highlighting the possible use of a hybrid loss function for classification.Outlining the conceptual basis of this framework, discussing its implications, and suggesting potential applications and future research directions.

## II. Introduction

Sleep stage classification plays a vital role in understanding sleep disorders and their impact on health. Traditional methods rely on time-domain, frequency-domain, or machine learning-based feature extraction techniques, which can sometimes struggle with the complexity and non-linearity of EEG signals.

Quantum walks, a quantum analog of classical random walks, have demonstrated significant potential in various domains such as optimization, search algorithms, and network analysis. Their inherent probabilistic and non-classical characteristics make them a promising candidate for modeling complex biological signals. This paper introduces a theoretical framework that applies quantum walk principles to extract features from EEG signals for sleep stage classification.

### *Background*

#### EEG and Sleep Stage Classification

EEG signals represent brain activity through electrical oscillations, typically analyzed in conjunction with other PSG signals like EMG, EOG and ECG found in various public sleep stage datasets . The classification of sleep stages—wake, NREM (stages N1, N2, N3), and REM—is essential for diagnosing sleep disorders. For example ,The lack of REM (Rapid Eye Movement) sleep is considered an onset indicator of Parkinson's disease. Specifically, individuals with REM sleep behavior disorder (RBD), which often occurs before the motor symptoms of Parkinson's manifest, may be at a higher risk for developing the disease later on.
The main challenge of analyzing these signals is that the transition between these stages is highly dynamic and context-dependent, posing challenges for traditional feature extraction methods.

**Quantum Walks**

Quantum walks are mathematical models describing the evolution of a quantum particle over a graph. Unlike classical random walks, quantum walks exhibit superposition, interference, and entanglement, allowing for richer representations of dynamic systems. A quantum walk is defined on a graph where nodes represent states and edges define transition probabilities, governed by unitary operators.

# Notation and Terminology

1. $\mathbf{U=S\cdot (I\otimes C)}$

   U: Unitary operator governing the evolution of the quantum walk.

   S: Shift operator acting on the position space.

   I: Identity operator acting on the position space.

   C: Coin operator acting on the internal degree of freedom.

   $\otimes$: Tensor product, combining the operations on the position and internal spaces.

2. $\mathbf{|\psi(t+1)\rangle = U|\psi(t)\rangle}$

   $|\psi(t)\rangle$ : Quantum state of the walker at time step t

   $|\psi(t+1)\rangle$ : Quantum state of the walker at the next time step, t+1

   U: The evolution operator that updates the state.

3. $\mathbf{|\psi(0)\rangle = \sum_{v\in V} \alpha_v |v\rangle}$

   $|\psi(0)\rangle$ : Initial quantum state of the walker at time step t=0

   V: Set of vertices (positions) in the space.

   $|v\rangle$ : Basis state corresponding to the position v

$\alpha_{\mathbf{v}}$ : Complex amplitude for position v , determining the probability of the walker being at that position

4.

$$\mathbf{C} = \sum_{\mathbf{f}} \mathbf{e}^{\mathbf{i}\phi(\mathbf{f})} | \mathbf{f} \rangle \langle \mathbf{f} | ,$$

C: Coin operator acting on the internal degree of freedom (e.g., spin, polarization).

$| f \rangle$ : Basis state corresponding to the internal degree of freedom (frequency component).

$\phi(f)$: Phase associated with the frequency component f

$\mathbf{e}^{\mathbf{i}\phi(\mathbf{f})}$ : Phase factor applied to the state $| f \rangle$ .

$\langle f |$ : Bra corresponding to the internal state $| f \rangle$ , used for projection.

5.

$$\mathbf{S} = \sum_{\mathbf{v}} | \mathbf{v} \rangle \langle \mathbf{v} | \otimes \mathbf{S}_{\mathbf{v}},$$

S: Shift operator acting on both position and internal spaces.

$| v \rangle \langle v |$ : Projector onto the position state v .

$S_v$ : Shift operation applied to the internal degree of freedom at position v .

$\otimes$: Tensor product, indicating that S acts on both position and internal spaces.

# III. Theoretical Framework

### 3.1 Quantum Walk Model for PSG Signals

To apply quantum walks to PSG signals, the proposal is to model the EEG time series as a discrete-time quantum walk on a graph. Each node in the graph represents a feature or a characteristic of the EEG signal, and the edges represent relationships or correlations between these features.

Let ( $G = (V, E)$ ) be a graph where ( $V$ ) is the set of nodes and ( $E$ ) is the set of edges. The quantum walker moves on this graph, with its state evolving according to the quantum walk dynamics.

#### 3.1.1 Initialization

The initial state of the quantum walker can be set based on the starting point in the EEG signal or predefined features. For example, it could begin at a node representing the dominant frequency band in the EEG.

#### 3.1.2 Coin Operator

The coin operator determines the direction of movement. In the context of PSG signals, this could be designed to reflect the spectral properties of the EEG. For instance, the coin operator could be constructed using the Fourier transform of the EEG signal, allowing the walker to explore different frequency components.

#### 3.1.3 Shift Operator

The shift operator moves the walker to adjacent nodes based on the outcome of the coin toss. In the PSG graph, this would correspond to transitioning between related features or signal characteristics.

### 3.2 Feature Extraction using Quantum Walks

The key idea is that the quantum walk can efficiently explore the feature space defined by the PSG signals, potentially identifying complex patterns and correlations that are not easily captured by classical methods.

#### 3.2.1 Probability Distribution Analysis

By analyzing the probability distribution of the quantum walker over the graph nodes after a certain number of steps, we can extract features that represent the significance of different signal characteristics.

For example, nodes with higher probabilities might correspond to prominent features in the EEG, such as sleep spindles or delta waves, which are indicative of specific sleep stages.

#### 3.2.2 Interference Patterns

Quantum interference can enhance or suppress certain paths in the graph, leading to more pronounced features in the probability distribution. This could help in highlighting relevant signal components for sleep staging.

### 3.3 Integration with Sleep Stage Classification

Once features are extracted using quantum walks, they can be used as input to machine learning classifiers for automated sleep stage classification.

Potential classifiers include support vector machines (SVM) or Sequential neural networks. The performance of these classifiers can be evaluated using standard metrics such as accuracy, precision, recall, and F1-score.

## Mapping EEG Signals to a Quantum Graph

1. **Node Representation**: EEG channels or signal features (e.g., power in frequency bands) are mapped to nodes of a graph.
2. **Edge Weights**: Transition probabilities between nodes are determined by signal dynamics, such as phase coherence or cross-correlation.
3. **Graph Structure**: Dynamic connectivity patterns in EEG signals (e.g., functional connectivity) define the topology of the graph.

## Quantum Walk Dynamics for Feature Extraction

1. **Initialization:** A quantum state is initialized, representing the signal's starting condition. The initialization process can be informed by the data, as demonstrated in the following hybrid quantum walk circuit implementation:

```python
# Example Implementation using Pennylane
import pennylane as qml
from pennylane import numpy as np

num_position_qubits = 3  # For 8 nodes
num_coin_qubits = 1
num_qubits = num_position_qubits + num_coin_qubits

dev = qml.device("default.qubit", wires=num_qubits)

@qml.qnode(dev)
def hybrid_quantum_walk_circuit(data, steps=3):
    # Initialize the quantum state based on input data
    for i in range(num_position_qubits):
        if data[i] > 0:
```

```
            qml.Hadamard(wires=i)
        else:
            qml.PauliX(wires=i)

    # Encode coin qubit based on signal properties
    qml.RY(np.pi * data[-1], wires=num_position_qubits)  # Example encoding

    # Perform the quantum walk for a given number of steps
    for _ in range(steps):
        # Apply the coin operator
        qml.Hadamard(wires=num_position_qubits)

        # Apply the shift operator based on graph structure
        # For example, conditional shifts based on coin state
        for control in range(num_position_qubits - 1):
            qml.CNOT(wires=[control, control + 1])

        # Potentially more complex shift operations based on graph adjacency

    # Measurement: expectation values for features
    return [qml.expval(qml.PauliZ(i)) for i in range(num_qubits)]

# Example data: binary vector representing signal presence and a feature for coin encoding
data = np.array([1, 0, 1, 0.5])  # Last value is for coin qubit

result = hybrid_quantum_walk_circuit(data)
print(result)
```

2. **Walk Evolution:** The quantum walk evolves over the graph according to predefined Hamiltonians or unitary operators.
3. **Measurement:** Probabilities of quantum states at nodes are measured after a specified number of steps, providing features reflecting the temporal and spatial dynamics of the signal.

## ***Addressing Class Imbalance in Hypnogram Data***

Class imbalance in sleep stage annotations is a common challenge in EEG sleep studies, where certain stages (e.g., N3 or REM) are underrepresented compared to others (e.g., wake or N2). Quantum walks **can possibly** inherently handle class imbalance through their probabilistic and superpositional nature:

1. **Weighted Initialization:** Nodes corresponding to underrepresented classes can be assigned higher weights during initialization, ensuring their contributions are amplified in the quantum state evolution.

2. **Path Exploration:** Quantum walks explore multiple paths simultaneously, allowing underrepresented patterns to emerge through constructive interference, thereby mitigating the dominance of overrepresented classes.
3. **Dynamic Measurement:** Probabilistic measurements can be tuned to emphasize rare transitions or underrepresented nodes, capturing features critical for balanced classification.

***Integration with Temporal Smoothness and Cross-Entropy Loss***

Combining temporal smoothness loss and cross-entropy loss has proven effective for EEG-based sleep stage classification by leveraging temporal dynamics and classification accuracy. Quantum algorithms can integrate with these losses in hybrid frameworks:

**1. Temporal Quantum Walks:** Quantum walks can explicitly model temporal relationships between sequential EEG segments, naturally incorporating smoothness constraints.
**2. Quantum Cross-Entropy Optimization:** Features extracted via quantum walks can be integrated into classical neural networks trained with cross-entropy loss. The interference properties of quantum walks can potentially highlight transitions between classes more effectively, enhancing classification boundaries.
**3. Hybrid Loss Functions:** A hybrid loss combining quantum-derived features with classical losses could leverage strengths from both domains, improving overall classification performance.

# IV. Mathematical Formulations

## Quantum Walk on Graphs

A discrete-time quantum walk is defined on a graph **( G = (V, E) )**, where:

**- Nodes ( V )** : Represent features or characteristics of the EEG signal.

**- Edges ( E)** : Represent relationships or correlations between these features.

The state of the quantum walker at time step **t** is represented by a quantum state vector $|\psi(t)\rangle$ . The evolution of this state is governed by a unitary operator:

$\mathbf{U=S\cdot (I\otimes C)}$ ,where

where:

- I is the identity operator acting on the position space of the walker.
- C is the coin operator, which acts on the internal degree of freedom (e.g., spin, polarization).
- S is the shift operator, which moves the quantum walker in the position space based on its internal state.

This unitary operator U describes the combined action of the coin toss (C) and the position shift (S) that dictates the evolution of the quantum walk.

- **Coin Operator ( C )**: Acts on the coin space and determines the direction of movement.

- **Shift Operator ( S )**: Moves the walker to adjacent nodes based on the coin state.

Mathematically, the quantum walk can be described as:

$$|\psi(t+1)\rangle = U|\psi(t)\rangle,$$

where the evolution operator **U** is defined as:

$$U = S\cdot(I\otimes C)$$

Here, ( I ) is the identity operator on the position space, and ( C ) is the coin operator the internal degree of freedom.

**Initialization**

The initial state $|\psi(0)\rangle$ can be defined based on the starting point in the EEG signal. For instance, if we consider the dominant frequency band in the EEG, the initial state could be a superposition over nodes corresponding to those frequencies.

$$|\psi(0)\rangle = \sum_{v\in V} \alpha_v |v\rangle$$

where $\alpha_v$ are coefficients determined by the signal's properties.

**Coin Operator**

The coin operator can be designed using the Fourier transform of the EEG signal, allowing the walker to explore different frequency components. For example:

$$C = \sum_{f} e^{i\phi(f)} |f\rangle\langle f|,$$

where **( f )** represents frequency components, and $\phi(f)$ is a phase associated with each frequency.

**Shift Operator**

The shift operator moves the walker to adjacent nodes based on the coin state. If the graph is defined such that edges connect related features (e.g., similar frequencies, spatially adjacent electrodes), the shift operator can be defined accordingly.

$$S = \sum_{v} | v \rangle \langle v | \otimes S_v,$$

where $S_v$ describes the movement from node **v** to its neighbors.

**Defining Nodes and Edges**

**Nodes**

- **Frequency Bands** : Nodes can represent different frequency bands (delta, theta, alpha, beta, gamma).

- **EEG Channels** : Each EEG channel can be a node, reflecting activity at specific scalp locations.

- **Feature Extracted from EEG** : Features such as power spectral density, wavelet coefficients, or entropy measures can be nodes.

**Edges**

- **Correlation between Frequency Bands** : Edges can represent the correlation or coherence between different frequency bands.

- **Spatial Connectivity** : For EEG channels, edges can represent functional connectivity or phase synchronization between electrodes.

- **Temporal Relationships** : Edges can model temporal dependencies or transitions between different signal states.

↠ **Specific Features Represented**

- **Power in Frequency Bands** : Nodes can represent the power in specific frequency bands, with edges indicating cross-frequency interactions.

- **Sleep Spindles and K-complexes** : Specific nodes can be dedicated to detecting these characteristic waveforms, with edges representing their co-occurrence or sequence.

- **Temporal Dynamics** : Nodes can capture temporal features such as signal derivatives or trends over time.

**Hybrid Loss Function**

A hybrid loss function combining temporal smoothness and cross-entropy losses can benefit from quantum walk integration in several ways:

1. **Feature Extraction** : Quantum walks can extract features that better capture the temporal dynamics of EEG signals, making them more suitable for *temporal smoothness loss*.

2. **Class Imbalance Handling** : As discussed earlier, quantum walks can inherently handle class imbalance through their probabilistic nature, which can be beneficial when combined with cross-entropy loss that is sensitive to class distribution.

3. **Optimization Landscape** : The interference patterns in quantum walks might lead to a more favorable optimization landscape for the hybrid loss function, potentially reducing overfitting and improving generalization.

**Number of Qubits**

Considering you have 8 different signals (let's say from a publicly available sleep stage dataset), the number of qubits required depends on how these signals are encoded into the quantum system:

- **Signal Features** : If each signal is represented by multiple features (eg., power in different frequency bands), more qubits would be needed.

- **Graph Size** : The size of the graph (number of nodes) determines the number of position qubits required.

Assuming a simple scenario where each of the 8 signals corresponds to a node, and you use additional qubits for the coin space, you might need:

- **Position Qubits** : $\mathbf{\log_2(8) = 3}$ qubits to represent **8** nodes.

- **Coin Qubits** : Depending on the dimensionality of the coin space; for a simple two-way walk, 1 qubit suffices.

Thus, for each walker, you might need **3 + 1 = 4 qubits.** If multiple walkers or more complex encodings are needed, the total number of qubits would increase accordingly.

***Robust experimental validation and optimization will be necessary to fully realize the potential of this approach.***

## V. Discussion

### Potential Applications

**1. Enhanced Feature Representation:** Quantum walk-based features could capture intricate dependencies in EEG signals, improving classification accuracy for sleep staging.
**2. Transferable Framework:** The same methodology may be applied to other biomedical signals (e.g., ECG, fMRI) where graph-like representations are meaningful.
**3. Real-Time Analysis:** Due to the parallel nature of quantum processes, the proposed framework could facilitate real-time feature extraction for portable sleep monitoring devices.

### Theoretical Implications

- **Complexity Modeling:** Quantum walks can model non-linear and stochastic patterns in EEG data, offering a new perspective on brain dynamics.
- **Interdisciplinary Potential:** Integrating quantum computing principles into neuroscience can spur cross-disciplinary innovations.
- **Computational Advantages:** Leveraging quantum computing hardware (if available) could make the proposed method computationally efficient compared to classical techniques.

### Challenges and Future Directions

1. **Validation and Verification :** The effectiveness of quantum walk-based feature extraction must be thoroughly validated against classical methods using large, diverse datasets.
2. **Scalability:** Extending the framework to multi-modal PSG signals and large datasets needs exploration.
3. **Quantum Hardware Integration:** Practical implementation requires advances in quantum computing hardware for scalable simulations.
4. **Mathematical Refinement:** Developing robust mathematical models for graph construction and quantum walk evolution tailored to EEG data.
5. **Signal-to-Noise Ratio (SNR)**: EEG signals are notoriously noisy due to various biological and environmental factors. The presence of noise in the input data can exacerbate the issues caused by quantum processing errors. Therefore, preprocessing steps to improve SNR may be necessary before applying quantum walks.

## VI. Conclusion

This theoretical framework introduces quantum walks as a novel methodology for feature extraction in EEG signal analysis, emphasizing their potential in sleep stage classification. By presenting a conceptual roadmap, we aim to inspire future research exploring the practical implementation and validation of this approach, paving the way for interdisciplinary advancements in neuroscience and quantum computing.

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