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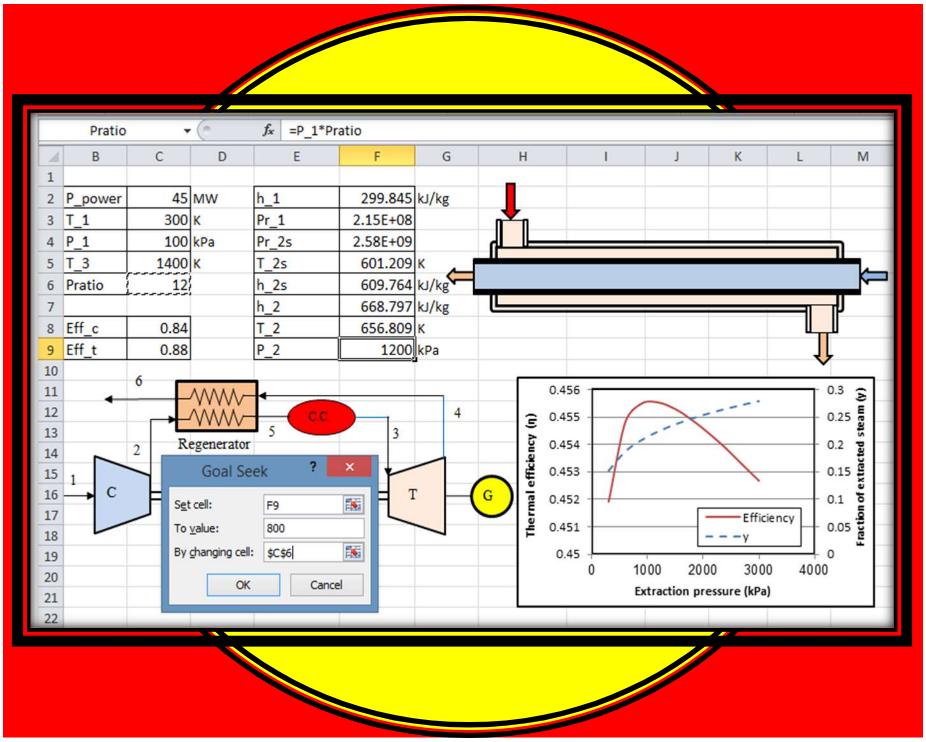
Computer-Aided Analyses and Optimisation of Fluid-Thermal Systems Using Excel



Mohamed M. El-Awad

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Preface

The three thermofluid subjects (thermodynamics, fluid mechanics, and heat-transfer), which are fundamental components of the curricula for most engineering specialisations, are traditionally taught by using data tables for fluid properties and charts for approximate analytical solutions of complex systems and design-based analyses. Although the repetitive use of tables and charts can help the students to acquire the three lower-order thinking skills (LOTS); Remember, Understand, and Apply, it is not suitable for conducting sensitivity and design optimisation type of analyses that help them to acquire the higher-order thing skills (HOTS); Analyse, Evaluate, and Create. In this regard, computer-aided learning (CAL) methods can be more effective as proved by research conducted at a number of top-class universities. Realising the benefits of CAL, most thermofluid textbooks now include computer-based exercises and mini-projects. CAL methods are also more suitable than traditional methods for online teaching that has now become a necessity rather than a choice. Although a number of commercial software are available and can be used for thermofluid subjects, these might not be available or affordable to many students and academic institutions particularly in developing countries. General-purpose spreadsheet applications, such as Microsoft Excel, can be the ideal solution to this problem.

With its wide availability on personal computers, Excel has been used as a graphical or computational tool in various engineering courses. Numerous papers on the use of Excel as a teaching aid have also been published in relevant journals or presented at specialised conferences over the past two decades. However, most of the previous experiences and efforts, if not all, focus on a particular type of analyses or a particular feature of Excel and do not utilise the full capability of Excel as an effective educational modelling platform. Moreover, the relevant papers are written in the form of short research articles and do not describe a general approach or provide sufficient information for using Excel as a modelling platform for the various types and levels of thermofluid analyses. This book complements the previous efforts in this regard by first describing an Excel-based modelling platform that is adequate for conducting various types of computer-aided thermofluid analyses and then showing how the platform can be used to deal with each type of these analyses.

The Excel-based modelling platform described in the book has four elements; (i) Excel's user-interface and built-in functions, (ii) the Solver add-in that comes with Excel, (iii) Microsoft Office Visual Basic for Applications (VBA) programming language, and (iv) an Excel add-in called Thermax that provides numerous VBA functions for fluid properties as well as a couple of useful numerical tools. While Excel's user-interface and Solver are adequate for most fluid-dynamics and heat-transfer analyses presented in this book, the Thermax add-in is used to perform the few thermodynamic analyses involved. VBA is needed for the development of custom functions when the analytical model cannot be executed by only using Excel's built-in functions and Thermax functions. Proper use of the Excel-based modelling platform minimises the effort of developing the analytical models for thermofluid analyses so that more attention can be paid to the application of the relevant principles.

For effective delivery of the material, this book adopts a learning-by-example approach and many of the examples given in the book have been adopted from standard thermofluid textbooks so that the students can verify their Excel solutions and look for any additional information if required. Exercises are given at the end of most chapters of the book to sharpen the students' skills related to the particular topic and more challenging exercises and mini projects are provided as an appendix. Certain topics that supplement the main chapters but require detailed treatments are also provided in the form of appendices. The material presented in the book provides sufficiently detailed and organised information for a stand-alone course on computer-aided analyses and optimisation of fluid-thermal systems. Such a course is commonly offered at an intermediate study stage of various engineering specialisations with the title "Computer Applications for Engineers". Individual chapters of the book can also be used to supplement the existing courses of fluid mechanics and heat-transfer.

Acknowledgements

The writing of this book would not have been possible without benefitting from the efforts of many colleagues who have made their information and software available in the open literature or on their websites. In this respect, a special gratitude goes to the Mechanical Engineering Department at the University of Alabama (USA) whose initiatives both inspired and helped me throughout the preparation of the book. I am also indebted to my colleagues and students at the University of Khartoum (Sudan) and the University of Technology and Applied Sciences (Oman) and hope that they find this book a worthy token of appreciation and gratitude. Last, but not least, I am grateful to my beloved family, Ghada, Ahmed, and Ula, for their unfailing support needed at various stages of this work.

CONTENTS

Nomenclature

A	Area, m^2
C	Friction coefficient in Hazen-Williams equation, Equation (1.18)
c_p	Specific heat at constant pressure, kJ/kg·°C
c_v	Specific heat at constant volume, kJ/kg·°C
D	Diameter, m
e	Energy per unit mass of the system
E_G	Total energy generated in the system
e_G	Energy generated per unit volume
f	Darcy friction factor, defined by Equation (1.3)
f	Manning friction factor, defined by Equation (1.9)
F_D	Drag force, defined by Equation (B.2)
g	Acceleration of gravity
h	Average heat-transfer coefficient
h	Enthalpy, kJ/kg
h_f	Major friction in a pipe system
k	Thermal conductivity, W/m·°C
k	Isentropic exponent, dimensionless ($k=c_p/c_v$)
K	Minor losses friction coefficient in a pipe system, defined by Equation (1.11)
L	Length, m
m	Mass, kg
$\dot{m}$	Mass rate of flow, kg/s
P	Pressure, usually in kPa
P_r	Relative pressure (for an ideal gas)
q	Heat-transfer per kg of the working fluid, usually kJ
Q	Heat, usually in kJ
Q	Volume flow rate, m^3/s
$\dot{Q}$	Rate of heat transfer, W or kW
r	Radius or radial distance
R	Gas constant, kJ/kg.K
R_{th}	Thermal resistance, usually °C/W
R_u	Universal gas constant kJ/kmol.K
s	Entropy
T	Temperature
u	Internal energy per unit mass, kJ/kg
U	Overall heat-transfer coefficient of a heat-exchanger
v	Specific volume, m^3/kg
V	Velocity, usually m/s
w	Work-done per kg of the working fluid, usually kJ
$\dot{W}$	Power, W or kW
x,y,z	Space coordinates in the Cartesian system

Greek Characters

γ	Specific weight of a fluid
δ	Thickness (e.g. of insulation)
Δ	Difference (e.g. temperature)
ε	Roughness of surface material
ε	Heat-exchanger effectiveness
η	Efficiency
μ	Dynamic viscosity, kg/m.s
v	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
τ	Time, annual operating hours of a system
τ	Mesh Fourier number in the finite-difference method, defined by Equation (7.6)

Dimensionless Numbers and Groups

Bi	Biot number
C_f	Friction coefficient, defined by Equation (B.3)
Fi	Fourier number
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number

Subscripts

f	Saturated liquid condition
fg	Difference in property between saturated liquid and saturated vapour
g	Saturated vapour condition
lm	Log-mean
s	Saturation temperature or pressure
s	Evaluated at the surface
∞	Evaluation at free-stream ambient conditions

1

Introduction

The electrical energy required to operate the refrigerators, air-conditioners, and many other essential appliances in our daily life mainly comes from power-generation plants that burn fossil fuels. Fossil fuels are also the main source of energy in the industrial and transport sectors. Apart from being non-renewable, large-scale combustion of fossil fuels is the main cause for the increasingly devastating effects of global warming at different parts of the world. Proper design of energy-conversion and utilisation systems, which is crucial for minimising these effects, is mainly based on the principles of the three thermofluid subjects: *thermodynamics*, *fluid mechanics*, and *heat transfer*. The two main principles that are commonly shared by the three thermofluids subjects are the conservation of mass and conservation of energy that are used to obtain mathematical equations for analysing the relevant systems and processes. While thermodynamic analyses usually assume steady-state and isotropic fluid properties, fluid mechanics and heat-transfer analyses have to take into consideration the temporal and spatial variations of the fluids properties. Therefore, they lead to multi-dimensional and/or non-linear equations which are difficult to solve without using computer-aided methods. This chapter reviews the main principles of fluid mechanics and heat transfer, highlights the advantages of computer-aided thermofluid analyses, and describes the Excel-based modelling platform used throughout this book for these analyses.

1.1. A brief review of fluid-dynamics and heat-transfer

The conservation of mass (the continuity equation) and the conservation of energy (the first-law of thermodynamics) that form the framework for thermofluid analyses take different mathematical forms depending on whether the system under consideration is open or closed and whether the flow is steady or unsteady. Since the flow can be laminar or turbulent, compressible or incompressible, etc., and the heat-transfer can be by conduction, convection, or radiation, the analyses need numerous auxiliary relationships in order to quantify the various parameters involved in the main equations such as pressure-variations, friction losses, and rates of heat-transfer. Considering the wide scope of the topic, the main concepts of fluid dynamics and heat-transfer are reviewed here by considering typical applications of each subject.

1.1.1. Fluid-dynamics

In addition to pipes and ducts, fluid-transporting systems require various equipment such as pumps, compressors, control valves, and flow-measuring devices. The principles of *fluid dynamics* help us to estimate the power needed for overcoming the friction in these systems and to determine the suitable types and sizes of their equipment. To illustrate the application of these principles, consider the pump-pipe system shown on Figure 1.1 that conveys a liquid between two non-pressurised tanks *A* and *B* through a pipe of known length *L*, diameter *D*, and roughness *ε*. Suppose that we want to determine the pump power needed for transporting the liquid at a certain flow rate *Q*. The required pump power ($\dot{W}$) can be determined from the following "power equation":

$$\dot{W} = \gamma \times Q \times h_p / \eta \qquad [\mathrm{W}] \qquad (1.1)$$

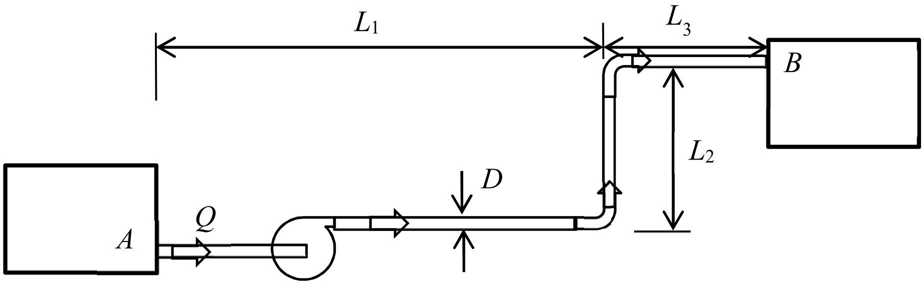

Figure 1.1. Schematic diagram of a simple pump-pipe system

where γ is the specific weight of the transported liquid (N/m^3), Q is the volume flow rate of the liquid (m^3/s), h_p is the pump head (m) needed to circulate the fluid through the pipe from A to B, and η is the combined efficiency of the pump and the electric motor. For a steady flow of an incompressible fluid, h_p can be determined from the "energy equation" as follows:

$$h_p = h_{f,total} + (Z_B - Z_A) + \frac{V_B^2 - V_A^2}{2g} \qquad \text{[m]} \tag{1.2}$$

where $h_{f,total}$ is the total head loss through the system due to friction (m), Z_A and Z_B are the elevations (m) at points A and B, respectively, and V_A and V_B are the corresponding fluid velocities (m/s). In case the two tanks are not open to the atmosphere, the energy equation should also include a term for the pressure difference between the tanks.

The total friction head loss $h_{f,total}$ consists of two parts: the *major friction loss* (h_f), which is the part that is lost in the pipe itself, and the *minor friction head loss* (h_c), which is the part lost in the equipment like nozzles, elbows, valves, etc. The major friction loss can be determined from the following Darcy-Weisbach equation [1]:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \qquad \text{[m]} \tag{1.3}$$

where f is the dimensionless Darcy friction factor, V the fluid velocity (m/s), L the total length of the pipe (m), and D the internal diameter of the pipe (m). The value of the friction factor, which depends on the roughness of the pipe surface and on whether the flow is laminar or turbulent, can be obtained from a Moody diagram [1] or calculated from a relevant formula. For laminar flows, f can be calculated from:

$$f = 64/\mathrm{Re} \qquad \mathrm{Re} < 2300 \tag{1.4}$$

where Re is the Reynolds number defined as:

$$\mathrm{Re} = VD/\nu \tag{1.5}$$

where ν is the kinematic viscosity of the flowing fluid (m^2/s). For a turbulent flow in rough pipes, f can be obtained from the following Swamee-Jain formula [1]:

$$f = 0.25/\left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2 \qquad \mathrm{Re} > 4000 \tag{1.6}$$

For more accuracy, the friction factor for a turbulent flow can be determined by using the following Colebrook-White formula (frequently referred to as the Colebrook equation):

$$\sqrt{\frac{1}{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1.7}$$

The Colebrook equation is an example of the implicit equations frequently met in thermofluid analyses that need to be solved iteratively. For turbulent flows in smooth tubes, f can be determined from the first Petukhov formula [1]:

$$f = \left(0.790\ln(\mathrm{Re}) - 1.64\right)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{1.8}$$

Chemical engineers usually determine the pipe friction by using the following Chezy-Manning equation instead of the Darcy-Weisbach equation:

$$h_f = 2f\frac{L}{D}\frac{V^2}{g} \qquad [\mathrm{m}] \tag{1.9}$$

where f is the Fanning friction factor. Comparison with Equation (1.3) reveals that the value of the Fanning friction factor is four times the corresponding value of the Darcy friction factor. Civil engineers determine the friction head loss in water-transporting pipes by using the following Hazen-Williams equation:

$$h_f = \frac{10.67\,LQ^{1.852}}{C^{1.852}\,D^{4.8704}} \qquad [\mathrm{m}] \tag{1.10}$$

where C is a coefficient that depends on the roughness of the pipe. Unlike Equations (1.3) and (1.9), Equation (1.10) is applicable for both laminar and turbulent flows.

The minor friction losses, h_c, can be determined from the following equation:

$$h_c = \sum_{1}^{n} K \frac{V^2}{2g} \quad \text{[m]} \tag{1.11}$$

where n is the total number of components in the fluid system and K is a coefficient the value of which can be obtained for each component from the relevant tables and charts.

Given the values of the pipe length and diameter, flow rate, fluid viscosity, and pipe material or roughness, Equation (1.1) can be used to determine the required pump power. The equation can also be used to determine the maximum flow rate of the fluid to be delivered via a pipe of a certain diameter such that the friction loss in the system or the needed pump power does not exceed a specified limit. Moreover, by adding another equation that determines the capital cost of the pump-pipe system, which increases with D, and the cost of electrical energy needed by the pump, which decreases with D, the two equations can be used to determine the economic pipe diameter D_{opt} that gives the lowest total owning cost for the system over its entire life-time. In general, the equations described above are also applicable for analysing and optimising pipe-networks.

The appropriate type and size of the pump for a given pump-pipe system can be selected by matching the "pump characteristic curve" with the "system curve". The pump characteristic curves are usually provided by the manufacturers and the system curve can be obtained by using Equation (1.2). In many situations a single pump or compressor may not be adequate for the required flow rate or delivery pressure and more than one pump or compressor have to be used. In this situation, the principles of fluid dynamics help us to decide whether to arrange the pumps/compressors in parallel or in series.

1.1.2. Heat-transfer

The principles of *heat transfer* enable us to quantify the rate of transferring thermal energy in energy-conversion equipment such as boilers, condensers, and heat exchangers. Heat-transfer analyses apply three independent physical laws to determine the rate of heat transfer between the surface of an object and its surroundings depending on whether the heat transfers by conduction (Fourier's Law), convection (Newton's law of cooling), or radiation (Stefan-Boltzmann law). The physical properties that determine the rate of heat transfer by conduction, radiation, and convection are the thermal conductivity (k), the surface emissivity (ε) and absorptivity (α), and the heat-transfer coefficient (h), respectively. While k, ε, and α are material or surface-specific and can be obtained directly form relevant tables or charts, h depends on both the fluid and the flow and has to be calculated by using empirical formulae except for simple configurations. The empirical formulae give the Nusselt number (Nu) which is related to h as follows [2,3]:

$$h = \frac{k}{D} Nu \qquad \text{[W/m}^2\text{.K]} \tag{1.12}$$

where D is the pipe diameter and k is the thermal conductivity of the transported fluid.

Many empirical formulae are used for determining the Nusselt number for forced or natural flows over single tubes, bank of tubes, plates, etc. For example, the following Dittus-Boelter equation is used for determining Nu inside a fluid-transporting pipe due to forced convection [2]:

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \tag{1.13}$$

where Re is the Reynolds number, Pr the Prandtl number, and n is a constant that takes a value of 0.4 when the pipe is being heated and 0.3 when it is being cooled.

Heat-transfer principles can be used to minimise the rate of heat-transfer between the system and its surroundings by using thermal insulation or to maximise it by using fins, heat-pipes, etc. To illustrate the use of these principles in thermal-insulation analyses, consider the metal pipe shown on Figure 1.2 that has an internal radius r_1 and external radius r_2. The pipe carries a fluid at a temperature T_i, while the surrounding air is at a different temperature T_∞.

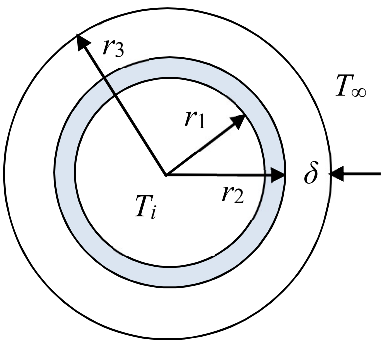

Figure 1.2. Schematic for an insulated metal pipe

The temperature difference between the pipe and the surroundings will cause heat gain or heat loss to/from the pipe and, in order to reduce this heat gain or heat loss, the pipe has to be covered by an insulating material. The reduced rate of heat transfer ($\dot{Q}$) can be calculated from [3]:

$$\dot{Q} = \frac{T_i - T_\infty}{R_{th}} \qquad [\mathrm{W}] \tag{1.14}$$

where R_{th} is the combined thermal resistance to heat-transfer by conduction, convection, and radiation, which is given by [2]:

$$R_{th} = \frac{1}{h_i A_1} + \frac{\ln(r_2 / r_1)}{2\pi L k_1} + \frac{\ln(r_3 / r_2)}{2\pi L k_2} + \frac{1}{h_o A_3} \qquad [\mathrm{K/W}] \tag{1.15}$$

where h_i and A_1 are the heat-transfer coefficient and surface area inside the pipe, respectively, h_o and A_3 are the heat-transfer coefficient and surface area outside the insulated pipe, respectively, L is the length of the pipe, and k_1 and k_2 are the thermal conductivities of the pipe and the insulation, respectively. To simplify the analysis, h_o in Equation (1.15) takes into account the heat-transfer by both convection and radiation to/from the insulation surface.

Equations (1.14) and (1.15) can be used to determine the thickness of insulation (δ) needed to reduce the rate of heat transfer to the required tolerance or the thickness needed to keep the surface temperature within a range that is dictated by safety or other practical considerations. Since the thickness of the metal pipe is usually small compared to its diameter, while its thermal conductivity is much higher than that of the insulation material, Equation (1.15) can be simplified by neglecting the second term on the right side that represents the thermal resistance due to conduction through the pipe.

Although the thicker the insulation the lower will be the rate heat transfer, from an economic point of view the cost of insulation increases with its thickness and, therefore, adding more insulation will not be economically profitable beyond a certain insulation thickness. By extending the above heat-transfer model so that the cost of insulation and the value of the saved thermal energy can be calculated and compared for different values of δ, the equations can also be used to determine the economically optimal thickness of insulation (δ_{opt}).

Another important application of heat-transfer principles is that related to the design and selection heat-exchangers. A heat-exchanger is a device that allows the transfer of thermal energy between two fluids through a separating surface usually a pipe, a duct, a tube, or a plate. Figure 1.3 and Figure 1.4 show two types of heat-exchangers commonly used in industrial and power-plants which are a shell-and-tube heat-exchanger and a cross-flow heat-exchanger, respectively.

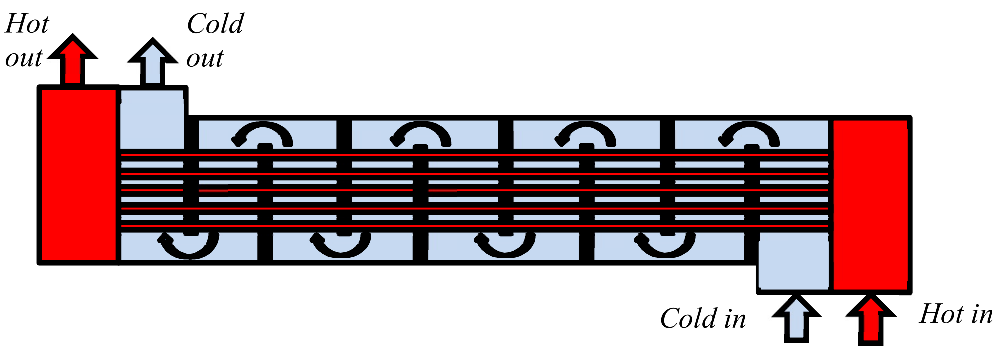

Figure 1.3. A parallel-flow shell-and-tube exchanger

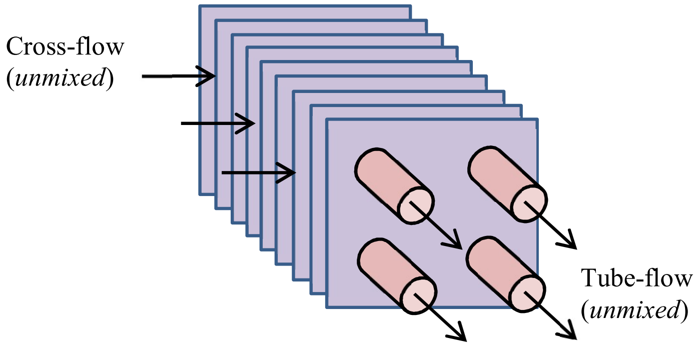

Figure 1.4. A cross-flow exchanger with both streams unmixed

Heat-exchanger analyses either aim at determining the required size (i.e., surface area) for a specified heat-transfer duty (called sizing analyses) or determining the exit temperatures of the two streams from a specified heat-exchanger type and size (called rating analyses). Two methods are used for these two types of analyses which are the log-mean temperature difference (LMTD) method and the effectiveness-number of transfer units (ε-NTU) method, respectively. Complex thermal systems use heat-exchanger networks (HENs) and finding the configuration that minimises the annual cost of the network is also based on the principles of heat transfer.

1.2. Advantages of computer-aided thermofluid analyses

Apart from saving time and eliminating possible human errors, computer-aided thermofluid of analyses have a number of advantages over traditional methods that use property tables and charts. Computer-aided methods can give more realistic results by avoiding unnecessary simplification of the models and by using more accurate estimations of fluid properties. Moreover, they offer reliable techniques for iterative solutions, optimisation analyses, and the analyses of complex fluid-thermal systems. In what follows, these advantages are illustrated by means of relevant examples.

A. Avoiding excessive simplification of the system's model

In many situations, traditional analytical methods adopt excessive simplifications of the analytical models which makes their results grossly deviate from the behaviour of real systems. A good example of this situation is given by the models of internal-combustion (IC) engines. Traditional air-standard models of IC engines, such as the Otto cycle and the Diesel cycle, neglect heat-transfer and friction losses, treat the combustion process as heat-addition from an external source, and use constant specific heats for the working fluids. These assumptions enable the engine processes to be represented by simple closed-form relations from which the amount of heat added to the engine and net work from the engine can be calculated [4]. However, air-standard models usually overestimate the engine's output and thermal efficiency. By comparison, computer-aided models of IC engines closely mimic the behaviour of actual IC engines by taking into consideration the geometrical as well as the thermodynamic characteristics of the engines [5].

Therefore, these models can be used to investigate the effect of important design and operation factors such the ignition or injection timing on the engine performance or the effect of engine speed on the specific fuel consumption. However, the formulation of these models leads to a set of ordinary differential equations that need to be solved simultaneously by using a numerical method such as the Newton-Raphson method.

B. Accurate representation of fluid properties and processes

A good example of this situation is the use of the ideal-gas law to model the *P-v-T* behaviour of a superheated vapour. It is known that when the temperature of the vapour approaches the saturation line, the value of the specific volume determined by the ideal-gas law departs significantly from the actual volume. More accurate estimates can be obtained by using the following Soave-Redlich-Kwong (SRK) equation of state [6]:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \tag{1.16}$$

Where P is the absolute pressure of the gas, $\tilde{v}$ is the molar specific volume, R_u is the universal gas constant, T is the absolute temperature of the gas, and the constants a, b and α are fluid-dependent parameters. Figure 1.5 shows the deviations from the tabulated values of those obtained by using the ideal-gas law and by using the SRK equation of state for refrigerant R134a at 0.2 MPa.

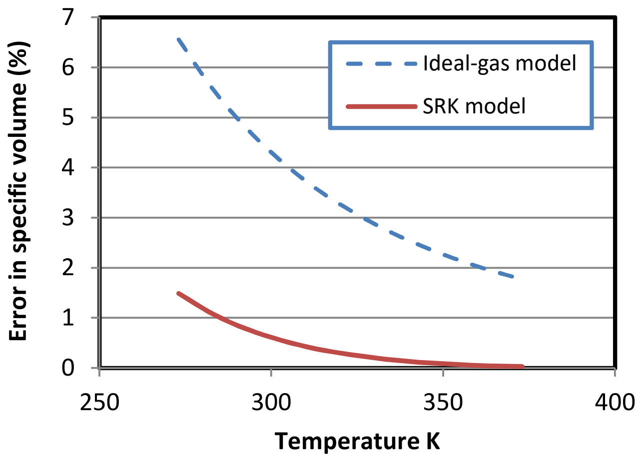

Figure 1.5. Errors in the specific volume of R134a by the ideal-gas law and the SRK equation of state

Figure 1.5 shows that the error of the ideal-gas law is more that 2% even at high temperatures and increases as the temperature approaches the saturation value, but the accuracy of the SRK equation remained higher than 99% even close to the saturation line. Since the SRK equation is implicit in $\tilde{v}$ and cannot be used directly to determine the specific volume, an iterative procedures (e.g. Newton-Raphson method) has to be used to solve it. Therefore, it is more suitable for computer-aided analyses than hand calculations.

Another important implicit equation used in thermofluid analyses is the Colebrook-White equation, Equation (1.7), that determines the friction factor (*f*) for turbulent pipe-flows. Since the equation involves *f* on both sides, it needs to be solved iteratively. There are many other nonlinear equations like the SRK equation and the Colebrook-White equation that give advantage to computer-aided thermofluid analyses by enabling more realistic and accurate estimations of fluid properties and behaviour.

C. Dealing with iterative solutions and optimisation analyses

Two good examples of thermofluid analyses that require iterative solutions are found in pipe-flow analyses. Pipe flow problems that require the friction head loss to be determined when both the diameter and flow rate are known can be solved in a straightforward manner by using Equation (1.3). However, in design analyses of pump-pipe systems we may need to find the flow rate in a given pipe that gives a specified head loss or to find a suitable pipe diameter for specified head loss, flow rate, and pipe length. In these two cases, the friction factor *f* cannot be determined in advance because it depends on the Reynolds number. Therefore, these two types of pipe-flow problems, referred to as type-2 and type-3 problems, need to be solved by iteration. It is much easier to carry out the iterative process to the required level of accuracy by using a computer-aided method than by doing it manually. Two other examples of thermofluid analyses that require iterative solutions are rating analyses of heat exchanger and the determination of the adiabatic flame temperature by first-law analysis of the combustion process.

Optimisation analyses are needed for determining the best arrangement for a fluid-thermal system such as the optimum intermediate pressure for a two-stage air-compression system, the optimum steam-extraction pressure for a regenerative Rankine cycle, and the economic thickness of insulation for a pipe. While simple single-objective optimisation analyses that involve a single design parameter can be performed by means of calculus techniques and graphic tools, the more complex optimisation analyses that involve multiple objectives and/or multiple design variables require the use of computer-aided techniques.

D. Analyses involving complex models

The complexity of modelling certain fluid-thermal systems makes their analyses only possible with the help of computer-aided methods. The model's complexity can be either due to the complexity of the physical structure of the system itself or due to the complexity of its mathematical representation. An example of physically complex systems is the pipe network shown on Figure 1.6 that consists of four pipe loops and four consumption points and fed by two water tanks; tank A and tank B. Suppose that the flow rates from the two supply tanks are specified together with the pipe diameters and lengths and it is required to determine the discharges at the four consumption points.

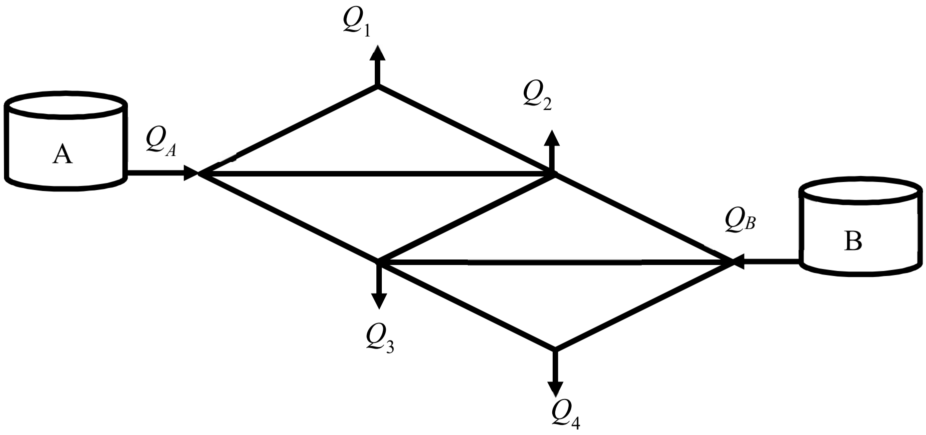

Figure 1.6. A looped pipe network supplied by two tanks

Although the solution of the problem is mainly based on the two well-known principles of conservation of mass and conservation of energy discussed earlier, it is difficult to obtain by using manual analytical methods especially when a minimum or a maximum pressure level is to be met at the discharge points. In this case, a computer-aided method, such as the Hardy-Cross method, has to be used [7, 8]. Analyses of heat-exchanger networks is another example of thermofluid analyses that involve physically complex systems [9]. The other type of model's complexity that need computer-aided numerical methods are found in multi-dimensional fluid-flow and heat transfer analyses. This type of models involves coupled and nonlinear partial differential equations that have to be solved by using computational fluid dynamics (CFD) methods such as the finite-volume method or the finite-difference method. Tempest (later Fluent), which was the first commercial CFD software with a graphical user-interface, appeared in the early 1980s. Thanks to more than 40 years of development, many other commercial CFD applications are available nowadays that offer great flexibility and user-friendliness [10].

1.3. The Excel-based modelling platform for thermofluid analyses

Microsoft Excel is a spreadsheet program and a component of Microsoft Office product group for business applications. Excel, which is widely used in industry for data analyses, have also been used to perform simple computer-based operations like matrix multiplications and solution of linear systems of equations [2,11]. However, Excel is equipped with other features that make it a capable modelling platform for a wide range of thermofluid and other engineering analyses [12-16]. In addition to its Goal Seek command and the Solver add-in, the "Developer" ribbon in Excel provides Visual Basic for Applications (VBA) which is a programming language that can be used for developing customised user-defined functions (UDFs) not provided by Excel. The Developer ribbon also allows the use of macros to remove the tedium of parametric studies and repetitive calculations. The lack of built-in functions for fluid properties that limits the usefulness of Excel for thermofluid analyses could be remedied by developing suitable add-ins by a number of academic and research institutions [17-19].

This book deals with various types of computer-aided analyses and design optimisation of fluid-thermal systems by using Excel as the modelling platform with an educational add-in for fluid properties called Thermax [20]. Thermax provides seven groups of property functions for ideal gases, saturated water and superheated steam, synthetic and natural refrigerants, atmospheric humid air for psychrometric analyses, two aqua solutions for vapour-absorption refrigeration, chemically-reacting substances, and air at standard atmospheric pressure. Thermax also provides a number of numerical tools and UDFs that enhance the usefulness of the Excel-based modelling platform for thermofluid analyses. Most of the analyses presented in the book can be performed by using Excel and Solver only, but certain analyses require the use of Thermax functions or VBA to develop UDFs. Table 1.1 summarises the roles of the four components of the Excel-based modelling platform as used in this book.

Table 1.1. Roles of the four components of the Excel-based modelling platform

Component	Role
Excel	• Provides the basic functions needed for developing the analytical models for fluid-thermal systems including the general mathematical functions and the matrix-operation functions • Provides the Goal Seek command that can be used to perform unconstrained iterative solutions involving a single parameter • Allows circular calculations which can be a convenient method for dealing with parameter dependency in certain analyses and can be used to solve a system of linear or nonlinear equations • Provides graphical tools needed for data visualisation and presentation • Allows macros to be recorded for repetitive calculations
Solver	• Allows constrained iterative solutions involving multiple parameters • Allows constrained optimisation analyses with single objectives but multiple design variables • Offers three solution options that suit different types of analyses
Thermax	• Provides the physical properties of various fluids • Provides two interpolation functions for tabulated data and a Newton-Raphson solver for non-linear equations such as the Colebrook-White equation and the SRK equation • Provides other UDFs needed by thermofluid analyses
VBA	• Can be used to develop additional functions not provided by Excel or the Thermax add-in, e.g. functions for the non-linear equations involved in iterative solutions and optimisation analyses • Can be used to develop numerical solvers for large systems of linear equations

1.4. Closure

The following four chapters describe the Excel-based modelling platform in more details and illustrate its use for the two basic types of computer-aided thermofluid analyses; iterative solutions and optimisation analyses. Chapter 2 focuses on the features of Excel that are mostly needed for computer-aided thermofluid analyses, such as its matrix functions and its Goal Seek command. Chapter 3 introduces the other three components of the modelling platform, gives examples of using the three solution methods offered by Solver, and describes the development of user-defined functions with VBA. Chapter 3 also shows how the property functions provided by Thermax can be used in Excel formulae. Chapter 4 gives examples of using Excel's Goal Seek command and Solver for the type of analyses that involve iterative solutions in the fields of fluid dynamics, heat-transfer, and thermodynamics while Chapter 5 shows how Solver can be used for optimisation analyses of fluid-thermal systems that involve a single design variable and multiple design variables.

Chapter 6 to 9 deal with fundamental analyses in heat-transfer and fluid-dynamics that require the use of computer-aided methods. Both Chapter 6 and Chapter 7 deal with the numerical solution of the heat-conduction equation by using the finite-difference (FD) method. While Chapter 6 focuses on the steady conduction equation, Chapter 7 focuses on the transient equation. Similarly, both Chapter 8 and Chapter 9 deal with hydraulic analyses of fluid systems. Chapter 8 focuses on hydraulic analyses of multi-pipe and pump-pipe systems and gives examples of using Excel's Goal Seek command and Solver for this type of analyses. Different pipe and pump arrangements are analysed in this chapter to determine the system's friction losses, power requirement, or operating point. Chapter 9 describes a method for using Solver for hydraulic analyses of gravity-driven pipe-networks with looped, branched, and mixed configurations.

The last three chapters of the book illustrate the use of the Excel-based platform for design analyses and optimisation of fluid-thermal systems. Chapter 10 considers four types of these systems, which are thermal insulation, a hot-water generation system for a power plant, a double-pipe heat-exchanger, and a pump-pipe system. Chapter 11 focuses on determining the optimum diameter for a pipeline and compares the result of the Excel-Solver method with that of the traditional method of using charts. Chapter 12 illustrates the use of the Excel-Solver method for optimisation analyses of gravity-fed and pump-driven pipe-networks.

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2

Excel

Excel's user-interface enables the development of mathematical models for engineering systems by providing a large set of built-in functions and a number of analytical tools. Together with its graphical tools and the "Developer" options, Excel forms the backbone of the modelling platform used in this book for thermofluid analyses. Excel's user-interface provides many functions for data analyses, but this chapter focuses on those needed for building analytical models for thermofluid analyses. The chapter highlights the use of "cell-labelling" for writing Excel's formulae instead of the usual referencing by location and illustrates the use of Excel's matrix functions for the solution of linear systems of equations. The chapter also illustrates the use of "Goal Seek" and "circular calculations" for the solution of nonlinear equations. Finally, the section on Excel's graphical tools illustrates the use of the "trendline" feature for curve-fitting of tabulated data. More detailed information about Excel can be found in, e.g., Walkenbach [1,2].

2.1. Elements of Excel's user-interface

Excel's user-interface allows the user to adjust the appearance of the workspace and present his/her primary data and analysis results in various forms. To allow easy access to the large number of functions, tools, and commands provided by Excel, the user-interface is divided into a number of elements with different purposes. For example, Figure 2.1 shows a screenshot of an Excel sheet that stores the scores obtained by a group of students in one semester.

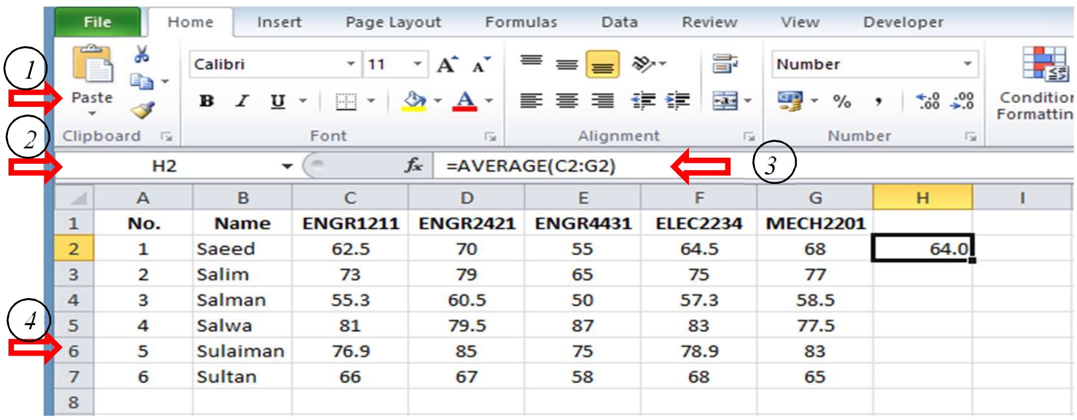


Figure 2.1. The main elements of Excel's user-interface

Figure 2.1 shows four elements of Excel's user-interface, which are:

1. The ribbon
2. The name box
3. The formula bar
4. The workspace

The **Ribbon**, which occupies the upper part of the sheet, organises the numerous commands provided by Excel into nine "tabs", e.g., the **File**, **Home**, and **Insert** tabs. Each tab consists of a number of command-groups that have a common purpose. For example, the **Developer** tab shown in Figure 2.2 allows the user to write customised functions using the Visual Basic for Applications (VBA) language and to record **Macros** together with other useful development tools.

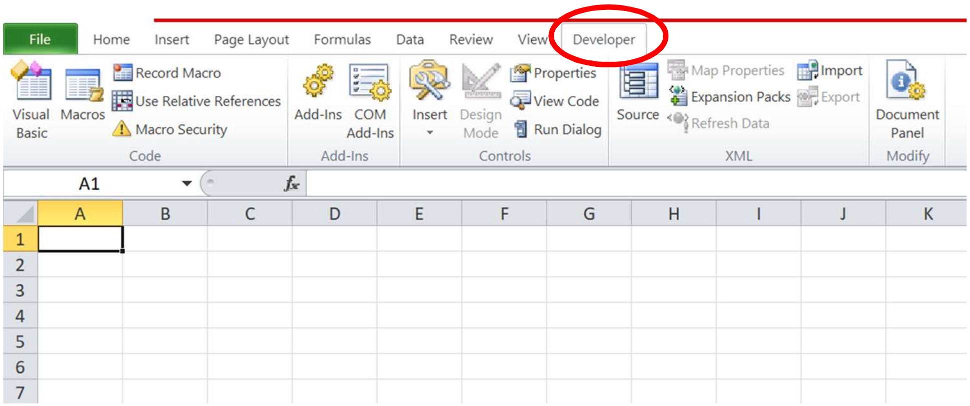


Figure 2.2. Elements of the Developer tab

The **Workspace**, which is the main part of the sheet, is divided into a grid of columns and rows that form separate "**cells**" at their intersections. A cell is referred to by a letter and a number, e.g., A1, B3, H2, etc. The letter represents the cell's column, while the number represents its row. The **Name box** shows the location of the current cell. As Figure 2.1 shows, a cell can simply contain a character data, such as "Saeed" and "Salim", or a numerical data, such as 62.5 and 70. A cell can also contain a formula for data manipulation using the numerous built-in functions provided by Excel. The formula bar in Figure 2.1 reveals the formula typed in cell H2 that uses the built-in function "**AVERAGE**" to determine the average score of the first student in the list (Saeed) in the five subjects as 64.0. Note that, unlike simple numerical or character cells, a cell that includes a formula must include the equal sign "=". The role of the **Formula bar** will be explained in more details in the following section.

2.2. Excel's formulae

In general, Excel's formulae include mathematical or logical operators, built-in functions, and cell references. Excel provides two ways to refer a particular cell in a formula; either by its location in the sheet, e.g., A2, C10, etc., or by giving it a relevant name, e.g., efficiency, diameter, etc. The two methods will be illustrated below.

2.2.1. Cell reference by location

To illustrate this method, let us write a formula that calculates the area of a circle that has a radius of 5 m. Open a new Excel sheet and type the number 5, which is the radius of the circle, in cell A1 as shown in Figure 2.3. Now, go to cell A2 and type the following formula:

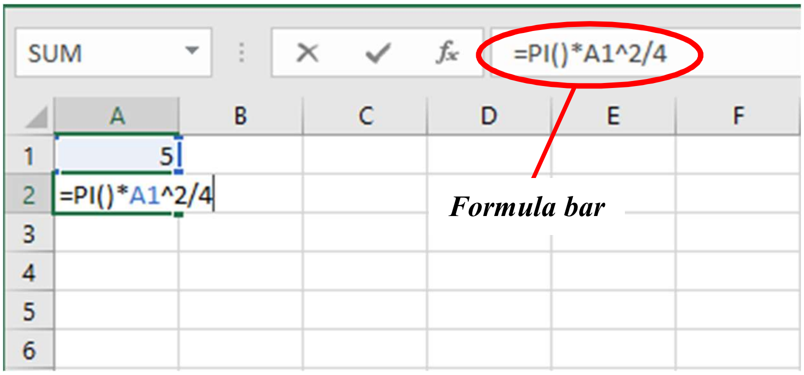


Figure 2.3. Writing an Excel formula to determine the area of a circle

=PI()*A1^2/4

The function "**PI()**" is a built-in function that returns the value of Archimedes' constant π. The formula also contains a reference to cell A1 that stores the value of the circle's radius, the multiplication operator *, the division operator /, the power operator ^, and the constants 2 and 4. Note that the formula is shown in the formula bar which can also be used to edit the formula. Pressing the **Enter** key after typing the formula, the result will be as shown in Figure 2.4; which is 19.63495 square metres. The following example shows how Excel's formulae and built-in functions can be used in a simple thermodynamic analysis.

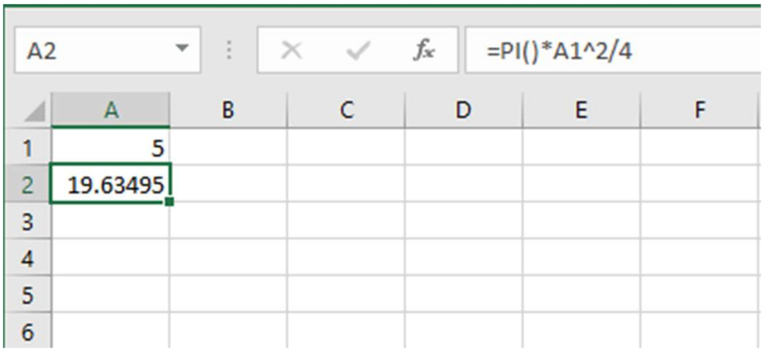


Figure 2.4. The Excel sheet with a formula that determines the area of a circle

Example 2-1. Determining the error in the specific volume of R134a calculated by using the ideal-gas law

Develop an Excel sheet that calculates the error in the specific volume (v) of superheated refrigerant R134a that results from applying the ideal-gas law at a pressure of 200 kPa (T_{sat} = -10.09°C) and temperatures in the range 0°C to 100°C (273 to 373 K).

Solution

Figure 2.5 shows the Excel sheet prepared for this example. The pressure (P), the gas constant (R), and the temperature (T) are stored in columns A, B, and C, respectively.

E2 f_x =B2*C2/A2

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal		
2	200	0.08149	273	0.10438	0.1112339		
3	200	0.08149	283				
4	200	0.08149	293				
5	200	0.08149	303				
6	200	0.08149	313				
7	200	0.08149	323				
8	200	0.08149	333				
9	200	0.08149	343				
10	200	0.08149	353				
11	200	0.08149	363				
12	200	0.08149	373				
13							

Figure 2.5. The sheet developed for Example 2-1

Column D stores the values of v obtained from relevant property tables and column E stores the corresponding values obtained from the ideal-gas law:

$$v = R \times \left(\frac{T}{P} \right) \tag{2.1}$$

Where, P and T are the absolute pressure and temperature, respectively, and R is the gas constant for R134a ($R = 0.08149$ kJ/kg.K). Note that the formula bar in Figure 2.5 reveals the formula in cell E2 that applies Equation (2.1). Since the pressure (P) and the gas constant (R) do not change in this example, Excel allows for these parameters to be stored in single cells instead of being repeated as shown in Figure 2.5 (see Section 2.4).

Figure 2.5 shows the tabulated value of the specific volume (v_table) and that determined by Equation (2.1) (v_ideal) at 273K. The percentage error of the ideal-gas law in estimating the specific volume is given by:

$$Error = \frac{v_{Ideal} - v_{Table}}{v_{Table}} \times 100 \tag{2.2}$$

To determine the percentage error at 273K, go to cell F2 as shown in Figure 2.6 and type the following formula, which is equivalent to Equation (2.2):

=(E2 – D2)/D2*100

Note that the formula bar in Figure 2.6 reveals the above formula for 273K. When you press the **Enter** key, the number **6.566** will appear in cell F2 as shown in the figure.

F2 f_x =(E2-D2)/D2*100

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal	error_v_ideal	
2	200	0.08149	273	0.10438	0.1112339	6.5662483	
3	200	0.08149	283	0.10922	0.1153084	5.5743911	
4	200	0.08149	293	0.11394	0.1193829	4.776944	
5	200	0.08149	303	0.11856	0.1234574	4.1306933	
6	200	0.08149	313	0.12311	0.1275319	3.5917878	
7	200	0.08149	323	0.12758	0.1316064	3.1559414	
8	200	0.08149	333	0.13201	0.1356809	2.7807363	
9	200	0.08149	343	0.13639	0.1397554	2.4674463	
10	200	0.08149	353	0.14073	0.1438299	2.2026931	
11	200	0.08149	363	0.14504	0.1479044	1.974869	
12	200	0.08149	373	0.14932	0.1519789	1.7806389	
13							

Figure 2.6.The completed sheet developed for Example 2-1

To find the percentage errors at other temperatures you can simply copy the formula in cell F2 and paste it on cells F3 to F12. Figure 2.6 shows the completed Excel sheet for the required temperature range. The calculated values of the errors show that the maximum error occurs at the lowest temperature, which is 273K. Note that the error decreases gradually as the temperature increases.

2.2.2. Use of cell labels

The usual reference to cells by their columns and rows suits perfectly statistical analyses in which the same formula is applied to a large body of data that is stored column-wise or row-wise. For example, we may want to determine the average value, maximum value, or minimum value of the tabulated data. However, thermofluid analyses usually involve numerous formulae but a small set of data, e.g. the diameter of a pipe, the density or viscosity of a fluid, the effectiveness of a heat exchanger, etc. For such analyses, it is more convenient to give the cell a meaningful name or "label" that matches its content. The cell can then be referred to by its label instead of its relative location. This method makes it easier to recognise the quantities involved in the Excel formulae.

For the purpose of illustration, let us develop an Excel sheet to compare the density of air before and after an isentropic compression process from an initial condition of P_1 = 100 kPa and T_1 =300K to a final pressure of P_2 = 800 kPa. The two air densities involved can be calculated from the ideal-gas law as follows:

$$\rho_1 = P_1 / RT_1 \quad (2.3)$$

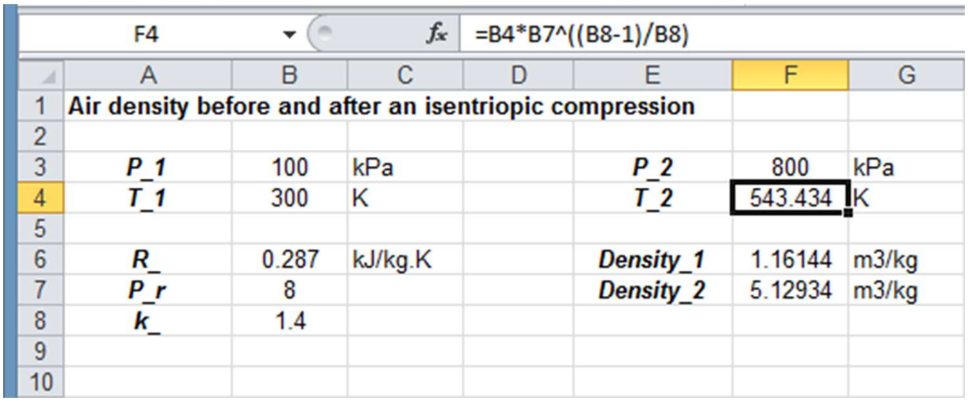
$$\rho_2 = P_2 / RT_2 \quad (2.4)$$

Where R is the gas constant (for air, R = 0.287 kJ/kg.K). For an isentropic process, T_2 is related to T_1 according to the following approximate relationship:

$$T_2 = T_1 \times \left(P_2 / P_1\right)^{\frac{k-1}{k}} \tag{2.5}$$

Where k is the ratio of specific heat at constant pressure (c_p) and at constant volume (c_v). At the given temperature range, k for air can be taken as 1.4. Note that k is also used for the thermal conductivity in Appendix A, Table A.1.

Figure 2.7 shows the sheet prepared for this analysis in which the respective cell labels are typed in the column to the left of the different pressures and temperatures, while the corresponding units are written in the column to the right of each quantity. This is also done to the other quantities in the calculations. The sheet also shows the units of the different properties involved for more clarification.


F4 fx =B4*B7^((B8-1)/B8)

	A	B	C	D	E	F	G
1	Air density before and after an isentriopic compression						
2							
3	P_1	100	kPa		P_2	800	kPa
4	T_1	300	K		T_2	543.434	K
5							
6	R_	0.287	kJ/kg.K		Density_1	1.16144	m3/kg
7	P_r	8			Density_2	5.12934	m3/kg
8	k_	1.4					
9							
10							

Figure 2.7. Excel sheet for calculating the air densities before and after compression

Placing the cursor on cell F4 makes the formula bar reveal the formula used in the calculation of the temperature T_2 according to Equation (2.5) which is:

=B4*B7^((B8-1)/B8)

The above formula can be made more understandable by using familiar labels to refer to the different cells involved. To do that, select the cells in columns A and B as shown in Figure 2.8, then go to **Formulas** and, at the **Name Manager**, select **Create** from **Selection**. When the form shown in Figure 2.8 appears to you, tick the "Left column" option. Pressing the "**OK**" button will make Excel create names for the different values in the selection box according to the labels written on the left column. The cell F3 that stores the value of P_2 can also be associated with its corresponding label in cell E3.

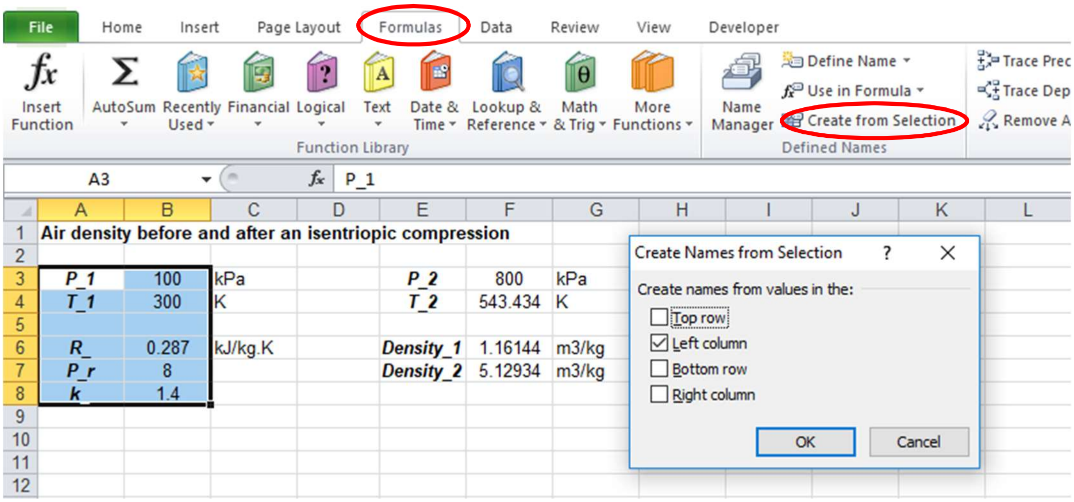


Figure 2.8. Creating names for a selected group of cells

Now, retype the formula in cell F4 that determines T_2 as follows:

=T_1*P_r^((k_-1)/k_)

The formula bar in the sheet shown in Figure 2.9 reveals the formula with the corresponding labels instead of location references.

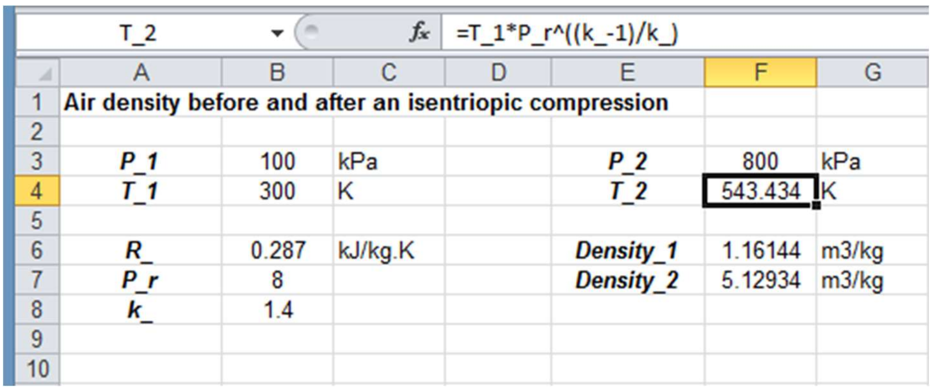


Figure 2.9. Formulae using cells labels instead of locations

Labelled formulae are easier to edit than those using location referencing particularly when intricate formulas are involved. Another advantage of cell-labelling is that if you copy a labelled formula and paste it in any other cell you will get the same answer, but if you copy a formula that uses the usual referencing by location in another cell you will get a different answer. To reveal or hide all the formulae in the sheet, press the control key (ctrl) with the tilde key (~). When naming your cells, choose suitable representative names for the variables involved, e.g. P_1 and T_1 for P_1 and T_1. Note that Excel does not accept "P1" or "T1" as labels since these can be confused with usual cell references by locations. Therefore, Excel automatically changes these labels to "P1_" and "T1_". More information about Excel's formulae can be found in Walkenbach [1].

2.3. Excel's built-in functions

Excel provides a large library of built-in functions for data manipulation, like the **AVERAGE** function, and other functions commonly used in engineering analyses like the **PI**, **SIN**, and **COS** functions. To view the full range of Excel's functions, type "=" in any Excel cell as shown in Figure 2.10 and then place the cursor on the **Insert Function** "*fx*" button in the formula bar and click it. The dialog box shown in Figure 2.11 will appear to you. List all the categories via the "**Select a category**" slot.

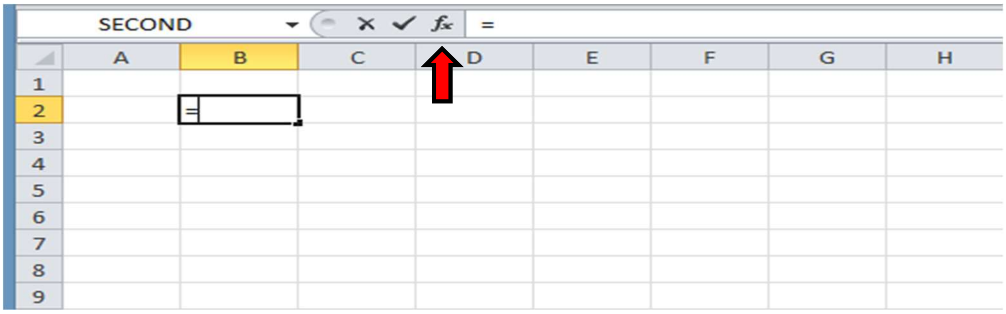


Figure 2.10. Exploring Excel's built-in functions

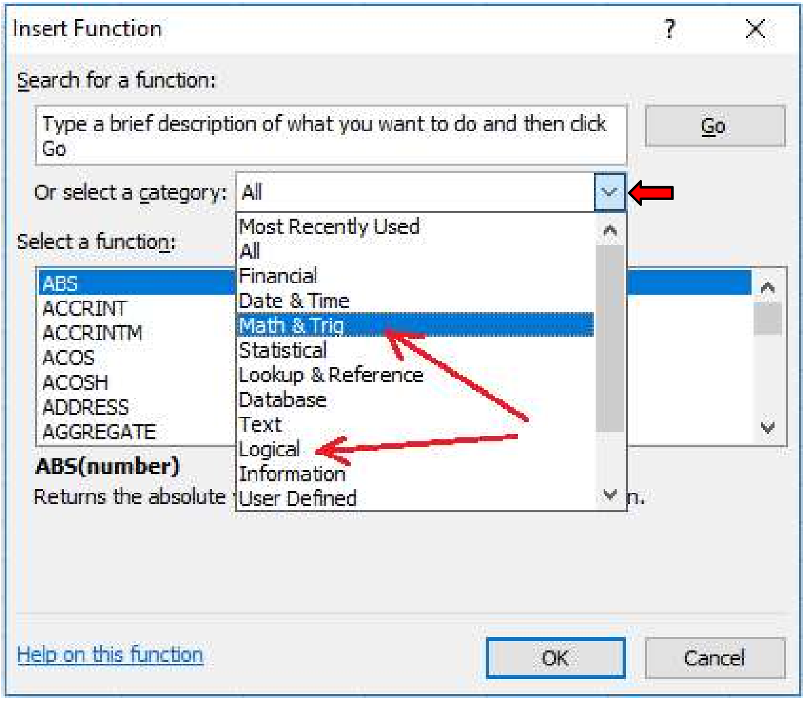


Figure 2.11. The various categories of Excel's built-in functions

The **Math & Trig** group includes the mathematical and trigonometric functions used in different types of engineering analyses. Figure 2.12 shows some of the numerous functions in this group. Note that the dialog box gives a brief explanation of each function. For example, the explanation given to the **ABS** function is that it returns the absolute value of a number. The functions **ACOS**, **ASIN**, and **ATAN** apply the familiar inverse trigonometric functions: $\cos^{-1}$, $\sin^{-1}$, and $\tan^{-1}$, respectively. By scrolling down the list, you can find many other familiar functions. The following discussion focuses on two types of functions that are needed for the development of analytical models in subsequent chapters of the book, which are (a) the logical functions and (b) the functions for matrix operations.

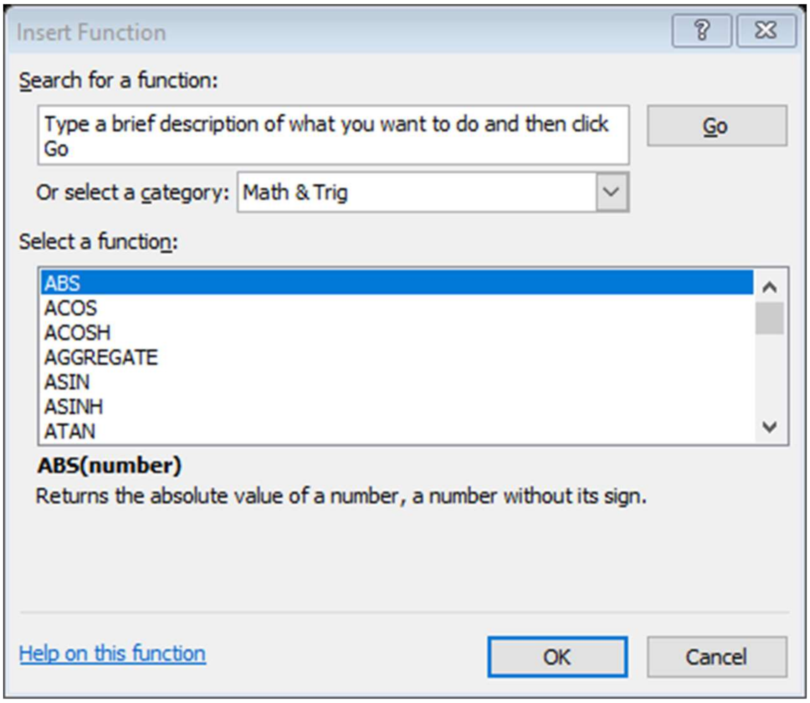


Figure 2.12. Common mathematical functions supported by Excel

2.3.1. Logical functions

To determine the major friction loss (h_f) in a pipe by using the Darcy-Weisbach equation we have to establish whether the flow is laminar or turbulent so as to select the relevant formula for the friction factor (f). The flow remains laminar before the Reynolds number (Re) reaches a certain value, which can be taken as 2,000, but the flow can only be considered fully turbulent beyond Re = 3000. There is a transitional region between laminar and turbulent flows when 2000 < Re < 3,000. Suppose that we want to write an Excel formula that determines the type of flow from the given value of the Reynolds number. Using a simple **IF** function, we can write the following formula:

=IF(Re<=2000, “Laminar”, “Turbulent or transitional”)

Note that the above **IF**-formula does not differentiate between a turbulent flow and a transitional flow. Therefore, we need to use a second logical test inside the first logical test. This can be done by using the following nested IF function:

=IF(Re<=2000, “Laminar”, IF(Re>=3000, “Turbulent”, “Transitional”))

Figure 2.13 shows an Excel sheet containing the above formula (shown in the formula bar) and the response of the formula when Re = 500, which is “Laminar”. Depending on the value of Re stored in cell C2, the result of the If-formula can be “laminar”, “Turbulent”, or “Mixed”. Excel supports six other logical functions; **AND**, **FALSE**, **IFERROR**, **NOT**, **OR** and **TRUE** that can be combined in the same formula so as to handle more intricate choices.

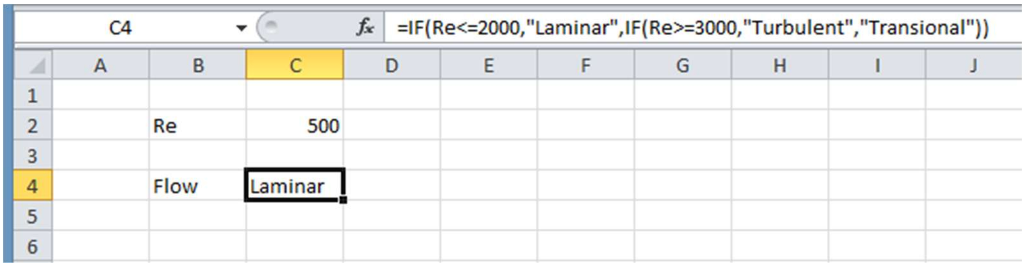


Figure 2.13. A formula using the nested IF function to determine the type of flow

2.3.2. Functions for matrix operations

Adjacent cells can be treated as a matrix or a vector and a group of Excel’s formulae allow for the addition, subtraction, and multiplication of these matrices and vectors according to the established rules of matrix operations. Figure 2.14 shows a 3x3 matrix (A) stored in the cells B3:D5 and a vector (b) stored in cells F3:F5.

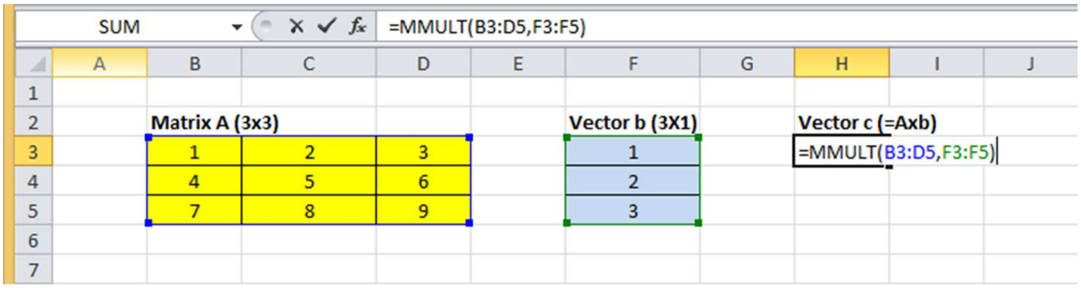


Figure 2.14. Step 1 of using the matrix multiplication function

Matrix (A) and vector (b) can be multiplied and the result stored in a third vector (c) by using the matrix function **MMULT**. The procedure is as follows:

1. After keying in the data of matrix (A) and vector (b) as shown in Figure 2.14, position the cursor at cell H3 and type the formula: =MMULT(B3:D5;F3:F5).

2. Now press ENTER key and cell H3 will take the value 14, which the result of multiplying the first row of the matrix with the vector (b) as Figure 2.15 shows.

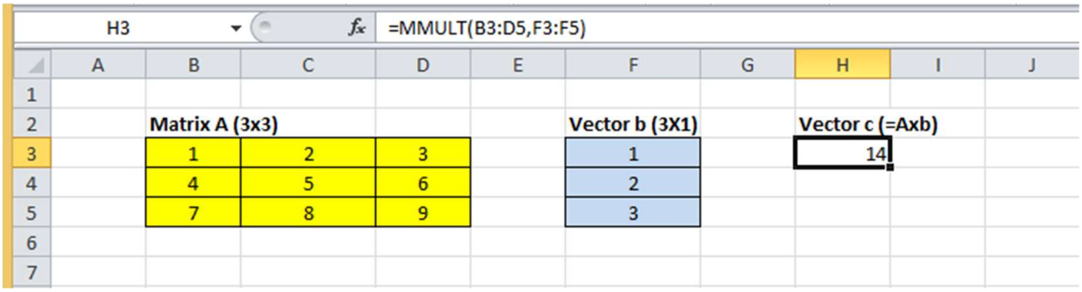


Figure 2.15. Step 2 of using the matrix multiplication function

The other two elements of the result vector will not appear automatically. To view the complete solution vector, do as follows:

1. Select the cells H3:H5 as shown in Figure 2.16,
2. Press the function key F2 once and then simultaneously hold the (**SHIFT** + **CONTROL**) keys together and press ENTER. The complete solution vector (c) will now appear as shown in Figure 2.17.

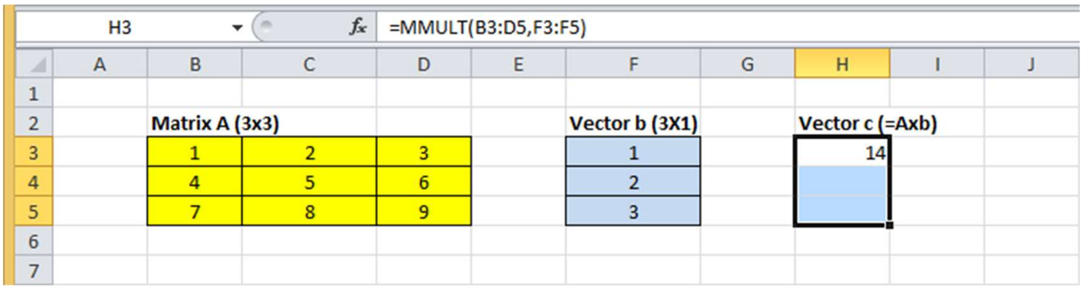


Figure 2.16. Step 3 of using the matrix multiplication function

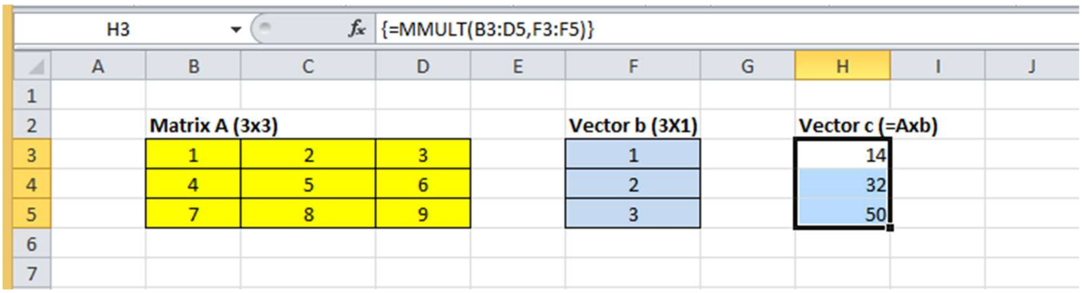


Figure 2.17. Step 4 of using the matrix multiplication function

An important matrix-operation function provided by Excel is the matrix-inversion function **MINVERSE** which is useful for the solution of linear systems of equations. The following example illustrates the use of this function.

Example 2-2. Using the matrix inversion function

By using the **MINVERSE** function, find the inverse of matrix [A] given by:

$$[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 6 \\ 7 & 0 & 5 \end{bmatrix}$$

Solution

The first step is to enter the elements of the matrix as shown in Figure 2.18. After entering the data, go to cell F2 and type the formula "=MINVERSE(B2:D4)". When you press **ENTER**, this cell will have the value -0.3125, which is the first element of the inverse matrix $[A]^{-1}$ shown in Figure 2.19.

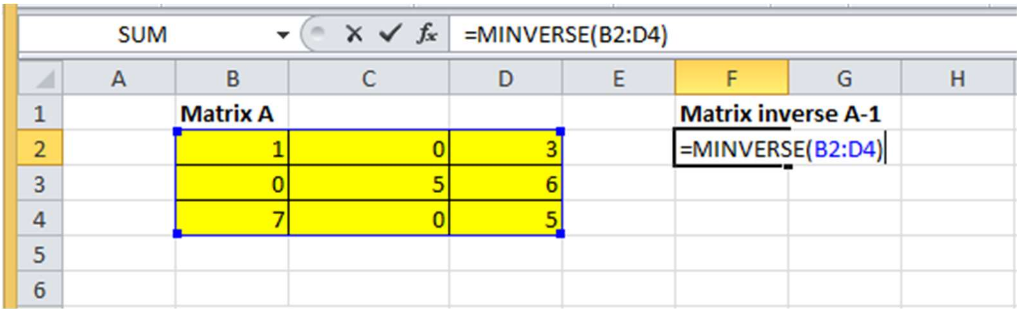


Figure 2.18. Step 1 of using the MINVERSE function

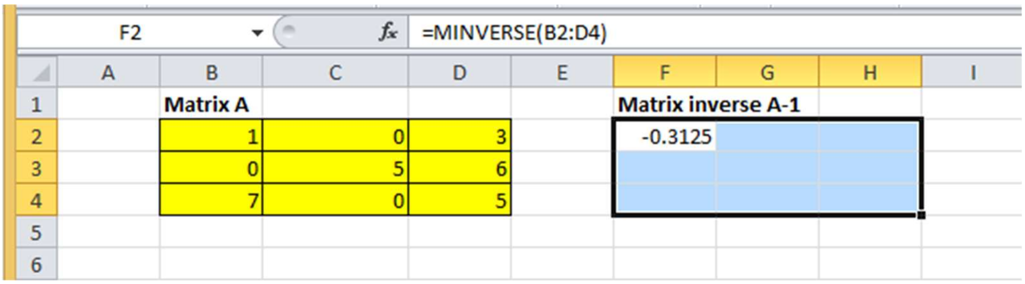


Figure 2.19. Step 2 of using the MINVERSE function

Starting with the formula in cell F2, select the range F2 to H4 as shown in Figure 2.19. Press and release the function key F2 and then simultaneously hold the **CTRL+SHIFT** keys and press **ENTER**. Other elements of the inverse matrix $[A]^{-1}$ will then appear as shown in Figure 2.20. You can check the solution by multiplying matrix [A] with its inverse by using the **MMULT** functions. The procedure is illustrated by Figures 2.21 to 2.23. As should be expected, Figure 2.23 shows that the resultant matrix is the identity matrix.

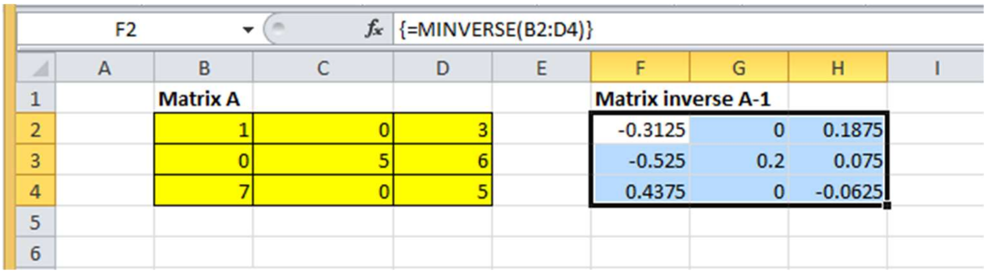


Figure 2.20. The complete inverse matrix $[A]^{-1}$

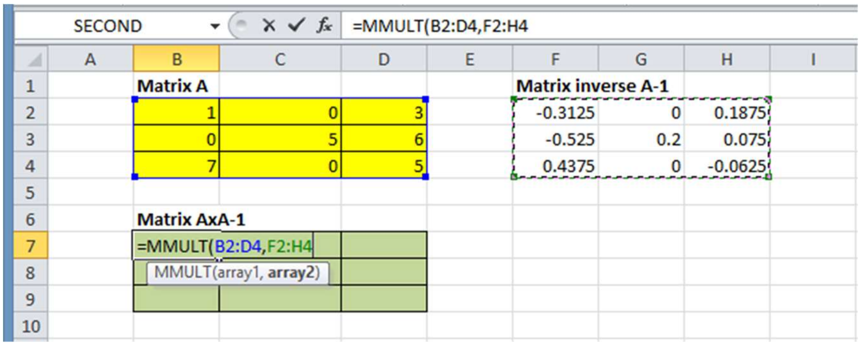


Figure 2.21. Multiplying matrix [A] by its inverse $[A]^{-1}$

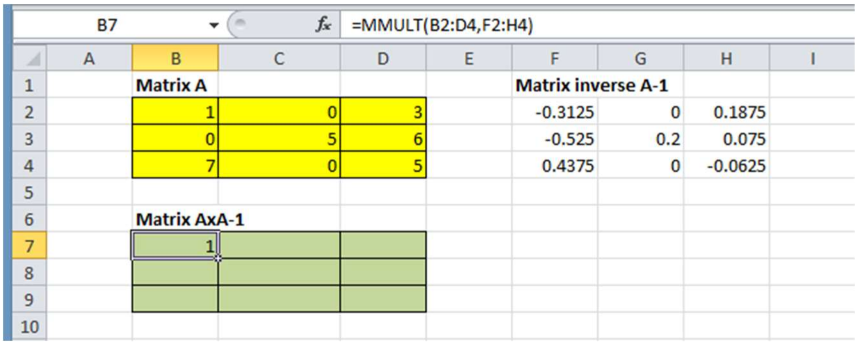


Figure 2.22. The first element of the product (identity) matrix

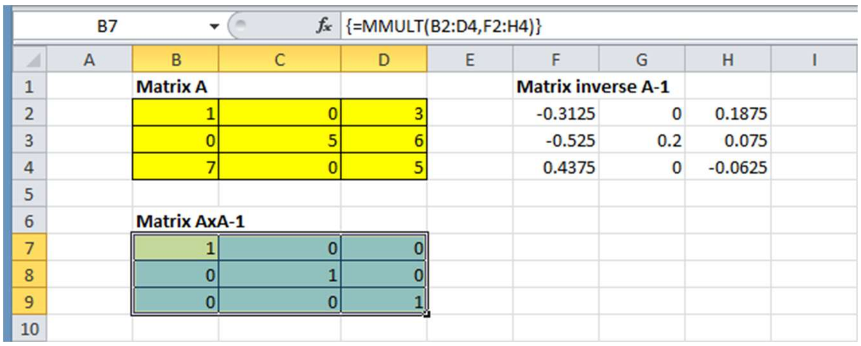


Figure 2.23. The complete solution which is the identity matrix

2.3.3. Solution of linear system of equations

An example of thermofluid analyses that involve systems of linear equations is the solution of the steady heat conduction equation by the finite-difference method. As shown in Chapter 6, the method leads to a system of linear equations the solution of which gives the unknown temperature values at the various predetermined nodes. The linear system of equations can be solved with Excel by applying the matrix-inversion method. For illustration, consider the following linear system written in matrix notation:

$$[A]\{x\}=\{y\} \tag{2.6}$$

Where [A] is the coefficient matrix, $\{x\}$ the vector of unknowns, and $\{y\}$ the right-side or "load" vector. By applying the matrix-inversion method, the solution vector $\{x\}$ can be obtained as follows:

$$\{x\}=[A]^{-1}\{y\} \tag{2.7}$$

Where $[A]^{-1}$ is the inverse of matrix [A]. The following example illustrates the procedure of applying the method by using Excel's matrix functions.

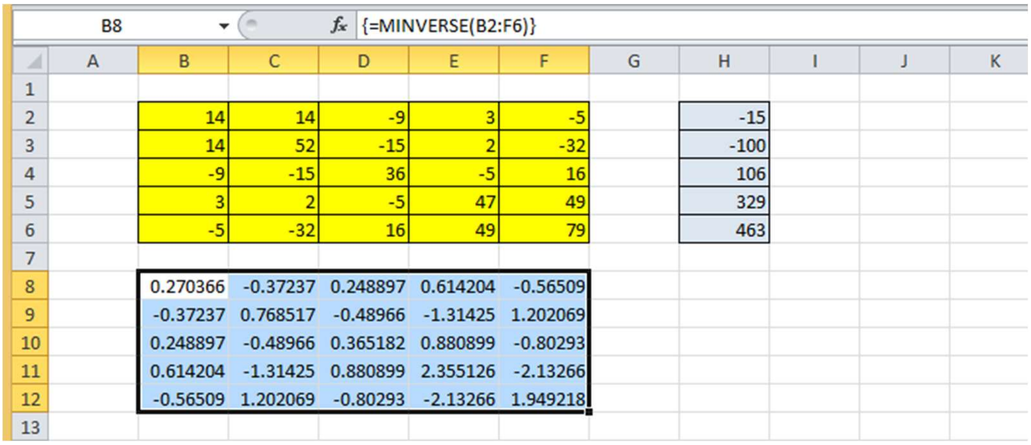
Example 2-3. Solution of a system of linear equations

Find the values of x_i in the following system of linear equations:

$$\begin{bmatrix} 14 & 14 & -9 & 3 & -5 \\ 14 & 52 & -15 & 2 & -32 \\ -9 & -15 & 36 & -5 & 16 \\ 3 & 2 & -5 & 47 & 49 \\ -5 & -32 & 16 & 49 & 79 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} -15 \\ -100 \\ 106 \\ 329 \\ 463 \end{Bmatrix} \tag{2.8}$$

Solution

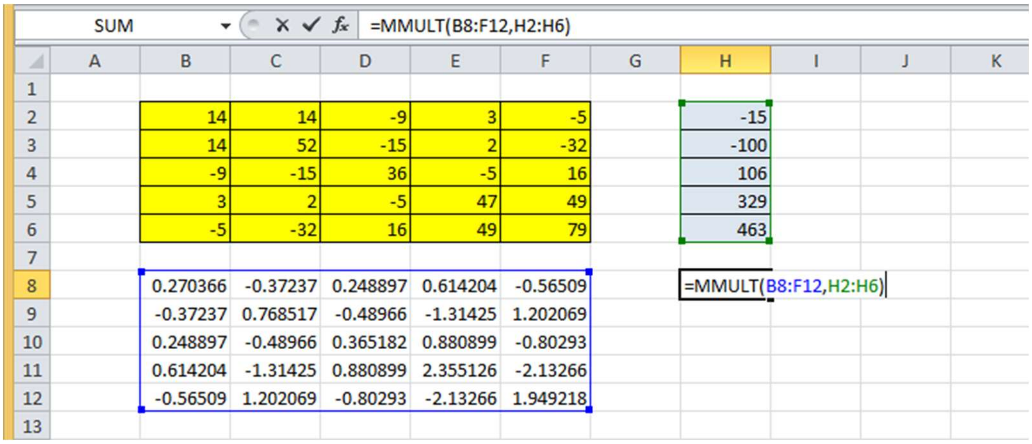
Note that the system is symmetric; which is typically the case with linear systems that arise in the solution of heat-conduction problems by the finite-difference method. For larger systems of equations, the symmetry of the system can be utilised for reducing the required computer memory by storing only one half of the coefficient matrix. However, this requires a complicated computer programming. For small systems like the one considered here, it is more convenient to use Excel's matrix inversion and multiplication functions. Figure 2.24 shows the Excel sheet that stores both the coefficient matrix [A] and the load vector $\{y\}$. The inverse matrix $[A]^{-1}$, which is obtained by following the procedure described in the previous section, is stored below the coefficient matrix as shown in the figure. The inverse matrix $[A]^{-1}$ is then multiplied with the load vector {y} and the result stored below the load vector as shown in Figure 2.25. The complete solution is shown in Figure 2.26. The first element is practically zero and, therefore, the solution vector is{x} = (0, 1, 2, 3, 4).



B8 {=MINVERSE(B2:F6)}

	A	B	C	D	E	F	G	H
1								
2		14	14	-9	3	-5		-15
3		14	52	-15	2	-32		-100
4		-9	-15	36	-5	16		106
5		3	2	-5	47	49		329
6		-5	-32	16	49	79		463
7								
8		0.270366	-0.37237	0.248897	0.614204	-0.56509		
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069		
10		0.248897	-0.48966	0.365182	0.880899	-0.80293		
11		0.614204	-1.31425	0.880899	2.355126	-2.13266		
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218		
13								

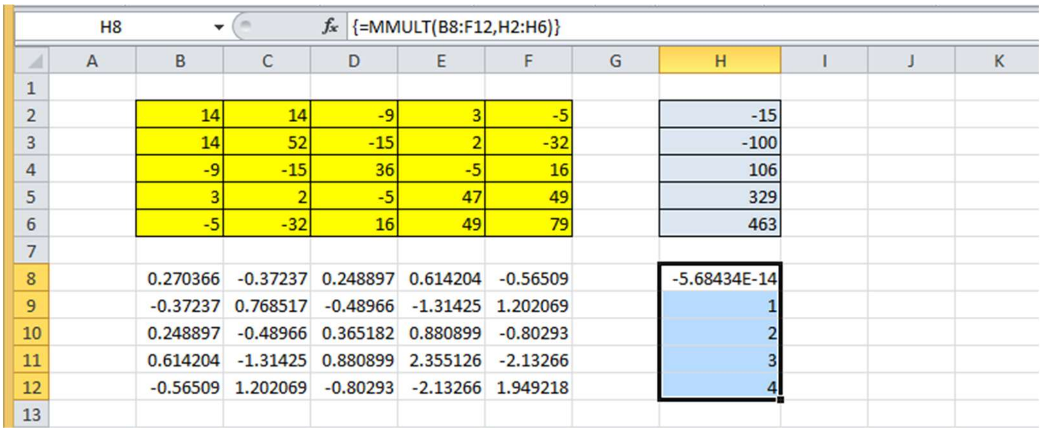
Figure 2.24. The coefficient matrix [A], load vector {y}, and inverse matrix $[A]^{-1}$



SUM =MMULT(B8:F12,H2:H6)

	A	B	C	D	E	F	G	H
1								
2		14	14	-9	3	-5		-15
3		14	52	-15	2	-32		-100
4		-9	-15	36	-5	16		106
5		3	2	-5	47	49		329
6		-5	-32	16	49	79		463
7								
8		0.270366	-0.37237	0.248897	0.614204	-0.56509		=MMULT(B8:F12,H2:H6)
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069		
10		0.248897	-0.48966	0.365182	0.880899	-0.80293		
11		0.614204	-1.31425	0.880899	2.355126	-2.13266		
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218		
13								

Figure 2.25. Multiplying the inverse matrix $[A]^{-1}$ with the load vector {y}



H8 {=MMULT(B8:F12,H2:H6)}

	A	B	C	D	E	F	G	H
1								
2		14	14	-9	3	-5		-15
3		14	52	-15	2	-32		-100
4		-9	-15	36	-5	16		106
5		3	2	-5	47	49		329
6		-5	-32	16	49	79		463
7								
8		0.270366	-0.37237	0.248897	0.614204	-0.56509		-5.68434E-14
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069		1
10		0.248897	-0.48966	0.365182	0.880899	-0.80293		2
11		0.614204	-1.31425	0.880899	2.355126	-2.13266		3
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218		4
13								

Figure 2.26. The complete solution vector {x}

It should be mentioned that the procedure described above may not suit two-dimensional and three-dimensional fluid-flow and heat-transfer analyses. This is because the linear systems generated in multi-dimensional analyses are usually too large to be solved efficiently by using the matrix-inversion method. Another method for solving small systems of linear equations by using Solver is described in Chapter 3.

2.4. Itearative solutions with Excel

Excel offers its user two methods to perform iterative solutions: (i) by using the **Goal Seek** command and (ii) by using **circular calculations**. In what follows the two methods will be illustrated with the help of simple examples.

2.4.1. Itearative solutions with Goal Seek

The **Goal Seek** command is used for finding the value of an independent variable (x) that yields a specified value of a dependent variable (y). It is a simple, yet very useful tool for "What-if" analyses. The following example illustrates how this command can be used to solve a nonlinear equation.

Example 2-4. Solution of a nonlinear equation by Goal Seek

A centrifugal pump is used for lifting water from the utility network at the ground level to a tank at the top of a building that is 30-m high as shown in Figure 2.27. The pump's characteristic curve can be represented by the following formula:

$$h_p = h_0 - aQ - bQ^2 - cQ^3 \qquad (2.9)$$

Where h_p and Q are the pump's head (m) and discharge (m^3/s), respectively, and h_0, a, b, and c are constants the values of which are 47.22, 2.985x10^3, 1.549x10^5, and 2.348x10^8, respectively.

Neglecting friction losses in the pipe, determine the water flow rate (m^3/s) that can be delivered by the pump.

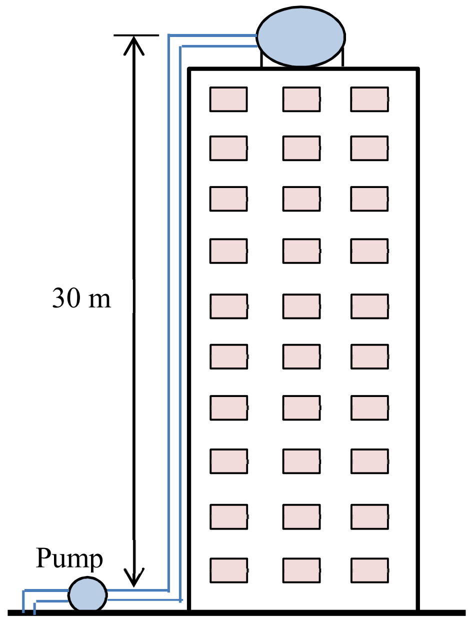


Figure 2.27. Schematic for Example 2-4

Solution

Figure 2.28 shows the Excel sheet prepared for this example in which the values of the four constants in Equation (2.9) are stored at the top of the sheet. The pump's head is calculated at various values of the discharge and plotted as shown in the figure. We can see from the plot that the value of Q that yield h_p = 30 m is approximately 0.003 m^3/s.

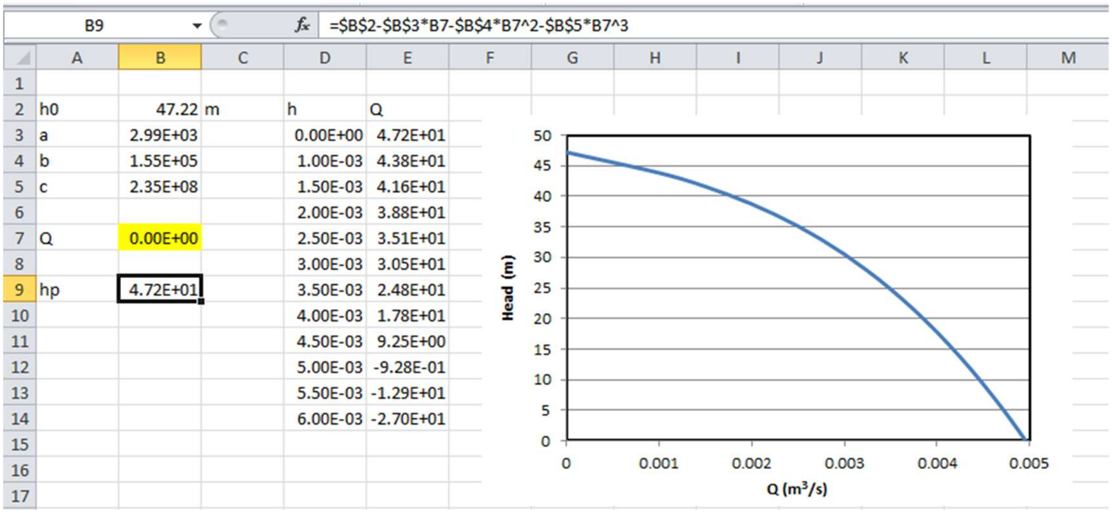


Figure 2.28. Excel sheet for Example 2-4

To solve the problem by using Goal Seek, enter an initial guess for Q in cell B7, say 0, and then enter the following formula that uses Equation (2.9) to calculate h_p in cell B9:

=B2-B3*B7-B4*B7^2-B5*B7^3

Note the dollar sign ($) that has been added to the references of the four constants, e.g. B2 has become B2. The formula bar in Figure 2.28 reveals the above formula by placing the cursor at cell B9.

To activate the Goal Seek command, go to the **Data** tab, select the **What-If-Analysis** option in the **Data Tools** group and then select **Goal Seek,** as shown in Figure 2.29. The Goal Seek dialog box shown in Figure 2.30.a will then appear to you. The dialog box asks you to select the "**Set cell**", i.e. the cell that contains the dependent variable, which is B9 in this case. You also have to specify the value sought for this cell and the adjustable cell that stores the parameter to be changed. In this case, we seek the value in the cell B9 to be 30 by changing the value of the cell B7. The completed form is shown in Figure 2.30.b.

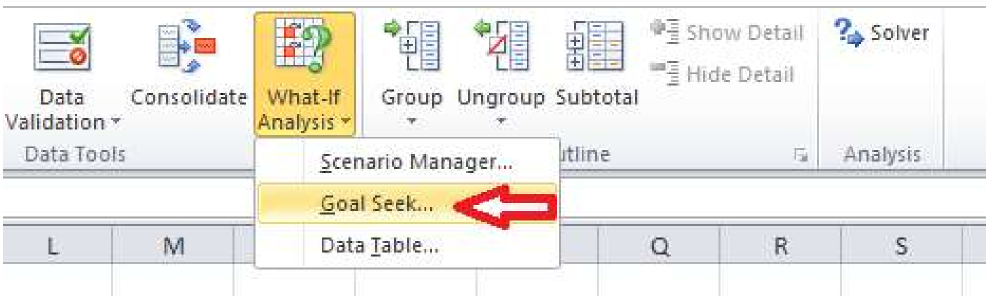
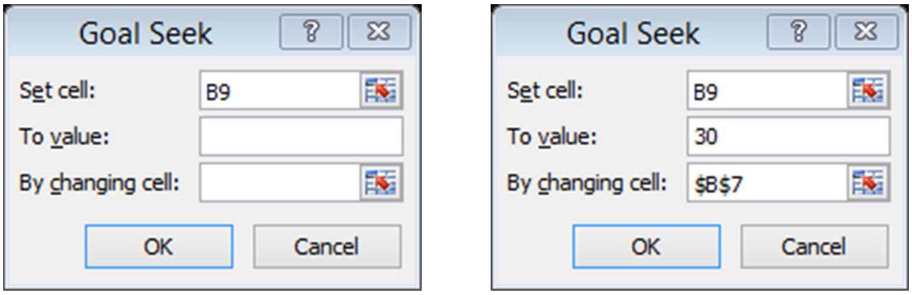


Figure 2.29. Activation of the Goal Seek command



(a) (b)

Figure 2.30. Goal Seek Set-up for Example 2-4: (a) before completion (b) the completed box

By pressing the “**OK**” button after completing the Goal Seek form, Excel will change the value in the adjustable cell (B7) until the Set cell (B9) acquires the required value. As shown in Figure 2.31, the answer obtained is Q = 0.003 m^3/s which agrees with the estimated value from the plot in Figure 2.28.

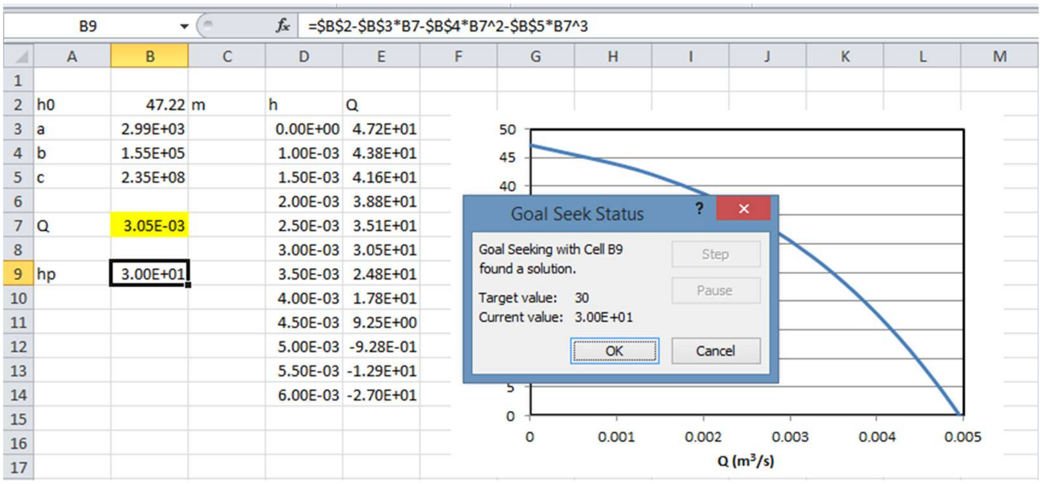


Figure 2.31. Goal Seek solution for Example 2-4

2.4.2. Iterative solution with circular calculations

A circular reference occurs when an Excel formula refers to its own cell in a direct or indirect manner. In standard programming this is not allowed, but Excel gives its user the option to use “**circular calculation**” if intended. In this case, Excel will itearate until all the formulae involved are satisfied. The following example illustrates this special feature which is useful for thermofluid analyses.

Example 2-5. Determining the final temperature of heated air

Heat is added to a piston-cylinder device that contains one kg of air initially at 300K. If 100 kJ of heat is added to the air at constant pressure, determine the final temperature of air taking into consideration that its molar specific-heat ($\tilde{c}_p$) varies with temperature according to the following formula:

$$\tilde{c}_p = a + bT + cT^2 + dT^3 \quad \text{[kJ/kmol]} \tag{2.10}$$

Where a = 28.11, b =1.97x10^{-03}, c = 4.80x10^{-06}, and d = -1.97x10^{-09}.

Solution
From the defenition of specific heat, the final temperature (T_2) is given by:

$$T_2 = T_1 + Q/\left(\tilde{c}_p / M\right) \tag{2.11}$$

Where T_1 is the initial temperature, Q is the amount of heat added, and M is the molar mass for air (M=29). If the variation of $\tilde{c}_p$ with temperature is ignored and its value at T_1 alone is used, Equation (2.11) determines T_2 as 399.73K. However, we can be more accurate by using Equation (2.10) to determine $\tilde{c}_p$ at the average temperature, T_{avr} = (T_1+T_2)/2. Figure 2.32 shows the Excel sheet developed for this method which reveals the formulae inserted in cells F2, F4, and F6.

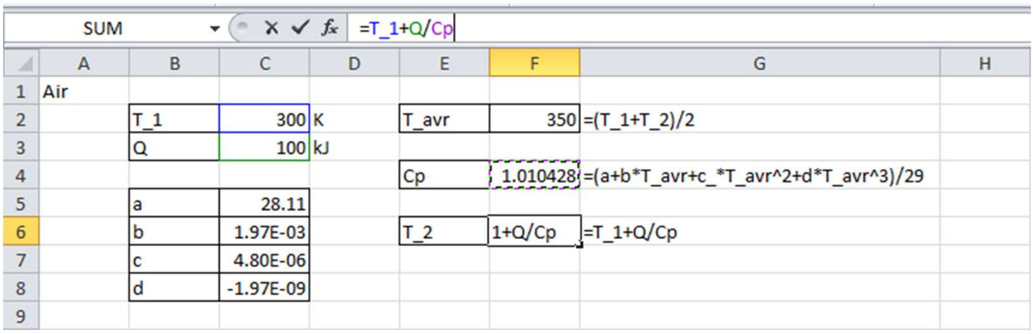


Figure 2.32. Excel sheet developed for Example 2-5

As soon as we type Equation (2.11) in cell F6, Excel will make the warning message that there is a circulare refernce as shown in Figure 2.33.

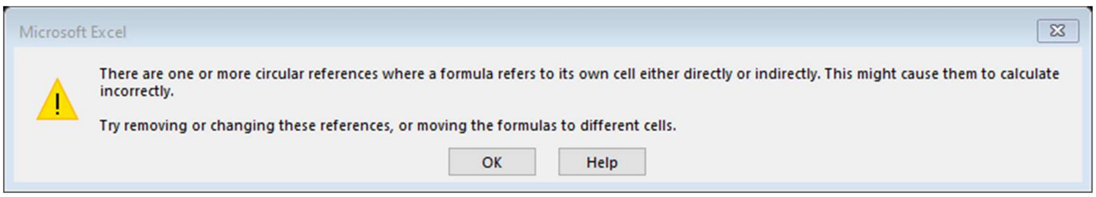


Figure 2.33. The circular-reference prompt

The circular reference occurs because T_2 depends on $\tilde{c}_p$ according to Equation (2.11) while $\tilde{c}_p$ itself depends on T_2 according to Equation (2.10). If we press the “OK” button shown in Figure 2.33, the cells involved in the circular reference whill be identified as shown in Figure 2.34. In this case, three cells are involved in the circular reference, which are F2, F4, and F6.

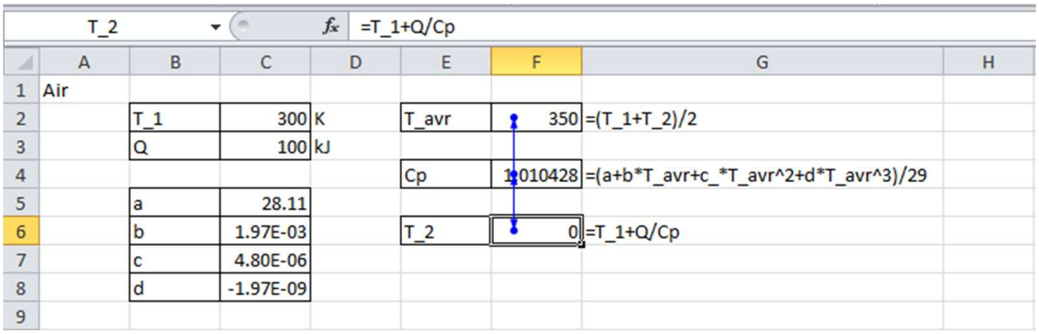


Figure 2.34. The cells involved in the circular reference

Excel can iterate to determine the values of both T_2 and $\tilde{c}_p$ that satisfy the relevant equations but the iterative-calculation option is not allowed by default. To allow it, go to **File** and select **Options**. The **Backstage View** form shown in Figure 2.35 will appear to you. Select **Formulas**, then the form will appear as shown in Figure 2.36. Enable iterative calculations by ticking (√) the box indicated in the figure and press the “OK” button. Excel can now iterate to find the values of T_2 and c_p that simultaneously satisfy Equations (2.10) and (2.11). Figure 2.37 shows the solution found by this method, which is T_2 =398.976K.

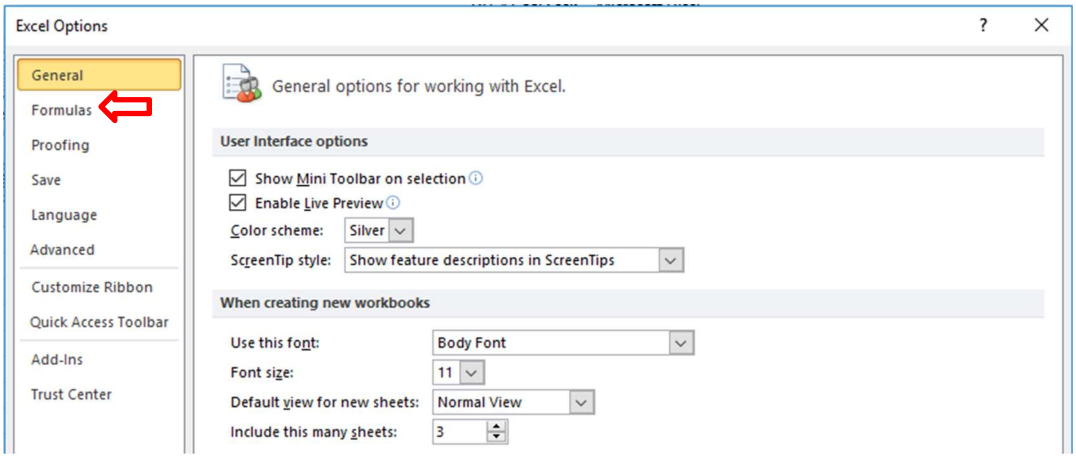


Figure 2.35. Selecting Excel's option (Formulas)

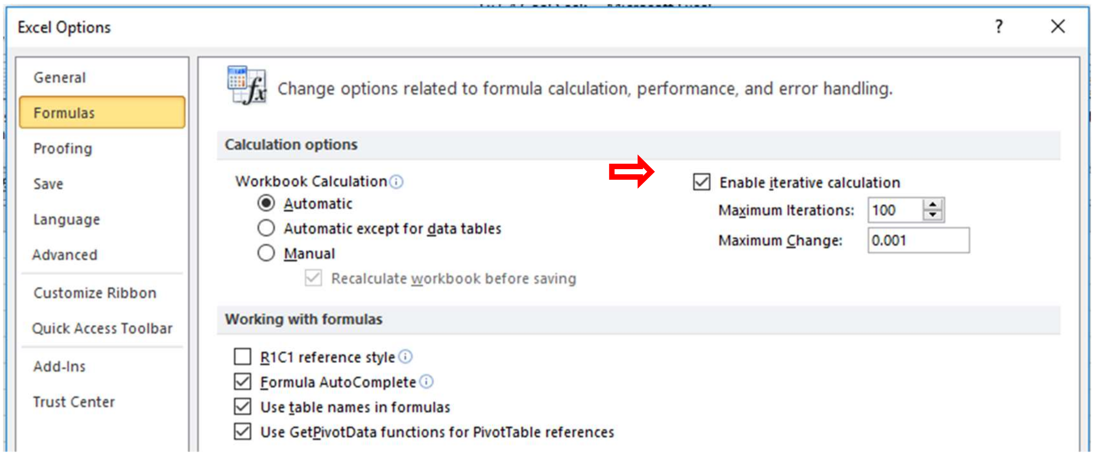


Figure 2.36. Enabling iterative calculations from Excel's Formulas option

T_2 =T_1+Q/Cp

	A	B	C	D	E	F	G	H
1	Air							
2		T_1	300	K	T_avr	349.488	=(T_1+T_2)/2	
3		Q	100	kJ				
4					Cp	1.010346	=(a+b*T_avr+c_*T_avr^2+d*T_avr^3)/29	
5		a	28.11					
6		b	1.97E-03		T_2	398.976	=T_1+Q/Cp	
7		c	4.80E-06					
8		d	-1.97E-09					
9								

Figure 2.37. Solution of Example 2-5 by circular calculations

This example can also be solved by using the Goal Seek command. In this case, we have to start the iterative solution by providing Excel with a guessed value for T_2, call it T_{2o}, based on which a new value for T_2 is calculated, called it T_{2c}. Since the guessed value T_{2c} is unlikely to be correct, it will be different from T_{2o}. Goal Seek is then used to adjust the value of T_{2o} until the difference (Diff = T_{2c} - T_{2o}) vanishes.

Most iterative solutions in this book are obtained by using the Goal Seek command. However, the circular-calculation option is more useful in certain situations as demonstrated in Chapters 6 and 7 that deal with the numerical solution of the heat-conduction equation with the finite-difference method.

2.5. Excel's graphical tools for data presentation and analysis

Excel has numerous graphical tools that can be used to present the stored data in a variety of charts. Figure 2.38 shows one type of Excel charts that displays the annual variation of temperature and relative humidity at one location in a certain day. The figure shows a line chart in which the temperature is scaled on the primary y-axis (on the left) while the humidity is scaled on the secondary y-axis (on the right). This arrangement is useful for displaying two or more types of data that differ significantly in magnitude such as the net specific work and thermal efficiency of a power cycle.

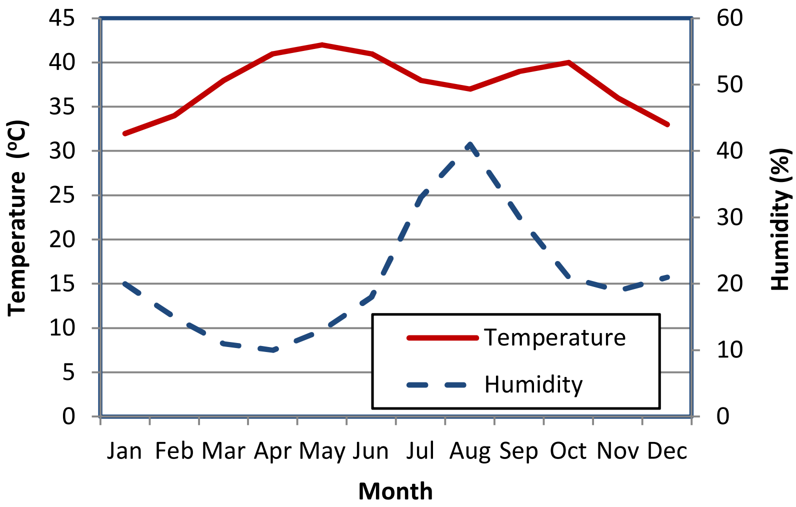


Figure 2.38. An example of line charts

Excel supports other types of charts that allow the user to select the most appropriate way to display his/her data in the form of bar, area, or scatter charts. For more information about the different types of Excel's charts, the reader can refer to specialised references such as Walkenbach [2]. A number of tutorials and videos that show how to create different types of charts can also be found in the internet.

Excel's charts provide a curve-fitting capability of numerical data by using the **Trendline** feature. This capability is particularly useful for computer-aided thermofluid analyses because it can be used to convert tabulated fluid-properties and other data into analytical equations that make the data more suitable for iterative solutions and optimisation analyses. To illustrate the use of this feature, consider Table 2.1 that shows properties of saturated water in the range 0.001ºC – 60ºC. These values of the saturation pressure (P_{sat}) and saturated liquid enthalpy (h_f) are used in psychrometric analyses of air-conditioning applications. For computer-aided analyses, it is useful to convert these data into relevant equations.

The trendline feature provides a number of options, which include exponential, linear, logarithmic, polynomial, and power equations as shown in Figure 2.39. To fit a trendline to the data, we have to create line charts for the two properties as shown in Figures 2.40.a and 2.40.b. Trendlines can then be added on the line charts. Figures 2.40.a and 2.40.b also show the corresponding trendline equations of the tabulated data as determined by using polynomial equations. As Figure 2.40.b shows, a linear equation is adequate for the h_f data since its variation over the given temperature range is mild. However, a third-order polynomial is required to represent the variation of P_{sat} with temperature as shown in Figure 2.40.a.

Table 2.1. Properties of saturated water at temperatures in the range 0ºC- 60ºC taken from Cengel and Boles [3]

TºC	P_{sat} [kPa]	v_f [m^3/kg]	v_g [m^3/kg]	u_f [kJ/kg]	u_g [kJ/kg]	h_f [kJ/kg]	h_g [kJ/kg]	s_f [kJ/kg.K]	s_g [kJ/kg.K]
0.01	0.6117	0.001000	206.00	0.000	2374.9	0.001	2500.9	0.0000	9.1556
5	0.8725	0.001000	147.03	21.019	2381.8	21.020	2510.1	0.0763	9.0249
10	1.2281	0.001000	106.32	42.020	2388.7	42.022	2519.2	0.1511	8.8999
15	1.7057	0.001001	77.885	62.980	2395.5	62.982	2528.3	0.2245	8.7803
20	2.3392	0.001002	57.762	83.913	2402.3	83.915	2537.4	0.2965	8.6661
25	3.1698	0.001003	43.340	104.83	2409.1	104.83	2546.5	0.3672	8.5567
30	4.2469	0.001004	32.879	125.73	2415.9	125.74	2555.6	0.4368	8.4520
35	5.6291	0.001006	25.205	146.63	2422.7	146.64	2564.6	0.5051	8.3517
40	7.3851	0.001008	19.515	167.53	2429.4	167.53	2573.5	0.5724	8.2556
45	9.5953	0.001010	15.251	188.43	2436.1	188.44	2582.4	0.6386	8.1633
50	12.352	0.001012	12.026	209.33	2442.7	209.34	2591.3	0.7038	8.0748
55	15.763	0.001015	9.5639	230.24	2449.3	230.26	2600.1	0.7680	7.9898
60	19.947	0.001017	7.6670	251.16	2455.9	251.18	2608.8	0.8313	7.9082

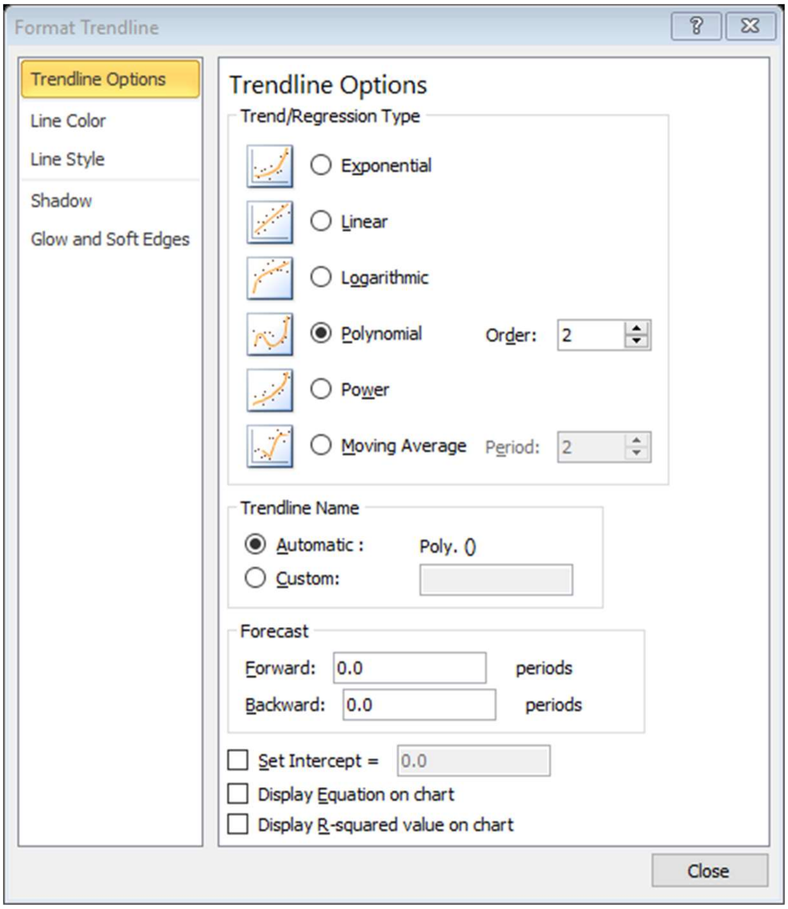


Figure 2.39. The Format Trendline window

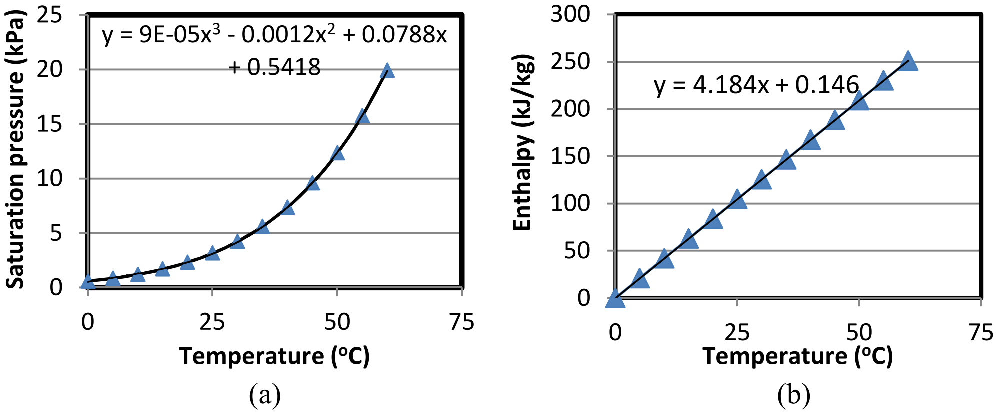


Figure 2.40. Fitting trendlines on tabulated data for water of (a) saturation pressure and (b) saturated liquid enthalpy

2.6. Closure

Volumes have been written about Excel and its tools for data analyses [1,2]. This chapter does not intend to describe all these capabilities, but focusses on the basic features of Excel that are mostly needed for thermofluid analyses. The chapter highlights the importance of using cell labelling with Excel's formulae, and illustrates the use of Excel's general mathematical functions, logical functions, and the functions for matrix operations. The chapter also demonstrates the use of Excel's two iterative tools: Goal Seek and circular calculations. As shown in later chapters of the book, the Goal Seek command is very useful in spite of its limitations for performing iterative solutions. Finally, the chapter illustrates the usefulness of Excel's charting tools for the presentation of tabulated data particularly the trendline feature.

Although not discussed in the chapter, it should be mentioned that the **Developer** tab of Excel's user-interface provides a number of useful features that can be utilised to enhance the effectiveness of Excel as modelling platform for thermofluid analyses. By providing these features, Excel becomes more effective as a modelling platform for thermofluid analyses than the available dedicated software. One of these features is the ability to develop user-defined functions with VBA; which will be discussed in Chapter 3. Another useful feature is the ability to record **macros** for conducting repetitive calculations and parametric analyses as illustrated in Chapter 7 and Appendix E.

References

[1] J. Walkenbach, *Excel 2010 Formulas*, Wiley Publishing Inc., 2010
[2] J. Walkenbach, *Excel 2007 Charts*, Wiley Publishing Inc., 2007
[3] Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007
[4] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*, 6th Edition, McGraw Hill, 2010

Exercises

1. The following table shows measured values of the temperature by two different methods compared to the correct corresponding values. Find the average error for each method.

***Correct T* (ºC)**	***Method* 1**	***Method* 2**
0	0.1044	0.1112
10	10.1092	10.1153
20	20.1139	20.1194
30	30.1186	30.1235
40	40.1231	40.1275
50	50.1276	50.1316
60	60.1320	60.1357
70	70.1364	70.1397
80	80.1407	80.1438
90	90.1450	90.1479
100	100.1493	100.1520

2. The following table shows the data for the saturation pressure of a certain fluid. Use a nested IF statement to develop an interpolation formula that determines the saturation pressure of the fluid at any temperature in the range 5ºC ≤ T ≤ 30ºC.

T(ºC)	P_{sat} (kPa)
5	0.872
10	1.228
15	1.705
20	2.339
25	3.169
30	4.246

3. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations to determine the values of the three unknowns a, b, and c. This exercise is based on Example 9.11 in Chapra and Canale [4]. The answer is: a = 8.5941, b=34.4118, and c = 36.7647.

4. Figure 2.P4 shows a triangular fin attached to the surface of a wall. Solution of the conduction heat transfer equation with the finite-difference method resulted in the

following system of linear equations the solution of which gives the temperatures in °C at different distances from the fin base as shown in the figure:

$-8.008\ T_1 + 3.5\ T_2 = -900.209$

$3.5\ T_1 - 6.008\ T_2 + 2.5\ T_3 = -0.209$

$2.5\ T_2 - 4.008\ T_3 + 1.5\ T_4 = -0.209$

$1.5\ T_3 - 2.008\ T_4 + 0.5\ T_5 = -0.209$

$T_4 - 1.008\ T_5 = -0.209$

Use Excel functions to solve the above system of linear equations.

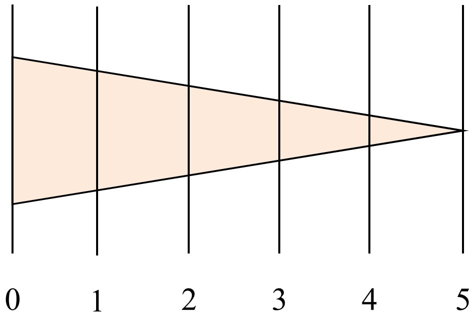


Figure 2.P4. Triangular fin

5. Adopting suitable names in your formulae, prepare an Excel sheet for calculating the frictional loss (h_f) in a circular pipe of diameter D, length L, and roughness k_s. Use your sheet to determine h_f in the following cases:

   (a) D = 25 cm, L = 150 m, V = 2 m/s, k_s = 0.045 mm, carrying water at 20°C.
   (b) D = 25 cm, L = 150 m, V = 0.2 m/s, k_s = 0.045 mm, carrying oil at 20°C.
   (c) D = 25 cm, L = 150 m, V = 7 m/s, k_s = 0.045 mm, carrying air at 20°C.

   Use the Darcy-Weisbach equation and determine the values of the kinematic viscosity from relevant property tables.
6. Using a line chart, plot the variation of sine θ for $-180 \leq \theta \leq 180$ in steps of 10°then add cosine θ on the same chart.
7. Using the data shown in Table 2.1, make a line chart for v_f and v_g. Add polynomial trendlines for both properties and comment on the trendlines equations.
8. The table below shows some of the thermo-physical properties of air at atmospheric pressure and different temperatures. Use Excel charts to show the variation of the

properties ρ, β, c_p, k, α, μ, ν, and Pr with temperature and use trendline to obtain suitable equations for these properties.

T (K)	ρ (kg/m^3)	$\beta\times10^3$ (1/K)	c_p (J/kg.K)	k (W/m.K)	α (m^2/s)	$\mu\times10^6$ (N S/m^2)	$\nu\times10^6$ (m^2/s)	Pr
273	1.252	3.66	1011	0.0237	19.2	17.456	13.9	0.71
293	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71
313	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71
333	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71
353	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71
373	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71
473	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71
573	0.596	1.75	1047	0.0429	68.9	29.322	49.2	0.71
673	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72
773	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72

9. The volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid by:

   $V = \pi h^2(3r - h)/3$

   Using the Goal Seek command, determine the value of h for the tank with r=1 m and V = 0.5 m^3.

   This exercise is based on Problem 8.9 in Chapra and Canale [4]. Answer: h = 0.431 m.

3

Solver, VBA, and Thermax

This chapter focuses on the three components that, together with Excel's user-interface, make up the modelling platform used in this book for thermofluid analyses: Solver, VBA, and Thermax. As discussed in the previous chapter, Excel's user-interface provides numerous built-in functions, two iterative tools, and the graphic tools. However, Excel needs the three other components in order to become particularly useful for thermofluid analyses. Solver offers three solution methods that can be used for performing optimisation analyses and solving systems of linear and nonlinear equations. The chapter illustrates the use of these methods by means of relevant examples. With respect to VBA and Thermax, the chapter shows how VBA can be used for developing custom functions and how Thermax functions can be used in Excel formulae.

3.1. Solver

Solver is a multi-purpose iterative tool developed by Frontline Systems [1] as an Excel add-in for "What-if" analyses. Compared to the Goal Seek command described in Chapter 2, Solver offers the following advantages:

1. While Goal-Seek can be used for simple problems that involve only one decision variable, Solver can deal with more difficult problems in which the objective cell is affected by numerous cells and decision variables.
2. Goal Seek allows only a required value of the objective cell to be achieved, but beside that Solver enables Excel to perform an optimisation analysis by finding the maximum or minimum value for the formula in the objective cell.
3. Solver allows constraints to be applied on the iterative solution. This ability, which is particularly important for performing iterative solutions and optimisation analyses, is not possible with Goal Seek.
4. Solver offers three solution methods that suit different types of problems and gives the user a number of numerical options for applying these methods.

3.1.1. Activation of Solver

Like the Goal Seek command, Solver is found in the **Data** tab as shown in Figure 3.1. If it doesn't appear in your **Data** tab, then go to **File**, click **Options**, and select **Add-Ins**. From the **Manage** option at the bottom of the menu select **Excel Add-ins** and then click "**Go**". The **Add-Ins** dialog box shown in Figure 3.2 will be shown. To add **Solver**, tick (√) on the "**Solver"** option and return to the **Data** tab. When you click the **Solver** button from the **Data** tab, the **Solver Parameters** dialog box shown in Figure 3.3 will be shown.

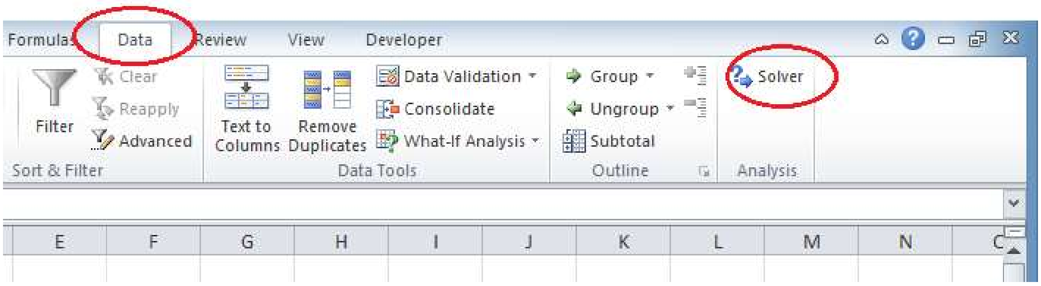


Figure 3.1. The Solver add-in in the Data tab

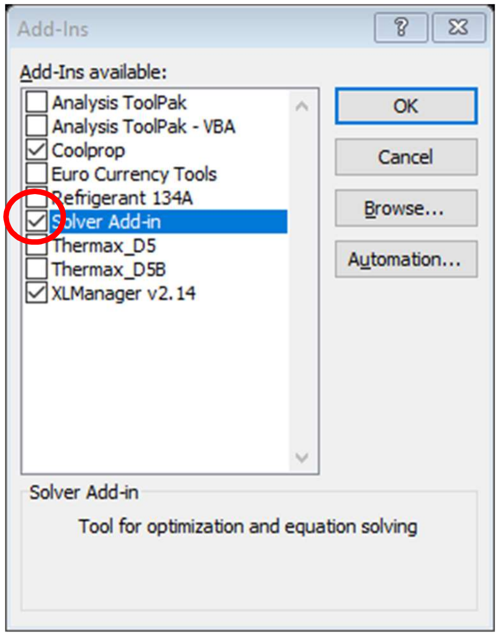


Figure 3.2. Activating Solver from the menu of Excel add-ins

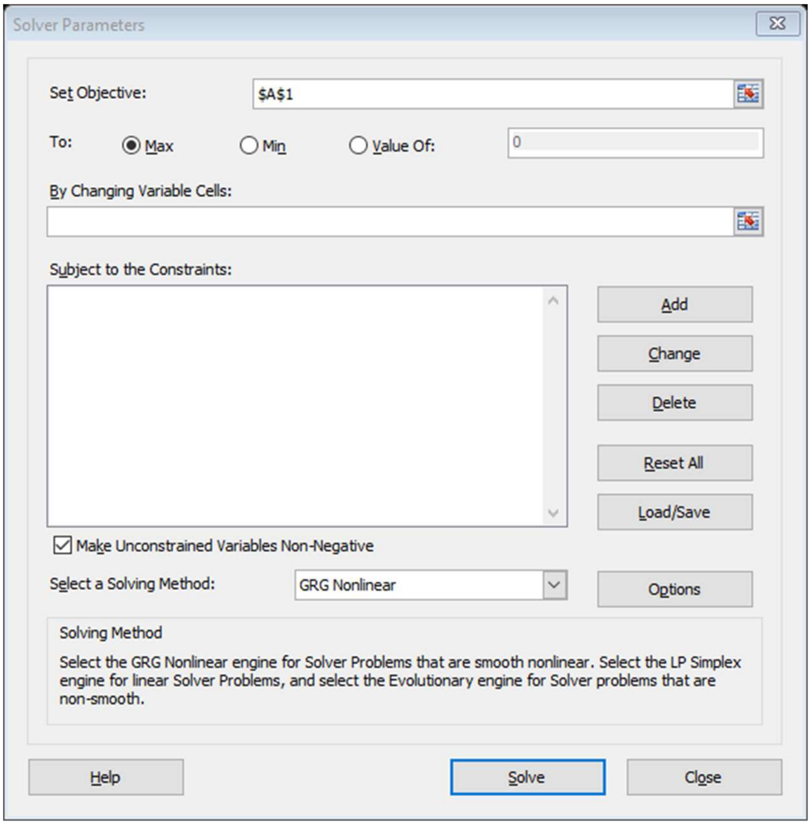


Figure 3.3. Solver Parameters dialog box

Solver Parameters dialog box helps the user to select a formula in one cell, called the **objective cell**, and a group of other cells, called **decision variables** or variable cells, that are directly or indirectly related to the formula in the objective cell. By adjusting the **decision variables**, the **objective cell** can be maximised, minimised, or made to acquire a certain value. As shown in Figure 3.3, **constraints** can be applied on the sought values of the decision variables. To suit different types of problems, Solver offers three solution methods which are:

1. The **GRG Nonlinear** method,
2. The **Evolutionary** method
3. The **Simplex LP** method.

The **GRG Nonlinear** method involves the determination of the function's gradient like the Steepest Descent method [2, 3]. The **Evolutionary** method adopts a variety of genetic algorithms and local search methods [4]. Both the **GRG Nonlinear** method and the **Evolutionary** method are suitable for non-linear problems. The **Simplex LP** method, which is a linear-programming method, is suitable for linear problems. As shown in Figure 3.3, Solver uses the **GRG Nonlinear** method by default. The following sections give examples of using the three solution methods.

3.1.2. The GRG Nonlinear method

In an optimisation analysis we may require the objective function to be maximised or minimised depending on the situation at hand. For example, the optimisation of pipe insulation requires its total cost to be minimised, while the optimisation of a central thermal power plant requires its thermal efficiency to be maximised. The following example illustrates the use of the **GRG Nonlinear** method in optimisation analyses.

Example 3-1. Finding the minimum value of a quadratic function

Find the minimum value of the following quadratic function in the specified range.

$$f(x) = x^2 - 2x - 1; \qquad -2 \le x \le 3 \tag{3.1}$$

Solution

Figure 3.4 shows the Excel sheet developed for this example. The line chart inserted in the figure shows the variation of f with x from which we can estimate that the minimum value of f is -2 and occurs at x =1. An initial value for x is entered in cell **B3** based on which the function $f(x)$ is calculated in cell **B6** according to Equation (3.1). Note the curser is placed on cell **B6** to reveal the formula typed in the cell, which is:

= B3^2-2*B3−1

We can now use Solver to determine the minimum value of the function. To do so, select **Solver** from the **Data** tab and fill its parameters dialog-box as shown in Figure 3.5 that shows the top part of the completed box.

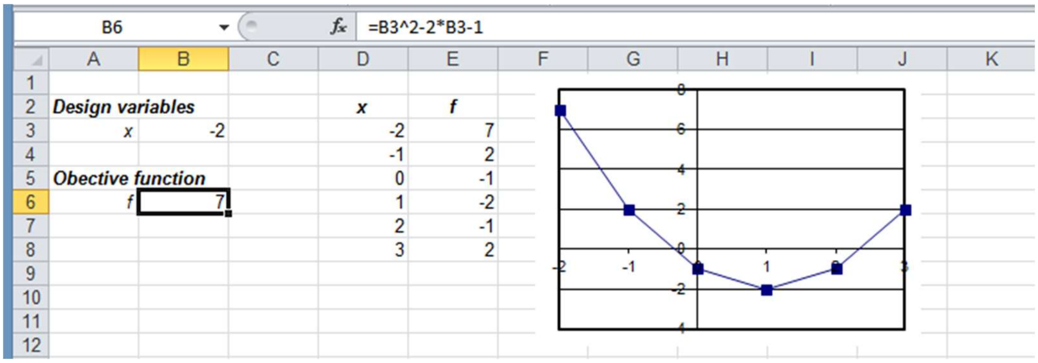


Figure 3.4. Excel sheet for determining the local minimum of the quadratic function

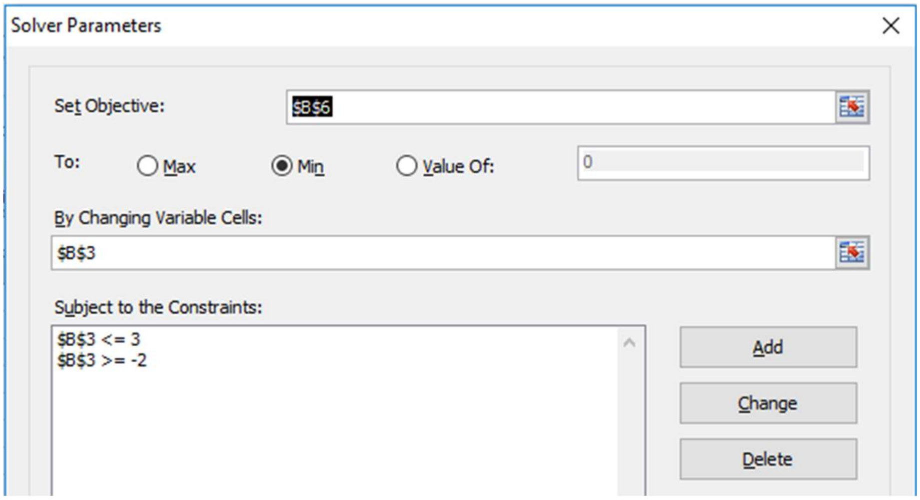


Figure 3.5. The completed Solver dialog box for Example 3-1

The dialog box in Figure 3.5 has been filled as follows:

Set Objective:	**B6** and **Min** have been selected for this option since want the value of the function in cell **B6** to be minimised.
By Changing Variable Cells:	**B3** which is the cell that stores the value of the independent variable x.
Subject to the Constraints:	Two constraints have been added that specify the minimum and maximum values of x as $x \geq -2$ and $x \leq 3$, respectively.
Select a Solving Method:	The **GRG Nonlinear** method (the default option).

Pressing the "**Solve**" button of the completed parameters box sparks Solver to find the required solution. As Figure 3.6, shows, the answer found by Solver, which is $x = 1$, $f = -2$, agrees with the graphical solution shown in Figure 3.4.

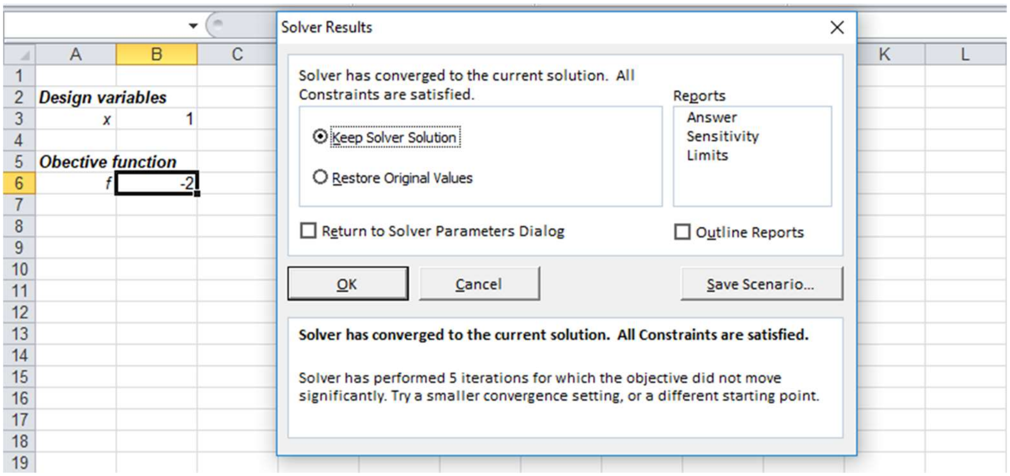


Figure 3.6. Solver solution for Example 3-1

3.1.3. The Evolutionary method

When the function to be optimised has more than one point of inflection, the solution found by the **GRG Nonlinear** method may only be a local minimum or maximum. The following example illustrates the capability of the **Evolutionary** method to find the global optimum solution in this situation for a simple function.

Example 3-2. Finding the global minimum of a function

Determine the global minimum value for the following function:

$$f(x) = x\cos(x) \qquad 3 \le x \le 14 \qquad (3.2)$$

Solution

Figure 3.7 shows the Excel sheet developed for solving this example. The insert shows that the function has two minima in the specified range of x; at $x \approx 5$ and $x \approx 11$.

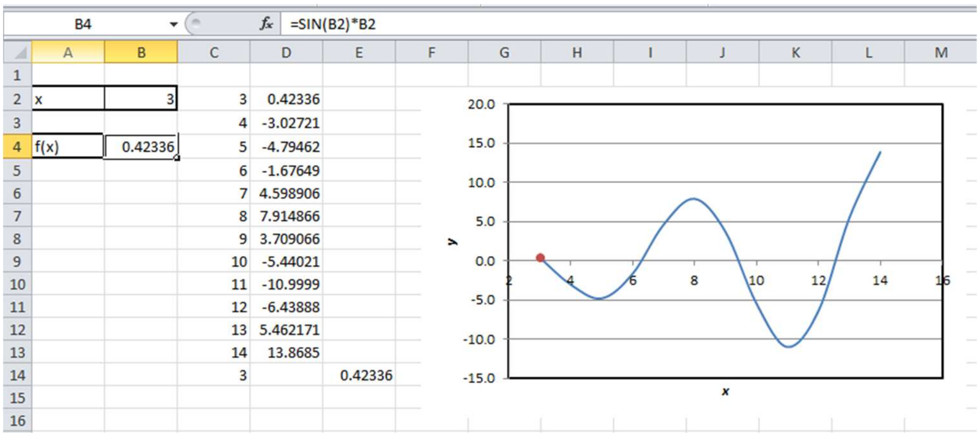


Figure 3.7. The Excel sheet for Example 3-2

Let us first try to solve the problem with the **GRG Nonlinear** method. An initial value for x has been entered in cell **B2** based on which the function f is calculated in cell **B4**. The formula bar in Figure 3.7 reveals the formula entered in the cell **B4**. Figure 3.8 shows the completed Solver parameters dialog-box with two constraints that specify the upper and lower limits for x. From Figure 3.9 that shows the solution found by Solver it is clear that it found the local minimum ($y = -4.81$) which is nearer to the initially specified value and not the global minimum.

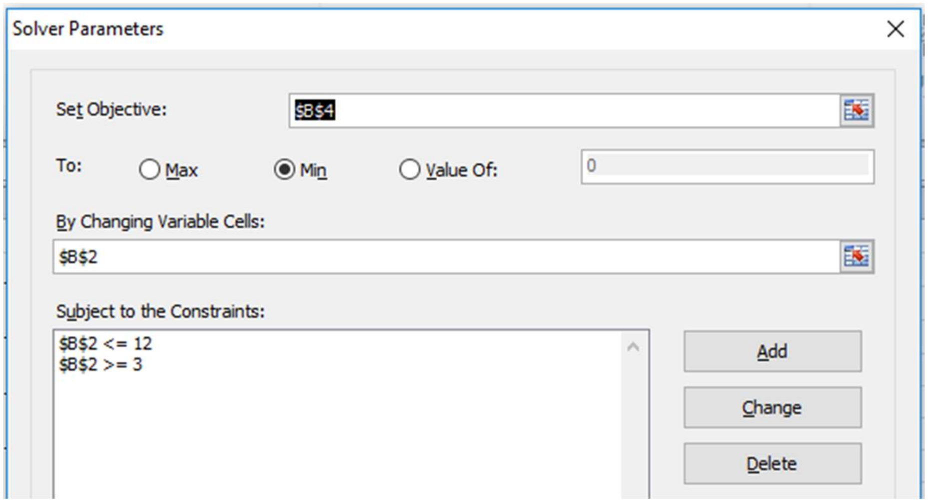


Figure 3.8. Solver set-up for Example 3-2 with GRG Nonlinear method

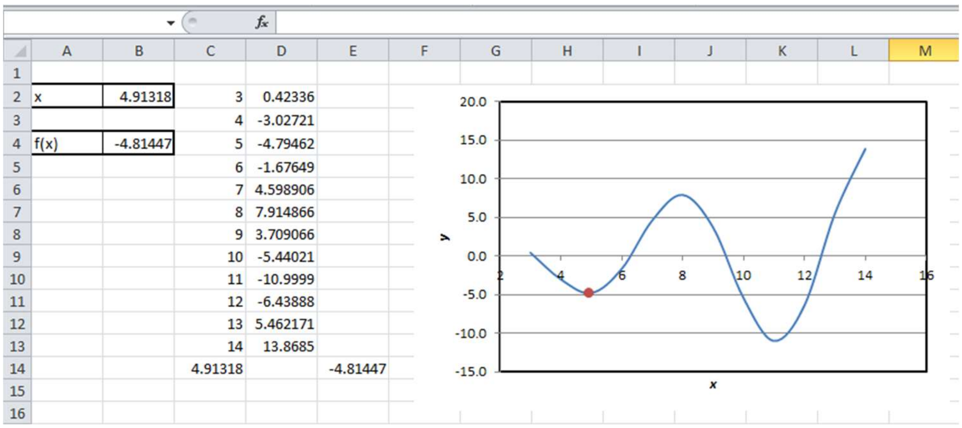


Figure 3.9. Solver solution for Example 3-2 with the GRG Nonlinear method

In order to locate the global minimum by the **GRG Nonlinear** method, the solution has to be started with an initial guess that is closer to the global minimum, e.g., $x = 9$. The advantage of the **Evolutionary** method is that such an arrangement is not required. To use this method, the set-up shown in Figure 3.8 only needs Solver's solution method to be changed to "**Evolutionary**". Figure 3.10 that shows the solution obtained by this method confirms that the method produced the global minimum ($y = -11.041$).

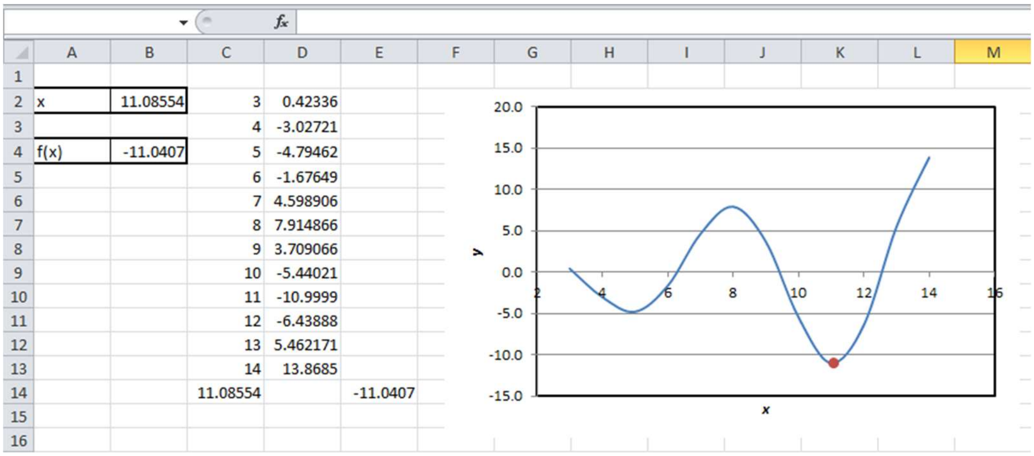


Figure 3.10. Solver solution for Example 3-2 with the Evolutionary method

The **Evolutionary** method is useful for optimisation analyses that involve non-smooth and discontinuous functions, which are difficult to solve with the **GRG Nonlinear** method. However, its disadvantage is that it takes long computer times. While the **GRG Nonlinear** method took less than a second to solve the above problem, the **Evolutionary** method took more than a minute with the same computer.

3.1.4. The Simplex LP method

This option provides a method for solving small systems of linear equations that can be used instead of the method described in the Chapter 2 by using Excel's matrix functions. The method will be illustrated by reconsidering the problem of Example 2-3. Figure 3.11 shows a new Excel sheet for solving the problem with the present method.

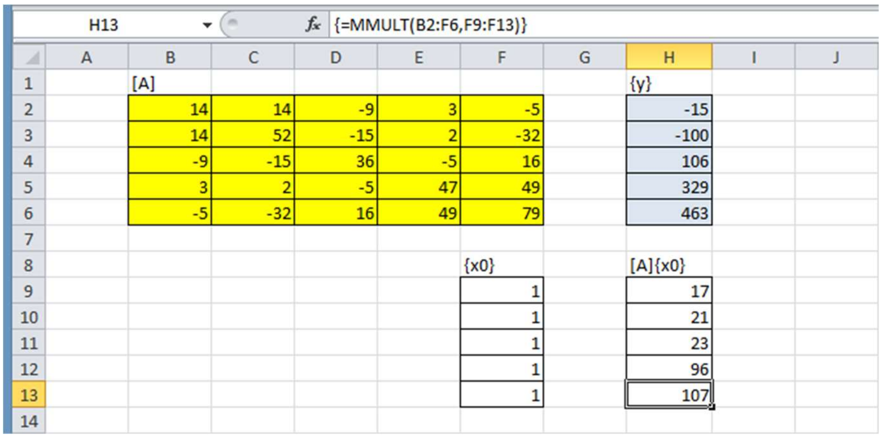


Figure 3.11. Excel sheet for solving Example 2-3 with Solver

The top part of the sheet stores the coefficient matrix [A] and the right-hand vector {y} of the system of linear equations to be solved. The procedure starts with a guessed solution which is stored as vector {x0} in cells **F9:F13**. All the elements of this vector are given a value of 1 as shown in Figure 3.11. The coefficient matric [A] is then multiplied by the guessed vector {x0} using Excel's "**MMULT**" function and the result stored in cells **H9:H13**. If this initial guess were the correct answer, the multiplication [A]{x0} would have been the same as the true right-hand side vector, i.e,:

$$[A]\{x0\} = \{y\} \qquad (3.3)$$

However, Figure 3.11 shows that the vector [A]{x0} is different form the true right-hand side vector {y} stored in cells **H2:H6**. Solver can now be used to adjust the variable cells **D9:D13** so that all elements of the vector [A]{x0} become equal to their counterparts in vector {y}, i.e:

H9 = H2
H10 = H3
H11 = H4
H12 = H5
H13 = H6

Solver set-up for this task is shown in Figure 3.12.

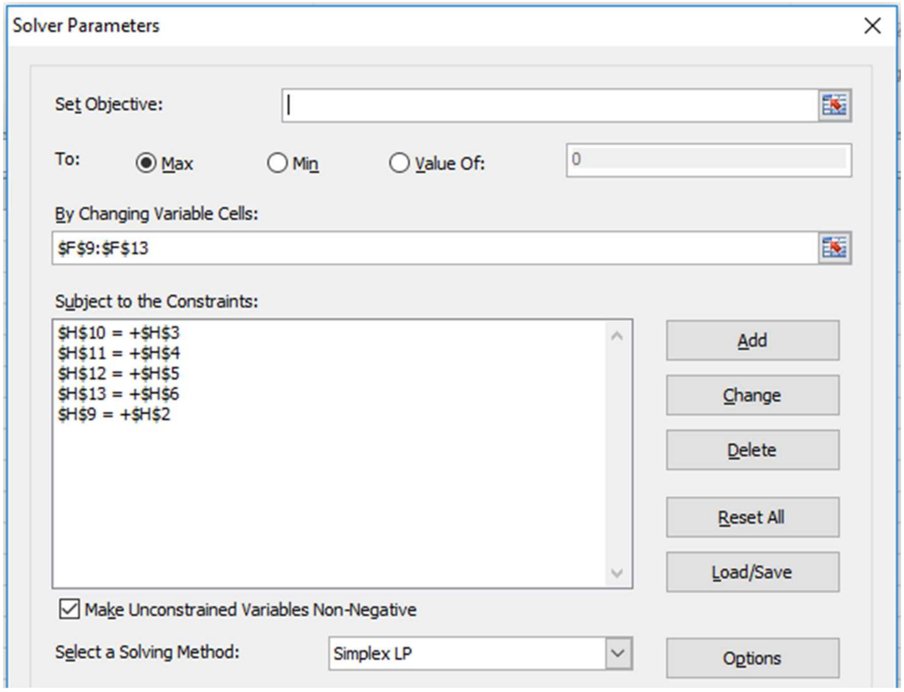


Figure 3.12. Solver set-up for Example 2-3 with the Simplex LP method

Note that the objective cell is left blank and the **Simplex LP** method is selected as the solution option. In this case, Solver will iterate to find the values of the decision variables that satisfy all the imposed constraints. The solution found by Solver using the above set-up is shown in Figure 3.13.

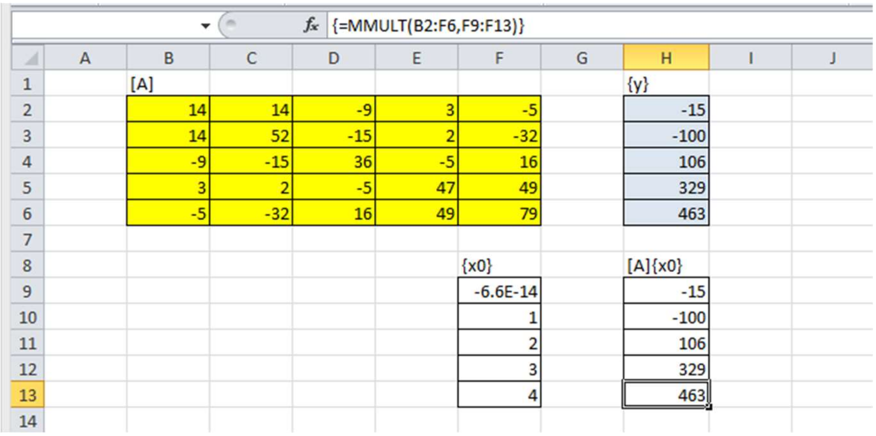


Figure 3.13. Solution of Example 2-3 with the Simplex LP method

All the elements of the [A]{x0} are now equal to their corresponding elements in the vector {y}. The first element of the solution vector, which is -6.6×10^{-16}, is practically zero. Therefore, the solution is [0,1,2,3,4], which is the same as that obtained in Example 2-3 by using the matrix-inversion method. The advantage of Solver compared to the matrix-inversion method is that it can be used for solving systems of nonlinear equations by following a similar procedure (Refer to Problem 3.5 in the Exercises).

3.1.5. The default settings of Solver options

Solver gives it user additional flexibility by allowing alternative options for monitoring and adjusting the precision and computer time of its three solution methods. While some options are common to all three solution methods, others are particular to the **GRG Nonlinear** or the **Evolutionary** method. By clicking the "**Options**" button in Solver's parameters dialog box shown in Figure 3.14, the dialog box shown in Figure 3.15 will be shown.

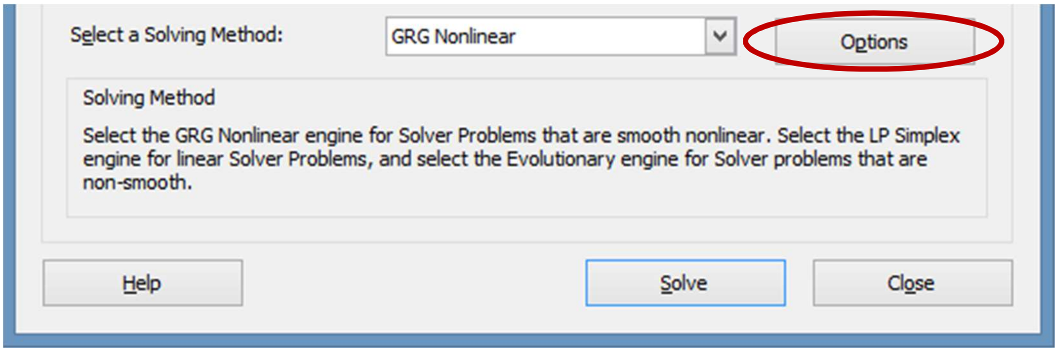


Figure 3.14. Solver options in the Properties dialog box

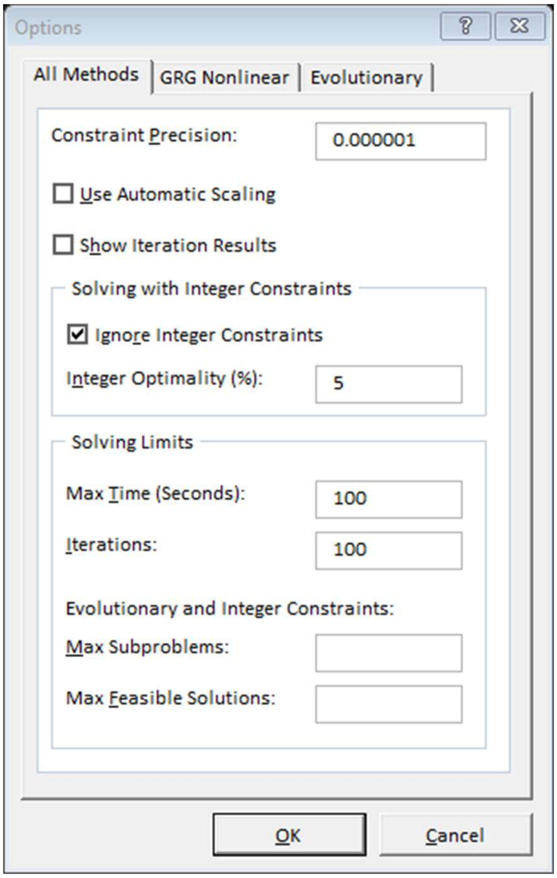
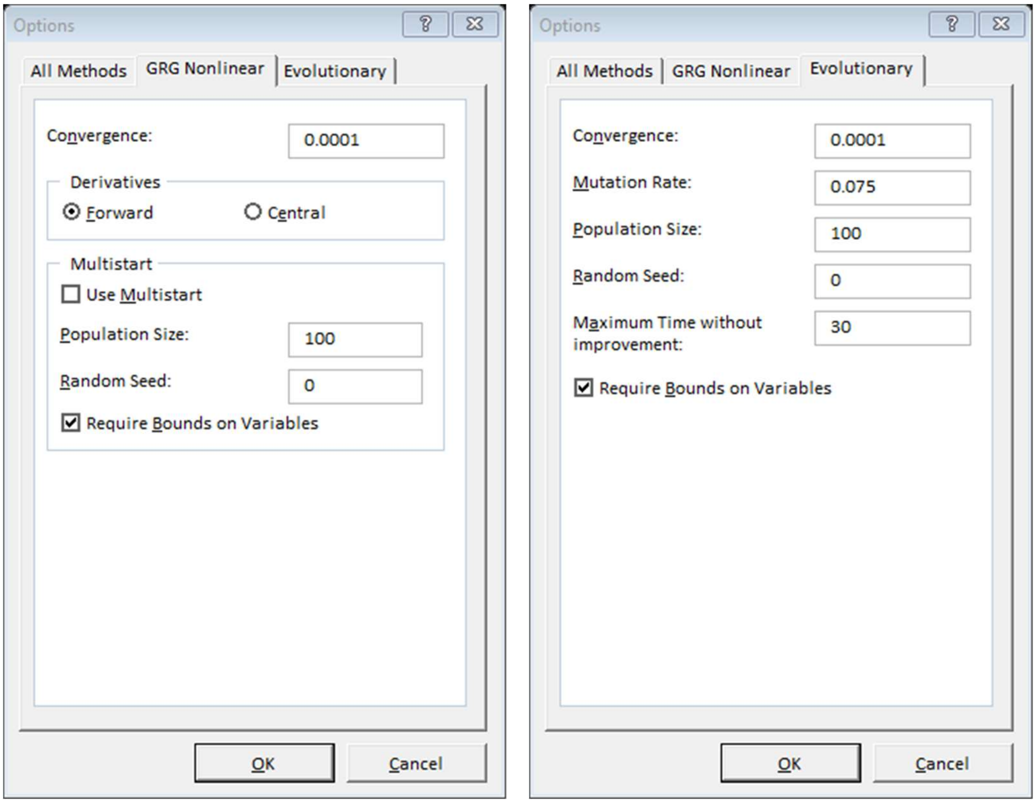


Figure 3.15. Default Solver options adopted in the analyses for all solution methods

Figure 3.15 shows the default settings of the options that are common to all three solution methods. In certain situations, some of these default settings may have to be changed in order to reduce the computation time or increase the precision of the solution. Sometimes, Solver fails altogether to find the solution if the default options are used. For example, the use of automatic-scaling (AS) is favourable in certain situations but not always. AS enables Solver to handle a poorly-scaled model, i.e., a model in which the values of the objective and constraint functions differ by several orders of magnitude. By using AS, the values of the objective and constraint functions are scaled internally in order to minimise the differences between them.

Figures 3.16.a and 3.16.b show the default settings which are particular to the **GRG Nonlinear** method and the **Evolutionary** method, respectively. The **GRG Nonlinear** method uses by default the forward difference (FD) approximation of derivatives. Most of the analyses presented in later chapters of the book used the **GRG Nonlinear** method with the default FD approximation. More information about Solver options for the **GRG Nonlinear** method can be obtained from the developer’s website [5].



(a) (b)

Figure 3.16. The default Solver options specific to: (a) the GRG Nonlinear method and (b) the Evolutionary method

Figure 3.16.b shows the default settings used by the **Evolutionary** method. Note that this method has more adjustable parameters than the **GRG Nonlinear** method. According to the default set-up, the population size is 100 and the maximum allowable time without improvement is 30 seconds. The few cases solved in this book with the **Evolutionary** method show that, with this set-up, the method needs long computer times. Although the time required by the **Evolutionary** method can be reduced by reducing the population size, number of iterations, or the maximum allowable time, the optimsation analyses presented in later chapters to do not show a clear advantage to this method over the **GRG Nonlinear** method which is easier to apply.

3.2. VBA and the development of user-defined functions

Although Excel's user-interface provides numerous built-in functions for data analyses in general, there are situations where the analytical model requires the development of a customised user-defined function (UDF) that is not provided by Excel. This arises, for example, in thermodynamic analyses that require functions that determine the properties of fluids at various pressures and/or temperatures. This section illustrates the process of activating VBA and using it to develop simple UDFs.

3.2.1. Activation of VBA

As shown in Figure 3.17, VBA is found on the left side of the **Developer** tab. If the **Developer** tab is not shown in the ribbon of your Excel sheet, then go to **File**, select **Options**, and then select **Customise Ribbon** from the **Backstage View** shown in Figure 3.18. From the **Main Tabs**, select the **Developer** check box and then click "**OK**". The **Developer tab** will now be shown in the ribbon of your Excel sheet.

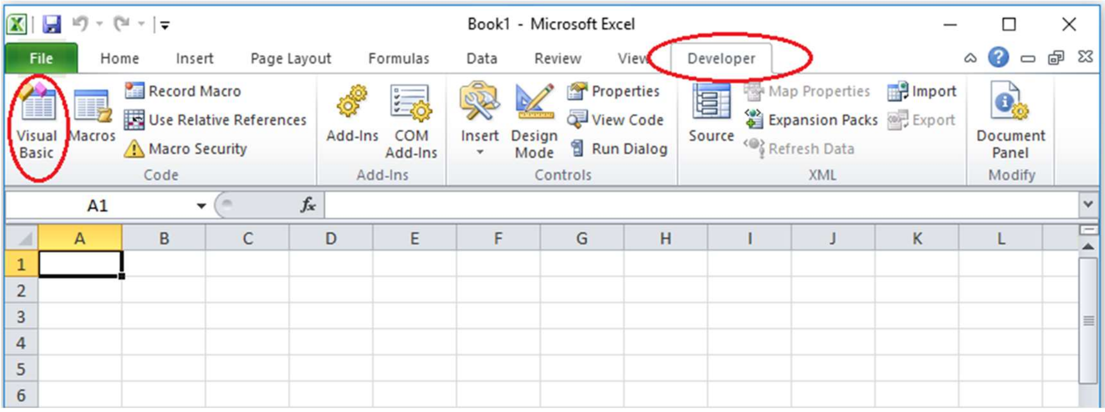


Figure 3.17. Selection of VBA from the Developer tab

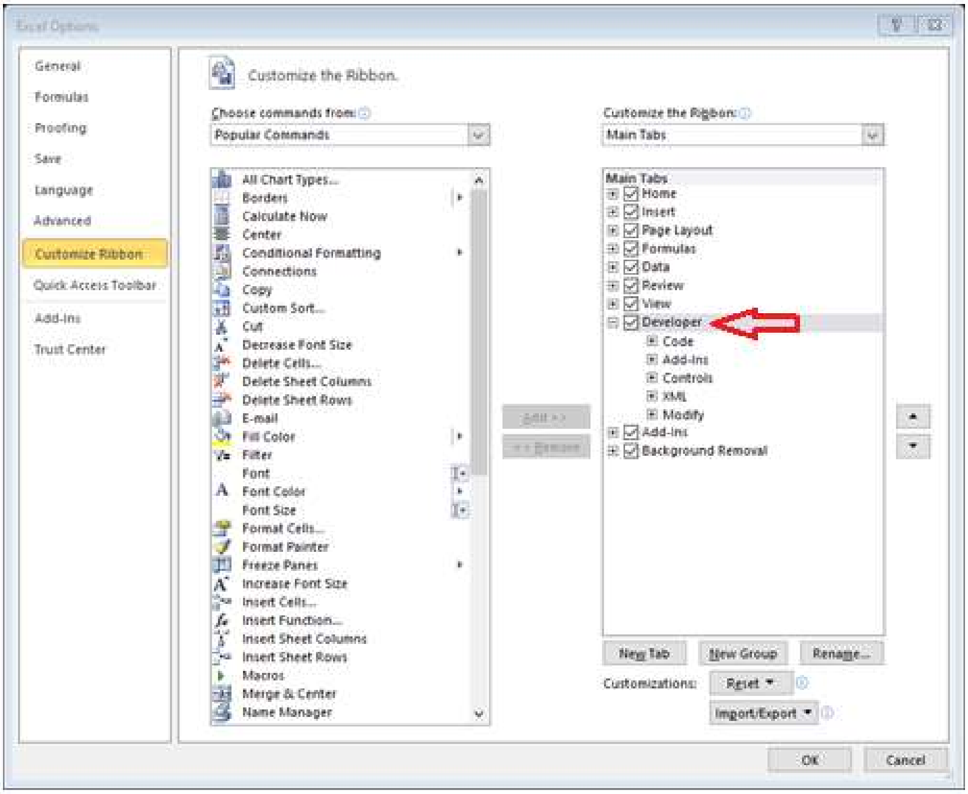


Figure 3.18. Adding VBA to the Developer tab

3.2.2. Development of UDFs

To start writing the UDF, go to **Developer** tab menu and select **Visual Basic**. The Visual Basic editor will appear to you as shown in Figure 3.19. Select **Insert** → **Module** and the blank page shown in Figure 3.20 will be open for you to type the VBA code.

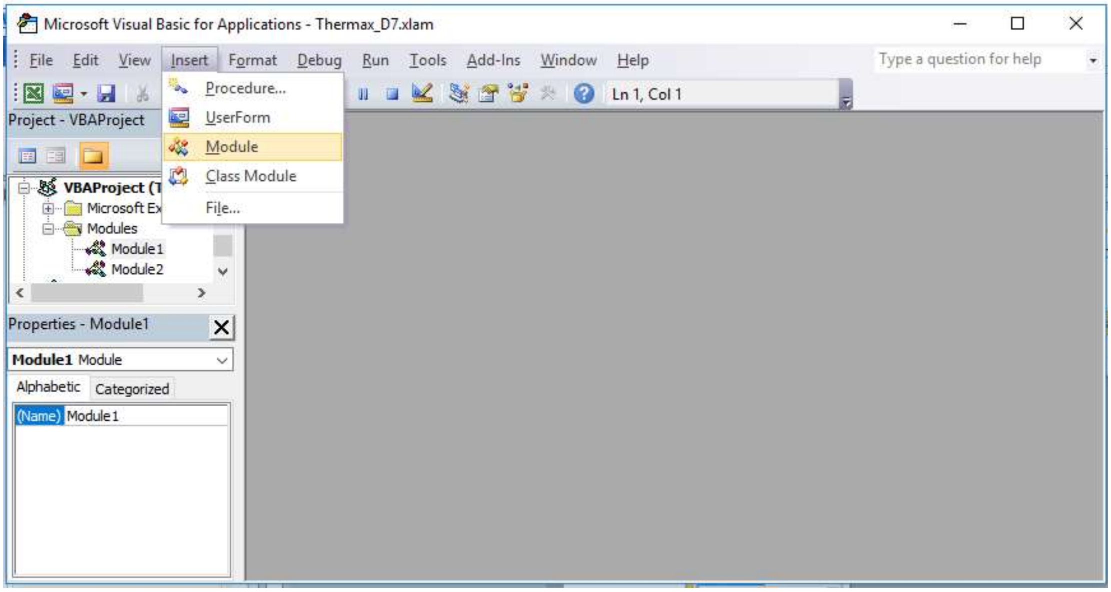


Figure 3.19. Inserting a new module

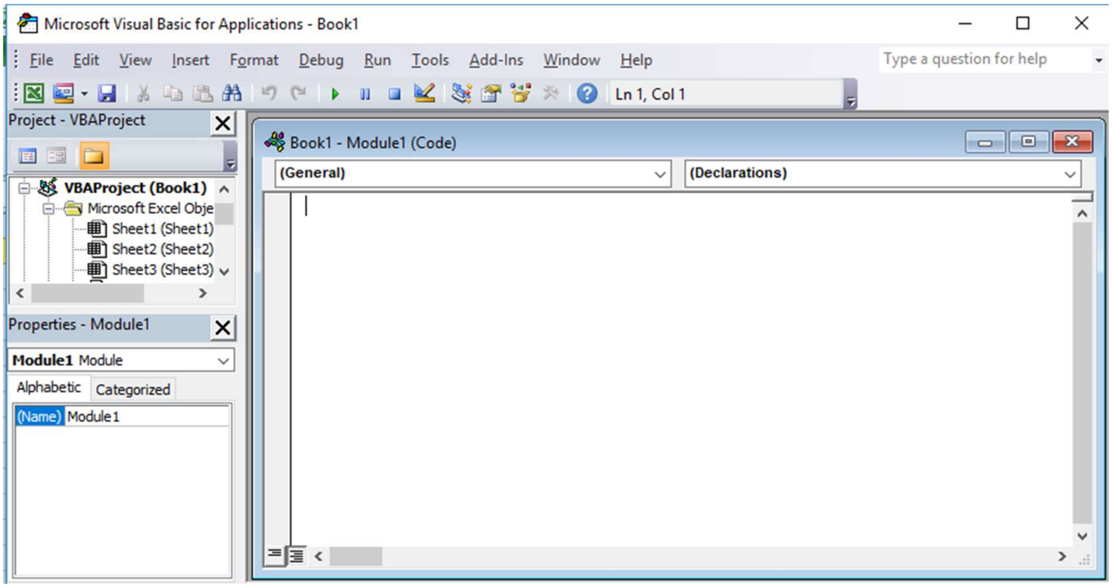


Figure 3.20. A new VBA module

As a first example let us write a VBA function for determining the area (A) of a circle given its diameter (D) using the following mathematical equation:

$$A = \pi D^2 / 4 \tag{3.4}$$

The following UDF determines the circle's area according to Equation (3.4):

```
Function Circ_area(Dia)
Pi = 3.141593
Circ_area = Pi * Dia ^2 / 4
End Function
```

Note that the first line in the code starts with the word "**Function**" followed by the name given to the function, which is "**Circ_area**". The required input parameters are specified between two brackets after the function name. The present function has only one input parameter, which is the diameter (Dia). As soon as you type the first line of the code and press the "**Enter"** key, the editor will automatically add the **End** line of the function. Now, type the rest of the code as shown in Figure 3.21.

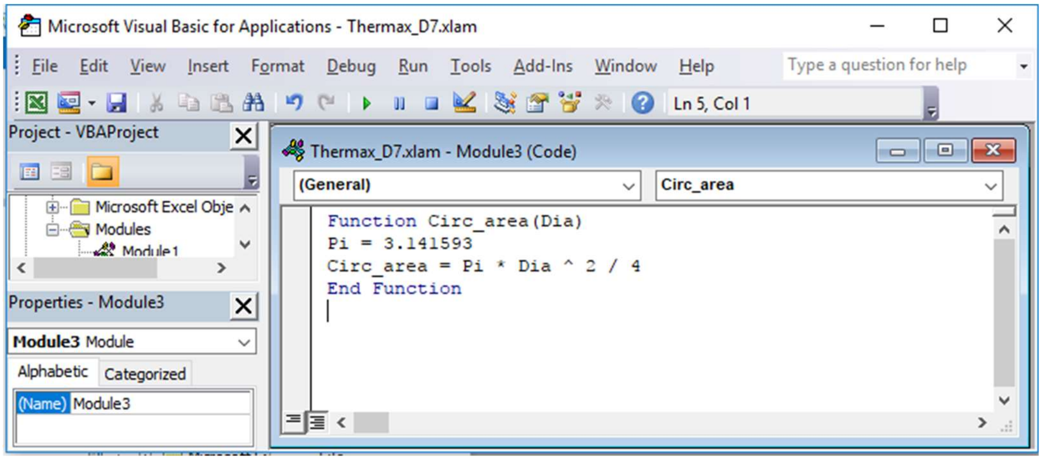


Figure 3.21. A UDF for caclcuating the area of a circle with a given diameter

After typing the code correctly, the function can be used via Excel UI just like any built-in function as shown in Figure 3.22. Note that the formula bar in Figure 3.22 reveals the formula in cell B2 as:

= Circ_area(10)

Where the number 10 refers to the diameter of the circle, which is the only input to the UDF. You can now check the answer of your user-defined function by calculating the circle's area with a normal Excel formula by typing in any cell "**=pi()*10^2/4**".

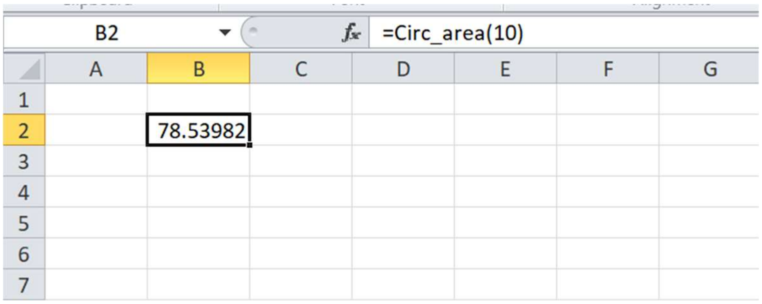


Figure 3.22. Using the “Circ_area” function in Excel

In writing the UDF shown in Figure 3.21 we assigned a value for the constant π because VBA does not provide a built-in function for it like Excel. However, it is possible to call the built-in functions **PI** provided by Excel within the VBA function as follows:

```
Function Circ_area(Dia)
Circ_area = Application.WorksheetFunction.Pi() * Dia ^ 2 / 4
End Function
```

This is very useful when built-in functions, like MIN and MAX, can be used to minimise the programming effort needed for developing the required UDF. More information about this and other features of the VBA language can be found in specialised references [7-9].

For thermodynamic analyses, VBA is useful for developing UDFs for fluid properties. As an example, let us develop a UDF that determines the molar specific-heat at constant pressure ($\tilde{c}_p$) for air. For an ideal gas, $\tilde{c}_p$ is given by the following polynomial [6]:

$$\tilde{c}_p = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad \text{[kJ/kmol.K]} \qquad (3.5)$$

Where T is the absolute temperature and a_0, a_1, a_2, and a_3 are constants that have different values for different gases. For air, the constants’ values are 28.11, $0.1967\text{x}10^{-2}$, 0.4802 $\text{x}10^{-5}$, and -1.966 $\text{x}10^{-9}$ in this order. The following VBA function, called “cp_air”, determines $\tilde{c}_p$ for air based on Equation (3.5):

```
Function cp_air(TempK)
a0 = 28.11
a1 = 0.00196
a2 = 0.000004802
a3 = -0.000000001966
M = 28.97
cpbar = a0 + a1 * TempK + a2 * TempK ^ 2 + a3 * TempK ^ 3
cp_air = cpbar / M
End Function
```

The only input for the function is the absolute temperature (TempK). Figure 3.23 shows the VBA function and the formula bar in Figure 3.24 shows how the function can be used in an Excel formula to determine $\tilde{c}_p$ for air at 300K. The value returned by the function is 29.0771 kJ/kmol.K. By making suitable extensions, the function can easily be used to determine the values of $\tilde{c}_p$ for other ideal gases in addition to air (Refer to Exercise 3.8). The various functions provided by Thermax for the thermo-physical properties of water, refrigerants, etc., have been developed by writing similar functions with VBA.

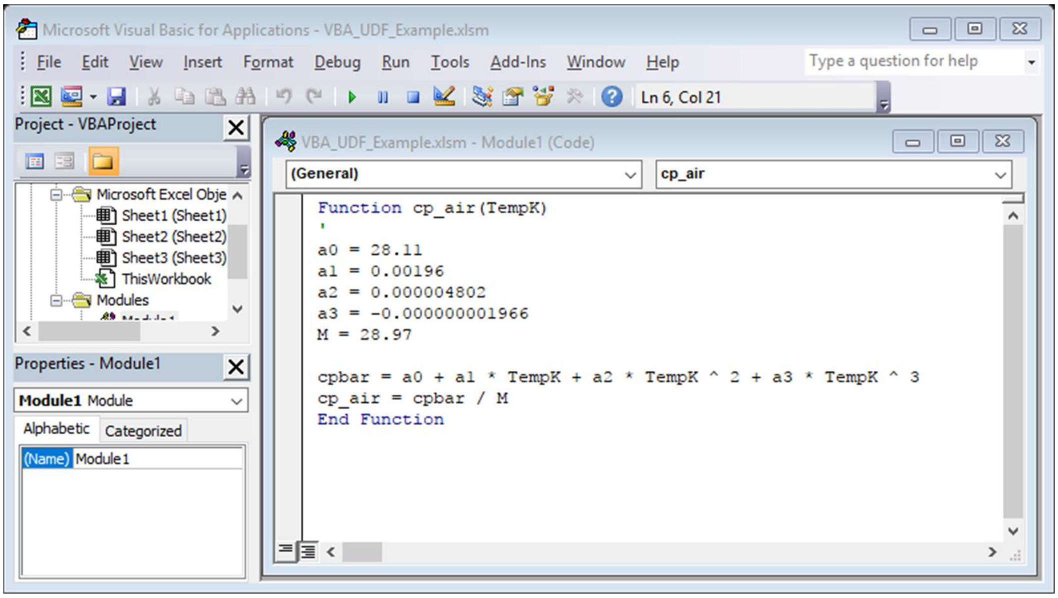


Figure 3.23. A UDF for caclcuating the molar specific-heat for air

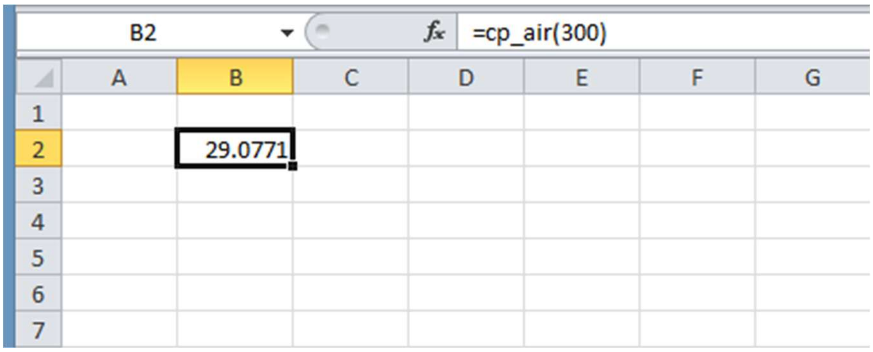


Figure 3.24. Using the cp_air function in Excel

3.3. Thermax installation and use

Thermax is an educational Excel add-in that provides property functions for a wide range of working fluids including 29 ideal gases, water, and 28 refrigerants for vapour-compression systems, two aqua solutions for vapour-absorption refrigeration systems, humid air for air-conditioning systems, combustion reactions, and atmospheric air at various temperatures [10]. Before it's functions can be used in Excel formulae you have to add it to the add-ins on your computer. To do that, open the **Thermax.xla** file and then save it as an "**Excel Add-in**". Recent Excel versions locate all add-ins in a certain folder in the computer and automatically direct you to that location when you want to save a new add-in. Save the Thermax add-in in the specified location and **restart** Excel in order to activate it. Open a new Excel sheet and then do the following:

1. Go to **File** and then click **Options**.
2. Select **Add-Ins**. From the **Manage** ribbon at the bottom of the menu select **Excel Add-ins** and then press **Go**. The pull-down menu shown in Figure 3.25 will appear to you.
3. To add **Thermax** to the add-ins menu, tick (√) the corresponding box.
4. If for any reason you saved the add-in in a location that is different from the default folder, then click on **Browse** and search for it in the destination folder and select it.

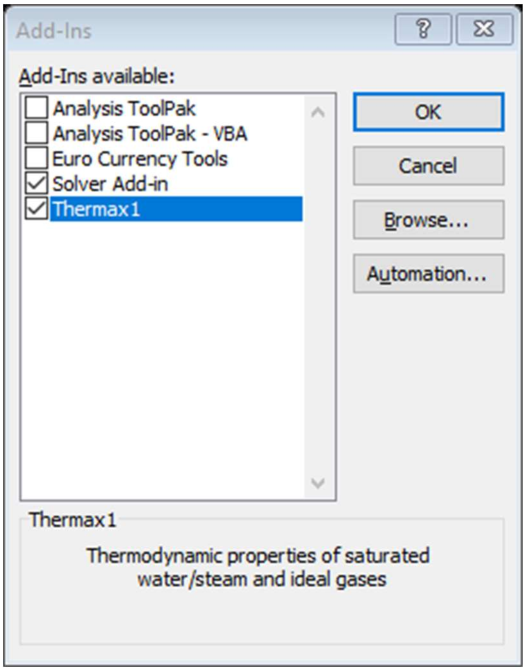


Figure 3.25. Adding Thermax to the menu of Excel add-ins

Once installed, Thermax functions can be used in Excel's formulae just like the built-in functions. For illustration, let us start a formula by entering the equal sign (=) in any cell (say cell B2). If you now press the *fx* button in the formula ribbon, the **Function Wizard** shown in Figure 3.26 will be shown. The Function Wizard first lists the various categories of built-in functions provided by Excel. Scroll down the list of function categories and select the **User-defined** functions. Then, all the functions provided by Thermax will be listed alphabetically as shown in Figure 3.27.

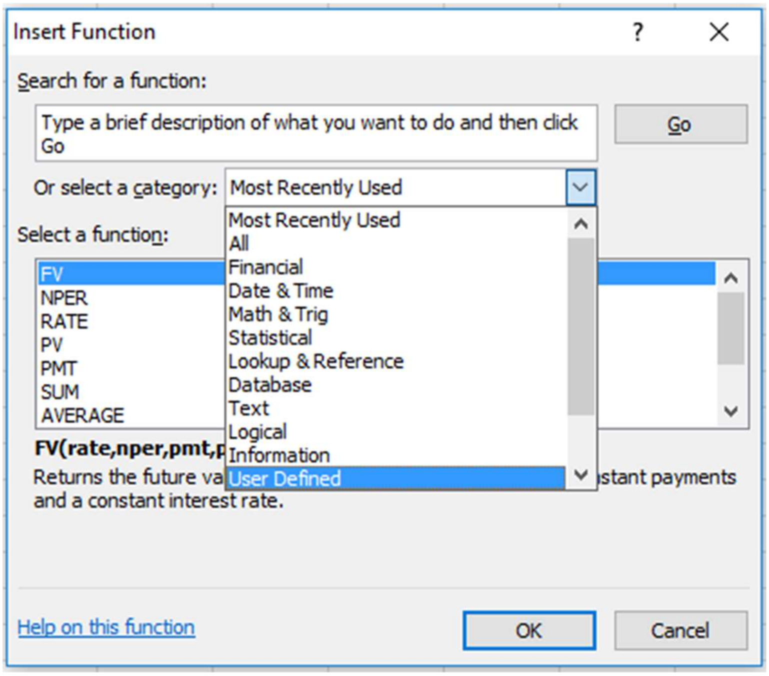


Figure 3.26. Finding the add-in user-defined functions in the Function Wizard

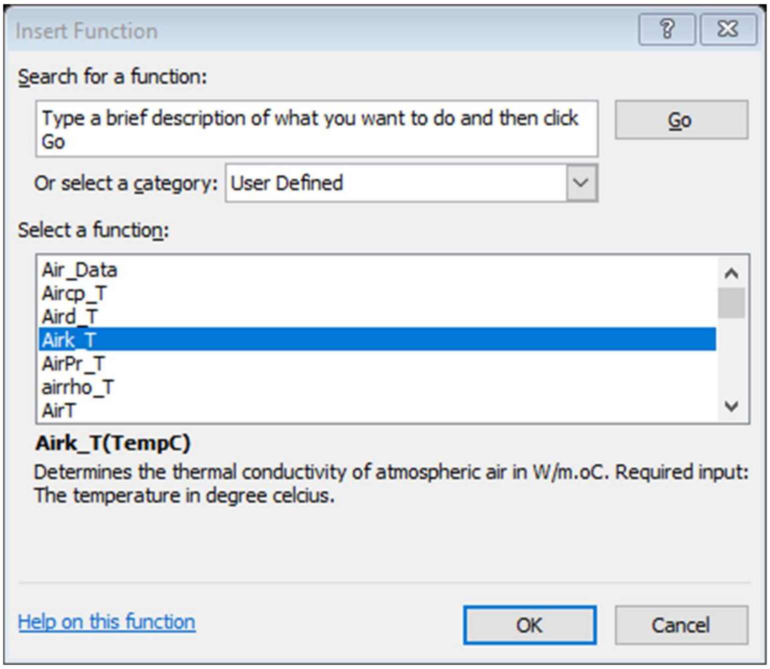


Figure 3.27. Thermax functions listed alphabetically in the User Defined category

The first function in the list, **Air_Data**, is the auxiliary function that stores the data for the thermo-physical properties of air at standard atmospheric pressure. This function is called by other functions in the same category to obtain the values of these properties at the required temperature. To start using the add-in functions, scroll down the list and select the function **Airk_T** that determines the thermal conductivity (k) of air at a given temperature. Upon pressing the **OK** button, the **Function Arguments** box shown in Figure 3.28 will be shown.

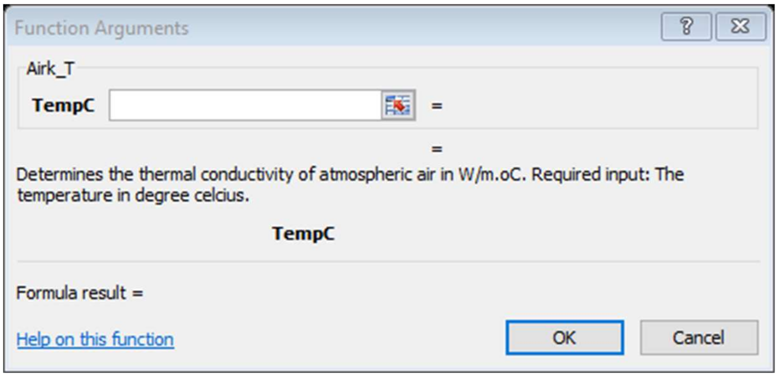


Figure 3.28. The Function Arguments box for the “Airk_T” function

Figure 3.28 shows that this function has one input parameter, which is the temperature in °C “TempC”, and gives a brief description of its intended use. Let us use the function to determine the thermal conductivity for air at 25°C. Fill the form by entering the value of the temperature, 25, as shown in Figure 3.29.

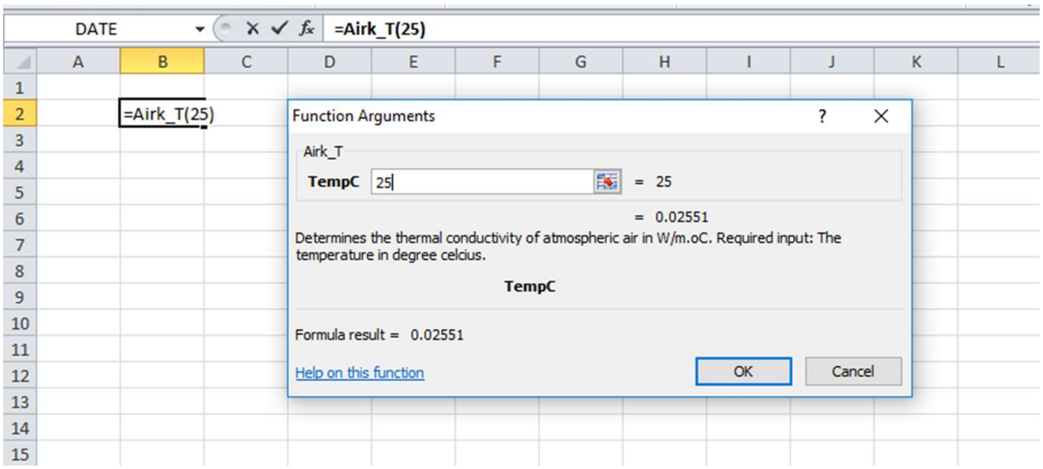


Figure 3.29. Using the function “Airk_T” to determine the thermal conductivity of atmospheric air at 25°C

Note that the formula ribbon now shows the formula in cell **B2**, which is **“=Airk_T(25)**”, and the function form shows the calculated value of k, which is 0.02551 W/m.°C. When you press the "OK" button, this value will appear in the cell **B2**. You can check this value with that given in Table A.1 of Appendix A.

In certain situations, the confinement of Excel's formula to one cell becomes too restrictive for developing the analytical model. This situation arises, for example, when an iterative process involves a non-linear equation such as the Colebrook-White equation or the Soave-Redlich-Kwong equation of state. In this case, a UDF is needed solve the nonlinear equation and return the result to Excel's formula like a built-in function. For such a case, Thermax provides a Newton-Raphson solver for nonlinear equations in addition to its property functions. Appendix B shows how to use this solver. Appendix B also describes two interpolation functions provided by Thermax for tabulated data which are useful for including additional fluid properties or other tabulated data needed in a thermofluid analysis.

3.4. Closure

This chapter introduces the three auxiliary components of the Excel-based modelling platform used in this book for thermofluid analyses; Solver, VBA, and Thermax. The chapter gives Examples of using the three solution methods provided by Solver to deal with different types of problems. It also shows how VBA can be used for developing user-defined functions not provided by Excel and explains the procedure for installing Thermax and using its functions in Excel formulae.

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Exercises

1. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations by using Solver to determine the values of the three unknowns a, b, and c. This exercise is based on Example 9.11 in Chapra and Canale [11]. The answer is: $a = 8.5941$, $b=34.4118$, and $c = 36.7647$.

2. Draw a line chart with Excel to show the variation of the following function in the range $0 \le x \le 4$:

$$f(x) = 2 \sin x - x^2/10$$

Use Solver to find the maximum of the function in the same range. Based on Example 13.1 in Chapra and Canale [11]. The answer is: $f(x) = 1.7757$ at $x = 1.4276$.

3. The curve shown in Figure 3.P3 is a plot of the function:

$$f = e^{\theta} \sin(2\theta)$$

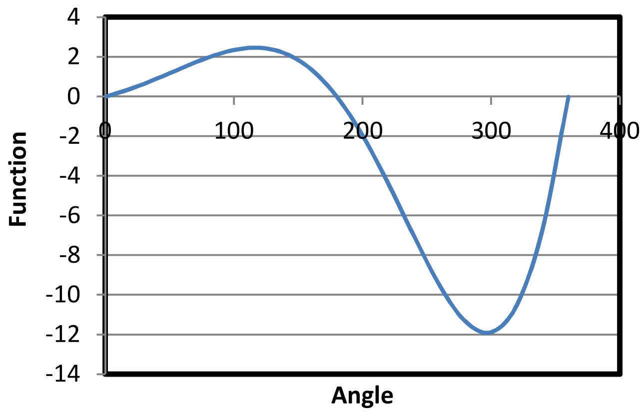

Figure 3.P3. A composite function

Use Solver to find:

a) The minimum value of the function and the corresponding angle
b) The maximum value of the function and the corresponding angle
c) The angle at which value of the function equals 4.0

4. Using the Excel sheet developed to solve Example 3-1 by the **GRG Nonlinear** method, study the effect of using central-difference approximation of derivatives instead of the default forward-difference approximation on the solution.

5. Consider the following set of simultaneous nonlinear equations:

$$x^2 + xy = 10 \quad \text{(A)}$$

$$y + 3xy^2 = 57 \quad \text{(B)}$$

To solve the system with Solver, rearrange the equations as follows:

$$u(x, y) = x^2 + xy - 10 = 0 \quad \text{(C)}$$

$$v(x, y) = y + 3xy^2 - 57 = 0 \quad \text{(D)}$$

Create two cells (**B1** and **B2**) to hold initial guesses for x and y. Enter the function values themselves, $u(x, y)$ and $v(x, y)$ into two other cells (**B3** and **B4**). The initial guesses may result in function values of u and v that are far from zero. Determine the sum of the function squares, i.e. $u^2 + v^2$, and store it in cell **B5**.

Use Solver to find the values of x and y in cells **B1** and **B2** (the Changing cells) that make the value in cell **B5** (the objective cell) equal to zero. Using this procedure, find the roots of the above system starting with initial guesses of x =1 and y = 3.5.

This exercise is based on Example 7.5 in Chapra and Canale [11]. The correct pair of roots are x=2 and y=3.

6. The volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid by:

$$V = \pi h^2(3r - h)/3$$

Using VBA, develop a user-defined function that determines h at any given values of r [m] and V [m^3]. Check your function with r=1 m and V = 0.5 m^3. Answer: h = 0.431 m.

7. Extend the UDF developed in Section 3.2 for determining the specific heat for air so that it can be used for other ideal gases as well. Note that in this case, the function will have two input parameter: the name of the gas and the temperature.
8. Using suitable formulae for the thermodynamic properties of superheated steam, develop user-defined functions with VBA for determining the specific enthalpy and entropy of superheated steam from its pressure and temperature.
9. Using suitable formulae for the thermodynamic properties of superheated refrigerant R134a, develop user-defined functions with VBA for determining properties, e.g., enthalpy and entropy, of superheated R134a from its temperature and pressure.
10. Using the data for properties of air at atmospheric pressure given in Appendix A, Table A.1, develop an Excel sheet that can be used to determine the kinematic viscosity of air at any given temperature in the range 200 – 1000K by using

    a. The trendline feature of Excel
    b. The linear-interpolation function (**Interpl**) provided by Thermax.

11. Develop a VBA function to determine the friction factor from the Colebrook-White equation and use it with the NRM solver provided by Thermax (refer to Appendix B) to determine the frictional losses (h_f) in a circular pipe that carries air at 20ºC with the following data:

D = 25 cm, L = 150 m, V = 7 m/s, k_s = 0.045 mm.