EQUATIONS THAT DESCRIBE THE GEOMETRY OF SUPERADOBE DOMED STRUCTURES

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Abstract

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"Superadobe" is a method for the construction of self-built shelters that has three main advantages: It's economical, it's easy to execute by common people, and produces buildings with good structural behavior against natural events such as earthquakes, floods, fire and high winds [1]. It consists in filling polypropylene sacks with a moist mixture of clay, sand, gravel and lime/cement; placing and compacting one on top of another in the form of concentric rings of decreasing radii and describing a double curvature monolith. (Similar to a paraboloid).

The construction of these domed monoliths follow a simple geometrical pattern based on the use of compasses, hence the form, and other building characteristics can be parametric. This is, a system of equations for these building's physical dimensions can be derived, based on initial parameters.

These equation's variables can be divided into independent ('constructive') and dependent ('observable'), and by defining the values of the independent parameters (sack dimensions, radius of base (internal compass), radius of roof (external compass), radius of skylight, dimensions of openings, mean material specific weight, mean material elastic modulus, mean friction coefficient between rows, etc.), resulting dimensions and mechanical behaviors, such as building height, interior habitable area, proportion of rows that rest upon the previous, or total building weight, maximum shear force amongst all rows, axial stresses, etc. can be determined.

This article deals with the resulting geometrical dimensions of the building, given corresponding initial parameters.

A note on the usability of this system of equations: If we were to set some dependent variables as fixed values, we could solve the system of equations for the missing independent parameters. This would allow for example, to set design values for building weight and height, and to determine the unknown values for the corresponding radiuses and necessary sack dimensions which will produce a building that meets this pre requisites.

This paper briefly explains a geometrical model of Superadobe domed structures.

1. INTRODUCTION

A Superadobe Structure's geometry consists of a set of concentric rings (circularly-arranged, earthen mixture-filled and tamped polypropylene bags), one on top of the other. These concentric rings have decreasing radii as they ascend, and the variation (decrease) of each ring's radius with respect to the previous is such, that if the plane *xz* is the ground plane (Fig. 1), a transversal section of the structure by the plane *xy* would be nearly an inverted parabola. The blue sketch on the *xy* plane, which is an arch of a circle, approximately describes one half of an inverted parabola over the *x* axis, and the dome can be modeled by revolving this half inverted parabola around the *y* axis.

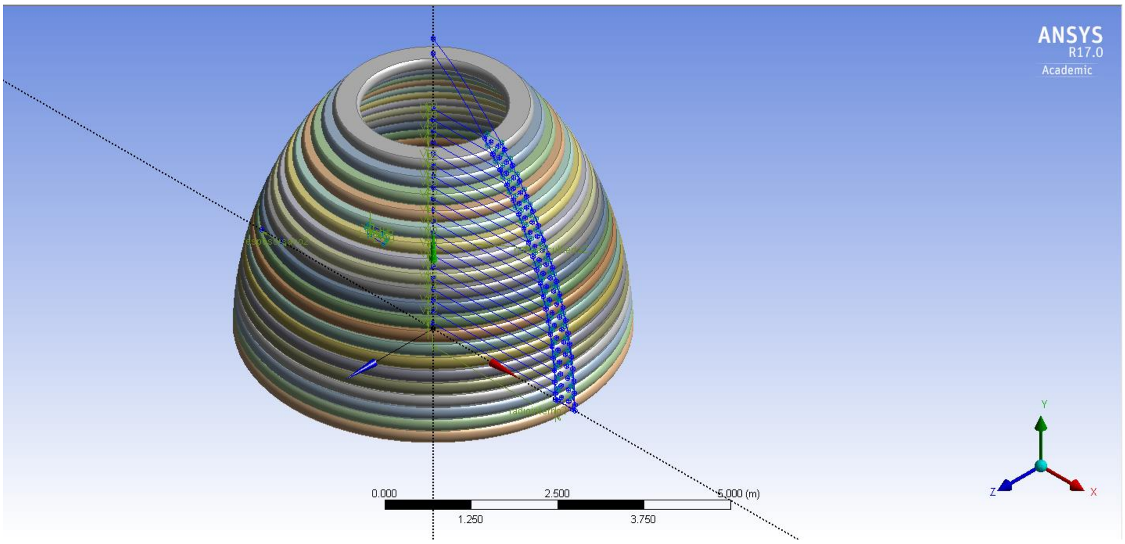

***Fig. 1*: Model of the geometry of a Superadobe structure.**

What follows, is the description of parameters of a Superadobe structure's geometry, and also of the existing relationships between them.

The blue 'generating sketch' of the Fig. 1 is the 'root' of a Superadobe structure's geometry. This sketch contains most of the main geometric parameters that can be varied to obtain different Superadobe geometries.

2. INDEPENDENT GEOMETRIC VARIABLES (CONSTRUCTIVE PARAMETERS), AND THE RELATIONSHIPS SATISFIED BY THEM ONLY IN CONVENTIONAL SUPERADOBE STRUCTURES

2.1. The first group of independent parameters. The Sack's transversal section width (sacks width, s_w), the Sack's transversal section height (height of sack, h_s) and the Sack's Exterior face length (the curved portion of the sack which does not rest on the previous sack, $Extf$)

These are parameters of the sack's transversal section. We can see below that this section is idealized as an oval. This idealization is based on empirical observations:

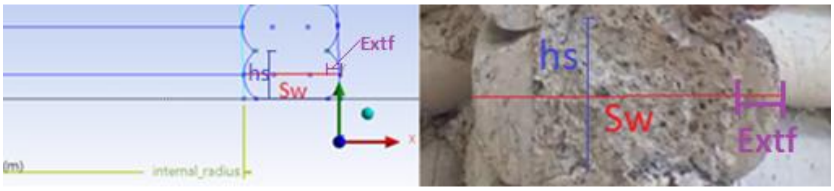

Fig. 2: Sack Transversal section Model vs Real Sack Transversal section.

The typically used sack's transversal section (filled and confined) is nearly an oval, and has approximate width $s_w = 0.4m$, and approximate height of $h_s = 0.2m$, but because of the variability of local sack's sizes, these dimensions will be considered variables. Also, defining L as the length of the variable rectangle inscribed in this oval, then the third independent sack parameter $\boldsymbol{Extf}$**,** can be interpreted as $s_w - L$, hence $\boldsymbol{Extf}$ **is independent from** s_w and from h_s.

2.2. The second group of independent geometric parameters: The internal compass or radius of the building's base, $\mathrm{r_b}$, the skylight radius ($\mathrm{t_r}$), the external compass or roof radius ($\mathrm{r_r}$) and the height of a cylindrical wall at the dome's base ($\mathrm{h_1}$):

The pragmatic steps in building a ***conventional*** Superadobe dome will illustrate these parameters, which can be later generalized. We begin by setting reference points on the ground as illustrated in Fig. 3:

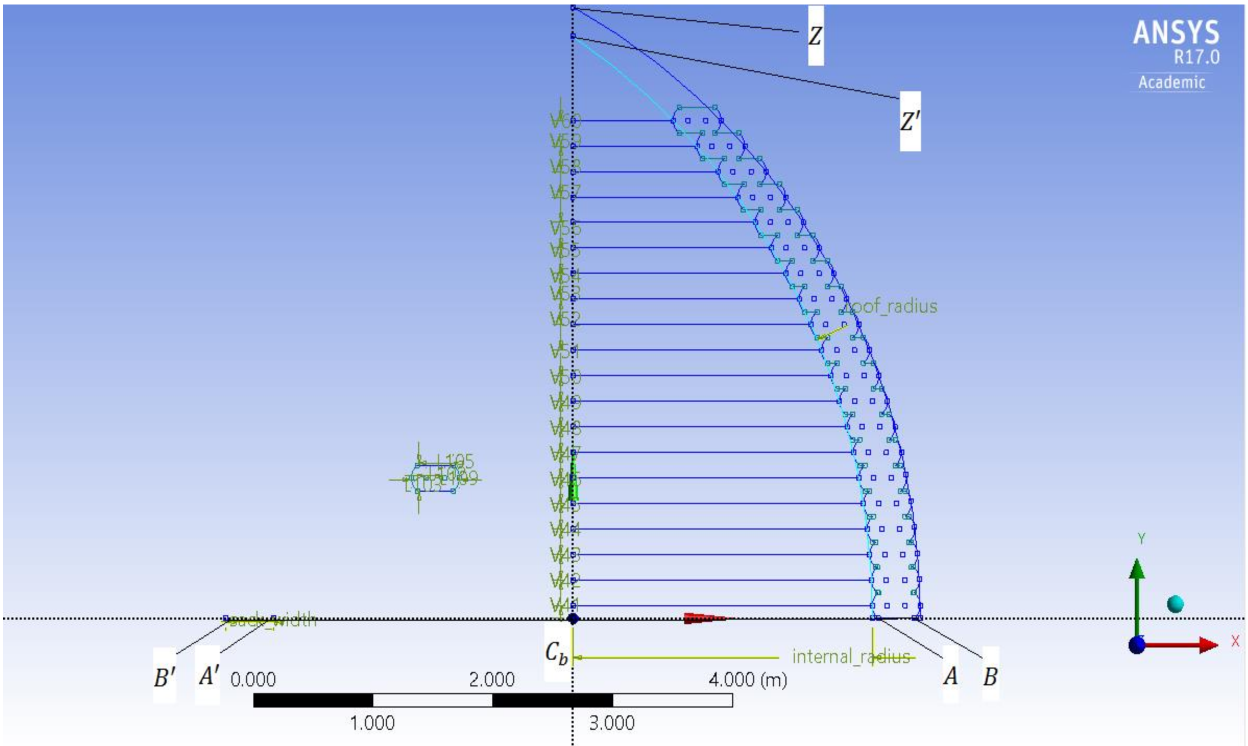

Fig.: 3 Definition of the center of the building C_b, definition of points A, B, A', B', Z and Z', and definition of the Base Radius r_b.

- **Definition of a Center point, C_b :**

Any point can be selected on the earth, as the center point C_b. This point, as seen on the above image, will be the center of the building's base circumference.

- **Definition of four points, A and B, and their symmetric points through the y axis, A' and B':**

Points *A* and *B* will be two points on the ground, such that the distance between them is s_w, the distance between C_b and *A* is r_b, and such that they both are collinear with $\boldsymbol{C_b}$**.** *A'* will be the symmetric of *A* through the y axis, and *B'* will be the symmetric of *B* through this same axis (see figure 3).

We can see that the distance between C_b and *A*, is equal to the distance between C_b and *A'*, the distance r_b .

Observation 1: A ***conventional*** Superadobe dome is one in which some relationships between $\mathbf{r_b}$, $\boldsymbol{s_w}$ and $\mathbf{r_r}$ are satisfied. It is a first recommendation and a general practice that the following geometrical relation between the sack's width s_w and the building's base radius r_b holds in conventional domes:

$$^{r_b}/_6 \leq s_w \tag{1}$$

The roof radius $\boldsymbol{r_r}$ is the radius of curvature of the roof. Equivalently, it is the radius of the arch passing through A and Z' in the Fig. 4.

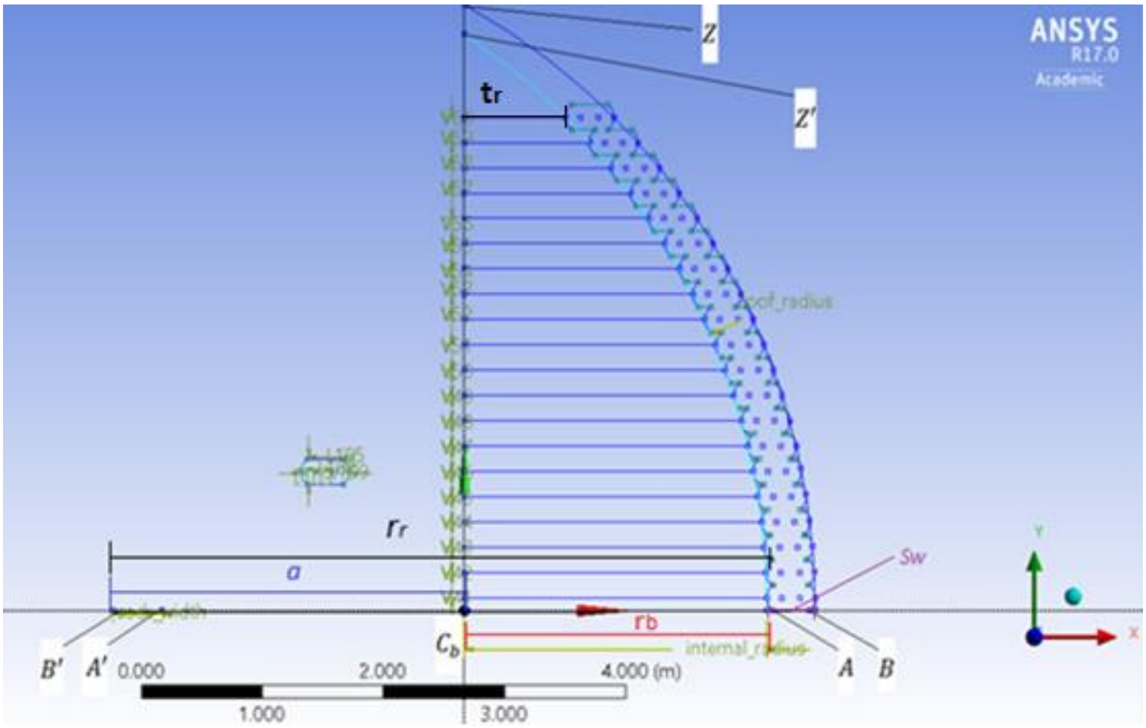

Fig.: 4 Roof radius r_r and skylight radius t_r in relation with other parameters.

$The\ left\ end\ point\ of\ segment\ \boldsymbol{r_r}$ has a distance ***a***, from C_b as shown in Fig. 4. $\boldsymbol{r_r}$ is the black segment on the figure, and we can see that the center of the circle described by the roof is exactly *B'*, and that the other end point of $\boldsymbol{r_r}$ is *A*. Hence, in the Fig. 4, and generally in all ***conventional*** Superadobe domes:

$$\boldsymbol{a} = r_b + s_w \text{ or equivalently } \boldsymbol{r_r} = 2r_b + s_w, \quad (2)$$

Observation 2: This last relation $a = r_b + s_w$, takes place **only** in Superadobe domes with ***pointed arch***, the type of arch which passes through A and Z' , and conventional domes follow this type of arch.

Observation 3: Since $The\ left\ end\ point\ of\ segment\ \boldsymbol{r_r}$ *is conventionally fixed in B', yielding*

$a = r_b + s_w$ in ***conventional*** domes of ***pointed arch,*** it is natural to consider the generalization in which a can be any positive number, which gives origin to a dome more ***general*** Superadobe dome with ***variable arch***.

Observation 4: In summary, only ***conventional*** Superadobe domes satisfy:

$$\begin{cases} {r_b}/{6} \le s_w \\ r_r = 2r_b + s_w \end{cases} \quad (1,\ 2)$$

And if we consider relation 1), ${r_b}/{6} \le s_w$ as an equality rather than an inequality, we are minimizing the possible sack width in this convention. The equation version of 1), ${r_b}/{6} = s_w$ together with 2), $r_r = 2r_b + s_w$, are linearly independent and they imply, and are in turn implied, by two linearly independent equations together with a third one, that arises from combining the first two:

$$\begin{cases} {r_b}/{6} = s_w \\ r_r = 2r_b + s_w \end{cases} \quad (1,2) \qquad \begin{cases} r_r = 13\ s_w \\ r_b = {6\ r_r}/{13} \\ 6\ s_w = r_b \end{cases} \quad (3,4,5)$$

Hence, the set (1,2) is equivalent to the set (3,4,5). Observe that the set of equations (3,4,5) relates each of the three possible pairs of geometrical parameters s_w, r_b and r_r, in ***conventional*** Superadobe geometries.

The skylight or "top" radius $\boldsymbol{t_r}$ is the distance between the *y* axis and the interior of the last (top) ring. It determines the openness of the dome skylight, as seen in figure 6.

The height $\boldsymbol{h_1}$ of a cylindrical base of the dome is desired to confer the building with sufficient space for habitability if r_r is small, providing additional height. Please note that in the Fig. 4, $\boldsymbol{h_1 = 0}$.

Observation 5: The Superadobe dome is formed by adhering the inner-most points of each of the ring's sections to the arch A Z'.

3. RELATIONSHIP SATISFIED BY THE INDEPENDENT PARAMETERS IN UNRESTRICTED (GENERAL) SUPERADOBE STRUCTURES

In general or "unrestricted" Superadobe geometries, $The\ left\ end\ point\ of\ segment\ \boldsymbol{r_r}$ need not be *B'*, it can in fact be any point *P* to the left of C_b, at any distance a from C_b, so only the equation $\boldsymbol{r_r} = \mathrm{a} + \boldsymbol{r_b}$, or equivalently, $\boldsymbol{r_r} \geq \boldsymbol{r_b}$ (6)

Is satisfied by unrestricted Superadobe geometries as shown in Fig. 5.

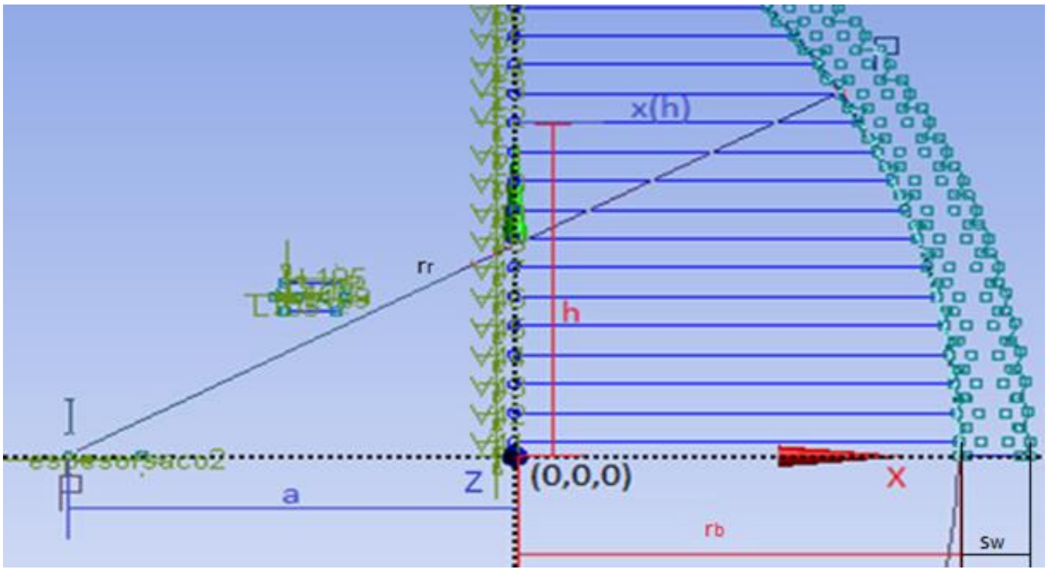

Fig.: 5 Variability of Parameter a, which is considered to be included in r_r, for it determines the length of r_r=a+r_b.

Observation 6: Thus far we have found three relations between $\boldsymbol{r_b}$, $\boldsymbol{r_r}$,$\boldsymbol{s_w}$, but (1) and (2) (3,4,5) only apply to ***conventional*** Superadobe geometries, while **(6)** does not tie $\boldsymbol{r_b}$, and $\boldsymbol{r_r}$ in any other way than simply asking for $\boldsymbol{r_r}$ to be greater tan $\boldsymbol{r_b}$. For this reason we can say that, **for general Superadobe geometries, there is no restriction or relation amongst independent parameters which must be satisfied, and hence these parameters (s_w, h_s, $Extf$, r_b, r_r, h_1 and t_r) can all vary independently from one another.**

4. MODELLING OF AN UNRESTRICTED SUPERADOBE DOME IN ANSYS WORKBENCH DESIGN MODELER

We will next see an example of how adjusting independent variables $\boldsymbol{r_b}, \boldsymbol{r_r}, \boldsymbol{s_w}$, and $\boldsymbol{h_s}$, Superadobe base geometries are generated from a 'generating sketch'.

Freely and independently setting the height of all ovals, h_s, the width of all ovals, s_w, the radius of curvature of the two parallel (blue) arches, r_r, the radius of curvature of the internal base circumference, r_b, $Extf$, h_1 and t_r,the Yellow 'generating sketch' dynamically adjusts to these values and rotates about the y axis (Fig. 6), conferring mass to the 3-d model.

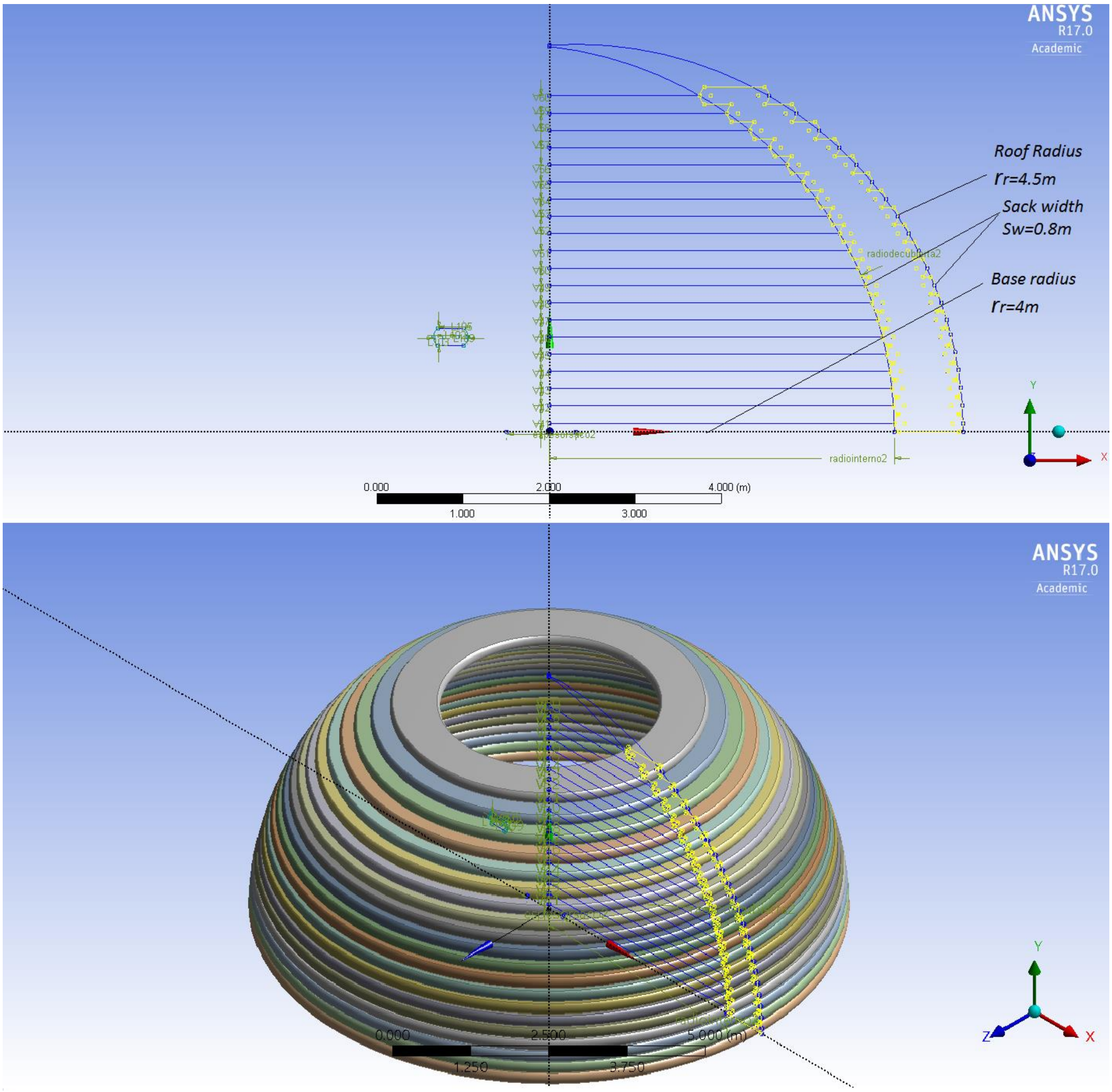

Fig. 6: Initial 3D Model of a Superadobe.

Then, apart from leaving a top opening, straight lateral windows, and main entrance will be opened.

The simplicity of these openings are sought to enable regular patterns in the numeric outcomes of the analyses.

For simplification, main entrance dimensions will be constant at 1.5m wide by 2.2 m tall, and lateral window dimensions will be constant at 1.5 m wide by 1.2 m tall. Top opening radius dimensions will be set to 0.5 m.

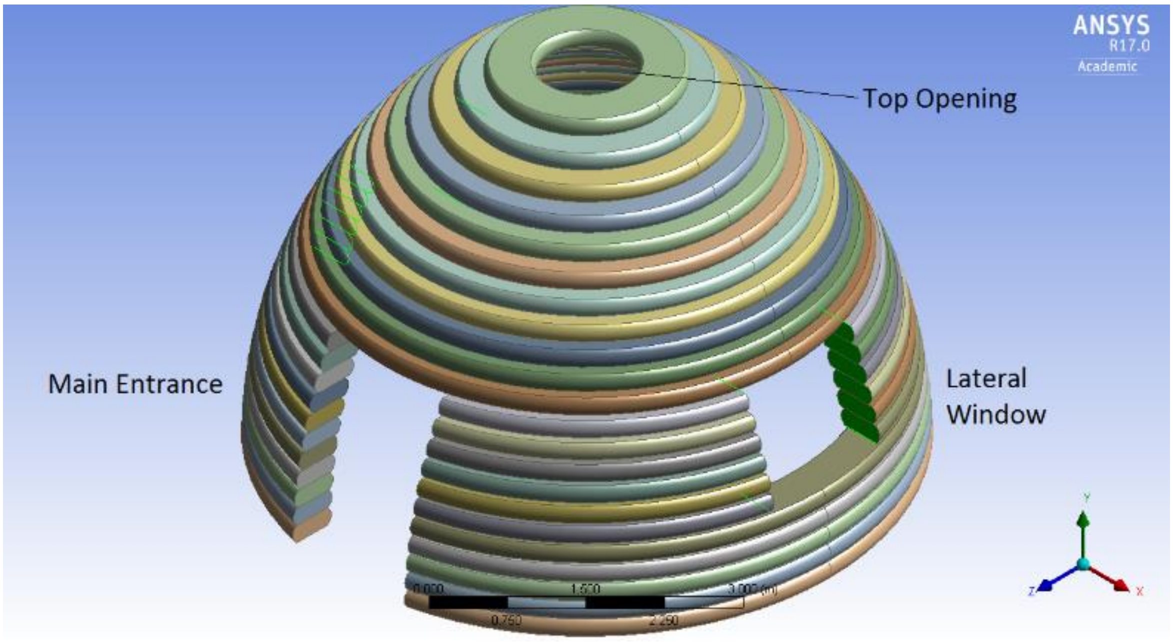

Fig. 7: Final 3D Model of a Superadobe.

Observation 7: By freely setting values for the independent (constructive) variables $(s_w, h_s, Extf, r_b, r_r, h_1 \text{ and } t_r)$, a Superadobe model can be generated, hence, we call these "constructive parameters", and also "independent from each other", and we can see how a Superadobe dome can be a parametric object. The independent parameters are sufficient to produce an unrestricted Superadobe model with no need to satisfy any relation involving them, save that all of them are positive, and $r_r \geq r_b$ (3).

5. DESIGN VARIABLES. GEOMETRIC MODEL OF UNRESTRICTED DOMED SUPERADOBE STRUCTURES

The geometric model of domed Superadobe structures is a system of equations. Each of these equations relate all or some of the independent variables seen previously, $(\boldsymbol{s_w}, \boldsymbol{h_s}, \boldsymbol{Extf}, \boldsymbol{r_b}, \boldsymbol{r_r}, \boldsymbol{h_1} \textbf{ and } \boldsymbol{t_r})$ with some "final", "observable", "dependent" or "design" variable, such as building height ***H***, habitable area ***A,*** total building mass ***M*** or weight ***W***. One of such equations is $\boldsymbol{H} = \sqrt{r_r{}^2 - (t_r + r_r - r_b)^2}$, and what follows is the deduction of this, and of some other equations of the model.

Supposing that the ground plane is the plane *xy*, a circular arch of radius $\boldsymbol{r_b} + \boldsymbol{a}$ rotates about the z axis (the axis perpendicular to the ground) forming a solid of revolution similar to an inverted parabola of revolution. Let us recall that $\boldsymbol{r_b}$ is the base radius, and ***a*** is any distance between the center of the base of the dome, and the center of the arch that describes the dome's roof, as can be seen in the Fig. 8.

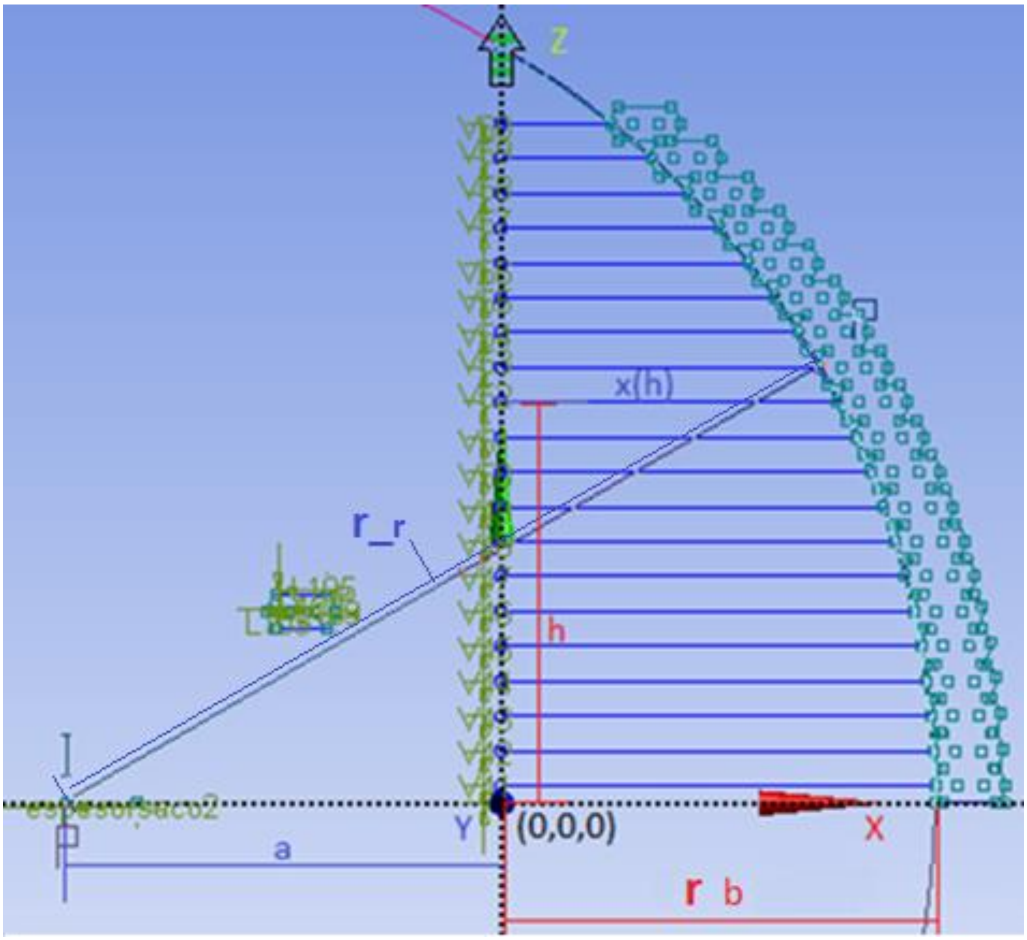

Fig. 8: Figure "Geometric Model Illustration": Parameters a and r_b, and dependent functions h and x(h) are illustrated. The ground level is represented by the dotted horizontal line. The ground plane is xy, and the Y axis is indicated in blue going out of the image.

From this suppositions, based on empirical practice in the construction of domes, initial equations can be derived:

$$x(h)\text{: } the\ variable\ radius\ x\ of\ the\ dome\ at\ each\ height\ h$$

$$x(h) = \begin{cases} \boldsymbol{r_b} \quad when \quad 0 \le h \le h_1, \\ +\sqrt{(\boldsymbol{r_b}+a)^2 - (h-h_1)^2} \; - a \quad when \;\; h_1 < h \le +\sqrt{\boldsymbol{r_b}^2 + 2\,\boldsymbol{r_b}a} \; + \; h_1 \end{cases} \qquad (1')$$

Where $0 \le h_1$, $0 \le \boldsymbol{r_b}$, $0 \le a$ and h_1 *being the height of a ciylindrical base, zero in the illustration above*

And taking into account that $\boldsymbol{a} = \boldsymbol{r_r} - \boldsymbol{r_b}$, (1') becomes:

$$x(h) = \begin{cases} \boldsymbol{r_b} \quad when \quad 0 \le h \le h_1, \\ +\sqrt{(\boldsymbol{r_r})^2 - (h-h_1)^2} \; + \boldsymbol{r_b} - \boldsymbol{r_r} \quad when \;\; h_1 \le h \le +\sqrt{\boldsymbol{r_b}^2 + 2\,\boldsymbol{r_b}(\boldsymbol{r_r} - \boldsymbol{r_b})} \; + \; h_1 \end{cases}$$

(1'')

Where $0 \le h_1$, $0 \le \boldsymbol{r_b}$, $0 \le a$

We can also have the height of a dome at any given point in its interior:

$$h(x,y)\text{: } the\ height\ of\ the\ dome\ at\ a\ point\ (x,y)\ in\ its\ interior\ is$$

$$h(x,y) = +\sqrt{(\boldsymbol{r_b}+a)^2 - \left(\sqrt{(x-x_0)^2 + (y-y_0)^2} + a\right)^2} \; + \; h_1 \qquad (2')$$

Where $0 \le h_1$ Where $0 \le \boldsymbol{r_b}$, $0 \le a$ and $+\sqrt{(x-x_0)^2+(y-y_0)^2} \le \boldsymbol{r_b}$

The equations above can be understood with the use of the Fig. 8. They were determined using the equation of a circle $x^2 + z^2 = \boldsymbol{r_r}^2$ as follows:

The expression for $x(h)$, comes from clearing the variable x in the equation of the circle centered in $(-a, h_1)$, considering z as h, and taking $\boldsymbol{r_r}$ to be $\boldsymbol{r_b}$+*a*. This equation, Equation (1'), gives the radius of each ring $x(h)$ at variable heights h. $\boldsymbol{r_b}$ is the radius of the base ring, while h_1 is the height of a cylindrical wall at the base of the building (note that in the Fig. 8, $h_1 = 0$).

The expression for the function of partial height *h*, equation (2'), follows from (1'). It is written in terms of x and y, by using the equation of the circle, $x^2 + z^2 = \boldsymbol{r_r}^2$, now centered in (-a,0), considering z as h, substituting $\boldsymbol{r_r}$ by $\boldsymbol{r_b}$+$\boldsymbol{a}$, substituting x by the module of the vector $(x-x_0, y-y_0)$, which is $+\sqrt{(x-x_0)^2+(y-y_0)^2}$, and finally adding h_1 (the height of a cylindrical wall at the base of the building) to the resulting expression of *h*.

Equation (2') gives the height of the building, $h(x,y)$, as a function of any point (x,y) of the ground plane enclosed by a dome centered in (x_0, y_0).

5.1. The First design variable H and the first equation of the model

After deriving equation (2'), the total height ***H*** of given Superadobe dome can be obtained assuming (x_0, y_0) = (0,0) and evaluating (2'), *h(x,y),* in the point $(x, y) = (\boldsymbol{t_r}, 0)$, since by definition $\boldsymbol{t_r}$ is the radius of the ring of maximum height. Doing so while taking into account that $\boldsymbol{r_r} - \boldsymbol{r_b} = \boldsymbol{a}$, yields, after some manipulation:

$$\boldsymbol{H} = \sqrt{r_r{}^2 - (t_r + r_r - r_b)^2} + h_1 \qquad (3')$$

Which is the first equation of our model, relating a dependent or design variable ***H***, with the subset of independent variables **(** $\boldsymbol{r_b}, \boldsymbol{r_r}, \boldsymbol{t_r}\ \boldsymbol{and}\ \boldsymbol{h_1}$ **)**

5.2. The Second design variable F and the second equation of the model

A very important fact to consider in a Superadobe's geometry, is the degree or amount of support (bearing, contact area) between two consecutive rings, being the minimum located between the **two top consecutive superblocks (rings).** When a minimum bearing area is not guaranteed in a Superadobe structure (see Fig. 9), it is not a valid dome in practice, and the probability of the simulation experiencing rigid body motion increases, as well as unnecessary stresses.

This amount of support ***F*** is an indicator of "geometrical soundness" of the building. If the designer sets ***F*** to a minimum and solves its corresponding equation for the independent parameters, the resulting dome will be "geometrically sound" up to the degree set by the designer. We proceed to define ***F*** in the ideas that follow:

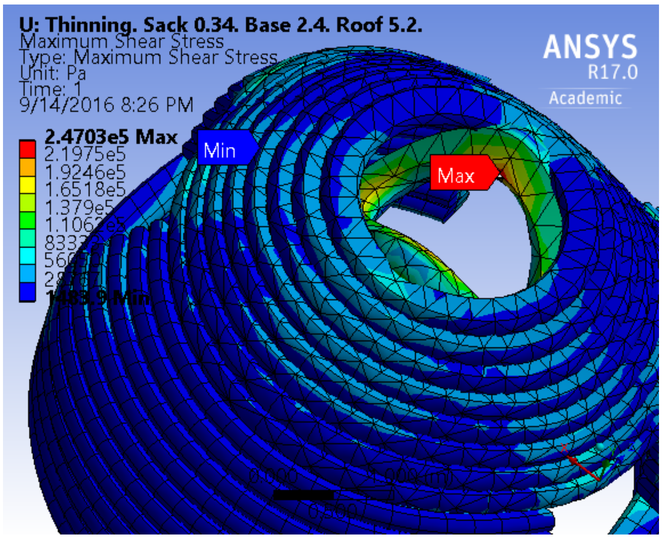

Fig. 9: Example of lack of appropriate support of the top superblock due to inappropriate configuration of sack width, base radius, roof radius and sack height creating a peak shear stress in the top superblock.

In order to avoid the situation illustrated above, a coherent set of values for independent parameters $(\boldsymbol{s_w}, \boldsymbol{h_s}, \boldsymbol{Extf}, \boldsymbol{r_b}, \boldsymbol{r_r}, \boldsymbol{h_1}\ \textbf{and}\ \boldsymbol{t_r})$ should be defined before commencing construction, such that the combination of these parameters results in a minimum sufficient bearing area between all superblocks up to the defined height.

Observation 8: By definition, the center of the transversal section of each ring, does not lie directly over the previous ring's transversal section center. This fact occurs between any pair of sections belonging to consecutive rings in order to achieve the dome's second curvature (the curvature of the roof), but in the case of the following illustration, the top superblock has very little support from the previous superblock.

Below, we can observe an idealized example in which the center of the superblock transversal sections are suspended from the 36th ring upwards, and from this row on, the bottom surface of each superblock has a very little contact portion resting on the previous.

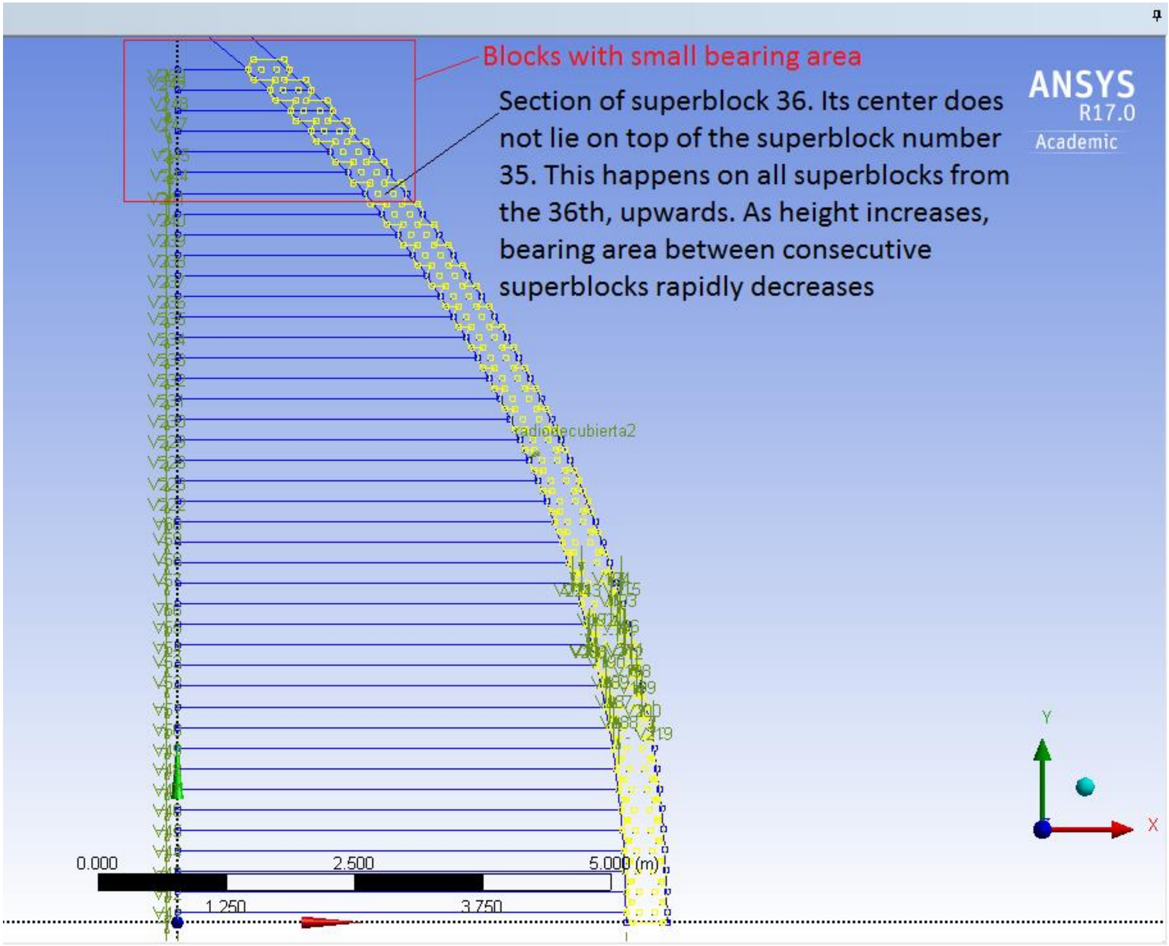

Fig. 10: Insufficient support of top Superblocks.

Observe in the Fig. 10, that block number 42 (the top block) has almost no support from block 41. Instability or excess stress can exist due to the rapid displacement of the blocks as the curve of the roof extends upwards and inwards towards the center of the structure. **The ratio of this bearing area to the weight of the structure from any point upwards, is also a factor contributing to excessive stress concentrations.**

This excess in displacement between consecutive rings from some height on, if it surpasses a certain limit, might also lead to failure of the Static Structural Analysis to converge due to rigid body motion, which indicates instability of the real dome if it were to have this dimensions.

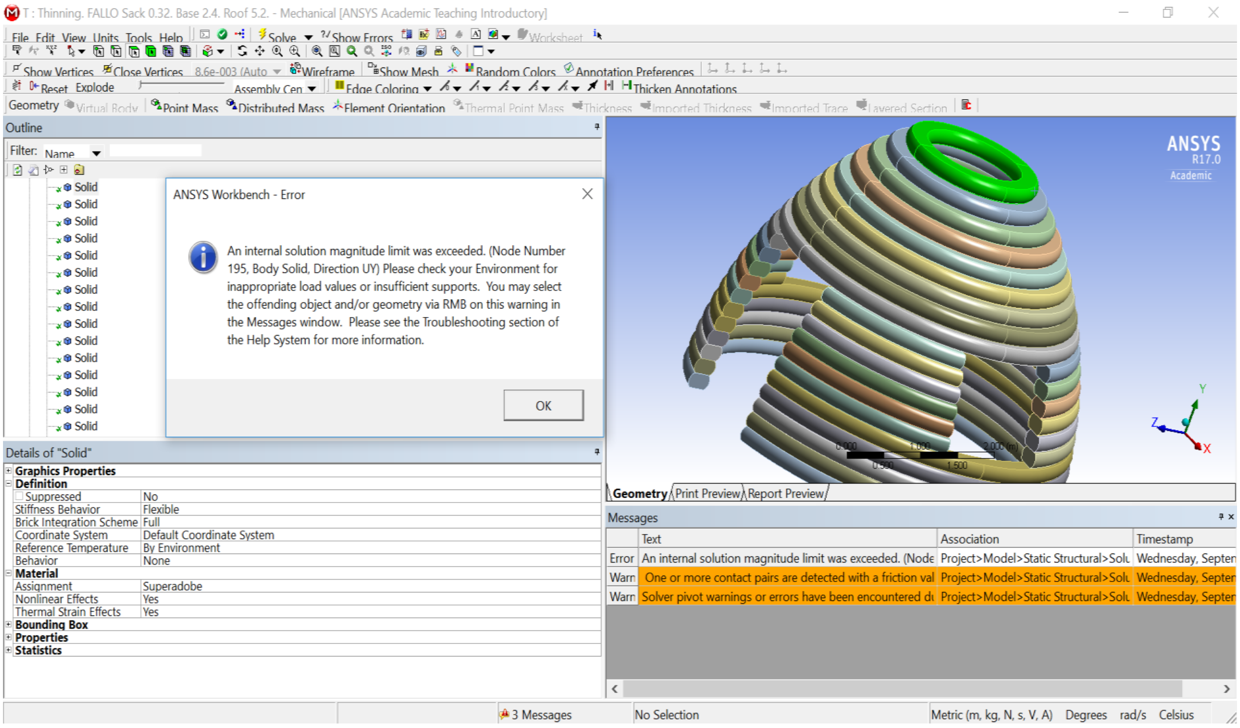

Fig. 11: Failure of the simulation to converge due to minimum contact between top Superblocks.

To guarantee a minimum portion of bearing area between all Superblocks of a Superadobe structure, a certain mathematical relation between the independent constructive parameters $(\boldsymbol{s_w, h_s, Extf, r_b, r_r, h_1} \textbf{ and } \boldsymbol{t_r})$ and a dependent indicator ***F***, must hold.

When this equation holds, we will call the structure “geometrically coherent” up to a desired value ***F***.

The above designation of “geometrically coherent”, responds to the fact that complete or mechanical stability of the structure is not yet guaranteed with this “geometrical coherency”. This is the reason why, of course, finite element analyses and other calculations are necessary. This visual (geometrical) indicator ***F***, when large enough, will imply that the building is "sound" in the geometrical sense, possessing visual support between its elements. **This "ideal" support will be correlated with the real stability of the dome.**

Guaranteeing a minimum value ***F***, representative of the contact area **between the top element and the superblock immediately below it**, would mean that the **minimum contact area** between any two consecutive sacks in the whole building is above a certain number, and we will be obtaining "geometrical coherency" up to this degree ***F***.

To do this, the horizontal distance from the center of the transversal section of the second –to- last ring, to the last ring's section center, must be less or equal than a certain number.

- **Deduction of the equation of *F***

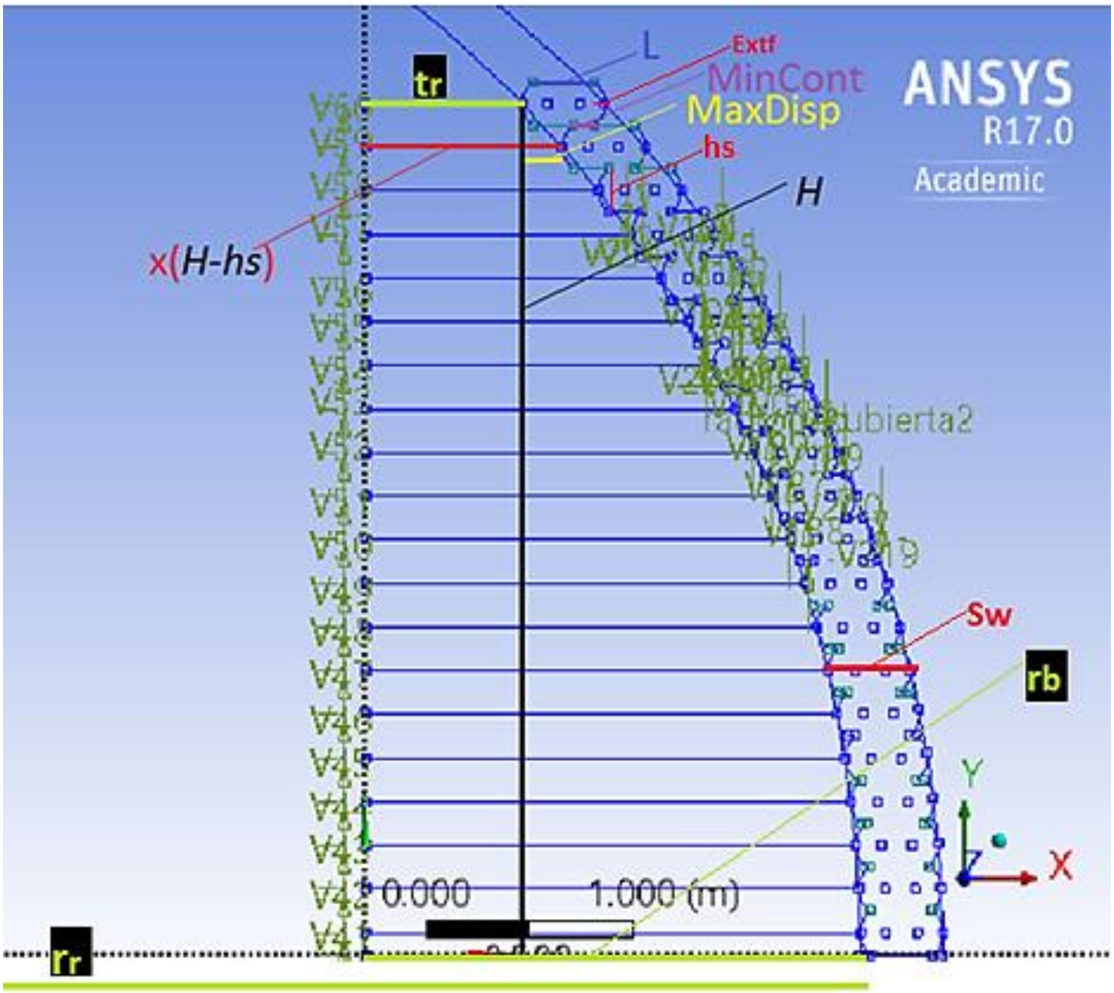

Fig. 12: Basis illustration for derivation of geometric coherency equation.

With the aid of the Fig. 12, suppose:

- ***L*** = The length of the rectangle inscribed in the sack cross section.
- ***MaxDisp*** = Displacement inwards in the x direction between the two top consecutive sacks.
- ***MinCont*** = Minimum contact length between the two top consecutive sack's sections (determines the contact surface area between the two highest sacks).
- $\boldsymbol{t_r}$ **=** The internal radius of the tallest ring / skylight (constructive - independent radius).
- $\boldsymbol{r_{r,}}, \boldsymbol{r_b}$ *and* $\boldsymbol{h_1}$ are: The radius of the roof, the radius of the base and the height of a cylindrical base respectively (constructive -independent radii and $\boldsymbol{h_1}$=0 in this case).
- $\boldsymbol{s_w}$ = as before, sack width (constructive independent property of sack).
- ***Extf*** $= \boldsymbol{s_w} - \boldsymbol{L}$, the Total length of the sack's transversal section width which is suspended (constructive independent property of sack).
- ***hs***=Height of a sack, (height of any superblock) (constructive independent property of sack.
- ***X(H-hs)*** = Radius of the ring at a height of ***H-hs***., *the second-to last ring.*
- ***H*** = The total height of the dome measured from ground level to the height of the midpoint of the tallest superblock, as defined before in equation (2'),
- ***H=*** *h* ($\boldsymbol{t_r}, \boldsymbol{0}$). (First design or dependent parameter).

We then define ***F*** as the minimum fraction of *L* to be the contact length between the two top consecutive sack's sections.

Having defined our parameters, the reasoning for the formula of ***F*** is as follows:

The minimum length of contact between two consecutive sack's sections (***MinCont***), is the length of contact between the two top sack sections. Then

MinCont = L – MaxDisp.

MinCont determines the minimum surface area of contact between any pair of consecutive sacks of the structure, and this length of contact must be greater or equal than some fraction ***F*** of ***L***. We can take this fraction to be ***F***= ¼, ½, etc., depending on the amount of contact we want to guarantee.

Using the above illustration and our last supposition, we then have:

MinCont = L - MaxDisp ≥ ***F*** * ***L***=>

- MaxDisp ≥ ***F*** * ***L*** - ***L*** =>

MaxDisp ≤ ***L - F*** * ***L*** =>

(To achieve the lowest possible building with geometric stability, we take the extreme case of the last inequality, which is the the equality).

MaxDisp = ***L - F*** * ***L*** =>

MaxDisp = ***L*** (1- ***F***) =>

(Using that ***MaxDisp*** = ***X(H-hs)*** – $\boldsymbol{t_r}$ and that ***L***=$\boldsymbol{s_w - Extf}$)

X(H-hs)- $\boldsymbol{t_r}$ = $(\boldsymbol{s_w - Extb})$(1- ***F***) =>

$$\left(\frac{\mathbf{1}}{\mathbf{1-F}}\right)(\boldsymbol{X}(\boldsymbol{H-hs}) - \boldsymbol{t_r}) - \boldsymbol{s_w} + \boldsymbol{Extf} = 0 \qquad (4')$$

What follows is the expansion of the term ***x(H-hs).*** To do this, we recall a past equation:

$$x(h): the\ variable\ radius\ x\ of\ the\ dome\ at\ each\ height\ h$$

$$x(h) = \begin{cases} \boldsymbol{r_b} \quad when \quad 0 \le h \le h_1, \\ +\sqrt{(\boldsymbol{r_r})^2 - (h-h_1)^2} + \boldsymbol{r_b} - \boldsymbol{r_r} \quad when \ h_1 \le h \le +\sqrt{\boldsymbol{r_b}^2 + 2\,\boldsymbol{r_b}(\boldsymbol{r_r} - \boldsymbol{r_b})} + h_1 \end{cases} \qquad (1'')$$

Where $0 \le h_1$, $0 \le \boldsymbol{r_b}$, $0 \le a$

With h_1 $being\ the\ height\ of\ a\ ciylindrical\ base$

Taking into account that ***H-hs*** ≥$\boldsymbol{h_1}$, and evaluating ***x*** at the point *h*=***H-hs,*** (1'') becomes:

$$\boldsymbol{x\ (H\text{-}hs)} = +\sqrt{(\boldsymbol{r_r})^2 - (\boldsymbol{H} - \boldsymbol{hs} - h_1)^2} - \boldsymbol{r_r} + \boldsymbol{r_b} \qquad (5')$$

Recalling that ***H*** is

$$\boldsymbol{H} = \sqrt{r_r^2 - (tr + r_r - r_b)^2} \quad + \quad h_1 \qquad (3')$$

And introducing ***H*** in (5'), we get, after manipulating terms:

$$\boldsymbol{x(H\text{-}hs)} = \sqrt{(Tr + r_r - r_b)^2 + 2\left(\sqrt{r_r^2 - (tr + r_r - r_b)^2}\right)(hs) - hs^2} + r_b - r_r \qquad (6')$$

We then introduce (6') in (4'), obtaining:

$$\left(\frac{1}{1-\boldsymbol{F}}\right)\left(\sqrt{(tr + r_r - r_b)^2 + 2\left(\sqrt{r_r^2 - (tr + r_r - r_b)^2}\right)(hs) - hs^2} + r_b - r_r - tr\right) - s_w + Extf = 0 \qquad (7')$$

Finally, clearing (7') for ***F***, we obtain:

$$\boldsymbol{F} = 1 - \frac{\left(\sqrt{(tr + r_r - r_b)^2 + 2\left(\sqrt{{r_r}^2 - (tr + r_r - r_b)^2}\right)(hs) - hs^2} + r_b - r_r - tr\right)}{s_w - Extf}$$

(8')

This is the second equation of our model, the equation for the design parameter ***F.***

A dome who's independent parameter values $(\boldsymbol{s_w}, \boldsymbol{h_s}, \boldsymbol{Extf}, \boldsymbol{r_b}, \boldsymbol{r_r}\ \textbf{and}\ \boldsymbol{t_r})$ satisfy (8') for a certain value of ***F***, will be the lowest (optimal) dome for a given $\boldsymbol{h_1}$ having *at least* a fraction ***F*** of all of its rings straight width (***L***) resting upon the previous ring. The minimum area of contact (that between the two top rings) of such dome will specifically be ***F* L****top superblock length.

5.3. The third design variable A and the third equation of the model

A is the sum of the areas of all the floors enclosed in the dome. This habitable surface area is an important value to request beforehand by a designer according to the needs of the family that will live in it. The idea behind its calculation is to take the number of habitable floors in the dome to the mathematical ceiling of ***H/n, where H is the height of the dome and n is the minimum height of each level except for the last.*** Taking ***n*** to be 3mts, what remains is the calculus of the area for each of the different $\lceil \boldsymbol{H}/\boldsymbol{3} \rceil$ **levels** according to the different radii that each level has. The formula is given by:

$$A = \sum_{i=0}^{\lceil H/3 \rceil} \pi x(3i)^2 \qquad (9')$$

Where $\boldsymbol{H} = \sqrt{{r_r}^2 - (t_r + r_r - r_b)^2} + h_1$ (3')

And x is the function:

$$x(h) = \begin{cases} \boldsymbol{r_b} \quad when \quad 0 \le h \le h_1, \\ +\sqrt{(\boldsymbol{r_r})^2 - (h - h_1)^2} + \boldsymbol{r_b} - \boldsymbol{r_r} \quad when\ h_1 \le h \le +\sqrt{\boldsymbol{r_b}^2 + 2\,\boldsymbol{r_b}(\boldsymbol{r_r} - \boldsymbol{r_b})} + h_1 \end{cases}$$

(1'')

Where $0 \le h_1$, $0 \le \boldsymbol{r_b}$, $0 \le a$

We can see that ***A*** depends upon the independent parameters $(\boldsymbol{r_b}, \boldsymbol{r_r},$ $\boldsymbol{t_r}\ \boldsymbol{and}\ \boldsymbol{h_1})$**.**

Observation 9: Note that main door and window openings as set before, do not come in conflict with floors set to be 3 m apart, as the first level will be above them.

5.4. The Fourth design variable V and the fourth equation of the model

V is the sum of the volumes of the filled superblocks of the dome. This measure is an important constraint to be requested beforehand by a designer that may respond to cost limits (it is a fairly good indicator of the costs involved), of the quantity of material needed, and corresponds to the weight limit that the building may have given the steepness of the terrain (***V=Mass/avg.density***). The idea behind its calculation is to take the number of rings in the dome to be the mathematical ceiling of ***H/n, where H is the height of the dome and n is the height of each superblock (ring) in their service state (h_s).*** It is necessary to calculate the total length of each of the different $\left\lceil \boldsymbol{H}/\boldsymbol{h_s} \right\rceil$ **rings** according to the perimeter that each one has and the existence of oppenings.

Let the height of each ring be $h(i) = \boldsymbol{h_s} * (\boldsymbol{i - 1/2})$

Let ***ihw, fhw*** and ***ww*** be the initial height, final height, and width of the windows, respectively (two identical window openings are proposed)

Let ***ihd, fhd*** and ***wd*** be the initial height, final height, and width of door, respectively.

Let the length of each ring be :

$$L_i = (2 * \pi * (x(h(i)) \ + \boldsymbol{0.5 s_w})) - 2 * j * \boldsymbol{ww} - k * \boldsymbol{wd}$$

Where $j = 1$ if ***ihw*** $\leq h(i) \leq$ ***fhw and*** $j = 0$ *when this is not so*

Where $k = 1$ if ***ihd*** $\leq h(i) \leq$ ***fhd and*** $j = 0$ *when this is not so*

And x is the function:

$$x(h) = \begin{cases} \boldsymbol{r_b} \quad when \quad 0 \leq h \leq h_1, \\ +\sqrt{(\boldsymbol{r_r})^2 - (h - h_1)^2} \ + \boldsymbol{r_b} - \boldsymbol{r_r} \quad when \ h_1 \leq h \leq +\sqrt{\boldsymbol{r_b}^2 + 2\,\boldsymbol{r_b}(\boldsymbol{r_r} - \boldsymbol{r_b})} \ + h_1 \end{cases}$$
(1'')

Where $0 \leq h_1$, $0 \leq \boldsymbol{r_b}$, $0 \leq a$

Then

$$\boldsymbol{V} = \sum_{i=1}^{\left\lceil \boldsymbol{H}/\boldsymbol{h_s} \right\rceil} L_i w_i h_s \qquad (10')$$

Where as we know $\boldsymbol{H} = \sqrt{{r_r}^2 - (t_r + r_r - r_b)^2} + h_1$
(3')

We can see that ***V*** depends upon the independent parameters **(** $\boldsymbol{r_b}, \boldsymbol{r_r}$, $\boldsymbol{t_r}$, $\boldsymbol{h_1}, \boldsymbol{s_w}$, $\boldsymbol{ihw}, \boldsymbol{fhw}$, $\boldsymbol{ww}$, $\boldsymbol{ihd}, \boldsymbol{fhd}$, $\boldsymbol{wd}$, $\boldsymbol{and\ h_s}$ **) and since the element inside the sum**

$$L_i w_i h_s$$

Is the volume of a parallelepiped, it is an over estimation (on the safe side) of the volume of the superblocks.

5. FINAL SECTION: THE GEOMETRIC MODEL AND CONCLUSIONS

Thus far our geometric model (system of equations) describing the geometry of domed-unrestricted Superadobe structures is:

$$\begin{cases} \boldsymbol{H} = \sqrt{{r_r}^2 - (t_r + r_r - r_b)^2} + h_1 \qquad (3') \\ \boldsymbol{F} = 1 - \dfrac{\left(\sqrt{(tr + r_r - r_b)^2 + 2\left(\sqrt{{r_r}^2 - (tr + r_r - r_b)^2}\right)(hs) - hs^2} + r_b - r_r - tr\right)}{s_w - Extf} \qquad (8') \\ \boldsymbol{A} = \displaystyle\sum_{i=0}^{\lceil \boldsymbol{H}/\boldsymbol{3} \rceil} \pi x (3i)^2 \qquad (9') \\ \boldsymbol{V} = \displaystyle\sum_{i=1}^{\lceil \boldsymbol{H}/\boldsymbol{h_s} \rceil} L_i w_i h_s \qquad (10') \end{cases}$$

Each time a designer sets fixed values for ***n*** of the four ***H, F, A*** or ***V***, and some independent parameters such that ***n*** unknowns remain, a compatible system of equations can arise. In this case, the rest of the unknown parameters can be found with the aid of computer algorithms, for example, one that imports the python mathematical libraries scipy.optimize and numpy to solve these nonlinear systems of equations implicitly.

An important ideas behind this first geometric model is that:

- Further equations must be included in a theoretical model of this parametric structures (describing not only geometrical dimensions such as ***H, A, F*** or ***V***, but also mechanical dependent parameters such as (masses over rows, total mass, max stresses, etc.) so that computer scripts could calculate many parameterizations. These results can be useful data that function as guidelines for theoretic exploration of Superadobe dome behavior, and once equations with mechanical variables are calibrated with some finite element analyses and empirical testing, a more complete description of these promising and overlooked structures will exist, as well as a practical means for their design.

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7. REFERENCES

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