

APPLICATIONS OF A “GEOMETRICAL SOUNDNESS EQUATION” FOR SUPERADOBE DOMES

Marco Aurelio López-Gómez ^a

^a Universidad Politécnica de Madrid (marcoarelio.lopez.gomez@alumnos.upm.es)

Abstract—A Superadobe dome is a parameterization in the sense that given some initial parameters, the structure can be completely determined by them [1]. While a set of such initial parameters can produce a safe and functional structure, a different set can result in an unsafe, unstable, or even geometrically senseless structure.

It is necessary for a Superadobe designer to predict the structure’s final dimensions and mechanical behavior from the initial parameters of his or her choice, so as to be able to choose them wisely. These initial independent numbers must satisfy some equations containing variables of both geometrical and mechanical nature, yet the parameters’ compliance with the single, purely-geometrical equation showcased in this article, is a necessary condition for all functional Superadobe domes.

Keywords— *Superadobe, Sand-bags, Self-construction, eco-friendly, emergency shelter, parametric analysis*

Introduction:

The following equation:

$$\left(\frac{1}{1-F}\right) \left(\sqrt{(tr + r_r - r_b)^2 + 2(\sqrt{r_r^2 - (tr + r_r - r_b)^2})(hs)} - hs^2 + r_b - r_r - tr \right) - s_w + Extf = 0 \quad (1)$$

consisting of independent parameters: (s_w , h_s , $Extf$, r_b , r_r and t_r), must be satisfied in any functional Superadobe dome. The development of the above equation, as well as the definition of each parameter is explained in [1].

A dome who’s initial parameter values satisfy (1) for a certain value of F , will be the lowest (optimal) dome for a given h_1 having a proportion F of its top row’s straight section resting upon the previous row, and if F is a **big enough positive number less than 1**, the dome will be geometrically coherent or ‘**sound**’.

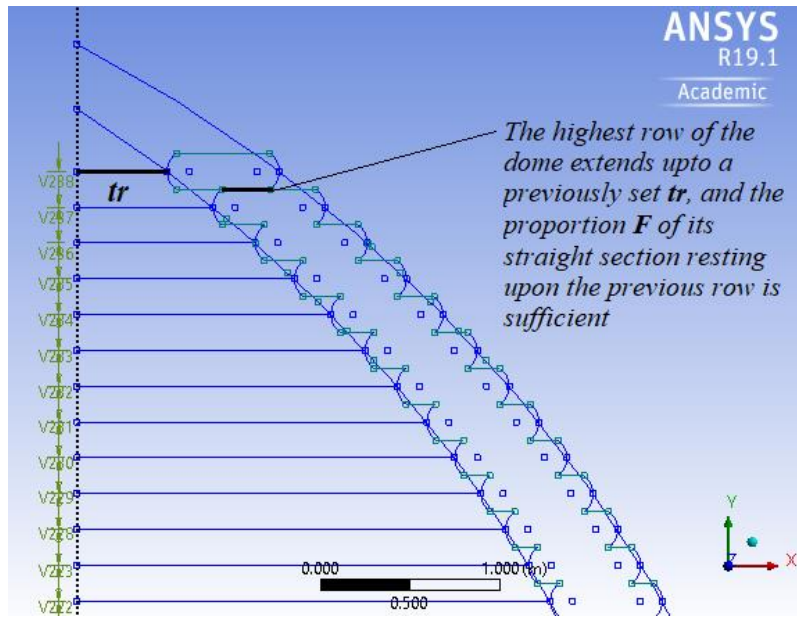


Figure 1. Section of a Superadobe dome with adequate set of initial parameter (s_w , h_s , $Extf$, r_b , r_r and t_r) resulting in a geometrically sound structure

Rewriting (1) we have:

$$F = 1 - \frac{\left(\sqrt{(tr+r_r-r_b)^2 + 2(\sqrt{r_r^2 - (tr+r_r-r_b)^2})(hs) - hs^2 + r_b - r_r - tr} \right)}{s_w - Extf} \quad (1')$$

For some combinations of values (s_w , h_s , $Extf$, r_b , r_r and t_r), F can result in a very small number, or even a negative one, and the respective dome would be geometrically 'unsound':

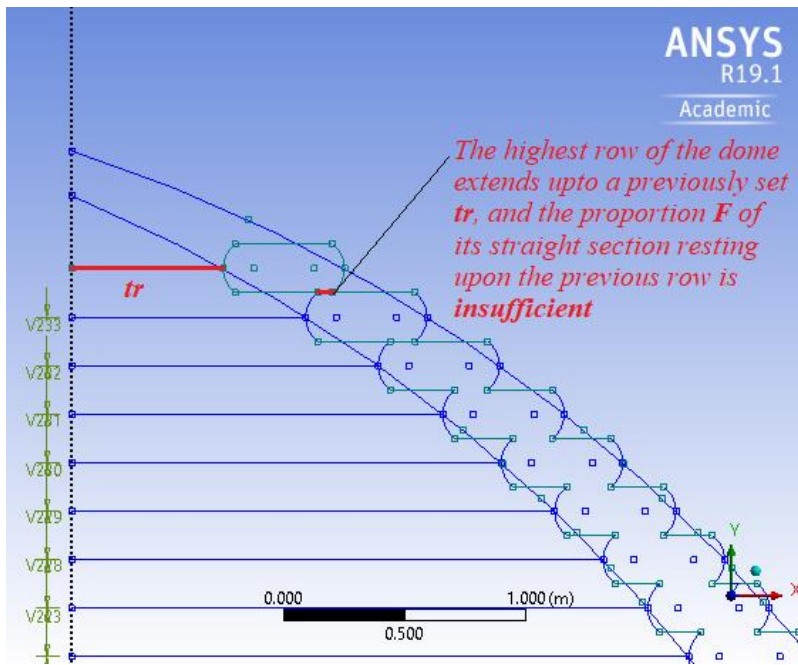


Figure 2. Section of a Superadobe dome with inadequate set of initial parameters s_w , h_s , $Extf$, r_b , r_r and t_r resulting in a geometrically unsound structure

The previous figures illustrate that the set s_w , h_s , $Extf$, r_b , r_r and t_r must be correctly set in the design of a dome.

Applications of Equation (1):

Equation (1) depends upon a **comprehensive** set of Superadobe dome initial geometric parameters, and relates them with a plausible indicator of behavior and stability F , geometrically observable as well. Hence, it alone constitutes a potent filter that will rule out inadequate designs.

When all variables save one in (1') are defined, this equation can be solved implicitly. A simple code, which solves the implicit equation resulting from defining all parameters in (1), except for one, was written in Python programming Language. This allows us to solve the equation for any variable, not only for F , but also for parameters such as tr, r_r or r_b , which cannot be isolated, and for which only implicit functions exist in subdomains determined by (1). With the aid of this tool, even F , which is the observable, isolated function in (1'), can be treated as another independent parameter.

Examples of these cases will be discussed in the upcoming section.

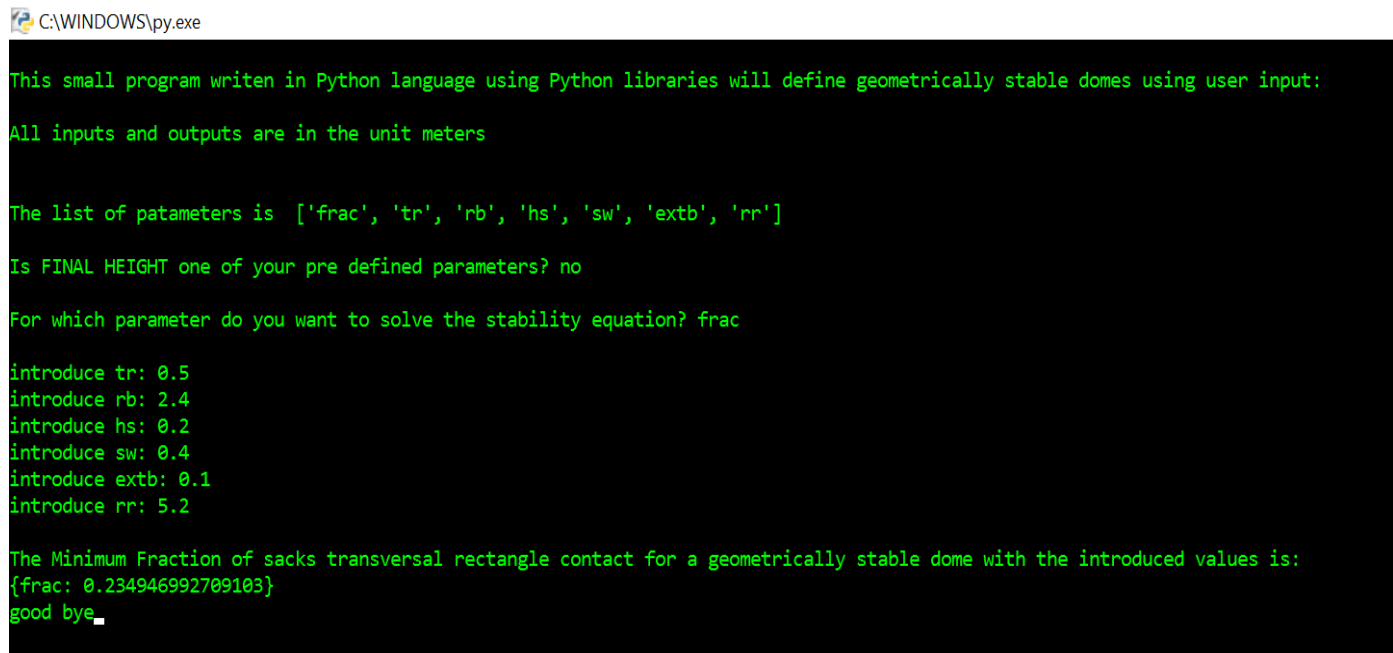
Example 1: What is the minimum degree of contact between rows that occurs in a **conventional** Superadobe dome?

If a Superadobe structure has geometric conventional initial parameters:

$r_r = 5.2m$, $r_b = 2.4m$, $s_w = 0.4m$, satisfying equations (3,4,5) in [1], and $hs=0.2m$, $Extb = 0.1m$, and we set the radius of opening at Max height H , to be for example, $0.5m$, then we can simply calculate F , using (1'), or equivalently solve for F in (1), discovering the minimum length of contact between the transversal sections of any two consecutive blocks for this design.

In the following figure, we see the program interface that solves the implicit equation (1) for parameter F . The result (0.234), means that if we take the **conventional** Superadobe sack, base radius and roof radius values, we have a minimum fraction of contact between transversal sack sections of 0.234 times 0.3m, (roughly 7 cm of contact between the top sections), up to $H(0.5m)=4.019m$ of height.

The program solves the equation behind the scenes, and the code for this program is attached in the appendix.



```

C:\WINDOWS\py.exe

This small program writen in Python language using Python libraries will define geometrically stable domes using user input:
All inputs and outputs are in the unit meters

The list of patameters is ['frac', 'tr', 'rb', 'hs', 'sw', 'extb', 'rr']
Is FINAL HEIGHT one of your pre defined parameters? no
For which parameter do you want to solve the stability equation? frac

introduce tr: 0.5
introduce rb: 2.4
introduce hs: 0.2
introduce sw: 0.4
introduce extb: 0.1
introduce rr: 5.2

The Minimum Fraction of sacks transversal rectangle contact for a geometrically stable dome with the introduced values is:
{frac: 0.234946992709103}
good bye_

```

Figure 3. Console Interface for solving implicitly eq (1) for F

Observation: F in the console is represented by 'frac'.

As a second example of the many applications of (1), we can test the maximum height up to which a conventional dome is geometrically stable.

Example 2: What is the maximum height of a **given** geometrically stable dome with set parameters? (This is proportional the maximum travel of the external compass arch [1], marked in red in the figure)

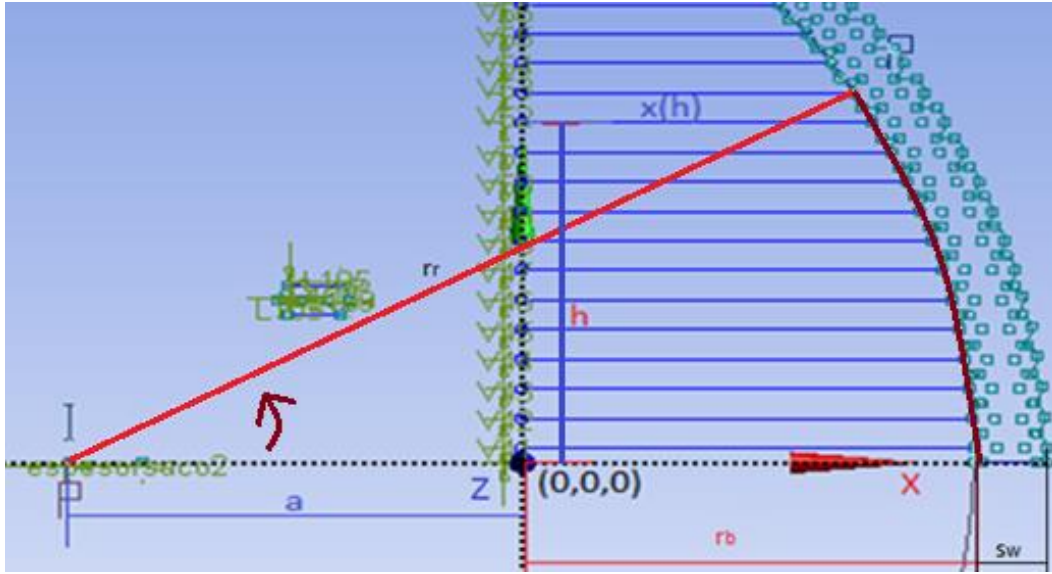


Figure 4. Travel of the external compass

If for example a Superadobe structure has parameters r_r , s_w and r_b satisfying equations (3,4,5) of [1]: $hs=0.2m$, $r_r = 5.2m$, $r_b = 2.4m$, $s_w = 0.4m$, $Extb = 0.1m$, and we set a **condition** for stability that minimum fraction of contact between top rows, $F = 0.25$, then we can solve (1) for Tr , obtaining a top opening (skylight) radius of $0.541m$, and we can know the building's height by calculating the corresponding H when $Tr=0.541m$. [1], equation (3')

Since $H(Tr) = \sqrt{r_r^2 - (Tr + r_r - r_b)^2}$ [1], then $H(0.541) = 3.98m$.

The geometrically stable building with this characteristics will not surpass $3.98+0.2m$ in height. The point of the arch which's distance from the y axis is $0.541m$ is $3.98m$ high, and does not coincide with the building's top sack section midpoint (by a measure of less than $0.2m$), because the resulting height is not divisible by hs . The following is an illustration of what was just said:

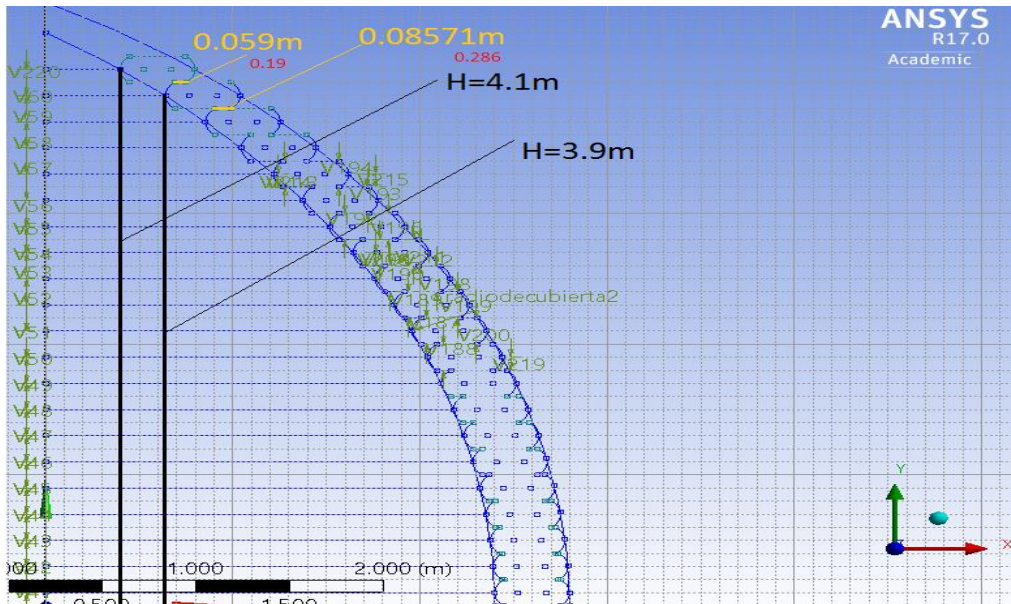


Figure 5. Verification of example 2: Max height for a Conventional Superadobe Structure, geometrically stable to a degree of $0.25 * 0.3m$ (The top sack surpasses the limit)

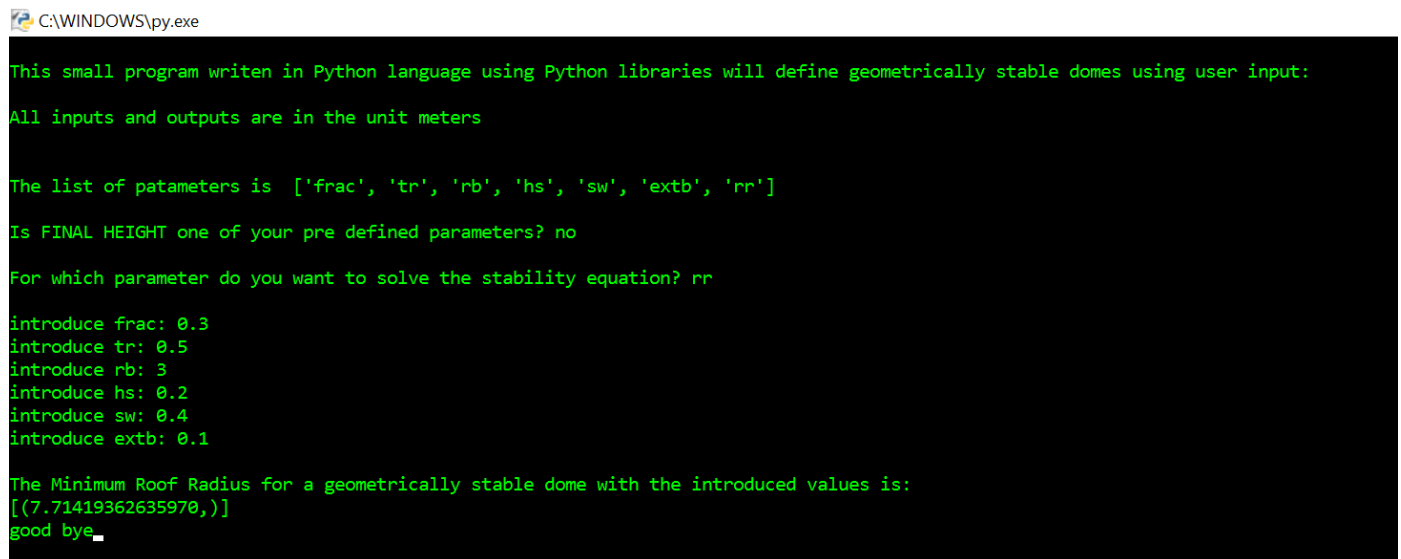
According to the illustration above, and as predicted by equation (1), a sack's section midpoint **above** 3.98m, will have a shared contact with the previous section of **just less** than $0.25 \cdot 0.3\text{m} = 0.075\text{m}$, (0.059m), and a sack's section midpoint **up to a height of 3.98m**, will have a shared contact with the previous section of **at least** $0.25 \cdot 0.3\text{m} = 0.075\text{m}$, (0.08571m).

Observations:

- Equation (1) optimizes a dome structure: It sets the parameters (s_w, r_r, Tr) at a **minimum** stable value when the equation is solved for them. When s_w is minimized, the structure is optimal in economy and weight, when r_r is minimized, the structure is the lowest possible stable structure (also optimal in economy and weight), and optimal in response to stresses from wind load and also excess mass coming from excess height and a steep roof radius. When Tr is minimized, the building will be as high as possible within this minimal r_r .
- Lastly, equation (1) maximizes r_b , $Extb$, and hs when the equation is solved for them. When r_b is maximized, the resulting building is geometrically stable with maximum possible base area. When $Extb$ is maximized, the bulks of the lateral extremes of the sack's transversal section are allowed to be as great as possible while the building maintains stability, and when hs is maximized, the sack is allowed to be as high as possible while the structure also remains stable, thus having to use less layers or courses to build it.

Example 3: Given the structure's width, what is the optimal height? (What is the optimal roof curvature/ smallest r_r for a Superadobe dome given its base area?)

If the builders have a predefined base surface area, minimum factor of contact, skylight radius and building sack characteristics, solving (1) for r_r will produce the lightest and lowest (optimal) geometrically sound structure.



```

C:\WINDOWS\py.exe
This small program written in Python language using Python libraries will define geometrically stable domes using user input:
All inputs and outputs are in the unit meters

The list of parameters is ['frac', 'tr', 'rb', 'hs', 'sw', 'extb', 'rr']
Is FINAL HEIGHT one of your pre defined parameters? no
For which parameter do you want to solve the stability equation? rr
Introduce frac: 0.3
Introduce tr: 0.5
Introduce rb: 3
Introduce hs: 0.2
Introduce sw: 0.4
Introduce extb: 0.1

The Minimum Roof Radius for a geometrically stable dome with the introduced values is:
[(7.71419362635970,)]
good bye_
  
```

Figure 6. Console interface for solving implicitly eq (1) for the parameter r_r . Calculation of optimal height given the structures width

The resulting structure in this experiment:

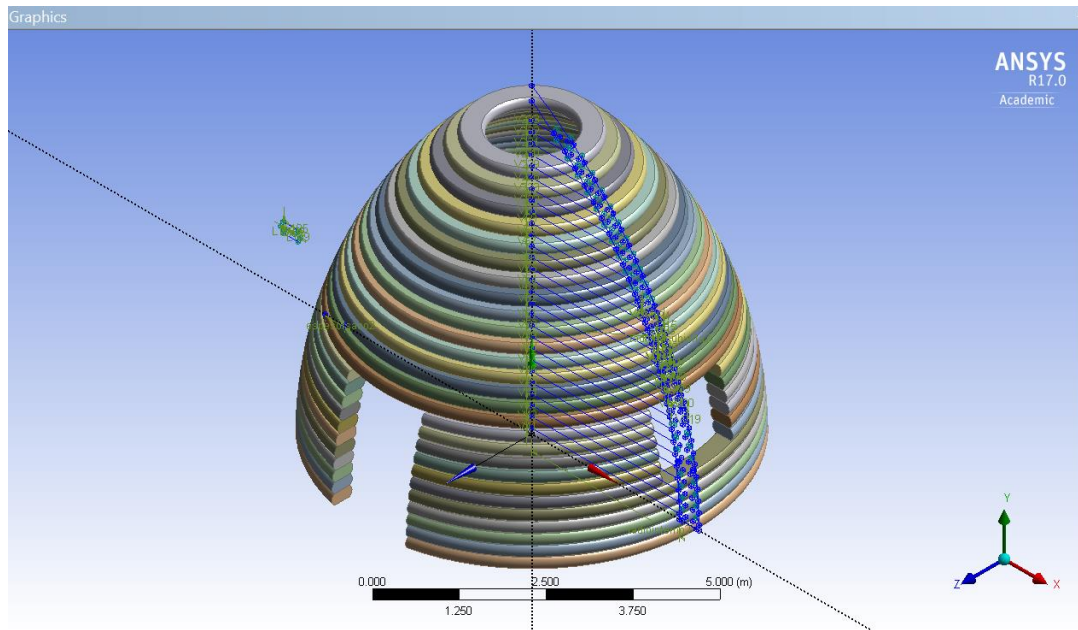


Figure 7. Solid of revolution. The resulting dome from the above experiment has 27 superblocks, an optimal height of 5.4m, a top opening of just over 0.5m, and all of its courses's transversal sections have at least 10cm of contact.

Example 4. What is the minimum Superblock width needed to build a stable Superadobe structure of a given area and height?

When builders have predefined the size of all three compasses: base area, the building's final height and skylight radius, they must know the minimum sack dimensions needed to guarantee a geometrically stable dome with this guidelines.

Since the height of the dome H depends upon the three compasses (r_b , r_r , Tr), the situation described in example 4 is similar to a predefined base radius r_b (since base area is dependent on this), skylight radius Tr and final dome height H [1], together with presetting minimum factor of contact F . solving (1) for s_w will produce the thinnest (optimal) geometrically stable structure, with the minimum width of sack needed for the building to be geometrically stable.

Note in the figure below of the execution of the program that H is set as a preset requisite by answering yes to the question: "Is final height one of your pre-defined parameters?"

```
C:\WINDOWS\py.exe

This small program written in Python language using Python libraries will define geometrically stable domes using user input:
All inputs and outputs are in the unit meters

The list of parameters is ['frac', 'tr', 'rb', 'hs', 'sw', 'extb', 'rr']
Is FINAL HEIGHT one of your pre defined parameters? yes
introduce the predefined HEIGHT: 7
introduce which height parameter you want to find, tr, rb or rr? rr
introduce rb: 5
introduce tr: 0.5

rr is:
[(7.694444444444444,)]
For which parameter do you want to solve the stability equation? sw

introduce frac: 0.25
introduce tr: 0.5
introduce rb: 5
introduce hs: 0.2
introduce extb: 0.1
introduce rr: 7.7

The Minimum Sack width for a geometrically stable dome with the introduced values is:
{sw: 0.640999102300943}
good bye_
```

Figure 8. Minimum required Superblock width given the structure's size

The resulting structure:

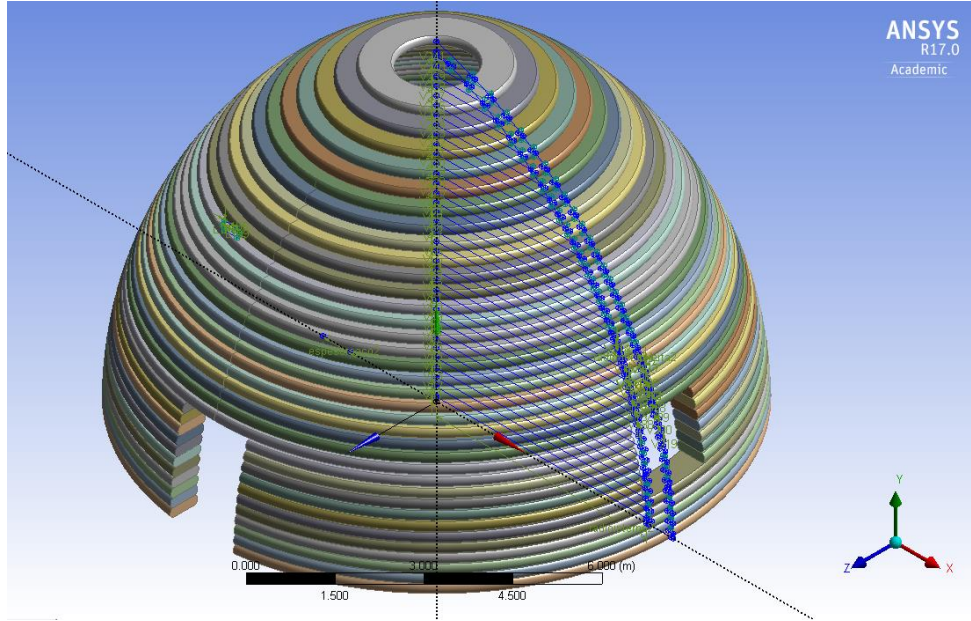


Figure 9. Solid of revolution. The resulting dome from the experiment above has 35 superblocks, a height of 7m, as required in the console, a top opening of just over 0.5m, it is built with sacks of 0.641m wide, and all of its course's transversal sections have at least 0.135m of their straight sections resting on the previous rows. (0.135m is $0.25 \cdot 0.64 - \text{Extb}$)

Observation: The final height of the structure, as we have seen in [1], depends on three parameters: Tr , r_r , and r_b . It is clear why H depends on Tr and r_r , but it can be less clear why H also depends on r_b .

The reason is H 's original dependence on a . This dependence on a means that, although one may know the length of the compass opening to trace the roof radius and the radius of the opening at the top (up to where the roof curve is traced), one must also know the location of the end point of the compass with respect to the dome's central axis to define H , and this is where r_b is needed, because $r_r - r_b = a$, which states the distance between the central axis and the center of the roof compass.

Equivalently, r_b is the distance between the central axis and the roof compass endpoint. When a designer has H pre-determined, he must also know two of r_r , r_b or Tr , to know the missing compass. This means that knowing H and two other radiuses rules out the need for the stability equation, unless this equation is solved for a parameter other than r_r , r_b or Tr , which is the case in the last example, where (1) was solved for s_w .

Example 5: What is the optimal (greatest) base area for a structure of a given roof curvature?

In this situation, designers have defined building's roof radius (external compass size: a pattern for gradual closing of the roof is already defined by pre conceived elements, for example) and skylight radius, with the rest of the sack-element characteristics also defined. They want to know the maximum base area permitted (maximum distance between central axis and roof compass endpoint) for a geometrically stable dome with these parameters and the defined degree of contact.

If the builders have a predefined roof radius and top opening radius, with determined sack characteristics and minimum factor of contact, solving (1) for r_b will produce the biggest (widest) geometrically stable structure, with the optimal base area permitted for the building to be geometrically sound up to the desired F level [1]. Note that as in **Example 3**, here the builder does not have the building height H predetermined, and it is a variable depending on r_b . All that is known is that the larger r_b , the higher the building will be.

Conversely, the longer the predetermined r_r is, the wider the building can be, while remaining geometrically stable.

```

This small program written in Python language using Python libraries will define geometrically stable domes using user input:
All inputs and outputs are in the unit meters

The list of parameters is ['frac', 'tr', 'rb', 'hs', 'sw', 'extb', 'rr']

Is FINAL HEIGHT one of your pre defined parameters? no

For which parameter do you want to solve the stability equation? rb

introduce frac: 0.25
introduce tr: 0.6
introduce hs: 0.2
introduce sw: 0.5
introduce extb: 0.1
introduce rr: 7

The Maximum Radius of base for a geometrically stable dome with the introduced values is:
[(3.86838653689950,)]
good bye_

```

Figure 10. Calculation of maximum building internal compass given the curvature of the roof

The resulting structure:

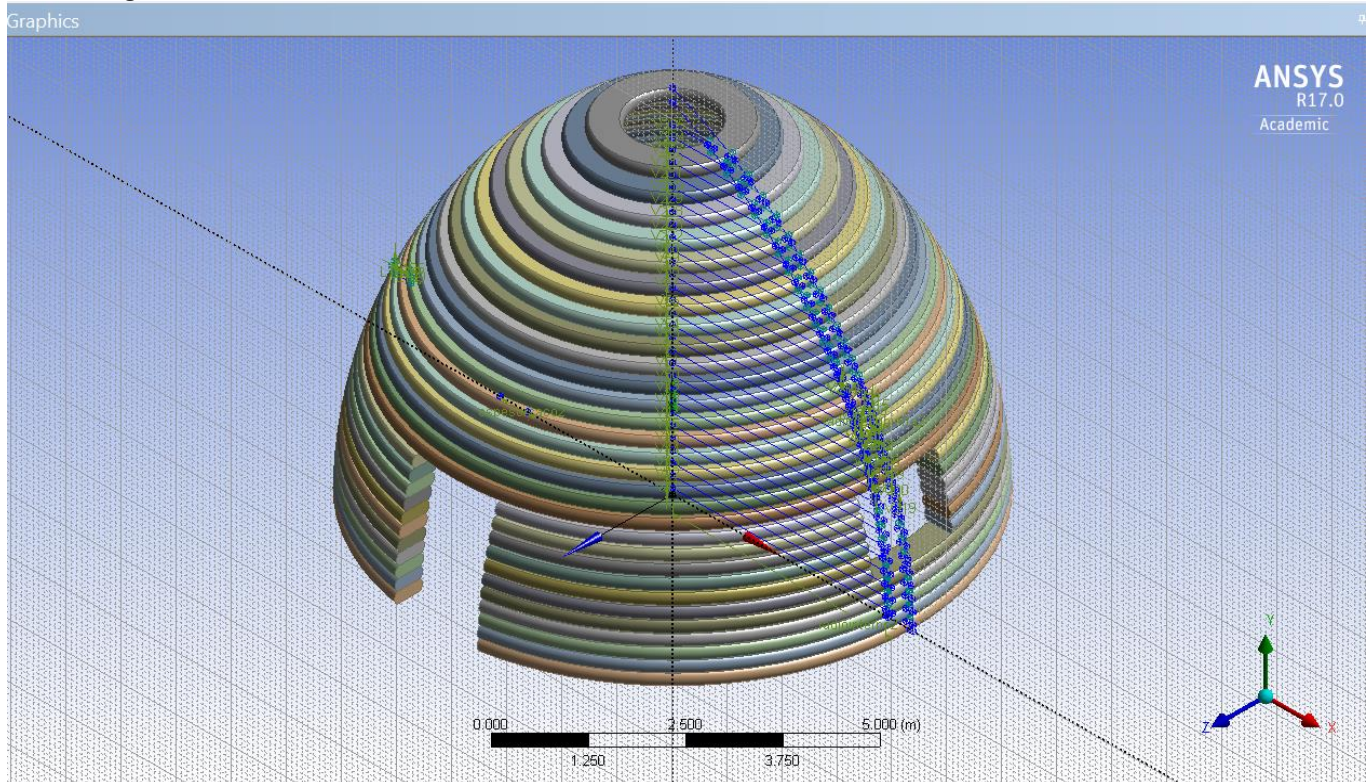


Figure 11. The resulting structure has 30 superblock levels, measuring 6 meters of height, and having a skylight radius of just over 0.6m, with sack width 0.5m and minimum degree of contact F of 0.25 (10cm)

Example 6: Unknown *Extb*:

Solving (1) for *Extb* will yield the maximum round (non-supportive) element parts, so that the building with given characteristics remains stable up to the desired *Frac* level. This case can be associated with maximizing possible sack undesired behavior while remaining with a stable building.

Example 7: Unknown **hs**: How does the height of the sacks affect dome stability or geometrical soundness?

It is perhaps evident, that the taller the sack, the greater the displacement between consecutive courses in a Superadobe dome is needed.

Given two different sack heights, a formal demonstration could be that if **h** is the height of the lowest one, the **displacement** between the last two sacks of greater height **kh**, which is given by:

$$\cos(\sin^{-1}(0.5kh)) - \cos(\sin^{-1}(1.5kh)),$$

Will be always greater than the displacement between the two last smaller sacks at the same building's height, given by $\cos(\sin^{-1}(1.5kh)) - \cos(\sin^{-1}(0.5h(3k + 2)))$.

Proving the inequality:

$$\cos(\sin^{-1}(1.5kh)) - \cos(\sin^{-1}(0.5h(3k + 2))) \leq \cos(\sin^{-1}(0.5kh)) - \cos(\sin^{-1}(1.5kh))$$

For any positive **h** and any **k** greater than 1, can prove the intuition that given two identical domes except for **hs**, that the one with lower **hs** will be more stable (have greater **F**), than its analogous and thus, solving (1) for **hs** will produce the greatest possible **hs** with which a given dome will remain geometrically stable up to the desired level **F**.

Example 8: Evaluation of (1) within a range of values.

We can keep all parameters identical to **the parameters of a given building**, solving equation (1) different times for a chosen parameter (changed defined parameter), but each time giving (forcing) a specific parameter within a range, as an input to the equation. This allows for comparisons between analogous buildings. This is equivalent to not knowing two parameters, feeding (1) with a range of values of one parameter, and solving for the other unknown, to devise graphs and patterns of how one parameter induces another.

References

- [1] López, M. González, M. Llaurodo, N. 2019. "EQUATIONS THAT DESCRIBE THE GEOMETRY OF SUPERADOBE DOMED STRUCTURES." engrXiv. March 14. doi:10.31224/osf.io/ptq47.