SCIENCE CHINA
Information Sciences

• **LETTER** •

Eigenvalue-based distributed target detection in compound-Gaussian clutter

Weijian LIU[1], Yuntao WU[2,3*], Qing JIANG[2,3*], Jun LIU[4], Shuwen XU[5] & Pengcheng GONG[2,3]

[1]*Air Force Early Warning Academy, Wuhan* 430019*, China;*
[2]*School of Computer Science and Engineering, Wuhan Institute of Technology, Wuhan,* 430205*, China;*
[3]*Hubei Key Laboratory of Intelligent Robot, Wuhan Institute of Technology, Wuhan,* 430205*, China;*
[4]*Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei,* 230027*, China;*
[5]*National Laboratory of Radar Signal Processing, Xidian University, Xi'an,* 710071*, China*

Target detection in Gaussian or compound-Gaussian clutter emerges as a particularly pertinent issue for a radar system [1, 2]. When a radar system employs pulsed Doppler processing to identify potential targets amidst such clutter, the target components often undergo fluctuations, which subsequently allows them to be embedded within the covariance matrix of the data under examination. In a recent study [3], the authors have devised innovative detectors for targets within compound-Gaussian clutter. These detectors leverage the eigenvalues of the covariance matrix, demonstrating superior detection performance compared to existing methods.

However, the detectors presented in [3] exhibit two notable limitations. Firstly, they are created without the capability to suppress clutter, making them vulnerable to intense clutter. Secondly, these detectors are tailored for point targets, whereas modern radar systems, with wide bandwidth, often encounter targets spanning multiple range bins, necessitating a different approach. To address these issues, we introduce eigenvalue-based detectors specifically designed for detecting distributed targets amidst compound-Gaussian clutter. These detectors incorporate a clutter suppression mechanism, enabling them to enhance detection performance. The effectiveness of our proposed detectors has been validated using both simulated and real-world data.

Problem formulation. For a distributed target, if present, occupying K range cells, the test data gathered by a radar system over a coherent processing interval (CPI) can be represented by $N \times 1$ column vectors $\mathbf{z}_l, l = 1, \ldots, K$, with N being the number of pulses in a CPI. $\mathbf{z}_l$ typically contains clutter $\mathbf{c}_l$ and noise $\mathbf{n}_l$. The covariance matrix of the clutter and noise is unknown. Estimation of this covariance matrix necessitates the use of training data, which are commonly acquired adjacent to the test data. We assume that the secondary data $\mathbf{z}_l$ s, $l = K+1, \ldots, K+L$, with L being the number of secondary data, share the same covariance matrix structure with the test data. Under hypothesis H_0, all the data only contain noise and clutter. Conversely, under hypothesis H_1, the test data also encompass signal component. Consequently, the detection problem can be formulated as

$$\begin{cases} \mathrm{H}_0 : \mathbf{z}_l = \mathbf{c}_l + \mathbf{n}_l, l = 1, \ldots, K+L \\ \mathrm{H}_1 : \begin{cases} \mathbf{z}_l = \mathbf{s}_l + \mathbf{c}_l + \mathbf{n}_l, l = 1, \ldots, K \\ \mathbf{z}_l = \mathbf{c}_l + \mathbf{n}_l, l = K+1, \ldots, K+L \end{cases} \end{cases} \tag{1}$$

where the signal has the form $\mathbf{s}_l = \beta_l \mathbf{p}$, $l = 1, \ldots, K$, β_l is the unknown target amplitude, $\mathbf{p} = \left[1, e^{-j2\pi f_d}, \ldots, e^{-j2\pi f_d (N-1)}\right]^T$ denotes the signal steering vector, and f_d is the normalized target Doppler frequency. The clutter $\mathbf{c}_l$ is modeled as a spherically invariant random vector (SIRV), described as $\mathbf{c}_l = \sqrt{\tau_l}\boldsymbol{\eta}_l, l = 1, \ldots, K+L$, where $\mathbf{c}_l$ is expressed as the product of the square root of the slowly varying texture τ_l and the quickly varying speckle η_l. Here, τ_l is a nonnegative real random variable, representing the local power of the clutter in the lth range cell. The texture τ_l is considered to be unknown and deterministic since the statistics of the texture are difficult to obtain in practice. The speckle $\boldsymbol{\eta}_l$ is characterized as an independent, zero-mean, complex circular Gaussian random vector with an $N \times N$ covariance matrix $\mathbf{R}$.

Detector design. To design effective detectors, we need the sample covariance matrix (SCM), given as $\hat{\mathbf{R}} = \frac{1}{L}\sum_{l=K+1}^{K+L} \mathbf{z}_l \mathbf{z}_l^H$. Then we whiten the data as $\tilde{\mathbf{z}}_l = \hat{\mathbf{R}}^{-\frac{1}{2}} \mathbf{z}_l$, resulting in the whitened test data $(l = 1, \ldots, K)$ and whitened training data $(l = K+1, \ldots, K+L)$. The data whitening process has the function of clutter suppression and hence enhances detection performance.

Note that if we whiten the data with the actual covariance matrix $\mathbf{R}$, resulting in the whitened data $\overline{\mathbf{z}}_l = \mathbf{R}^{-\frac{1}{2}} \mathbf{z}_l$, then the covariance matrix of $\overline{\mathbf{z}}_l$ can be written as

$$\mathbf{R}_{\overline{\mathbf{z}}_l} = \mathrm{E}\left[\overline{\mathbf{z}}_l \overline{\mathbf{z}}_l^H\right] = \begin{bmatrix} r_{0,l} & \cdots & r^*_{N-1,r} \\ \vdots & \ldots & \vdots \\ r_{N-1,r} & \cdots & r_{0,r} \end{bmatrix}, \tag{2}$$

where $r_{m,l}$ is the correlation coefficient, expressed as

$$r_{m,l} = \mathrm{E}\left[\bar{z}_{q,l} \bar{z}^*_{q+m,l}\right], 0 \leqslant m \leqslant N-1, 0 \leqslant q \leqslant N-m-1. \tag{3}$$

* Corresponding author (email: ytwu@sina.com, qqzjiang@163.com)
This paper is accepted by Science China: Information Sciences.

In (3), $\bar{z}_{m,l}$ is the mth element of $\bar{\mathbf{z}}_l$. Moreover, the data in the lth range cell $\mathbf{z}_l$ can be represented by $\mathbf{z}_l = \left[z_{0,l}, \ldots, z_{N-1,l}\right]^T$, $l = 1, \ldots, K+L$.

The matrix $\mathbf{R}_{\bar{z}_l}$ in (2) is unknown in practice. It is assumed that the data received by the radar are wide-sense stationary (WSS). It follows that $\mathbf{R}_{\bar{z}_l}$ has a Toeplitz Hermitian positive-definite (HPD) structure. Hence, to estimate $\mathbf{R}_{\bar{z}_l}$, we use the whitened data in all the range cells. The ergodicity of the stationary Gaussian process permits us to estimate the correlation coefficient $r_{m,l}$ by averaging the whitened data, namely,

$$\widehat{r}_{m,l} = \frac{1}{N} \sum_{q=0}^{N-1-m} \widetilde{z}_{q,l} \widetilde{z}^*_{q+m,l}, 0 \leqslant m \leqslant N-1, l = 1, \ldots, K+L, \tag{4}$$

where $\widetilde{z}_{q,l}$ is the qth element of $\widetilde{\mathbf{z}}_l$. Moreover, the estimated whitened covariance matrix is denoted as $\widehat{\mathbf{R}}_{\widetilde{\mathbf{z}}_l}$.

Eigenvalues play a pivotal role in extracting information about potential targets from the test data. The maximum eigenvalue of the test data, signifies the most significant information pertaining to the potential target [4]. Adopting the maximum, minimum, harmonic mean (HM), arithmetic mean (AM), and geometric mean (GM) of eigenvalues of an HPD matrix, we propose the following eigenvalue-based detector for distributed targets (EDDT)

$$t_{\text{EDDT}} = \frac{\frac{1}{K}\sum_{l=1}^{K} \widetilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L} \left[\frac{1}{N}\sum_{n=1}^{N} \widetilde{\lambda}^a_{n,l}\right]^{1/a}} \Lambda \tag{5}$$

where $\widetilde{\lambda}_{\max,l} = \max\{\text{eig}(\widehat{\mathbf{R}}_{\widetilde{\mathbf{z}}_l})\}$, $l = 1, \cdots, K$, $\widetilde{\lambda}_{n,l}$ is the nth eigenvalue of $\widehat{\mathbf{R}}_{\widetilde{\mathbf{z}}_l}$, $l = 1, \cdots, K+L$, Λ is the detection threshold, and a is a tunable parameter. We can adjust the power parameter a to obtain different kinds of EDDT. As $a = -1, 1, 0, -\infty, \infty$, the EDDT in (5) can be recast as

$$t_{\text{EDDT}} = \begin{cases} \frac{\frac{1}{K}\sum_{l=1}^{K} \tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L} \left[N/\sum_{n=1}^{N}\left(1/\tilde{\lambda}_{n,l}\right)\right]}, & a=-1 \\ \frac{\frac{1}{K}\sum_{l=1}^{K} \tilde{\lambda}_{\max,l}}{\frac{1}{LN}\sum_{l=K+1}^{K+L}\sum_{n=1}^{N} \tilde{\lambda}_{n,l}}, & a=1 \\ \frac{\frac{1}{K}\sum_{l=1}^{K} \tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L}\left(\prod_{n=1}^{N} \tilde{\lambda}_{n,l}\right)^{1/N}}, & a=0 \\ \frac{\frac{1}{K}\sum_{l=1}^{K} \tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L} \tilde{\lambda}_{\min,l}}, & a=-\infty \\ \frac{\frac{1}{K}\sum_{l=1}^{K} \tilde{\lambda}_{\max,l}}{\frac{1}{L}\sum_{l=K+1}^{K+L} \tilde{\lambda}_{\max,l}}, & a=\infty \end{cases} \tag{6}$$

where $\widetilde{\lambda}_{\min,l} = \min\{\text{eig}(\widehat{\mathbf{R}}_{\widetilde{\mathbf{z}}_l})\}$. For convenience, the proposed EDDT in (5) or (6) with $a = -1, 1, 0, -\infty, \infty$ is referred to as maximum eigenvalue to harmonic mean detector (MHM-D), maximum eigenvalue to arithmetic mean detector (MAM-D), maximum eigenvalue to geometric mean detector (MGM-D), maximum eigenvalue to minimum eigenvalue detector (MME-D), and maximum eigenvalue to maximum eigenvalue detector (MEM-D), respectively.

Experiments. To evaluate the detection performance of the detectors, we compare the probabilities of detection (PDs) of the proposed detector with distributed target version of the MGM detector in [3], which is shown to have the best detection performance therein, referred to as the MGM detector with no whitening (MGM-D-nW) for convenience. Precisely, the MGM-D-nW can be obtain from the MGM-D in (8) in [3] when the numerator $\lambda_{\max,\text{cut}}$ is replaced by $\frac{1}{K}\sum_{l=1}^{K} \lambda_{\max,\text{cut},l}$, with $\lambda_{\max,\text{cut},l}$ being the maximum eigenvalue of the estimated covariance matrix for the lth test data, $l = 1, ..., K$. For the simulation results, the covariance matrix of the speckle component has the structure of $\mathbf{R} = \mathbf{R}_0 + p_0\mathbf{I}_N$, where $\mathbf{R}_0$ represents the clutter covariance matrix, p_0 represents the thermal noise power, and the (b,n)th element of $\mathbf{R}_0$ is $\mathbf{R}_0(b,n) = \sigma_c^2 \rho^{|b-n|}$, where $b, n = 1, ..., N$ and $\rho = 0.9$. We set probability of false alarm (PFA) as 10^{-3}. The texture component of the compound-Gaussian clutter follows an inverse gamma distribution, with a shape parameter of $\nu = 0.9$ and a scale parameter of $\mu = 1.3$. For the real data, the dataset TFA17_014.03.mat is used [5]. The results in Figure 1 show that the new detector MHM-D has the highest PD, and when the PD is 0.8, the performance improvement of the MHM-D in terms of output signal-to-clutter ratio (SCR) exceeds 5dB compared to the MGM-D-nW.

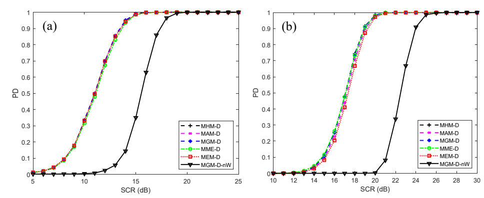

Figure 1 PDs of the detectors under different SCRs. (a) Simulated data; (b) Real data

Conclusions. The EDDT first implement a whitening operation to the test data to reduce the impact of the clutter. Then, tt performs eigenvalue decomposition to the whitened data to effectively utilized the energy of potential target embedded in the clutter. Finally, it uses these eigenvalues to form effective detectors. Numerical results with simulated and real data showed that the proposed detectors outperform the existing ones.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 62071482, 62471485 and 62471450) and the Natural Science Foundation of Anhui Province (Grant No. 2208085J17).

Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

References

1 W. Liu, J. Liu, C. Hao, Y. Gao, and Y.-L. Wang, "Multichannel adaptive signal detection: Basic theory and literature review," *Science China: Information Sciences*, vol. 65, no. 2, p. 121301, 2022.

2 T. Jian, J. He, Y. Liu, Y. He, C. Xu, and Z. Xie, "Persymmetric adaptive detection of range-spread targets in subspace interference plus Gaussian clutter," *Science China: Information Sciences*, vol. 66, p. 152306, 2023.

3 W. Zhao, M. Jin, G. Cui, and Y. Wang, "Eigenvalues-based detector design for radar small floating target detection in sea clutter," *IEEE Geoscience and Remote Sensing Letters*, vol. 19, p. 3509105, 2022.

4 W. Zhao, C. Liu, W. Liu, and M. Jin, "Maximum eigenvalue-based target detection for the K-distributed clutter environment," *IET Radar, Sonar and Navigation*, vol. 12, no. 11, pp. 1294–1306, 2018.

5 P. L. Herselman and C. J. Baker, "Analysis of calibrated sea clutter and boat reflectivity data at C-and X-band in South African coastal waters," in *2007 IET International Conference on Radar Systems*, 2007.

[1]Weijian LIU

SCIENCE CHINA
Information Sciences

• Supplementary File •

Eigenvalue-based distributed target detection in compound-Gaussian clutter

Yuntao WU[2,3*], Qing JIANG[2,3*], Jun LIU[4], Shuwen XU[5] & Pengcheng GONG[2,3]

[1]*Air Force Early Warning Academy, Wuhan* 430019*, China;*
[2]*School of Computer Science and Engineering, Wuhan Institute of Technology, Wuhan,* 430205*, China;*
[3]*Hubei Key Laboratory of Intelligent Robot, Wuhan Institute of Technology, Wuhan,* 430205*, China;*
[4]*Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei,* 230027*, China;*
[5]*National Laboratory of Radar Signal Processing, Xidian University, Xi'an,* 710071*, China*

1 More information

In this appendix, the evaluation of the detection performance of the detectors proposed in this letter under more parameter settings is presented to reflect the effectiveness of the proposed methods in a more comprehensive manner.

Figure A1 shows the variations of the PFAs of the detectors proposed in this letter and the MGM-D-nW proposed in [3] under different clutter correlation characteristics (i.e., one-lag correlation coefficients). It can be seen from the figure that the PFAs of the proposed detectors in this letter do not change with the variations of the clutter correlation characteristics, while for the MGM-D-nW, its PFA increases as the clutter correlation parameters increase. The reason is that the detectors proposed in this letter contain a data whitening process, which realizes clutter suppression and thus enables the detectors to have the constant false alarm rate (CFAR) characteristic.

The detection performance of adaptive detectors with clutter suppression capabilities is affected by the output signal-to-clutter ratio (SCR) [1]. The detectors proposed in this letter also belongs to the category of adaptive detectors and has clutter suppression capabilities. The essence of the superior detection performance of the proposed detectors for highly correlated clutter is that the proposed detectors can improve the output SCR. This factor is confirmed in Figure A2, which shows the input and output SCRs under different one-lag correlation coefficients. The input and output SCRs are defined as

$$\mathrm{SCR}_{\mathrm{in}} = \frac{1}{N}\frac{\sum\limits_{l=1}^{K}|\beta_l|^2\mathbf{p}^H\mathbf{p}}{\mathrm{tr}(\mathbf{R})} \tag{1}$$

and

$$\mathrm{SCR}_{\mathrm{out}}=\frac{1}{N}\sum_{l=1}^{K}|\beta_l|^2\mathbf{p}^H\mathbf{R}^{-1}\mathbf{p} \tag{2}$$

respectively. The results in Figure A2 show that the output SCR is significantly higher than the input SCR, especially when the clutter correlation characteristics are relatively strong. In other words, the proposed detectors can suppress the clutter quite effectively for highly correlated clutter, thus improving the detection performance.

Figure A3 presents the PDs of the detectors proposed in this letter and the detector MGM-D-nW in [3] under different PFAs. This type of figure is usually called the receiver operating characteristic (ROC) curve. It can be seen from the figure that the PDs of the detectors proposed in this letter are obviously higher than that of the MGM-D-nW. This is because the proposed detectors have the function

* Corresponding author (email: ytwu@sina.com, qqzjiang@163.com)

of clutter suppression, which realizes the improvement of the target detection performance in the clutter environment.

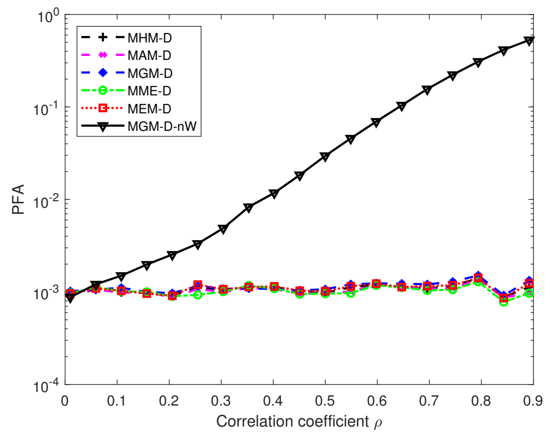

Figure A1 The PFAs of the detectors under different clutter correlation characteristics. $N = 8$, $K = 4$, $L = 16$, $\nu = 0.9$, $\mu = 1.3$, and $f_d = 0.05$.

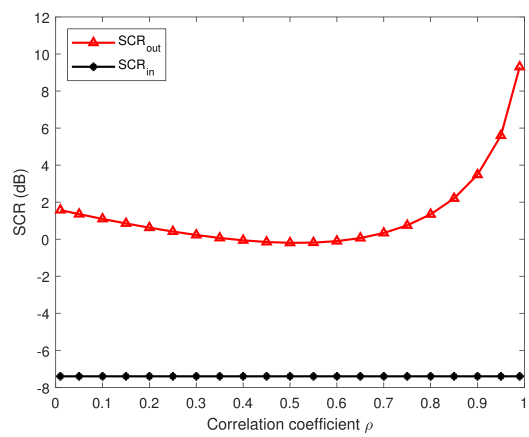

Figure A2 The input and output SCRs. $N = 8$, $K = 4$, $\beta_k = 2$, $k = 1, \ldots, K$, and $f_d = 0.05$.

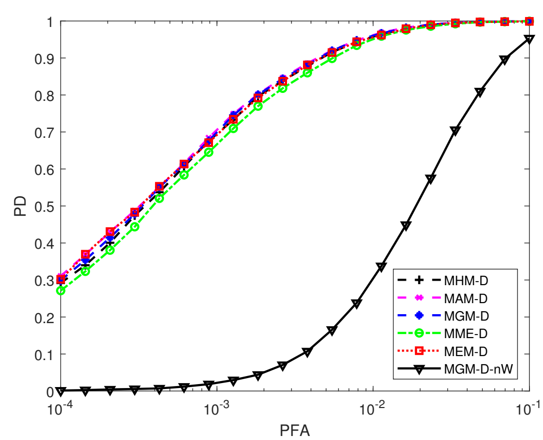

Figure A3 The ROC curves of the detectors. $N = 8$, $K = 4$, $\rho = 0.9$, $L = 16$, $\nu = 0.9$, $\mu = 1.3$, $\mathrm{SCR}_{\mathrm{out}} = 12\mathrm{dB}$, and $f_d = 0.05$.