

Development of Flight Envelope of a Twin Turboprop Aircraft (ATR-72) at ISL+20°C

Introduction and Problem Statement

The ATR 72 is a twin-engine turboprop, short-haul regional airliner developed and produced in France and Italy by aircraft manufacturer ATR (Aerei da Trasporto Regionale or Avions de transport régional), a joint venture formed by French aerospace company Aérospatiale (now Airbus) and Italian aviation conglomerate Aeritalia (now Leonardo S.p.A.). The number "72" in its name is derived from the aircraft's standard seating configuration in a passenger-carrying configuration, which could seat 72–78 passengers in a single-class arrangement.

There are two possible level flight speeds $V_{\min}$ and $V_{\max}$ for full throttle setting at different altitudes considering drag (D) or thrust available (TR) and thrust available (TA) curves.

Using this data ($V_{\min}$ and $V_{\max}$), aircraft flight envelope (speed vs altitude) for turbojet powered aircraft was drawn. For jet powered aircraft TA is nearly constant with speed and it was convenient to consider TR and TA curves.

For piston engine aircraft, the engine power is nearly constant with forward speed and we consider PR and PA curves to get two possible level flight speeds $V_{\min}$ and $V_{\max}$, at different altitudes.

Using $PR = \{AV^3 + B/V\}/\eta_P$, $P_A = P_{ASL} \times \sigma$ and level flight condition $P_R = P_A$ it is possible to get $V_{\min}$ and $V_{\max}$ at different altitude and draw the flight envelope. (η_P - propeller efficiency).

PROBLEM:

Draw Flight Envelope in ISA+ 20°C for a turboprop aircraft ATR72' for the following cases:

- a) $W = W_{TO}$ and engine set at i) full throttle and ii) 80% throttle
- b) $W = 0.85 W_{TO}$ and engine set at i) full throttle and ii) 80% throttle
- c) $W = 0.65 W_{TO}$ and engine set at i) full throttle and ii) 80% throttle
- d) Show in the above flight envelope plots the stall boundary for above 3 cases of aircraft weight
- e) In the speed (V) vs altitude (h) domain showing flight envelope, draw constant Mach number lines for Mach numbers ranging from 0.2 to 0.8 in steps of 0.1

Use aircraft and engine data given below for your calculations on flight envelope

Maximum takeoff mass of the aircraft $m_{TO} = 23 \text{ T}$

Wing area = 61 m²

Sea level max shaft power per engine = $P_{S-SL} = 2050.7 \text{ kW}$

No of Engines = 2

Propulsive efficiency = 0.85

Variation of shaft power with altitude = $P_S = P_{S-SL}\sigma$

Drag characteristics = $C_D = 0.0165 + 0.0313C_{L2}$

Maximum lift coefficient in flight = $C_{L\max} = 1.2$

Formulae Used

$$P_A = P_{ASL} \times \sigma$$

$$P_R = \{AV^3 + B/V\}/\eta_P$$

$$P_R = P_A$$

SOLUTION:

Flight Envelope: It is the plot of $V_{\min}$ and $V_{\max}$ for a given weight and given engine conditions.

To find the values of Sigma (density ratios) for altitude ranging from 0 to absolute ceiling for ISA+20

To do so I used Simulink Block called ISA Atmosphere Model.

Temperature values are changed parameter to 278+20 and gave input of $H=0:100:20000$.

Subsequently density is multiplied by a factor of 1/1.225 to find sigma.

Later the entire array to is transferred on MATLAB workspace.

TABULATION AND GRAPHS

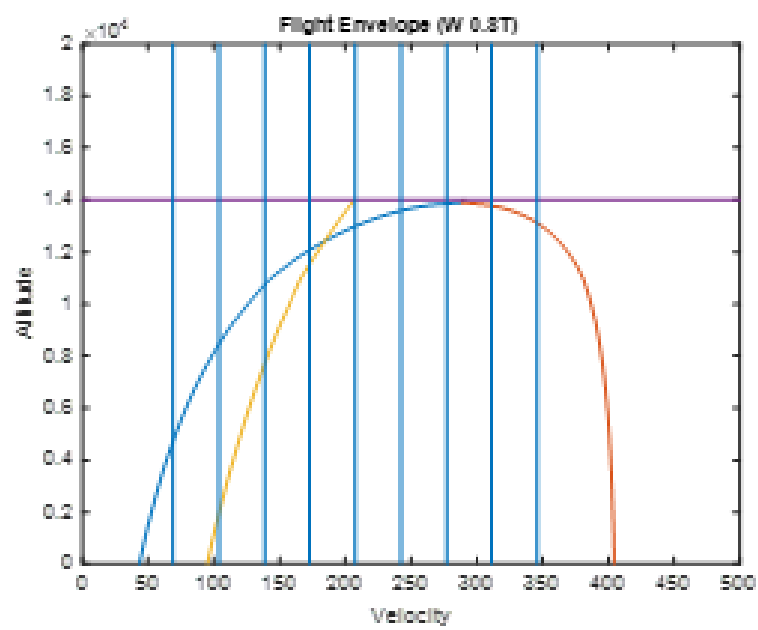
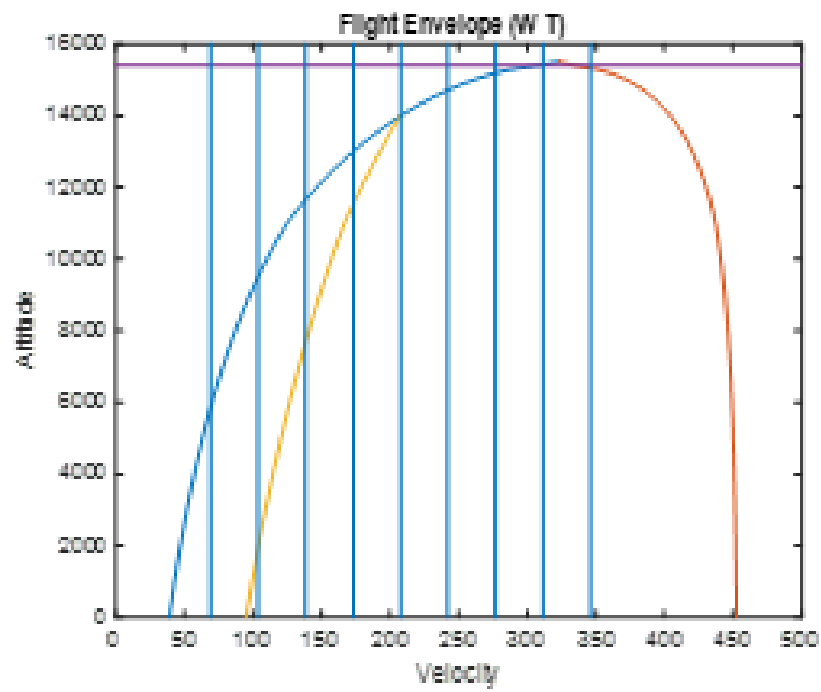
A) $W=W_{TO}$ ENGINE SET AT

Full throttle

Altitude (m)	Temperature(K)	Density(Rho)	Density Ratio	Power Required	V min	V max
	288	1.225				
0	308	1.448877474	1	4101400	9.05915	165.335
1000	301.5	1.373640733	0.948072392	3888424.109		
2000	295	1.300798146	0.897797205	3682225.455		
3000	288.5	1.230323765	0.849156528	3482730.584		
4000	282	1.162191354	0.802132254	3289865.226		
5000	275.5	1.096374377	0.756706069	3103554.269	15.8311	162.819
6000	269	1.032845994	0.712859446	2923721.733		
7000	262.5	0.971579043	0.670573641	2750290.732		
8000	256	0.912546035	0.62982968	2583183.448		
9000	249.5	0.855719136	0.590608351	2422321.093		
10000	243	0.801070158	0.552890201	2267623.87	29.7942	157.137
11000	236.5	0.748570541	0.516655517	2119010.939		
12000	230	0.698191343	0.481884324	1976400.368		
13000	223.5	0.649903219	0.448556369	1839709.092		
14000	217	0.603676408	0.41665111	1708852.864		
15000	210.5	0.559480713	0.386147706	1583746.202	64.3239	139.135
16000	204	0.51728548	0.357025	1464302.334	79.3459	129.037

80% throttle

Altitude (m)	Temperature(K)	Density(Rho)	Density Ratio	Power Required	V min	V max
	288	1.225				
0	308	1.448877474	1	3281120	11.3625	152.428
1000	301.5	1.373640733	0.948072392	3110739.287		
2000	295	1.300798146	0.897797205	2945780.364		
3000	288.5	1.230323765	0.849156528	2786184.467		
4000	282	1.162191354	0.802132254	2631892.181		
5000	275.5	1.096374377	0.756706069	2482843.415	19.8127	149.144
6000	269	1.032845994	0.712859446	2338977.386		
7000	262.5	0.971579043	0.670573641	2200232.586		
8000	256	0.912546035	0.62982968	2066546.759		
9000	249.5	0.855719136	0.590608351	1937856.874		
10000	243	0.801070158	0.552890201	1814099.096	37.5568	141.328
11000	236.5	0.748570541	0.516655517	1695208.751		
12000	230	0.698191343	0.481884324	1581120.294		
13000	223.5	0.649903219	0.448556369	1471767.273	59.5606	129.258
14000	217	0.603676408	0.41665111	1367082.291		
15000	210.5	0.559480713	0.386147706	1266996.962		
16000	204	0.51728548	0.357025	1171441.867		
17000	197.5	0.477059577	0.329261505	1080346.509		
18000	191	0.438771376	0.30283539	993639.2563		
19000	184.5	0.402388718	0.277724463	911247.2888		
20000	178	0.367878896	0.253906147	833096.5361		



B)

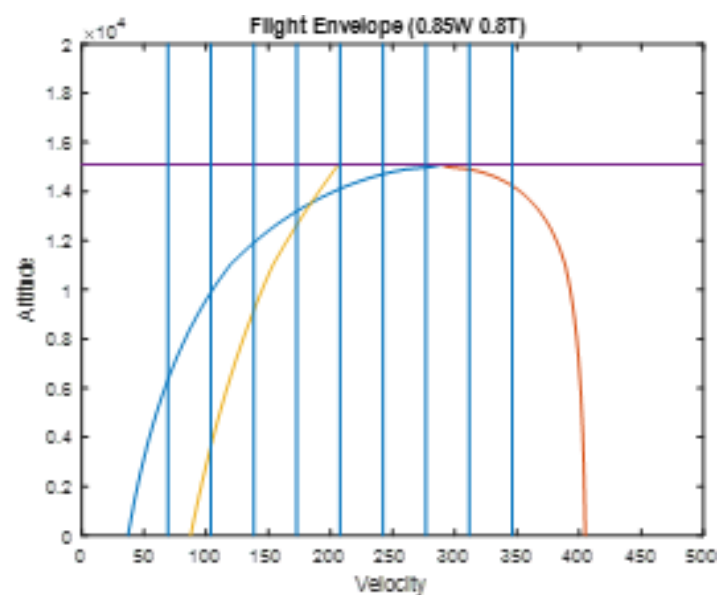
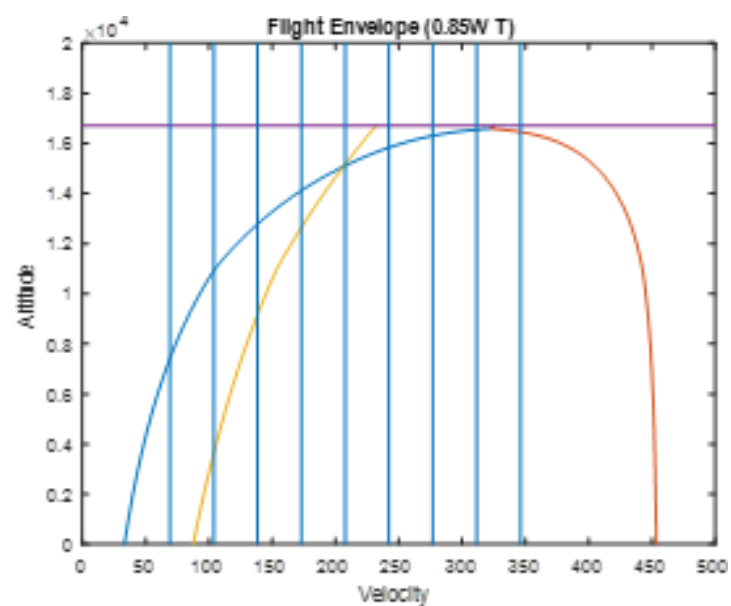
$$W=0.85W_{TO}$$

Full throttle

Altitude (m)	Temperature(K)	Density(Rho)	Density Ratio	Power Required	V min	V max
	288	1.225				
0	308	1.448877474	1	4101400	6.5442	166.229
1000	301.5	1.373640733	0.948072392	3888424.109		
2000	295	1.300798146	0.897797205	3682225.455		
3000	288.5	1.230323765	0.849156528	3482730.584		
4000	282	1.162191354	0.802132254	3289865.226		
5000	275.5	1.096374377	0.756706069	3103554.269	11.4318	164.465
6000	269	1.032845994	0.712859446	2923721.733		
7000	262.5	0.971579043	0.670573641	2750290.732		
8000	256	0.912546035	0.62982968	2583183.448		
9000	249.5	0.855719136	0.590608351	2422321.093		
10000	243	0.801070158	0.552890201	2267623.87	21.4515	160.628
11000	236.5	0.748570541	0.516655517	2119010.939		
12000	230	0.698191343	0.481884324	1976400.368		
13000	223.5	0.649903219	0.448556369	1839709.092		
14000	217	0.603676408	0.41665111	1708852.864		
15000	210.5	0.559480713	0.386147706	1583746.202	44.7239	150.117
16000	204	0.51728548	0.357025	1464302.334	52.9872	145.773

80% throttle

Altitude (m)	Temperature(K)	Density(Rho)	Density Ratio	Power Required	V min	V max
	288	1.225				
0	308	1.448877474	1	3281120	8.18095	153.566
1000	301.5	1.373640733	0.948072392	3110739.287		
2000	295	1.300798146	0.897797205	2945780.364		
3000	288.5	1.230323765	0.849156528	2786184.467		
4000	282	1.162191354	0.802132254	2631892.181		
5000	275.5	1.096374377	0.756706069	2482843.415	14.2963	151.3
6000	269	1.032845994	0.712859446	2338977.386		
7000	262.5	0.971579043	0.670573641	2200232.586		
8000	256	0.912546035	0.62982968	2066546.759		
9000	249.5	0.855719136	0.590608351	1937856.874		
10000	243	0.801070158	0.552890201	1814099.096	26.8958	146.204
11000	236.5	0.748570541	0.516655517	1695208.751		
12000	230	0.698191343	0.481884324	1581120.294		
13000	223.5	0.649903219	0.448556369	1471767.273		
14000	217	0.603676408	0.41665111	1367082.291		
15000	210.5	0.559480713	0.386147706	1266996.962	57.7714	130.352
16000	204	0.51728548	0.357025	1171441.867		
17000	197.5	0.477059577	0.329261505	1080346.509		
18000	191	0.438771376	0.30283539	993639.2563		
19000	184.5	0.402388718	0.277724463	911247.2888		
20000	178	0.367878896	0.253906147	833096.5361		



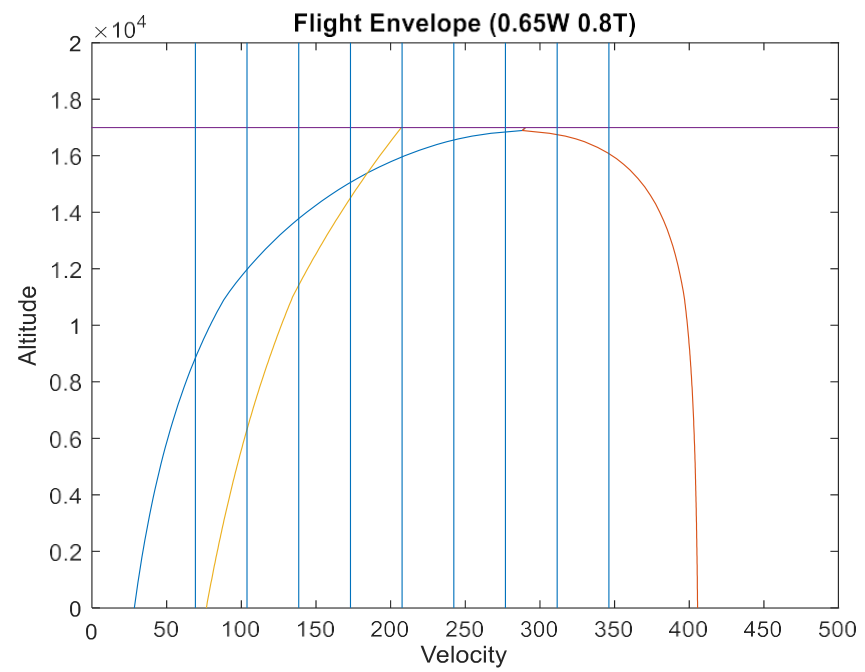
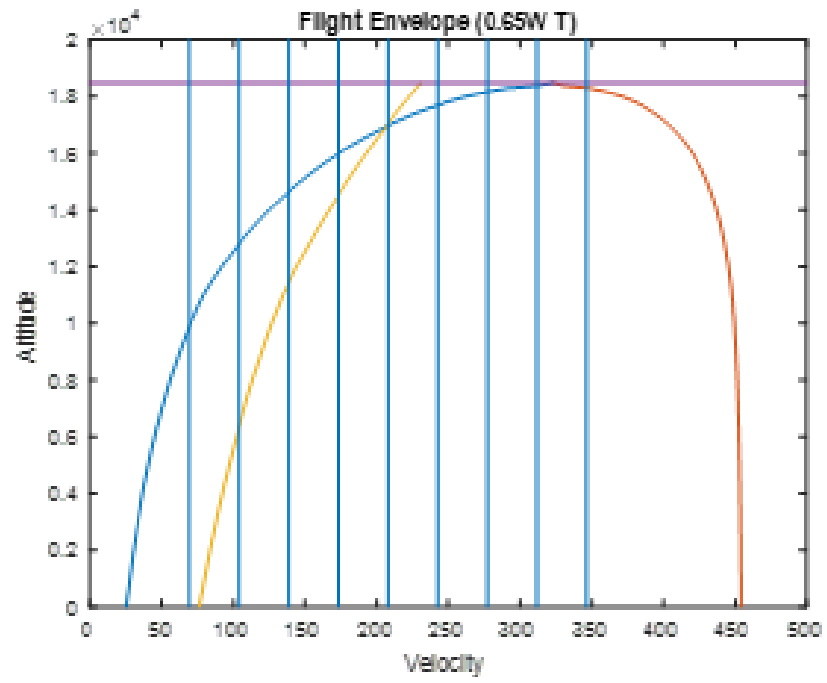
C) $W=0.65W_{TO}$

Full throttle

Altitude (m)	Temperature(K)	Density(Rho)	Density Ratio	Power Required	V min	V max
	288	1.225				
0	308	1.448877474	1	4101400	3.82686	167.174
1000	301.5	1.373640733	0.948072392	3888424.109		
2000	295	1.300798146	0.897797205	3682225.455		
3000	288.5	1.230323765	0.849156528	3482730.584		
4000	282	1.162191354	0.802132254	3289865.226		
5000	275.5	1.096374377	0.756706069	3103554.269	6.68346	166.171
6000	269	1.032845994	0.712859446	2923721.733		
7000	262.5	0.971579043	0.670573641	2750290.732		
8000	256	0.912546035	0.62982968	2583183.448		
9000	249.5	0.855719136	0.590608351	2422321.093		
10000	243	0.801070158	0.552890201	2267623.87	12.5236	164.073
11000	236.5	0.748570541	0.516655517	2119010.939		
12000	230	0.698191343	0.481884324	1976400.368		
13000	223.5	0.649903219	0.448556369	1839709.092		
14000	217	0.603676408	0.41665111	1708852.864		
15000	210.5	0.559480713	0.386147706	1583746.202	25.7557	158.848
16000	204	0.51728548	0.357025	1464302.334		
17000	197.5	0.477059577	0.329261505	1350433.136		
18000	191	0.438771376	0.30283539	1242049.07	42.4032	151.288
19000	184.5	0.402388718	0.277724463	1139059.111		

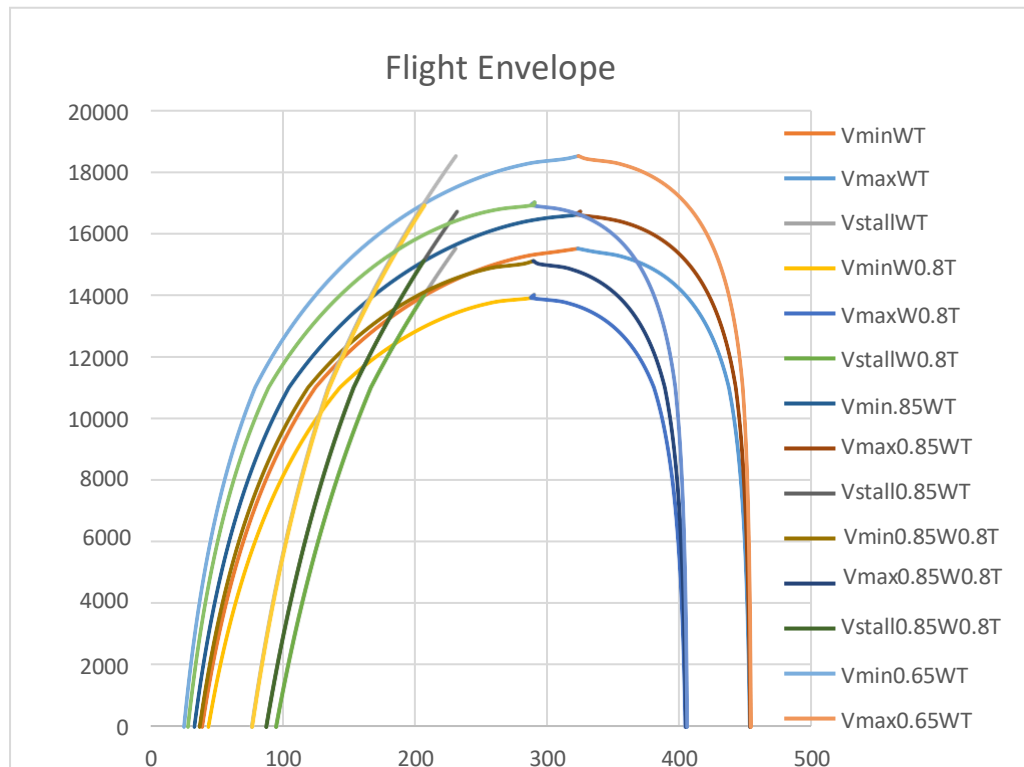
80% Throttle

Pressure	Speed of Sound	V at Mach 0.2	V at Mach 0.3	V at Mach 0.4	V at Mach 0.5	V at Mach 0.6	V at Mach 0.7	V at Mach 0.8
128074.9732	351.7874358	70.35748716	105.5362307	140.7149743	175.8937179	211.0724615	246.2512051	281.4299487
118861.8194	348.055599	69.6111198	104.4166797	139.2222396	174.0277995	208.8333594	243.6389193	278.4444792
110132.075	344.2833136	68.85666271	103.2849941	137.7133254	172.1416568	206.5699881	240.9983195	275.4266509
101870.1926	340.469235	68.09384701	102.1407705	136.187694	170.2346175	204.281541	238.3284645	272.375388
94060.79502	336.6119427	67.32238855	100.9835828	134.6447771	168.3059714	201.9671656	235.6283599	269.2895542
86688.67744	332.7099337	66.54198675	99.81298012	133.0839735	166.3549669	199.6259602	232.8969536	266.167947
79738.80926	328.7616158	65.75232315	98.62848473	131.5046463	164.3808079	197.2569695	230.133131	263.0092926
73196.33618	324.7652999	64.95305997	97.42958996	129.9061199	162.3826499	194.8591799	227.3357099	259.8122399
67046.58231	320.7191918	64.14383836	96.21575755	128.2876767	160.3595959	192.4315151	224.5034343	256.5753535
61275.05234	316.6213827	63.32427655	94.98641482	126.6485531	158.3106914	189.9728296	221.6349679	253.2971062
55867.43387	312.4698385	62.49396771	93.74095156	124.9879354	156.2349193	187.4819031	218.728887	249.9758708
50809.59975	308.2623882	61.65247765	92.47871647	123.3049553	154.1311941	184.9574329	215.7836718	246.6099106
46087.61052	303.9967105	60.7993421	91.19901315	121.5986842	151.9983553	182.3980263	212.7976974	243.1973684
41687.71702	299.6703189	59.93406377	89.90109566	119.8681275	149.8351594	179.8021913	209.7692232	239.7362551
37596.36303	295.2805446	59.05610891	88.58416337	118.1122178	147.6402723	177.1683267	206.6963812	236.2244357
33800.18806	290.8245175	58.16490351	87.24735526	116.329807	145.4122588	174.4947105	203.5771623	232.659614
30286.03027	286.2991443	57.25982885	85.88974328	114.5196577	143.1495721	171.7794866	200.409401	229.0393154
27040.9295	281.7010827	56.34021654	84.51032481	112.6804331	140.8505414	169.0206496	197.1907579	225.3608662
24052.13049	277.0267135	55.4053427	83.10801405	110.8106854	138.5133568	166.2160281	193.9186995	221.6213708
21307.0862	272.2721065	54.45442131	81.68163196	108.9088426	136.1360533	163.3632639	190.5904746	217.8176852
18793.46131	267.4329823	53.48659645	80.22989468	106.9731929	133.7164911	160.4597894	187.2030876	213.9463858



E) Mach numbers

Altitude	Pressure	Speed of Sound	V at Mach 0.2	V at Mach 0.3	V at Mach 0.4	V at Mach 0.5	V at Mach 0.6	V at Mach 0.7	V at Mach 0.8
0	128074.9732	351.7874358	70.35748716	105.5362307	140.7149743	175.8937179	211.0724615	246.2512051	281.4299487
1000	118861.8194	348.055599	69.61111198	104.4166797	139.2222396	174.0277995	208.8333594	243.6389193	278.4444792
2000	110132.075	344.2833136	68.85666271	103.2849941	137.7133254	172.1416568	206.5699881	240.9983195	275.4266509
3000	101870.1926	340.469235	68.09384701	102.1407705	136.187694	170.2346175	204.281541	238.3284645	272.375388
4000	94060.79502	336.6119427	67.32238855	100.9835828	134.6447771	168.3059714	201.9671656	235.6283599	269.2895542
5000	86688.67744	332.7099337	66.54198675	99.81298012	133.0839735	166.3549669	199.6259602	232.8969536	266.167947
6000	79738.80926	328.7616158	65.75232315	98.62848473	131.5046463	164.3808079	197.2569695	230.133131	263.0092926
7000	73196.33618	324.7652999	64.95305997	97.42958996	129.9061199	162.3826499	194.8591799	227.3357099	259.8122399
8000	67046.58231	320.7191918	64.14383836	96.21575755	128.2876767	160.3595959	192.4315151	224.5034343	256.5753535
9000	61275.05234	316.6213827	63.32427655	94.98641482	126.6485531	158.3106914	189.9728296	221.6349679	253.2971062
10000	55867.43387	312.4698385	62.49396771	93.74095156	124.9879354	156.2349193	187.4819031	218.728887	249.9758708
11000	50809.59975	308.2623882	61.65247765	92.47871647	123.3049553	154.1311941	184.9574329	215.7836718	246.6099106
12000	46087.61052	303.9967105	60.7993421	91.19901315	121.5986842	151.9983553	182.3980263	212.7976974	243.1973684
13000	41687.71702	299.6703189	59.93406377	89.90109566	119.8681275	149.8351594	179.8021913	209.7692232	239.7362551
14000	37596.36303	295.2805446	59.05610891	88.58416337	118.1122178	147.6402723	177.1683267	206.6963812	236.2244357
15000	33800.18806	290.8245175	58.16490351	87.24735526	116.329807	145.4122588	174.4947105	203.5771623	232.659614
16000	30286.03027	286.2991443	57.25982885	85.88974328	114.5196577	143.1495721	171.7794866	200.409401	229.0393154
17000	27040.9295	281.7010827	56.34021654	84.51032481	112.6804331	140.8505414	169.0206496	197.1907579	225.3608662
18000	24052.13049	277.0267135	55.4053427	83.10801405	110.8106854	138.5133568	166.2160281	193.9186995	221.6213708
19000	21307.0862	272.2721065	54.45442131	81.68163196	108.9088426	136.1360533	163.3632639	190.5904746	217.8176852
20000	18793.46131	267.4329823	53.48659645	80.22989468	106.9731929	133.7164911	160.4597894	187.2030876	213.9463858



ASSUMPTIONS AND GENERAL CONSIDERATIONS

We get the sigma value from the international standardized atmosphere chart and hence get the various points.

In doing so we find the value of sigma for which $V_{\min}=V_{\max}$ and thus the corresponding value of altitude gives us the absolute ceiling.

The temperature also drops 6.5 degrees per 1000 m of height.

We used MATLAB to do the calculations by writing a code, and using the above given data, we got the values and tabulated them for proper values.

CONCLUSION

Hence we conclude that from the above set of values we can find the flight envelope of the turboprop dual engine ATR 72 flight.

We can infer the following points:

- With decrease in Weight for same Thrust the value of Absolute Ceiling Increases.
- $V_{\max}$ at Lower Velocities Depend majorly on Thrust and not on Weight.
- V_{stall} depends only on Weight.
- $V_{\min}$ tends to converge at lower velocities.
- With decrease in Thrust Absolute Ceiling Decreases.

References

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