Theoretical Investigation of Temperature-Dependent Resistivity in Metallic Conductors

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Abstract

The electrical resistivity of metallic conductors is a fundamental property that depends on temperature. This paper presents a theoretical investigation into the temperature-dependent resistivity of metallic conductors using both classical and quantum mechanical models. The derivation of resistivity as a function of temperature is provided, along with a discussion on electron-phonon interactions that contribute to scattering mechanisms. Furthermore, a simple experiment to measure resistivity variations with temperature is described. While classical models provide a general understanding, quantum mechanics further refines the predictions, particularly at low temperatures. This paper is written for high school and early university students, ensuring accessibility to fundamental concepts with a rigorous mathematical foundation.

Keywords: Electrical resistivity, temperature dependence, Drude model, free electron model, electron-phonon interaction, Bloch-Grüneisen theory

Introduction

Electrical resistivity is a fundamental property of materials that describes how strongly a material opposes the flow of electric current. It is denoted by the symbol ρ and is mathematically defined as:

$$\rho = R\frac{A}{L}$$

Where R is the electrical resistance of a conductor, A is its cross-sectional area, and L is its length. This equation shows that resistivity depends on both the material's intrinsic properties and its dimensions.

Resistivity is an essential concept in electrical and material sciences because it determines how efficiently a material conducts electricity. Materials with low resistivity, such as copper and silver, are excellent conductors, while materials with high resistivity, such as rubber and glass, are insulators. The resistivity of a material is not constant; it changes with temperature due to interactions between free-moving electrons and the atomic structure of the material.

Understanding Electrical Resistance

Resistance, denoted by R, is a measurable property that quantifies how much a material resists the flow of electric current. Ohm's law relates resistance to voltage V and current I as:

$$V = IR$$

where V is the applied voltage, I is the current flowing through the material, and R is the resistance. Resistance is dependent on the material's resistivity and its shape, making it crucial to distinguish between resistance and resistivity. While resistance can change based on the size and shape of a conductor, resistivity is an intrinsic property of the material itself.

Temperature Dependence of Resistivity

In metallic conductors, resistivity increases with temperature. This occurs because as temperature rises, the metal's atomic lattice vibrates more intensely, increasing the likelihood of collisions between free electrons and the lattice atoms. These collisions disrupt the smooth flow of electrons, causing increased resistance. This relationship can be approximately expressed as:

$$\rho(T) = \rho_0(1 + \alpha T)$$

where ρ_0 is the resistivity at a reference temperature (usually room temperature), T is the temperature in degrees Celsius or Kelvin, and α is the temperature coefficient of resistivity, a material-dependent constant. This equation is valid for moderate temperature ranges but requires more complex models at extremely high or low temperatures.

Microscopic View: Electron Movement and Scattering

In a metal, electrical conduction is carrier out by free electrons, which move through the atomic structure of the material. These electrons behave similarly to gas molecules in an enclosed space, bouncing around and colliding with atomic nuclei. This concept is modeled by the Drude Model, which treats electrons as classical particles moving through a lattice. When an electric field is applied, these electrons gain a drift velocity, allowing them to contribute to an electric current. However, as they move, they experience scattering, which disrupts their motion and contributes to electrical resistance. Scattering occurs due to two main mechanisms:

Electron-Ion scattering

Atoms in a metal are arranged in a regular lattice structure. As electrons travel, they collide with these positively charged atomic cores, causing their paths to deflect and lose energy.

Electron-Phonon Scattering

At non-zero temperatures, atoms in the lattice vibrate. These vibrations, known as **phonons**, interact with electrons and further disrupt their movement. As temperature increases, phonon activity intensifies, increasing the rate of electron scattering and, consequently, resistivity.

Why Study Temperature-Dependent Resistivity?

The temperature dependence of resistivity is a crucial factor in designing electrical systems. In applications such as power transmission lines, temperature fluctuations affect efficiency, requiring engineers to account for resistivity changes to prevent energy loss. In microelectronics, understanding resistivity variations is essential for designing semiconductors, where precise control of electrical properties is necessary for optimal performance.

Furthermore, studying resistivity at different temperatures allows scientists to explore phenomena like **superconductivity**, a state where materials exhibit zero electrical resistance below a critical temperature. This field has vast technological implications, including applications in MRI machines, maglev trains, and energy-efficient power grids.

Outline of the Paper

This paper will explore the theoretical basis of temperature-dependent resistivity in metallic conductors. It will begin with an analysis of classical models, including Ohm's Law, the Drude Model, and the Free Electron Model, which provide foundational explanations for electron behavior in metals. This will be followed by a discussion of quantum mechanical corrections, particularly the impact of the **Fermi surface**,

electron degeneracy, and **Bloch-Grüneisen theory** in refining our understanding of resistivity.

In addition, the paper will examine material science considerations, including how mechanical deformation, heat treatment, and annealing influence resistivity. A simple experiment to measure resistivity as a function of temperature will be outlined, providing a practical approach to verifying theoretical predictions.

By the end of this study, we aim to provide a comprehensive understanding of the principles governing temperature-dependent resistivity, its theoretical framework, and its applications in engineering and technology.

Classical Models of Electrical Resistivity

The Drude model, formulated in 1900 by Paul Drude, provides a classical explanation of electrical conductivity in metals. It assumes that conduction electrons behave like a gas of free electrons moving randomly and colliding with fixed positive ion cores. The force acting on an electron due to an applied electric field E is given by:

$$m\frac{dv}{dt} = -eE - \frac{mv}{\tau}$$

where m is the mass of an electron, e is the elementary charge, and τ is the mean time between successive collisions known as the Relaxation Time.

The relaxation time τ represents the average time interval between successive collisions of conduction electrons with impurities, lattice atoms, or phonons. It is a key parameter in determining the electrical resistivity of a material. A longer relaxation time implies fewer scattering events and, consequently, higher electrical conductivity. Conversely, a shorter relaxation time results in more frequent scattering, increasing resistivity. Mathematically, relaxation time is related to the average velocity of an electron and the mean free path λ (the average distance traveled between collisions) by:

$$\tau = \frac{\lambda}{v_F}$$

where v_F is the Fermi velocity, the typical speed of conduction electrons near the Fermi level. Since scattering mechanisms depend on temperature, impurities, and lattice vibrations, τ is temperature dependent.

Under steady-state conditions, where acceleration is zero, the equation simplifies to:

$$v = -\frac{e\tau}{m}E$$

Since electric current density J is given by:

$$J = nev$$

where n is the number density of conduction electrons, substituting v yields:

$$J = \left(\frac{ne^2\tau}{m}\right)E$$

From Ohm's Law, $J = \sigma E$, which allows us to identify electrical conductivity σ as:

$$\sigma = \frac{ne^2\tau}{m}$$

Since resistivity is the inverse of conductivity, we obtain:

$$\rho = \frac{m}{ne^2\tau}$$

This equation demonstrates how resistivity depends on the number of charge carriers and their scattering frequency. The Drude model successfully explains the general trends in electrical conductivity but fails at low temperatures and does not account for quantum mechanical effects.

Free Electron Model and Density of States

The Free Electron Model improves upon the Drude model by incorporating quantum mechanical concepts. It considers conduction electrons as a free electron gas confined within the metallic structure. Unlike Drude's purely classical approach, this model introduces the concept of energy quantization.

The total number of available electronic states per unit energy range is given by the density of states function, $g(E)$, in three-dimensional space:

$$g(E) = \frac{(2m)^{3/2}}{2\pi^2\hbar^3}E^{1/2}$$

where $\hbar$ is the reduced Planck's constant. The number of electrons occupying these states follows Fermi-Dirac statistics:

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

where E_F is the Fermi energy, k_B is Boltzmann's constant, and T is temperature. At absolute zero, all states below E_F are occupied, and those above are empty.

The Free Electron Model successfully explains why metals are good conductors and accounts for the high electron mobility observed in experiments. However, it does not consider electron scattering by the atomic lattice (phonons), which is necessary for understanding temperature-dependent resistivity.

Limitations of Classical Models

While both the Drude and Free Electron Models provide insights into conductivity they fail to explain the previse temperature dependence of resistivity, which is expected to be linear at high temperatures, and T^5 dependence at low temperature is not predicted accurately.

Neither model accounts for the influence of lattice vibration on electron scattering. In addition, the quantum mechanical effects at very low temperatures – the behaviour of electrons near absolute zero requires Fermi-Dirac statistics and Bloch wavefunctions to be explained fully.

To address these shortcomings, quantum mechanical refinements such as the Bloch-Grüneisen theory and electron degeneracy effects will be discussed in later sections.

Temperature Dependence of Resistivity

Electron-Phonon Scattering and Temperature Effects

Electrical resistivity in metallic conductors is significantly influenced by temperature, primarily due to electron-phonon scattering. In a metal, conduction electrons move through the lattice, experiencing collisions with vibrating ions. These vibrations, known as phonons, increase in amplitude as temperature rises, leading to a higher probability of electron scattering and, consequently, increased resistivity.

At absolute zero ($T = 0$), the lattice is perfectly ordered, and phonon activity is minimal, resulting in very low resistivity. As temperature increases, phonon density rises, and scattering events become more frequent, disrupting the orderly motion of conduction electrons. This increased scattering reduces the mean free path of electrons, leading to higher resistivity.

Mathematical Derivation of Resistivity Dependence

Resistivity due to electron-phonon interactions can be expressed as:

$$\rho(T) = \rho_{imp} + \rho_{phonon}(T)$$

where ρ_{imp} represents the temperature-independent contribution due to impurities and structural defects, while $\rho_{phonon}(T)$ accounts for phonon-induced scattering.

For temperatures well above the Debye temperature (Θ_D) all phonon modes are excited, and the resistivity follows a linear dependence on temperature:

This behavior is observed in most metals at high temperatures and can be derived from the classical approximation of phonon populations. The increased number of phonons at elevated temperatures enhances scattering events, reducing the relaxation time and increasing resistivity.

At temperatures much lower than the Debye temperature, only long-wavelength phonons contribute significantly to electron scattering. The resistivity in this regime follows a power law:

This dependence arises from the quantum mechanical treatment of lattice vibrations, where the density of phonon states and the probability of electron-phonon interactions follow specific temperature-dependent scaling laws. The dependence, known as the Bloch-Grüneisen law, is particularly accurate for pure metals at low temperatures.

Quantum Mechanical Treatment of Electron Scattering (Further Reading)

Free Electron Model and Wave Nature of Electrons

The free electron model extends the classical Drude model by incorporating quantum mechanics. It assumes that conduction electrons behave as wave-like particles rather than classical point-like entities. In this framework, electrons are described by the Schrödinger equation, which defines their wave-like properties within the periodic potential of the metal lattice. The energy of a free electron is given by:

$$E = \frac{\hbar^2 k^2}{2m}$$

where $\hbar$ is the reduced Planck's constant, k is the electron wavevector, and m is the electron mass. This equation implies that electrons in metals can be treated as free particles, except for occasional scattering events that alter their trajectory.

Electron Degeneracy and Its Role in Conductivity

At low temperatures, most conduction electrons reside near the Fermi surface, an energy boundary that separates occupied and unoccupied electronic states. The distribution of electrons is governed by the Fermi-Dirac distribution:

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

where E_F is the Fermi energy, k_B is Boltzmann's constant, and T is temperature. Due to the Pauli exclusion principle, each electron state can be occupied by at most two electrons (one with spin up and one with spin down). This principle leads to electron degeneracy, meaning that at low temperatures, only electrons near the Fermi energy can participate in conduction. Electrons in fully occupied lower energy states cannot contribute to conduction because all available states at those energy levels are already filled.

The concept of electron degeneracy plays a crucial role in resistivity. At very low temperatures, electron-phonon scattering decreases significantly, leading to a reduction in resistivity. However, impurity scattering remains temperature-independent, contributing to residual resistivity even as T approaches zero.

Bloch's Theorem and Periodic Potential Effects

In a crystalline metal, electrons experience a periodic potential due to the regular arrangement of atomic nuclei. Bloch's theorem states that electron wavefunctions in such a periodic potential take the form:

$$\psi_k(r) = e^{ik \cdot r} u_k(r)$$

where $\psi_k(r)$ is the total wavefunction of the electron, $e^{ik\cdot r}$ represents a plane wave component associated with the electron's momentum, and $u_k(r)$ is a function that has the same periodicity as the crystal lattice. The wavevector k describes the electron's momentum and position within the energy band structure, while r is the position coordinate of the electron. The function $u_k(r)$ encapsulates the influence of the periodic lattice potential on the electron's behavior.

This theorem has significant implications for electrical conduction. Because the electron wavefunction retains a periodic structure, electrons in a solid do not scatter off individual atoms as in a classical picture. Instead, they behave as if they are moving through a medium that allows nearly free motion, except at certain critical points in the band structure where scattering becomes significant. This behavior leads to the formation of distinct energy bands, which determine whether a material is a conductor, semiconductor, or insulator.

Bloch-Grüneisen Formula for Resistivity

A more refined quantum mechanical treatment of resistivity due to electron-phonon interactions is given by the Bloch-Grüneisen formula:

$$\rho(T) = A\left(\frac{T}{\Theta_D}\right)^5 \int_0^{\Theta_D/T} \frac{x^5 e^x}{(e^x - 1)^2} dx$$

Where $\rho(T)$ is the resistivity as a function of temperature, A is a material-dependent constant, and Θ_D is the Debye temperature, which represents the characteristic temperature associated with phonon vibrations in the lattice. The integral term accounts for the quantum mechanical effects of phonon interactions, leading to different resistivity behaviors at varying temperatures.

At high temperatures, where $T \gg \Theta_D$, the resistivity simplifies to a linear dependence: $\rho(T) \propto T$. This is because at high temperatures, phonon density increases proportionally to temperature, enhancing electron-phonon scattering events. At low temperatures, where $T \ll \Theta_D$, the resistivity follows a power law with a dependence of T^5, as derived from the integral form. This occurs because only low-energy phonons are excited in this temperature regime, limiting the available scattering pathways for electrons.

Significance of Quantum Corrections in Resistivity

Quantum mechanical effects refine our understanding of resistivity by considering factors that are absent in classical models. The concept of electron degeneracy explains why only electrons near the Fermi surface contribute to conduction, as lower-energy electrons are already in filled states and cannot undergo transitions. Bloch's theorem provides a framework for understanding how electron wavefunctions interact with the periodic structure of a solid, leading to band formation and modified scattering behaviors. The Bloch-Grüneisen function accurately models how phonon interactions influence resistivity, capturing the observed deviations from classical predictions at both high and low temperatures.

These quantum principles enable the development of materials with tailored resistivity properties, essential for modern electronic and superconducting applications. Understanding resistivity at a quantum level is crucial for optimizing materials used in electrical conductors, semiconductors, and emerging nanotechnology devices.

Material Science Considerations

Permanent Deformation and Its Effects on Resistivity

The electrical resistivity of metallic conductors is influenced not only by temperature but also by structural changes within the material. Permanent deformation, which includes plastic deformation and work hardening, alters the arrangement of atoms within the lattice, leading to increased electron scattering.

Plastic deformation occurs when a material is subjected to stress beyond its elastic limit, causing irreversible changes in its crystal structure. Dislocations, which are defects within the crystal lattice, increase in density as plastic deformation progresses. These dislocations act as additional scattering centers for conduction electrons, thereby increasing resistivity. The increased resistivity results from a reduction in the mean free path of the electrons, similar to the effect of phonon scattering at high temperatures.

Work hardening, also known as strain hardening, is a process in which a metal becomes stronger and more resistant to deformation as it is plastically deformed. During this process, the number of dislocations increases, and their interactions make further movement more difficult. The increased density of dislocations enhances electron scattering, leading to an observable rise in resistivity.

In practical applications, the effect of plastic deformation on resistivity is significant in industries where metals undergo mechanical processing, such as wire drawing, rolling, and extrusion. Engineers must consider these changes in resistivity when designing electrical components that undergo significant mechanical stress.

Heat Treatment and Annealing

Heat treatment processes, including annealing, quenching, and tempering, modify the microstructure of metals and significantly impact their electrical resistivity. Annealing is a controlled heating and cooling process that reduces dislocation density and restores the material's original crystal structure. By heating a deformed metal to a specific temperature and allowing it to cool slowly, atoms have sufficient energy to rearrange themselves into a more ordered structure, reducing the number of defects and, consequently, lowering resistivity.

In contrast, quenching is a rapid cooling process that locks atoms in a high-energy, disordered state. This process can increase resistivity by introducing residual stresses and defects into the metal. Tempering, a process typically performed after quenching, involves reheating the metal to an intermediate temperature to relieve internal stresses while maintaining desirable mechanical properties. The impact of tempering on resistivity depends on the extent to which the material's structure is stabilized.

The effect of annealing on resistivity is particularly important in electrical applications where maintaining low resistance is crucial. For example, copper and aluminum conductors used in power transmission lines are often annealed to ensure high conductivity and flexibility. Understanding the relationship between heat treatment and resistivity is essential for designing materials that balance electrical performance and mechanical strength.

Implications for Material Selection in Conductors

The selection of materials for electrical conductors requires careful consideration of both intrinsic resistivity and mechanical stability. While high-purity metals such as copper and silver offer excellent conductivity, their mechanical properties may limit their use in demanding environments. In applications where both high conductivity and mechanical strength are required, alloys are often used.

For example, aluminum alloys are widely employed in electrical transmission lines due to their combination of low weight, moderate conductivity, and high strength. Although pure aluminum has lower resistivity than its alloys, the slight increase in resistivity is outweighed by the enhanced mechanical properties that alloys provide. Similarly, copper alloys are used in high-performance electrical contacts where increased strength and wear resistance are necessary.

In summary, the electrical resistivity of metallic conductors is not solely a function of temperature but is also influenced by structural changes induced by mechanical deformation and thermal processing. Plastic deformation increases resistivity due to enhanced electron scattering by dislocations, while annealing restores the lattice structure and reduces resistivity. These material science considerations play a critical role in optimizing the performance of electrical conductors in industrial and technological applications.

Simple Experiment: Measuring Temperature-Dependent Resistivity

Objective

This experiment aims to measure how the resistivity of a metallic conductor changes with temperature by heating and cooling a copper wire submerged in water. By recording resistance variations with temperature, the relationship between resistivity and temperature can be analyzed and compared with theoretical predictions.

Materials and Setup

A 5m long coated copper wire with a diameter of 0.2mm is used. Additional materials include a digital multimeter, a digital thermometer, a power supply, a beaker filled with water, a heater, a gauze mat, a tripod, and an ice bath. The wire is coiled and submerged in water to ensure uniform heating. A circuit is constructed where voltage and current are measured to determine resistance using Ohm's Law:

$$R = \frac{V}{I}$$

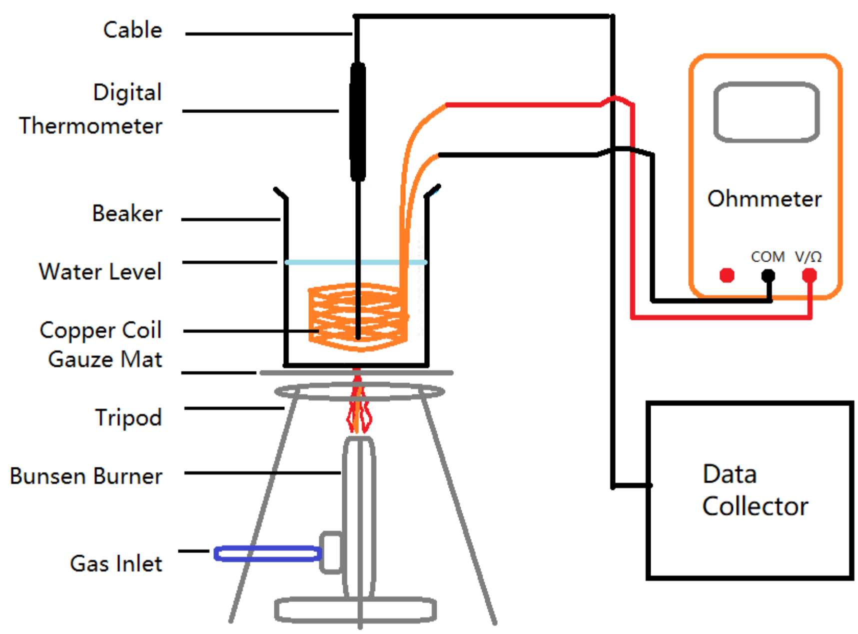
The resistivity ρ is calculated using:

$$\rho = R\frac{A}{L}$$

where A is the cross-sectional area of the wire and L is its length.

Figure 1

Diagram of the experimental setup illustrating the placement of the copper wire, thermometer, and measuring devices.



Procedure

1. Coil the copper wire and place it in the beaker filled with water.
2. Connect the wire to the multimeter and measure initial resistance at room temperature.
3. Clamp a digital thermometer into the water to monitor temperature changes.
4. Gradually cool the water using an ice bath and record resistance at various temperatures.

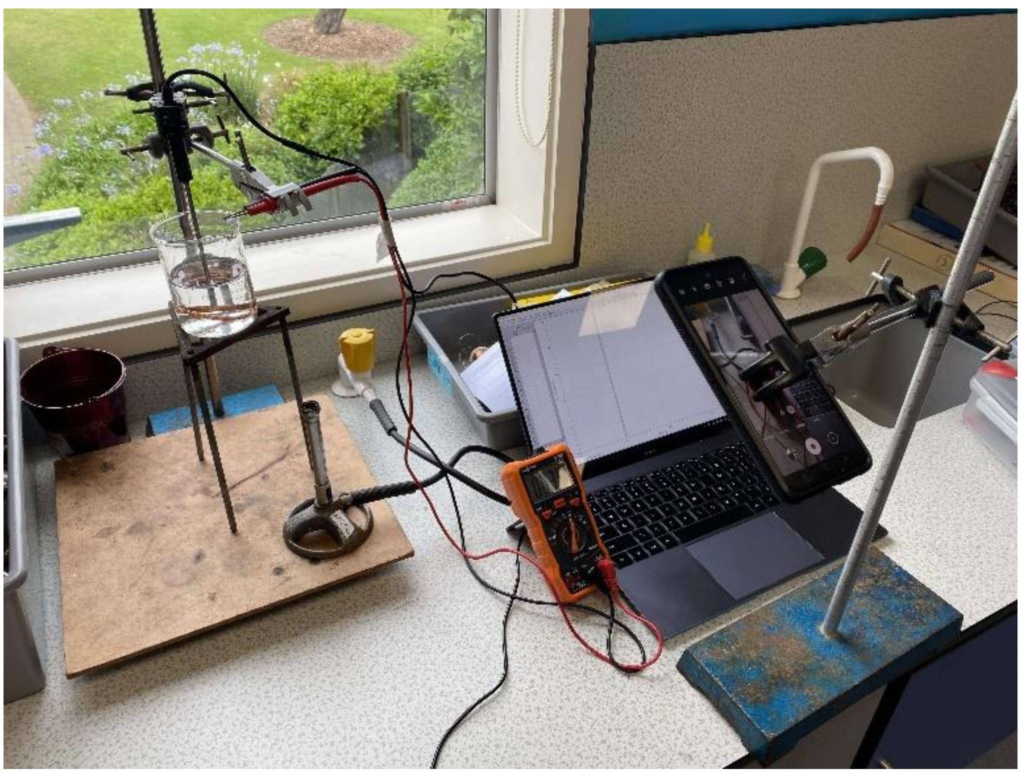
5. Heat the water using a heater while continuously measuring resistance and temperature.

6. Heat until the water reaches boiling temperature (100°C) while recording data.

7. Repeat the experiment for five trials to ensure consistency.

Figure 2

Photograph of the experimental setup, showing the real-life arrangement of equipment used in the experiment.



Expected Results and Errors

The resistivity is expected to increase linearly with temperature due to increased electron-phonon scattering. At lower temperatures, resistivity stabilizes due to impurity scattering effects. Potential sources of error include imperfect thermal equilibrium, contact resistance variations, and thermal expansion of the wire. Accuracy can be improved by using high-precision instruments and ensuring uniform temperature distribution in the water bath.

Discussion and Implications

Analysis of Results

The results from the experiment should confirm the theoretical predictions that the resistivity of a metallic conductor increases with temperature. This occurs because as temperature increases, atomic vibrations intensify, leading to a higher frequency of electron-phonon scattering. This increased scattering reduces the mean free path of electrons, thereby increasing resistivity. The linear relationship observed at higher temperatures aligns with the classical model of resistivity, while at lower temperatures, deviations may occur due to impurity scattering and quantum effects.

A comparison between experimental values and theoretical calculations using the Bloch-Grüneisen formula provides insight into discrepancies. Any observed differences may be attributed to factors such as material imperfections, measurement inaccuracies, or changes in wire dimensions due to thermal expansion. These discrepancies highlight the importance of careful calibration and precision in experimental setups.

Practical Implications

Understanding how resistivity varies with temperature is essential in designing electrical and electronic systems. In power transmission, variations in resistivity must be considered to minimize energy loss and maintain efficiency. Transmission lines, for instance, experience resistance changes due to fluctuating environmental temperatures, necessitating compensatory designs to mitigate power loss.

In semiconductor and microelectronics industries, precise control of resistivity is crucial. Devices such as microprocessors and sensors require materials with stable resistivity characteristics over a range of operating temperatures to ensure consistent performance. Additionally, research into materials exhibiting superconductivity at low temperatures relies on understanding temperature-dependent resistivity trends. Superconductors, which exhibit zero resistivity below a critical temperature, have vast applications in medical imaging (MRI machines), maglev trains, and energy-efficient power grids.

Limitations and Recommendations

One of the main limitations of this experiment is the difficulty in maintaining uniform temperature throughout the wire. Variations in temperature along the wire length may introduce inconsistencies in resistance measurements. Additionally, contact resistance at the wire-multimeter connection may introduce small but significant errors in readings. Ensuring proper thermal equilibrium before taking measurements and using well-calibrated instruments can minimize these issues.

Another source of error is the assumption that the wire's cross-sectional area remains constant throughout the experiment. Thermal expansion may slightly alter the wire dimensions, affecting resistivity calculations. Future experiments could incorporate expansion coefficients to correct for this effect.

To improve accuracy, future experiments could use more precise temperature sensors, controlled thermal environments, and automated data collection systems. Exploring different materials, such as alloys or superconducting materials, could provide further insight into resistivity behavior in various conductors.

Conclusion

This experiment successfully demonstrates the temperature dependence of resistivity in metallic conductors. The observed trends align with theoretical predictions, reinforcing the fundamental concepts of electron-phonon interactions and scattering mechanisms. These findings have practical implications in electrical engineering, semiconductor technology, and materials science. Future studies can expand on these results by exploring advanced materials and refining measurement techniques to enhance accuracy and applicability in real-world applications.

Conclusion

Summary of Findings

This study explored the temperature-dependent resistivity of metallic conductors through both theoretical analysis and experimental validation. The theoretical foundation was established using classical models, such as the Drude model and the free electron model, along with quantum mechanical refinements, including the Bloch-Grüneisen formula. These models predict that resistivity increases with temperature due to enhanced electron-phonon scattering, a trend confirmed by experimental observations.

The experimental approach involved measuring resistance variations in a metallic wire submerged in a controlled thermal environment. The results demonstrated a clear relationship between temperature and resistivity, consistent with theoretical expectations. Discrepancies between theoretical predictions and experimental results were attributed to material imperfections, thermal expansion, and measurement limitations, reinforcing the importance of precision in experimental physics.

Significance of the Study

Understanding temperature-dependent resistivity is essential for numerous scientific and engineering applications. In electrical power transmission, accounting for resistivity variations with temperature can enhance energy efficiency and prevent excessive power loss. In semiconductor technology, precise control of resistivity is crucial for optimizing device performance and reliability. Furthermore, advancements in superconducting materials rely on understanding resistivity behavior at low temperatures, contributing to the development of energy-efficient technologies such as maglev trains and medical imaging systems.

Future Directions

Further research can expand on this study by incorporating more advanced materials, including high-temperature superconductors and composite conductors. Investigating resistivity in different environmental conditions, such as extreme pressures or varying humidity levels, could provide additional insights into material performance in real-world applications. Moreover, integrating automated data collection techniques and high-precision thermal control systems could enhance measurement accuracy and reduce experimental uncertainties.

Final Remarks

This study has reinforced fundamental principles of solid-state physics and electrical engineering by demonstrating how resistivity in metallic conductors varies with temperature. The results align with classical and quantum mechanical theories, providing a deeper understanding of electron transport in materials. As technology continues to evolve, knowledge of temperature-dependent resistivity will remain integral to developing efficient electrical and electronic systems, paving the way for innovations in energy transmission, microelectronics, and materials science.

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