

# PA-SLNR Multi-user MIMO Precoding Based on GSVD

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*Abstract*- In this work, a linear precoding scheme for Multi-user Multiple Input Multiple Outputs (MU-MIMO) system is introduced. The proposed method optimizes Per-Antenna Signal to Leakage plus Noise Ratio (PA-SLNR) performance criteria, and makes use of the Generalized Singular Value Decomposition (GSVD) computation to develop a simple precoding algorithm. However, the linear technique available in the literature so far uses Per-User Signal to Leakage plus Noise Ratio (PU-SLNR) performance criteria and develop a closed form precoding matrix solutions based on Generalized Eigenvalue Decomposition (GEVD) and simple algorithm based on GSVD. The PU-SLNR objective function neglects to take the intra-user antennas interference cancellation into account. In this work, a new cost function based on PA-SLNR is defined, GSVD based solution is explained and simple algorithm is developed to compute the precoding matrices for multiple users with multiple antennas. Simulation results show that the proposed precoding method has a better BER and output SINR outage performance than the conventional methods.

*Index Terms*— Generalized Singular Value Decomposition (GSVD), Generalized Eigenvalue Decomposition (GEVD), Multi-user Multiple Input Multiple outputs (MU-MIMO), Precoding, Per-Antenna Signal to leakage Plus Noise Ratio (PA-SLNR).

## I. INTRODUCTION

Multi-user multiple input multiple output broadcast channel (MU-MIMO-BC) precoding design has received significant attention in the last decade [1-3]. This high thoughtfulness is due to the high predictable performance/capacity gain of both MIMO and the multi-user spatial dimensions. In the literature, due to the complexity of nonlinear precoding methods [4-7], several linear precoding techniques were proposed. In these methods both joint and independent optimization techniques were used to mitigate the multiuser interference (MUI) [8-12]. Given that the channel state information is available at the transmitter and receiver, various conditions and objectives were used to study this problem. In [8, 9, 13-15] various closed form solutions based on minimum mean square error (MMSE) were proposed. The methods in [8, 15] impose dimensionality restrictions that the number of base station antennas should be greater than or equal to the total numbers of receive antennas for all users. Conversely, the methods in [9, 14] relax the dimensionality constraint but with some performance loss at high SNR values when the total number of receive antennas at all users exceeds the number of base station antennas. In [10] an iterative sum mean square error (SMSE) method also called joint transceiver design was developed to compute both the precoding and decoding matrices. In spite of its high performance, computational complexity and restrictions on the number of antennas are the main demerits of this method. The work by [11] propose a beam-forming precoding design utilizing Per-User Signal to Leakage and Noise Ratio (PU-SLNR) cost function. In this method the precoding vectors for all users are obtained by solving a series of optimization

problems using generalized eigenvalue decomposition (GEVD). In [12] PU-SLNR precoding matrix design based on GEVD for MU-MIMO spatial Multiplexing is proposed. This work shows that the precoding matrix computation using GEVD is simple but it's sensitive to the singularity in the leakage plus noise power matrix, thus there is some performance loss at high SNR values. To solve the singularity problem in matrix computation, Fukunaga-Koontz transforms (FKT) and GSVD based computation were independently utilized in [16] and [17] respectively. The precoding design based on PU-SLNR cost function introduced in [12, 16, 17] were totally neglects to take into account the interference between streams multiplexed to one user (intra-user antenna interference).

In this paper, an objective function based on the PA-SLNR criteria is defined to account for the intra user antennas interference cancelation. Moreover, GSVD is utilized to develop a new precoding algorithm. Using both bit error rate (BER) and the received out SINR outage as performance metrics, simulation result shows that the combination of the proposed PA-SLNR optimization method and the GSVD computation algorithm generate numerically better results than the conventional PU-SLNR optimization with GSVD computation. The rest of this work is organized as follows: section II describes the system model and the precoding problem description is stated in section III. Section IV describes the algorithm development and the simulation results are given in section V. Section VI concludes the paper.

The superscripts  $\mathbf{Q}^T, \mathbf{Q}^H$  denote the transpose and conjugate transpose of matrix  $\mathbf{Q}$  respectively.  $tr(\mathbf{Q})$  and  $\|\mathbf{Q}\|_F^2$  are denoting the trace and the Frobenious norm respectively.  $\mathbf{I}_{N_T}$  is an  $N_T \times N_T$  identity matrix.  $\mathbf{D} = diag\{\lambda_1 \lambda_2 \dots \lambda_r\}$  Denotes the diagonal matrix with diagonal elements  $\{\lambda_1 \lambda_2 \dots \lambda_r\}$ .

## II. MU-MIMO BROADCASTING SYSTEM MODEL

We consider a downlink MU-MIMO-BC channel with a central node Base station/ Access point communicating with  $B$  number of users simultaneously at the same time and frequency. The base station is equipped with  $N_T$  transmit antennas and each individual user  $k$  is equipped with  $M_k$  receive antennas,  $k = 1, \dots, B$  as depicted in figure 1. The central node spatially multiplexes and sends  $B$  data vectors to the  $B$  number of users simultaneously. We assume perfect channel state information (CSI) is available at both the transmitter and receivers.

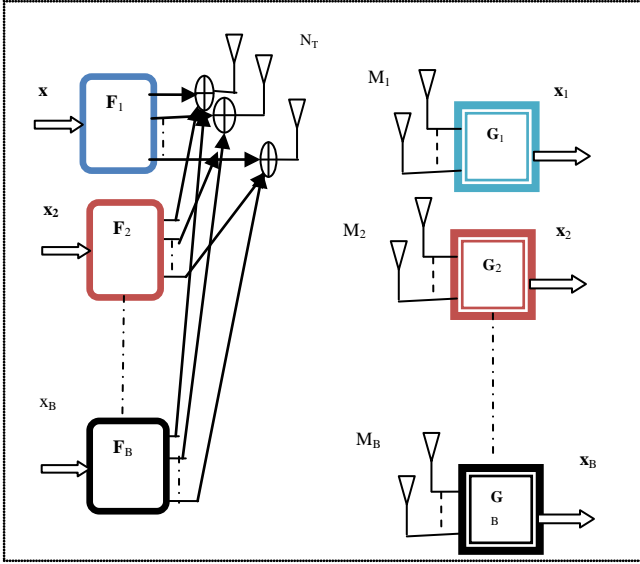


FIG. 1: MU-MIMO BROADCAST SYSTEM BLOCK DIAGRAM.

The channel from the central node to each user is assumed to be frequency flat fading and constant over all transmission block duration. The element of any user channel  $\mathbf{H}_k \in \mathbf{C}^{M_k \times N_T}$  is assumed to be complex Gaussian variables with zero mean and unit-variance. The combined channel matrix for all the  $B$  users in the system is given by:

$$\mathbf{H}_{\text{com}} = [\mathbf{H}_1^T \mathbf{H}_2^T \dots \mathbf{H}_B^T]^T \quad (1)$$

and the received signal at the  $k^{\text{th}}$  user is given by:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{F}_k \mathbf{x}_k + \mathbf{H}_k \sum_{i=1, i \neq k}^B \mathbf{F}_i \mathbf{x}_i + \mathbf{n}_k \quad (2)$$

Where  $\mathbf{F}_k \in \mathbf{C}^{N_T \times M_k}$  is the  $k^{\text{th}}$  user precoding matrix, while  $\mathbf{x}_k \in \mathbf{C}^{M_k \times 1}$  is the  $k^{\text{th}}$  user data vector. The entries of receiving noise vector  $\mathbf{n}_k$  are assumed to be independent complex Gaussian variables with zero-mean and variance equal to  $\sigma_k^2$ . Also we assume that the noise variances at all antenna elements as well as the noise at the all users are equal such that:  $\sigma_1^2 = \sigma_2^2 = \dots = \sigma_{M_k}^2$ . The entries of the  $M_k \times N_T$  channel  $H_K$  are denoted by:

$$\mathbf{H}_k = \begin{bmatrix} h_k^{(1,1)} & \dots & h_k^{(1,N_T)} \\ \vdots & \ddots & \vdots \\ h_k^{(M_k,1)} & \dots & h_k^{(M_k,N_T)} \end{bmatrix} \in \mathbf{C}^{(M_k \times N_T)} \quad (3)$$

Also we consider that for any  $k^{\text{th}}$  user, the data vector  $\mathbf{x}_k$  and the precoding matrix  $\mathbf{F}_k$  are normalized such as that:

$$E \|\mathbf{x}_k\|_f^2 = \mathbf{I}_{M_k} ; \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) = 1 \quad (4)$$

for  $k = 1, \dots, B$

And for each  $k^{\text{th}}$  individual user, the decoding matrix  $\mathbf{G}_k$  will be constructed at the  $k^{\text{th}}$  user terminal to decode the received signal.

### III. PA-SLNR MAXIMIZATION PRECODING

The objective function originally proposed in [11] maximizes the PU-SLNR of each user. Thus, the precoder designed cancels only the inter-user interference. This work, however, consider a new cost function that maximizes the SLNR for each individual receive antenna (PA-SLNR). This cost function would help to minimize the intra-user antennas interference and leads to a better precoder that maximizes the overall SLNR per user more efficiently. This is justified because PA-SLNR as explained in fig 2, takes into account the intra-antenna interference mitigation.

For each  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user, The  $PA-SLNR$ ,  $\gamma_k^j$ , is defined as ratio between the individual  $j^{\text{th}}$  received antenna desired signal power to the interference introduced by the  $j^{\text{th}}$  receiving antenna to all other antennas plus the noise power at the  $j^{\text{th}}$  receive antenna front end. So for the  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user, the signal to leakage plus noise ratio  $\gamma_k^j$  is given by:

$$\gamma_k^j = \frac{\|\mathbf{h}_k^j \mathbf{f}_k^j\|_F^2}{\sum_{i=1, i \neq k}^B \|\mathbf{H}_i \mathbf{f}_k^j\|_F^2 + \sum_{i=1, i \neq j}^{M_k} \|\mathbf{h}_k^i \mathbf{f}_k^j\|_F^2 + \sigma_k^2} \quad (5)$$

Where  $\mathbf{h}_k^j \in \mathbf{C}^{1 \times N_T}$  is the  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user, received row vector. If we define an auxiliary matrix  $\mathbf{H}_k^j$  as the matrix contains all the  $k^{\text{th}}$  user receive antennas row vectors except the  $j^{\text{th}}$  antenna row vector as follows:

$$\mathbf{H}_k^j = \begin{bmatrix} h_k^{(1,1)} & h_k^{(1,2)} & \dots & h_k^{(1,N_T)} \\ \vdots & \vdots & \vdots & \vdots \\ h_k^{(j-1,1)} & h_k^{(j-1,2)} & \dots & h_k^{(j-1,N_T)} \\ h_k^{(j+1,1)} & h_k^{(j+1,2)} & \dots & h_k^{(j+1,N_T)} \\ \vdots & \vdots & \vdots & \vdots \\ h_k^{(M_k,1)} & h_k^{(M_k,2)} & \dots & h_k^{(M_k,N_T)} \end{bmatrix} \in \mathbf{C}^{((M_k-1) \times N_T)} \quad (6)$$

And the combined channel matrix for all the system received antennas except the  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user desired receive antenna row as:

$$\tilde{\mathbf{H}}_k^j = [\mathbf{H}_k^j \mathbf{H}_1^T \dots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \dots \mathbf{H}_B^T]^T \quad (7)$$

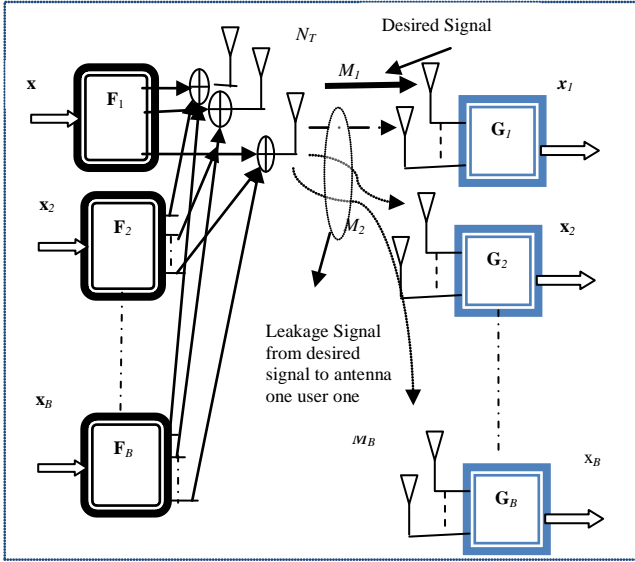


FIG.2: SYSTEM MODEL DEPICTING ALL VARIABLES.

From equation (6) and (7), the expression in (5) can be reduced to:

$$\gamma_k^j = \frac{\|\mathbf{h}_k^j \mathbf{f}_k^j\|_F^2}{\|\tilde{\mathbf{H}}_k^j \mathbf{f}_k^j\|_F^2 + \sigma_k^{2j}} \quad (8)$$

Problem statement: For any  $j^{\text{th}}$  receive antenna, at the  $k^{\text{th}}$  user, select the precoding vector  $\mathbf{f}_k^j$ , where  $k = \{1, \dots, B\}$ ,  $j = \{1, \dots, M_k\}$  such that the per-antenna signal to leakage plus noise ratio  $\gamma_k^j$  is maximized:

$$\mathbf{f}_k^j = \arg \max_{\mathbf{f}_k^j \in \mathbf{C}^{N_T \times 1}} \frac{\mathbf{f}_k^{jH} (\mathbf{h}_k^j \mathbf{h}_k^j) \mathbf{f}_k^j}{\mathbf{f}_k^{jH} (\tilde{\mathbf{H}}_k^j \tilde{\mathbf{H}}_k^j + \sigma_k^{2j} \mathbf{I}_{N_T}) \mathbf{f}_k^j} \quad (9)$$

subject to

$$\begin{aligned} \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) &= 1 \\ \mathbf{F}_k &= [\mathbf{f}_k^1, \dots, \mathbf{f}_k^{M_k}] \end{aligned}$$

The optimization problem in the equation (9) deals with the  $j^{\text{th}}$  antenna desired signal in the numerator and a combination of total leaked power from the  $j^{\text{th}}$  antenna of the  $k^{\text{th}}$  user to all other antennas plus noise power at the  $j^{\text{th}}$  antenna front end in the denominator. To calculate the precoding matrix for each user we need to calculate the precoding vector for each receive antenna in the system independently. This requires solving the linear fractional optimization problem in the equation (9)  $M_k \times B$  times using GEVD[11]. In this work, we develop a GSVD based algorithm to solve the PA-SLNR objective function of equation 9 for multiple users with multiple antennas.

#### IV. GSVD BASED PRECODING ALGORITHM

First formulation and generalization of singular value decomposition was introduced by Charles F. and Van Loan in [18]. There initial formulation was restricted to the dimension of the matrix pairs. More general and less restrictive version of GSVD is developed by Paige and Saunders in [19] which can be summarized as follows:

For any two matrices have the same numbers of columns i.e.

$\Psi_A \in \mathbf{C}^{P \times L}$  and  $\Psi_B \in \mathbf{C}^{Z \times L}$  then for:  $\Psi = \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix}$  and  $\text{rank}(\Psi) = t$  there exists an orthogonal matrices such as  $\mathbf{U} \in \mathbf{C}^{P \times P}$ ,  $\mathbf{V} \in \mathbf{C}^{Z \times Z}$ ,  $\mathbf{W} \in \mathbf{C}^{t \times t}$  and  $\mathbf{Q} \in \mathbf{C}^{L \times L}$  such that :

$$\mathbf{U}^H \Psi_A \mathbf{Q} = \Sigma_A (\mathbf{W}^H \mathbf{R}, \mathbf{0}) \quad (10)$$

$$\mathbf{V}^H \Psi_B \mathbf{Q} = \Sigma_B (\mathbf{W}^H \mathbf{R}, \mathbf{0}) \quad (11)$$

where:

$$\Sigma_A = \begin{pmatrix} \mathbf{I}_A & & \\ & \mathbf{D}_A & \\ & & \mathbf{0}_A \end{pmatrix}, \quad \Sigma_B = \begin{pmatrix} \mathbf{0}_B & & \\ & \mathbf{D}_B & \\ & & \mathbf{I}_A \end{pmatrix}$$

$\mathbf{R} \in \mathbf{C}^{t \times t}$  is nonsingular matrix with its singular valves equal to the nonzero singular value of  $\Psi$ . The matrices  $\mathbf{I}_A^{r \times r}$  and  $\mathbf{I}_B^{(t-r-s) \times (t-r-s)}$  are identity matrices with value of  $r$  and  $s$  depend on the matrices  $\Psi_A$  and  $\Psi_B$ . The matrix  $\mathbf{D}_A = \text{diag}(\alpha_{r+1}, \dots, \alpha_{r+s})$  and  $\mathbf{D}_B = \text{diag}(\beta_{r+1}, \dots, \beta_{r+s})$  are diagonal matrices with the diagonal elements satisfies the following:

$$1 \geq \alpha_{r+1} \geq \dots \geq \alpha_{r+s} \geq 0 \quad (12)$$

$$0 \leq \beta_{r+1} \leq \dots \leq \beta_{r+s} \leq 1 \quad (13)$$

and

$$\alpha_i^2 + \beta_i^2 = 1 \quad \text{for } i = r+1, \dots, r+s \quad (14)$$

This analysis reveals that the computation of the GSVD starts with the complete orthogonal decomposition of the matrix  $\Psi$  followed by simple SVD of the orthogonal sub-matrices. Mapping the optimization problem in the equation (9) to the Paige and Saunders GSVD formulation, we can observe that for any  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user, the PA-SLNR  $\gamma_k^j$  is a qualified GSVD problem and the eigenvector corresponding to the  $(r+1)$  singular value will be the best vector in the direction of the  $j^{\text{th}}$  receive antenna of the  $k^{\text{th}}$  user (equation 14). Another notable observation is that the matrix the overall interference domain is constant for all users and receive antenna. These facts will allow the designer to reduce the computational load by making use of the matrix permutation principle. Algorithm 1, summarize

the complete steps to calculate the precoding matrices for multiple users in MU-MIMO-BC.

**Algorithm 1:** PA-SLNR MU-MIMO precoding based on GSVD Computation for multiple independent MU-MIMO users.

**Input:** all the users combined channel matrix and the received noise variance.

$$\mathbf{H}_{\text{com}} = [\mathbf{H}_1^T \mathbf{H}_2^T \cdots \mathbf{H}_B^T]^T, \sigma_k^{2j}.$$

**Output:** multiple users precoding matrices  $\mathbf{F}_k$  such that,

$k = 1, \dots, B$

1. Let  $\boldsymbol{\psi}_k^j = [\mathbf{h}_k; [\tilde{\mathbf{H}}_k^j; \sigma_{N_T}^j]]$
2. Compute the QRD of the matrix  $\boldsymbol{\psi}_k^j$  such that

$$\mathbf{Q}\boldsymbol{\psi}_k^j = \mathbf{R}$$

3. For  $k = 1$  to  $B$

4. For  $j = 1$  to  $M_k$

    Compute the vector  $\mathbf{v}_k^j$  from the

    SVD of  $\mathbf{Q}((k-1)M_k + j : (k-1)M_k + j, 1 : N_T)$

    Solve for  $\mathbf{R}\mathbf{f}_k^j = \mathbf{v}_k^j$

5. End

    The  $k^{\text{th}}$  user precoding matrix is

$$\mathbf{F}_k = [\mathbf{f}_k^1 \cdots \mathbf{f}_k^{M_k}]$$

6. End

## V. SIMULATION AND RESULTS

The performance of the MU-MIMO system is measured in terms of the received BER and received output SINR outage. We evaluate the performance of the proposed PA-SLNR MU-MIMO precoding scheme based on the GSVD algorithm through Monte-Carlo simulation for different MU-MIMO broadcast system configurations. In each simulation setup, the entry of each  $k^{\text{th}}$  user MIMO channel  $\mathbf{H}_k$  is generated as complex white Gaussian entries with zero mean and unit variance. Furthermore, in all simulations experiments, user data symbols vector is 4-QAM modulated and spatially multiplexed at the base station. At the receivers, matched filter is used for the decoding

Figure 3, compares the received BER performance of the proposed PA-SLNR precoding which denoted in the figure By PA-SLNR-GSVD and the literature reference results of PU-SLNR-GSVD [17, 20] precoding schemes for the MU-MIMO-BC system configurations of  $N_T = 12$   $B = 3$   $M_k = 3$ .

In this MU-MIMO configuration, the base station modulates, spatially multiplexes and precodes a vector of length 3 symbols to each user. The BER is calculated over 5000 MU-MIMO channel realization for each algorithm. The proposed method outperforms PU-SLNR-GSVD. At BER equal to  $10^{-4}$  there is approximately 2dB performance gain between PA-SLNR-GSVD and PU-SLNR-GSVD.

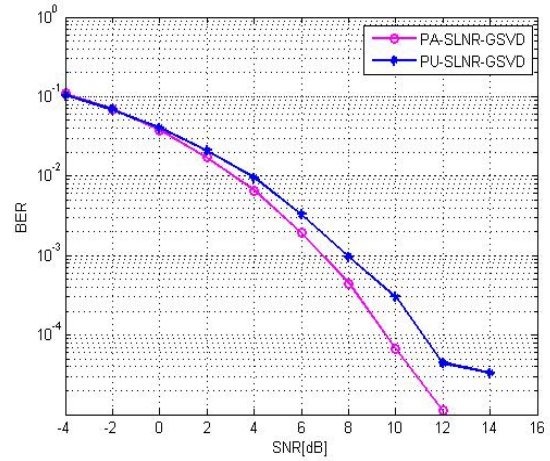


Fig 3: Shows the compression of the un-coded BER performance for PA-SLNR-GSVD, PU-SLNR-GSVD[17] precoding methods under system configuration of  $B = 3$   $M_k = 3$   $N_T = 12$ , 4-QAM modulated signal.

Figure 4, compares the received BER performance of the proposed PA-SLNR precoding which denoted in figure By PA-SLNR-GSVD and the literature reference results of PU-SLNR-GSVD precoding schemes for the MU-MIMO broadcast system configurations of  $N_T = 10$   $B = 3$   $M_k = 3$ . In this MU-MIMO configuration, the base station modulates, spatially multiplexes and precodes a vector of length 3 symbols to each user. The BER is calculated over 5000 MU-MIMO channel realization for each algorithm. The proposed method outperforms PU-SLNR-GSVD. At BER equal to  $10^{-3}$  there is approximately 2 dB performance gain between PA-SLNR-GSVD and PU-SLNR-GSVD.

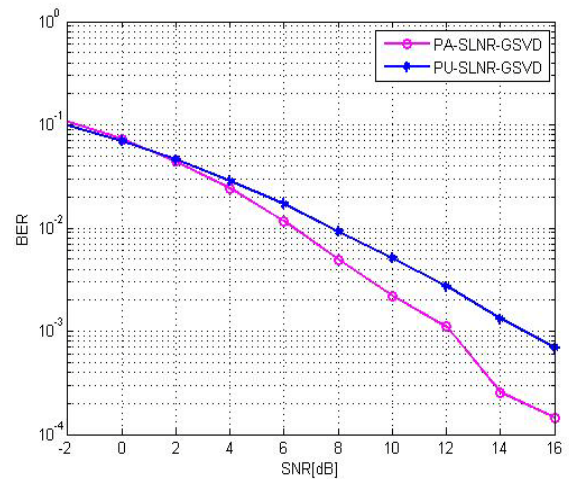


Fig 4: Shows the compression of the un-coded BER performance for PA-SLNR-GSVD, PU-SLNR-GSVD precoding methods under system configuration of  $B = 3$   $M_k = 3$   $N_T = 10$ , 4-QAM modulated signal.

Figure 5, compares the received output SINR outage performance of the proposed PA-SLNR-GSVD precoding and the reference PU-SLNR-GSVD. MU-MIMO system configuration of  $B = 3$   $M_k = 4$   $N_T = 12$  and 4dB input SNR is considered in the simulation. The proposed method outperforms the PU-SLNR-GSVD method by 1 dB gain at 10% received output SINR outage.

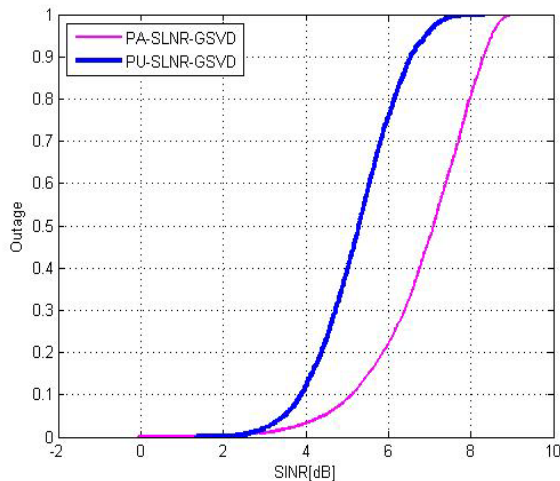


Fig 5: Shows the comparison of the un-coded output SINR outage performance PA-SLNR-GSVD and PU-SLNR-GSVD precoding methods under system configuration of  $B = 3, M_k = 4, N_T = 12$ , 4-QAM modulated signal at 4 dB SNR.

Figure 6, compares the received output SINR outage performance of the proposed PA-SLNR-GSVD precoding and the reference PU-SLNR-GSVD. MU-MIMO system configuration of  $B = 3, M_k = 4, N_T = 12$  and 12dB input SNR is considered in the simulation. The proposed method outperforms the PU-SLNR-GSVD method by 2.5 dB gain at 10% received output SINR outage.

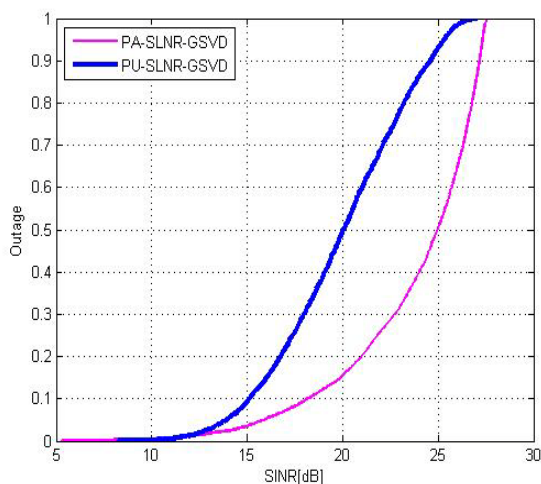


Fig 6: Shows the comparison of the un-coded output SINR outage performance PA-SLNR-GSVD and PU-SLNR-GSVD precoding methods under system configuration of  $B = 3, M_k = 4, N_T = 12$ , 4-QAM modulated signal at 12dB SNR.

## VI. CONCLUSIONS

In this paper, a new MU-MIMO precoding method based on the combination of the PA-SLNR optimization criteria and GSVD computation algorithm is developed. Unlike the conventional method in literature which does not take into account the individual user intra-antenna interference cancellation, the proposed method transforms the MU-MIMO channel into a set of parallel single antenna channels

to effectively cancel intra-antenna interference and multiplex multiple data stream to each user. Simulation results show that the proposed method outperforms the conventional method.

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