A general PID controller tuning method based on state space pole placement

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Abstract. This paper presents a universally applicable analytic tuning method for PID control. The method begins with a second-order dead time model derived from step response analysis. Optimal poles are determined using a Linear Quadratic (LQ) regulator. State feedback for a non-zero setpoint and load disturbance, combined with a reduced-order observer, results in a two-degree-of-freedom PID structure with a first-order filter. Using the poles from state feedback, PID controller parameters are calculated via pole placement. A robust PID controller is achieved with M constraint optimal integral gain (MIGO) through the optimal selection of the LQ weighting factor and observer pole for load disturbance. Benchmark examples demonstrate the method's applicability, providing both good regulation and servo behaviour.

Keywords: PID control; LQ regulator; Reduced order observer; Pole placement; MIGO

1 Introduction

In the realm of control systems, the Proportional-Integral-Derivative (PID) controller stands as a cornerstone due to its simplicity and effectiveness [1]. In a recent review paper, Hägglund substantiates the importance of PID control in real-life applications and control engineering, highlighting its significance over the past century [2]. However, tuning PID parameters to achieve optimal performance remains a critical task. A comprehensive handbook collects more than 500 PI and PID tuning rules [3] underscoring the complexity of this endeavour. Tuning the PID algorithm for accurate and robust stable closed-loop control is a challenging problem [4]. In essence, PID controllers can be tuned either for optimal disturbance rejection (regulation) or for optimal setpoint tracking (servo), as demonstrated by Sato et al [5]. In traditional process control, the primary focus has been on regulation. However, modern industrial applications increasingly demand fast servo control with minimal overshoot. To address both regulation and servo requirements, two-degree-of-freedom (2DOF) PID controllers have been developed [6]. These controllers aim to balance the trade-offs between the two modes, maintaining effective disturbance rejection while obtaining as good as possible setpoint tracking with restricted overshoot. Arrieta and Vilanova [7] propose a different approach, employing sub-optimized PID settings for both regulation and servo modes. Their approach minimizes performance degradation by using intermediate tuning settings, ensuring a smoother transition between regulation and servo operation.

This paper presents a general approach to analytical PID controller tuning by combining principles of linear optimal control [8] and advanced PID control techniques [9,10] with the aim to obtain inherent good regulation and servo control. The foundation of this general PID tuning method is a second-order model, which can be derived e.g. from the “half rule” model reduction method of Skogestad [11]. Here we show how a second order model can be derived from a step response. By leveraging the results of state feedback control with a reduced-order observer, we establish the structure for the PID controller. The parameters are calculated through pole placement, with the regulator poles determined by a linear quadratic weighting factor and an observer pole related to the integral action. This method requires the selection of two key parameters: a weighting factor and an observer pole. The calculation of the PID parameters is done using analytical expressions. To streamline the tuning process, we employ the M-constrained Integral Gain Optimization (MIGO) method proposed by Åström and Hägglund [10]. Illustrative examples demonstrate the applicability and effectiveness of the proposed PID tuning method.

2 Step response models

Estimation of transfer function models for PID controller design can be done with different techniques [9]. Least squares curve fitting is often used, but here we restrict ourselves to simple graphical methods for three typical step responses with second order model approximation.

2.1 Monotone step response

In the PID control literature [9,10,11], a transfer function P(s) of a first order with dead time is often used as approximation model of the process

$$P(s) = \frac{Ke^{-L_o s}}{1 + T_o s} \tag{1}$$

Where K is the gain, T_o the apparent time constant and L_o the apparent dead time. The (unit) step response S(t) of this model is given by

$$S(t) = K\left(1 - e^{-(t-L_o)/T_o}\right) \tag{2}$$

The graphical determination of these parameters is illustrated in Figure 1.

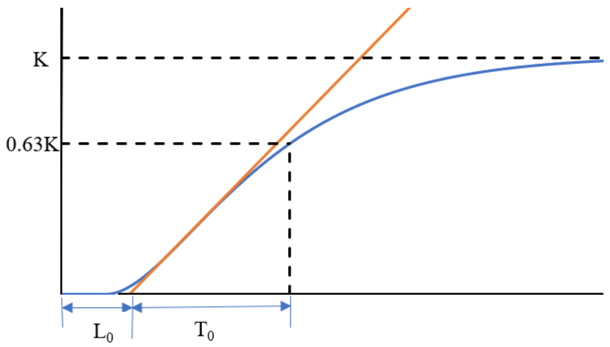

Figure 1. First order model parameters determination from unit step response.

The maximum slope tangent to the step response intersects the time-axis at $t=L_o$. At 63 % of the step response $t=L_o+T_o$. Note that the apparent dead time is larger than the “real” dead time. Also note that the time constant is determined with the 63 % value and not with the tangent line intersection of the final value, which leads to a too big estimate of the time constant [9]. The higher order dynamics are partly lumped in L_o and partly in T_o. As indicated by Aström and Hägglund [9] a better approximation of the S-shaped step response is obtained with the second order transfer function

$$P(s) = \frac{Ke^{-Ls}}{(1 + Ts)^2} \tag{3}$$

The (unit) step response S(t) of this transfer function is given by

$$S(t) = K\left[1 - \left(1 + \frac{t - L}{T}\right) e^{-(t-L)/T}\right] \tag{4}$$

Note that L is smaller than L_o because part of the bending of the step response is taken up by (4) with the part

$$P_o(s) = \frac{K}{(1 + Ts)^2} \tag{5}$$

When (5) is approximated with (1) it can be calculated that $L_o=(3-e)T=0.28T$ and $T_o=1.86T$. So, the other way around, given that a step response is approximated with (1), a second order approximation (3) is then given by

$$L = L_o - 0.15T_o \quad T = 0.54T_o \tag{6}$$

Remark:
The model (3) should only be applied with a clear S-shaped step response. A check for this could be the step response value at $t = L_0$. If this value is very small the first order with dead time model (1) is a better choice. A reasonable criterion for using model (3) could be that $S(t=L_0)/K > 0.03$.

2.2 Integrating process

The transfer function of a second order integrating process is given by

$$P(s) = \frac{Ke^{-Ls}}{s(1+Ts)} \tag{7}$$

With (unit) step response S(t) given by

$$S(t) = K\left[t - L - T\left(1 - e^{-(t-L)/T}\right)\right] \tag{8}$$

The parameters are determined graphically with figure 2 [9].

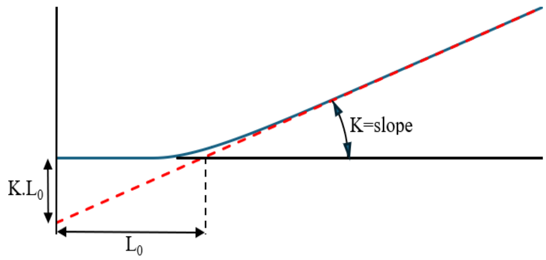

Figure 2. Integrating model parameters determination from unit step response.

Where

$$T = 2.72S(t = L_o)/K \qquad L = L_o - T \tag{9}$$

2.3 Oscillatory process

The transfer function of a second order oscillatory process can be written as

$$P(s) = \frac{Ke^{-Ls}}{T^2s^2 + 2\vartheta Ts + 1} \tag{10}$$

with K the gain, L the dead time, T the inverse of the natural frequency and ϑ the relative damping. The parameters can be derived from the (unit) step response figure 3.

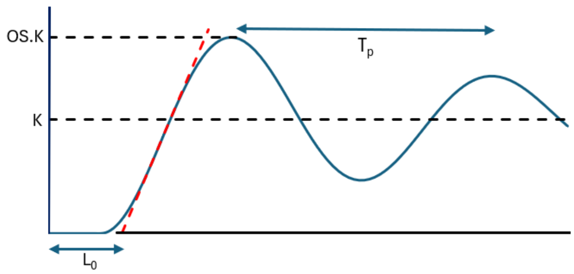

Figure 3. Oscillatory model parameters determination from unit step response.

The overshoot ratio OS is related to ϑ as

$$\mathrm{OS} = e^{-\pi\vartheta/\sqrt{1-\vartheta^2}} \tag{11}$$

and the period of oscillation T_p is related to T and ϑ as

$$T_p = 2\pi T/\sqrt{1-\vartheta^2} \tag{12}$$

And so ϑ and T are estimated from the step response as

$$\vartheta = \sqrt{\ln^2 \mathrm{OS}/(\pi^2 + \ln^2 \mathrm{OS})} \tag{13}$$

$$T = \frac{1}{2\pi} T_p \sqrt{1-\vartheta^2} \tag{14}$$

The tangent of maximum slope intersects the time axis at the apparent dead time L_o. Following the same reasoning as with the monotone response, the “real” dead time L is smaller than L_o. Regression gives than as good approximation

$$L = L_0 - (0.09 - 0.075\vartheta)T_p \tag{15}$$

3 Controller design

The controller design starts with the selection of a state model which serves as the basis for the regulator design. The unmeasured states are reconstructed with an observer. Finally, the complete controller is formed by the combination of regulator and observer.

3.1 Model for controller design

$$Y(s) = P(s)[U(s) + V(s)]$$

$$P(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{16}$$

Where U(s) is the input, Y(s) the output, P(s) the process model, s Laplace variable and V(s) the load disturbance; b0≠0.
A process with a dead time will be approximated with model (16) by using the first order Padé approximation

$$e^{-Ls} \approx 1 - Ls \tag{17}$$

As state space realisation for (16) we take the system

$$\begin{aligned} \frac{dx_1}{dt} &= -a_1 x_1 + x_2 + b_1 u + b_1 v \\ \frac{dx_2}{dt} &= -a_0 x_1 + b_0 u + b_0 v \\ \frac{dv}{dt} &= 0 \\ y &= x_1 \end{aligned} \tag{18}$$

3.2 Regulator design

Assuming that v(t) is a constant or slowly varying disturbance, the state space controller with constant disturbance rejection and nonzero constant setpoint w is given by [8]

$$u = -f_1 x_1 - f_2 x_2 - v + hw \tag{19}$$

With

$$h = \frac{a_0}{b_0} + f_1 - \frac{a_0 b_1 f_2}{b_0} - a_1 f_2 \tag{20}$$

The closed loop regulator poles of the controlled system (18) and (19) are defined with the characteristic polynomial:

$$s^2 + (a_1 + b_1 f_1 + b_0 f_2)s + (a_0 - a_0 b_1 f_2 + b_0 f_1 + a_1 b_0 f_2) = 0 \tag{21}$$

With pole placement the characteristic polynomial coefficients are fixed, and the regulator gains f_1 and f_2 can be calculated by equating the characteristic polynomial coefficients with the coefficients of equation (21). With a linear quadratic regulator the poles are fixed for a single input single output system and vice versa. So, if the control criterion is given by

$$\mathrm{Min!}\, J = \int [Ry^2 + u^2]\, dt \tag{22}$$

where R is a weighting factor for compromising between the squared error of the controlled variable y and of the control variable u. Then the closed loop regulator poles are the left half s-plane zeros [8] of

$$(s^2 + a_1 s + a_0)(s^2 - a_1 s + a_0) + R(b_1 s + b_0)(-b_1 s + b_0) = 0 \tag{23}$$

The closed loop regulator poles are then given with the zeros of

$$s^2 + r_1 s + r_0 = 0 \tag{24}$$

With

$$r_0 = \sqrt{a_0^2 + Rb_0^2} \tag{25}$$

$$r_1 = \sqrt{2r_0 + a_1^2 + Rb_1^2 - 2a_0}$$

3.3 Observer design

The controller is given with equations (19), (20), (21), (24) and (25). In equation (19) only x_1 is directly measured with y, but x_2 and v must be reconstructed from the measurement y and the state space model (18). We use a reduced order observer with the method given by Kwakernaak and Sivan [8] which leads to

$$\begin{gathered}
\frac{dq_1}{dt} = -k_1 q_1 + (b_0 - k_1 b_1)q_2 + (b_0 - k_1 b_1)u + (b_0 k_2 - a_0 + k_1 a_0 - k_1^2 - b_1 k_1 k_2)y \\
\frac{dq_2}{dt} = -k_2 q_1 - k_2 b_1 q_2 - k_2 b_1 u + (k_2 a_1 - k_1 k_2 - b_1 k_2^2)y \\
x_{2e} = q_1 + k_1 y \\
v_e = q_2 + k_2 y
\end{gathered} \tag{26}$$

Where x_{2e} is the estimate for x_2 and v_e is the estimate for v. The observer poles are given with the characteristic polynomial:

$$s^2 + (k_1 + b_1 k_2)s + b_0 k_2 = 0 \tag{27}$$

The pole related to the reconstruction of x_2 should be fast with respect to the regulator poles e.g. 3x faster than the fastest regulator pole. Because v is a constant or slowly varying the related pole is chosen in the neighbourhood of the regulator poles or slower. If one observer pole $\rightarrow -\infty$ it follows from (27) that $|k_2| \rightarrow \infty$. Also $|k_1| \rightarrow \infty$ except if the other pole lies on the zero of (16) i.e. lies on $-b_0/b_1$.

3.4 Complete controller

The complete controller is the combination of the regulator (19) and observer (26). With straightforward but tedious algebra it can be shown that the controller has the following structure

$$U(s) = \frac{1}{1+\tau s}\{k_p[\beta.W(s) - Y(s)] + \frac{k_i}{s}[W(s) - Y(s)] - k_d.s.Y(s)\} \qquad (28)$$

The term between the accolades has the two-degree-of-freedom (2DOF) PID structure, where $0< \beta <=1$ and the D-action for the setpoint is zero [6]. The PID parameters, β, k_p, k_i and k_d are expressions in the model parameters and the parameters f_1, f_2, k_1 and k_2. The time constant τ of the first order filter is given by

$$\tau = \frac{1}{k_1(1 - f_2 b_1) + f_2 b_0} \qquad (29)$$

In general, $\tau \rightarrow 0$ if the observer pole for $x_2 \rightarrow -\infty$, because then $k_1 \rightarrow \infty$. There are two exceptions:

a. $f_2 b_1 = 1$, then $\tau = b_1/b_0$. This case occurs when 1 regulator pole is placed on the system zero $-b_0/b_1$, see by substitution in equation (21).
b. $k_1 = b_0/b_1$, then also $\tau = b_1/b_0$. This case occurs when the observer pole for v is placed on the system zero $-b_0/b_1$, see by substitution in equation (27).

The PID Parameters can be directly calculated from the closed loop poles with the identity

$$(1+\tau s)(s^3 + a_1 s^2 + a_0 s) + (b_1 s + b_0)(k_p s + k_i + k_d s^2) \equiv (s^2 + r_1 s + r_0)(s + z_1)(s + z_2) \qquad (30)$$

Where the regulator poles polynomial coefficients (25) and observer pole $-z_1$ for x_2 and observer pole $-z_2$ for v are given. The solution of (30) for the PID parameters is given by

$$\tau = (b_0 - a_1 b_1 + a_0 b_1^2/b_0)/(a_0 b_1 - a_1 b_0 + b_1^2 p_1/b_0 - b_1^3 p_0/b_0^2 + b_0 p_3 - b_1 p_2)$$

$$k_p = [-a_0 + \tau(p_1 - b_1 p_0/b_0)]/b_0 \qquad (31)$$

$$k_i = p_0 \tau / b_0$$

$$k_d = (\tau p_3 - 1 - a_1 \tau)/b_0$$

In (31) p_0, p_1, p_2 and p_3 follow from the right hand side of (30) as

$$p_0 = r_0 z_1 z_2$$

$$p_1 = r_1 z_1 z_2 + r_0(z_1 + z_2) \qquad (32)$$

$$p_2 = z_1 z_2 + r_1(z_1 + z_2) + r_0$$

$$p_3 = z_1 + z_2 + r_1$$

When a regulator pole is on or close to the system zero a too fast observer pole $-z_1$ leads to non feasible high k_p and k_d. When the regulator pole is not too close to the system zero the observer pole $-z_1$ can go to $-\infty$ and $\tau = 0$. In that case (30) is simplified to the closed loop identity

$$(s^3 + a_1 s^2 + a_0 s) + (b_1 s + b_0)(k_p s + k_i + k_d s^2) \equiv (s^2 + r_1 s + r_0)(s + z_2) \qquad (33)$$

The solution of (33) is given by

$$
\begin{aligned}
A &= b_0\,(r_1+z_2)-b_1(r_0+z_2r_1)+b_1^2z_2r_0/b_0 \\
k_d &= (a_1b_0-a_0b_1-A)/(Ab_1-b_0^2) \qquad (34)\\
k_i &= z_2r_0(1+b_1k_d)/b_0 \\
k_p &= [(r_1+z_2)(1+b_1k_d)-a_1-b_0k_d]/b_1
\end{aligned}
$$

k_d is not defined if $Ab_1-b_0^2=0$, which means that one closed loop pole is located on the system zero $-b_0/b_1$, then the PID parameters should be calculated with (31).

To summarize the calculation of the PID parameters:

a. Choose weighting factor R.
b. Regulator poles are defined with (24) and (25).
c. Observer pole $-z_1$ for x_2 is chosen 3x faster than fastest regulator pole.
d. Observer pole $-z_2$ for load disturbance is chosen in the neighbourhood of the slowest regulator pole or slower.
e. PID parameters are calculated with (31) and (32).
f. With the factor β in (28) the servo behaviour to changes in setpoint w could be optimized e.g. with respect to overshoot or rise time.

More often than not, the regulator poles are not close to the system zero and the PID parameters calculations can be simplified to:

a. Choose weighting factor R.
b. Regulator poles are defined with (24) and (25).
c. Observer pole $-z_2$ for load disturbance is chosen in the neighbourhood of the slowest regulator pole or slower.
d. PID parameters are calculated with (34).
   With the factor β in (28) the servo behaviour to changes in setpoint w could be optimized e.g. with respect to overshoot or rise time.

Remark:
If a regulator pole is close to the system zero and it is required to use a PID controller (28) with τ=0 and parameters $\tilde{k}_p$, $\tilde{k}_i$ and $\tilde{k}_d$, an approximation could be obtained with

$$
\begin{aligned}
\widetilde{k_p} &= k_p-k_i\tau \\
\widetilde{k_i} &= k_i \qquad (35)\\
\widetilde{k_d} &= k_d-k_p\tau+k_i\tau^2
\end{aligned}
$$

4 PID controller tuning

The PID controller parameters are determined with the choice of the weighting factor R and $-z_2$ as the observer pole for v. Both parameters have an intuitive meaning. A higher value for R gives a higher weight on deviations of the output y. So, a higher value R gives faster control by faster regulator poles. However, a too high value for R makes the control sensitive to variations in the process parameters. A good practice is to choose R with a gain normalized value, i.e. tune R with R_n for a non integrating process as:

$$R=R_n.(a_0/b_0)^2 \qquad (36)$$

With $R_n=1$ as starting point.

A faster pole $-z_2$ gives more integral action so a higher value of k_i. Default could $-z_2$ be chosen equal to the real part of the slowest regulator pole.

Instead of using the intuitive tuning with R_n and z_2 we could use the MIGO method of advanced PID control techniques [10, chapter 4]. MIGO stands for M-constrained Integral Gain Optimization. The goal of MIGO is to maximize the integral gain of the PID controller while ensuring that the system remains robust for model inaccuracies. The mathematical formulation of MIGO is:

$$\begin{gathered} \text{Max! } k_i \text{ subject to:} \\ \left|\frac{1}{1+PC(\omega)}\right| \le M_s \\ \left|\frac{PC(\omega)}{1+PC(\omega)}\right| \le M_t \end{gathered} \tag{37}$$

Where $PC(\omega)$ is the loop transfer function of process and controller, M_s the maximum sensitivity and M_t the maximum complementary sensitivity. In the original MIGO design additional constraints are needed to avoid poor damping. In the here proposed method, we use as optimization parameters R and z_2 instead of directly k_p, k_i and k_d and with no additional constraints.

The MIGO optimization problem can easily been solved with the solver add-in of Microsoft Excel.

5 Examples

In this section the proposed PID tuning method is applied to various process models and compared, if possible, with PID settings in literature. In case MIGO is used for optimisation the constraints are taken as M_s=M_t=1.6.

5.1 Unstable process

Consider a process with the transfer function

$$P(s) = \frac{s+1}{(s-1)^2} \tag{38}$$

So, the parameters of (16) are b_1= 1, b_0 = 1, a_1 = -2, a_0 = 1. Note that this process is unstable with 2 poles at s = 1. The zero at s = -1 is the mirror image of the poles. One regulator pole will always be located on the system zero irrespective the choice of the weighting factor R. (For details about the regulator poles root loci see Kwakernaak and Sivan [8]). In this case the PID parameters have to be calculated with (31), with τ = 1.

Suppose we choose R=1, the regulator poles according to equation (24) and (25) are s_1=-1, s_2=-1.41. The observer pole $-z_1$ for x_2 cannot be placed too far in the left half s-plane because then the PID parameters will be too large. So, we choose z_1=4 ($\approx$ 3x1.41). The observer pole for the load disturbance is chosen as z_2=0.5. Equations (31) and (32) give k_p=7.36, k_i=2.83, k_d=7.91; Equation (35) gives $\tilde{k}_p = 4.54$, $\tilde{k}_i = 2.83$ and $\tilde{k}_d$ = 3.38. For both PID settings we use β = 1.

Figure 4 shows the responses of the system to changes in setpoint and load disturbance.

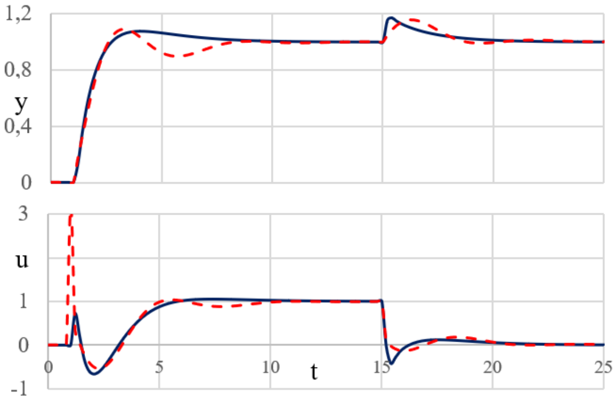

Figure 4. Responses to unit step change in setpoint (t=1) and load (t=15) for process $(s+1)/(s-1)^2$ with complete PID controller design (31), (32) (line) and approximation (35) (dashed line).

Both controller settings stabilise the system and have les than 10 % overshoot with the setpoint change. The approximation controller settings (35) give 10 % undershoot with the setpoint change whereas the controller with filter (31) has no undershoot. Initially the load disturbance rejection of the controller settings (35) seems better, but overall, the performance is worse.

5.2 Second order system with two time constants and dead time

Consider the system

$$P(s) = \frac{K.\,e^{-Ls}}{(1 + T_1 s)(1 + T_2 s)} \tag{39}$$

For the PID parameters calculation the dead time will be approximated with the Padé approximation (17) so that we can represent the process with equation (16). In this case the zero will be in the right half plane so the (stable) closed loop poles will not be close to this zero and we can use equation (34), with τ=0. The choice for R and z_2 is determined by optimization (37) with the choice of M_s=M_t=1.6. Note that in the calculation of the sensitivities the exact dead time (no Padé approximation) is used. The optimization has been performed for different L, T_1 and T_2 values. It appeared that the results can be fitted with the formulas given by Åström and Hägglund [10, p264] but with different parameter values ($T_1 \geq T_2$, $L \leq T_2$):

$$\begin{aligned} k_p &= \frac{0.18}{K} + \frac{0.39T_1 + 0.38T_2}{KL} + \frac{0.045T_1T_2}{KL^2} \\ k_i &= \frac{0.43}{KL} + \frac{0.023T_1 + 0.0077T_2}{KL^2} + \frac{0.0056T_1T_2}{KL^3} \\ k_d &= \frac{T_1 + T_2}{K(T_1 + T_2 + L)}\left(-0.031L + 0.12T_1 + 0.28T_2 + \frac{0.37T_1T_2}{L}\right) \end{aligned} \tag{40}$$

As illustration gives table I the PID settings (40) for a second order system with: K=1, L=20, T_1=100, T_2=40. For comparison also the PID settings of Skogestad [11] and Åström and Hägglund [10, p264] are given.

Table I. Comparison of PID settings for second order system with K=1, L=20, T_1=100, T_2=40

	kp	ki	kd	β	IAE setpoint	IAE load
Skogestad	3.50	0.025	100	1.0	83.5	40.0
Åström Hägglund	2.60	0.032	75.1	0	138.5	38.0
Proposed method	3.34	0.031	84.5	0.8	81.3	33.2

The choice for β=0.8 in the proposed method is such that the overshoot with setpoint step change is not higher than with Skogestad PID settings. The choice for β=0 in Åström and Hägglund settings was chosen for reducing the overshoot with setpoint change. Note that highest gain and lowest integral action are obtained with settings of Skogestad. The lowest gain and highest reset are obtained with MIGO of Åström and Hägglund. The settings of the proposed method for gain and reset are between. Finaly the integral absolute error of w-y with setpoint and load unit step change is given in table I. Figure 5 gives the setpoint and load change responses for the three PID settings.

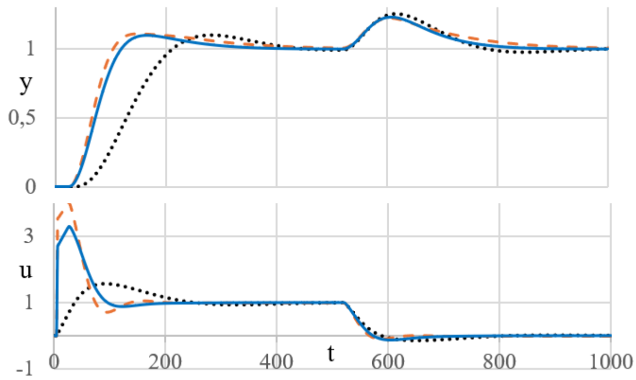

Figure 5. Responses to unit step change in setpoint (t=5) and load (t=500) for process $e^{-20s}/[(1+100s)(1+40s)]$ with PID controller settings from Skogestad (dashed line), Åström and Hägglund (dots) and proposed method (line).

Skogestad PID settings give the fastest response to setpoint change, although the settling time is longer than with the proposed method settings. Åström and Hägglund setpoint response is much slower because the setpoint factor $\beta=0$. The load IAE has the lowest value for the proposed method. Also, for load change Skogestad settling is slower, while Åström and Hägglund settings give some undershoot.

5.3 Underdamped process

Consider the process system with transfer function (10) where K=1, L=50, T=100 and relative damping $\vartheta=0.1$. In this case MIGO with optimisation parameters R and z_2 leads to the undesirable result of R=0 and z_2=-1,86. These parameter values give poor damping. Therefor we use MIGO with only R as optimisation parameter and where the pole $-z_2$ is chosen equal to the real part of the slowest regulator pole. The PID controller with this MIGO optimisation is compared with PID controller settings proposed by Hu et al [12] with the intention of minimisation of the integral time absolute error. Table II gives the PID parameter values and the IAE performances.

Table II. Comparison PID settings for underdamped process with K=1, L=50, T=100, $\vartheta=0.1$

	kp	ki	kd	β	IAE setpoint	IAE load
Hu et al	-0.032	0.00315	88.9	1.0	358.0	353.2
Proposed method	0.223	0.00352	73.6	1.0	283.7	287.7

The proposed method has better IAE performances. Also the setpoint and load disturbance responses are quite different (see figure 6).

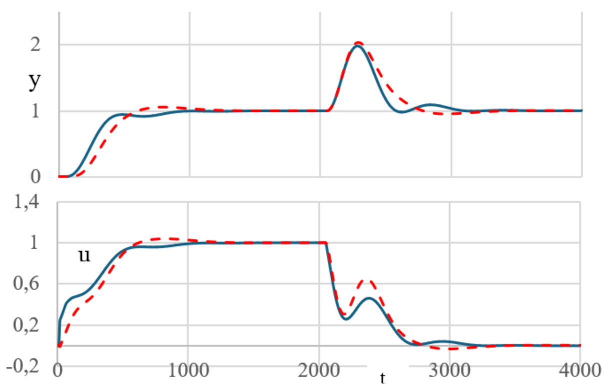

Figure 6. Responses to unit step change in setpoint (t=10) and load (t=2000) for process $e^{-50s}/(10000s^2+20s+1)$ with PID controller settings of Hu et al [12] (dashed line) and proposed method (line).

5.4 4th order process

Consider a process with the transfer function

$$P(s) = \frac{1}{(s+1)^4} \tag{41}$$

Graphical fitting of the model (1) to the step response of process (41) gives K=1, L_0=1.42 and T_0=2.9. This process was analysed for PID control with MIGO design [13]. With (6) the model (3) becomes K=1, L=0.98, T=1.56. The PID parameters are calculated with (40). Åström and Hägglund [13] determined the PID parameters directly with MIGO optimisation for the process given by (41). Table III gives the PID parameter values and the IAE performances.

Table III. Comparison of PID settings for process $1/(s+1)^4$

	kp	ki	kd	β	IAE setpoint	IAE load
Åström	1.19	0.54	1.43	0	5.20	2.33
Proposed method	1.52	0.50	1.15	0.7	3.29	1.99

The choice for β=0 was proposed in [13]. The choice for β=0.7 was such that the overshoot with a setpoint step change was less than with Åström and Hägglund PID settings. Again, the gain is higher, and the integral action is lower with the proposed method. Figure 7 gives the setpoint and load change responses for the two PID settings.

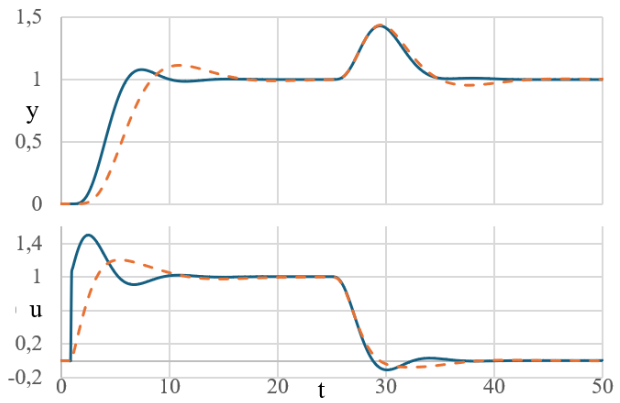

Figure 7. Responses to unit step change in setpoint (t=1) and load (t=25) for process $1/(s+1)^4$ with PID controller settings of Åström and Hägglund [13] (dashed line) and proposed method (line).

6 Conclusions

This paper introduces a novel analytic tuning method for PID control, utilizing a second-order dead time model which could e.g. be derived from graphical step response analysis. By employing a Linear Quadratic (LQ) regulator and combining state feedback with a reduced-order observer, we achieve a two-degree-of-freedom PID structure with prefilter. The prefilter can often be removed but is necessary with pole-zero cancellation. The PID controller parameters are calculated via pole placement through the selection of the LQ weighting factor R and observer pole $-z_2$ for load disturbances. The selection of these two parameters can be done with default values or with M constraint integral gain optimisation (MIGO).

Benchmark examples validate the method's effectiveness, demonstrating good control by a low IAE for both load disturbance as well for setpoint step changes. Overshoot with setpoint changes are minimised with a setpoint weighting factor β close to 1 resulting in a faster settling than with the traditional MIGO method.

Future research could explore extensions to state feedback PID tuning for the second order dead time system without use of a Padé approximation.

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