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Fence ownership laws and organic parcel mosaics: towards a complete, algorithmic regulation

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Abstract

Building laws establish responsibilities and limitations for various building activities that occur on the geometric mosaics of land parcels, such as the construction and maintenance of fences. Fence ownership can be determined by agreements between neighbors, local or national regulations based on the geometric properties of parcels, established customs, or cadastral parcel registries. Despite their advantages, geometric rules have lost popularity in the 20th century due to existing policies being incomplete and lacking clear guidelines for irregularly shaped parcels. In this paper, we identify the requirements for an effective fence ownership policy, including clarity, cost-effectiveness, robustness, and alignment with established rules. We also develop a family of geometric algorithms based on a historical regulation once widespread across Europe, which is still part of the building laws in several countries. The proposed algorithms meet all these requirements and can serve as the foundation for efficient future regulations.

Keywords
land law, fencing, algorithm, parcel

Introduction

Parcels are basic units of land ownership in modern societies (Lifshitz 2021) and the building blocks of urban fabric. The geometry of land lots depends heavily on the use and the historical development of an area (Forejt et al. 2018). Measuring geometric characteristics of parcel mosaics (Demetriou et al. 2013; Asami and Niwa 2008), tipology of parcel systems (Bobkova et al. 2021), their roles in land use (Wu et al. 2009; Song et al. 2021), and improvement of land use by changing parcel mosaics (Kwinta and Gniadek 2017; Leń 2018) are all fields of intensive research. New parcel allocations designed by engineers typically follow simple, repetitive patterns, even though topography and other constraints often complicate the picture. On the other hand, organic development of historical areas often results in highly irregular mosaics with imprints of historical development, transformation of land usage, and individual decisions (Dahal and Chow 2014) as illustrated by Fig. 1.

Building laws regulate various building activities, which take place on land parcels. For example, building regulations determine where buildings can be placed within a parcel, and how existing parcels can be subdivided or merged. Building laws should ideally treat all types of parcel mosaics within a common framework. The geometric variability of parcels makes the formulation of a good policy challenging. On the other hand, good laws have fundamental impact on the quality of the built environment and human life (Imrie 2004; Van der Heijden and De Jong 2009), as they protect resources like access to sunlight, and view, safety (e.g. fire regulations), and various rights (e.g. private property).

This paper examines the regulation of fence ownership in urban areas as a representative example of the application of law to to situations involving complex geometric structures. In many areas of the built environment, physical barriers (fences) between lots have beneficial effect on the quality of land use. Accordingly, the owners are often required to erect fences at parcel boundaries. Historically, the first fencing laws were constructed in agricultural areas in which plant crops had to be protected from the damage caused by grazing livestock and two types of regulation became widespread: in fence-in regulations, owners of livestock are responsible for preventing damage in adjacent crops, whereas fence-out regulations oblige plant producers to protect their areas (Kantor and Kousser 1993; Sanchez and Nugent 2000; Centner 1997).

Today, lots within urban areas in many parts of the world are also fenced for the sake of protecting privacy and private property, and different types of regulation exist (Table 1). In Section 2.1, we review common types of regulation, and argue that the optimal regulation is based on a geometric algorithm, which assigns each lot boundary to one of the two adjacent parcels. However it is found that existing algorithmic regulations are incomplete and not applicable to irregular parcel mosaics. One of the existing algorithmic Fence ownership laws (used in Hungary), originating from the 18th century Prussian Land Law is reviewed in Section 2.2. Then we finish Section 2 by proposing a set of properties that fence ownership regulations should ideally possess.

Section 3 is devoted to the construction of a family of algorithms, all of which are applicable to any mosaic including highly irregular ones. We compare the

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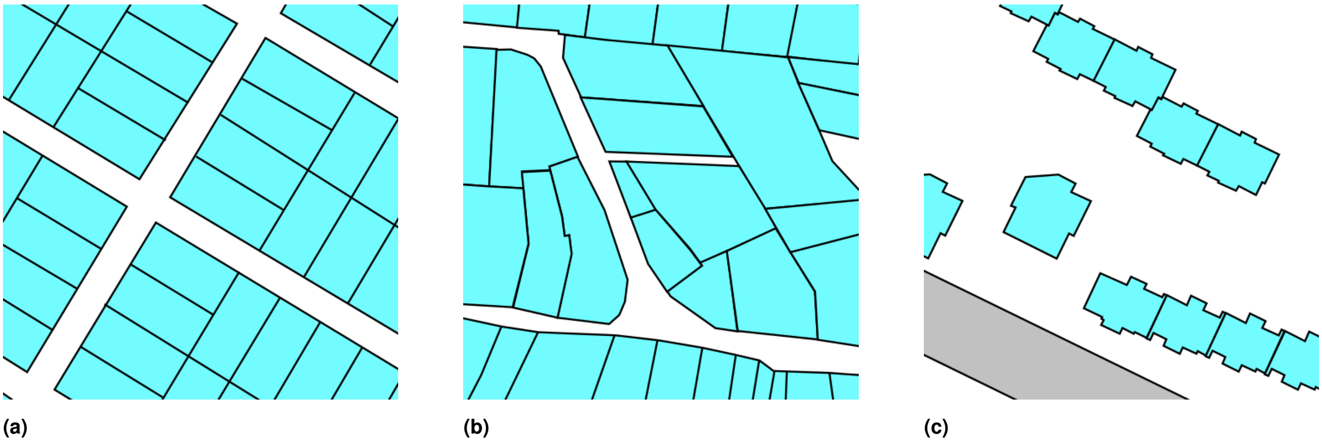


Figure 1. Examples of parcel mosaics in Solymár, Hungary: a) an engineered suburban residential area; b) an organic parcel mosaic in the historic city center; c) a housing estate in which parcel boundaries follow the contours of individual buildings, hence fence regulations are irrelevant.

performances of these rules by statistical examination of the parcel system in several parts of the town of Solymár in Central Hungary. Finally, the concluding section of the paper collects other problems in building law, where geometric algorithms or other mathematical techniques are required.

Table 1. Types of fence ownership regulation with examples of countries where they are applied.

Regulation	States
Modified versions of Prussian Land Law	Hungary, Austria and the states of Brandenburg, Berlin and Lower Saxony in Germany
Compulsory erection upon the request of either neighbor. Shared ownership, repair and maintenance costs.	Thailand, France, Germany (excluding 3 federal states)
Voluntary erection by either neighbor. Shared ownership and costs possible upon agreement.	USA, United Kingdom, Turkey

Existing laws and evaluation methods

Fence ownership regulations of urban areas

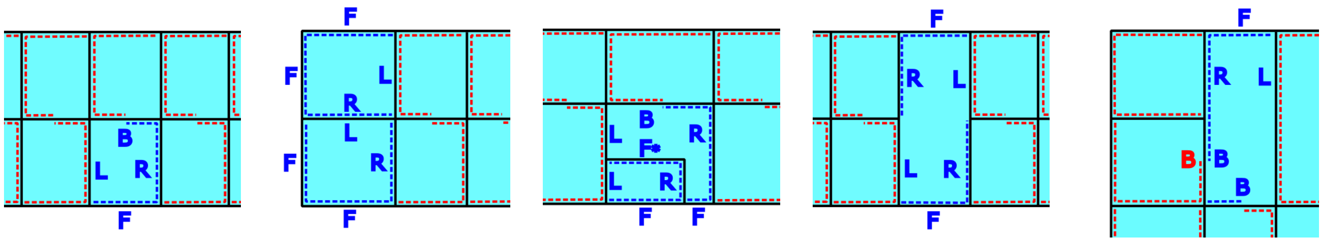
In many countries building law requires neighbors to erect and maintain fences jointly if requested by any of the two parties. Joint decision making and sharing of costs are common sources of conflicts and disputes, which often require costly and complicated legal action. In other countries fencing is not obligatory, and each owner is allowed to build a fence at their own cost within the boundary of the parcel. Even though the possibility of voluntary action resolves many disputes, the neighbors are often motivated to act against their joint benefit, by waiting for each other. Moreover the ownership of existing fences is often subject to debates. To avoid this issue, an extensive cadastral database is used in the United Kingdom where established fence ownership is recorded. However the database remains incomplete despite more than 60 years of continuous operation.

The apparent weaknesses of these approaches call for a simple, and efficient regulation. As first step towards algorithmic regulation, the land use law of Kingdom of Prussia effective in 1794 introduced a simple law based on geometric properties of land parcels. The law considered the common arrangement of rectangular parcels in a grid, where each parcel borders a public land along one of its edges (Figure 2). Every owner was ordered to build and maintain fences along the right side (viewed from the adjacent public land). The back of the lot was jointly owned by the two neighbors. The *Right Hand Rule* (RHR) of fence ownership became widespread over Europe in the 19th century. Later it was discontinued in most countries except Austria, Hungary, and three states within Germany, where different variants of the regulation are still effective. In many other countries, the Right Hand Rule still serves as a rule of thumb, albeit not legally binding. Clearly, the RHR is incomplete as it is limited to grid-like networks of rectangular parcels, and requires agreement between neighbors in other situations. Moreover in practice the law is often applied in borderline cases such as parcels with approximately rectangular shapes, but the exact limits of applicability are undefined, which may again lead to unresolvable disputes and legal consequences.

The Right Hand Rule of Hungary

Among the countries using the Right Hand Rule, Hungary uses the most detailed regulation. In Hungary, fencing lots in urban areas is compulsory upon the request of any of the two adjacent land owners. The regulation assumes that parcels are rectangular, and their edges are divided to four parts: front (F), right (R), back (B), and left (L) as in Fig. 2. Then the following rules are applied:

1. Each land owner is responsible for fencing the front and the right side.
2. the back part is divided to two sections of equal length, and land owners are responsible for fencing the section adjacent to the right (R) edge of their plot.
3. corner parcels have two front edges, as well as one right and one left side as in Fig. 2b.



(a) (b) (c) (d) (e)

Figure 2. Illustration of the Right Hand Rule (RHR) of fence ownership. Dashed lines mark the owner of a fence. (a): in a regular rectangular grid, owners build fences on the front (F) and right (R) edges. Fences of backward (B) edges are either built jointly (historical Prussian Land Law) or divided to two equal parts (current Hungarian law) as indicated in the figure. (b)-(d): extensions of the regulation of Hungary to corner plots, flag plots, and plots opening to two parallel streets. (e): extension of the rule to cases where the backward (B) edge of a plot meets another plot's lateral (R or L) edge.

4. in the case of a flag plot, the frontal edge of the main part (F*) belongs to the neighbor of the flag plot (Fig. 2c).
5. if a plot is adjacent to two parallel streets, then its internal edges should be divided to two parts (R and L) fitting in with adjacent plot boundaries or at the midpoint in the absence of such boundaries (Fig. 2d).
6. if a boundary is flagged as L or R in one parcel, but B in the other, then it should be treated as a B edge.
7. in those areas where zoning laws order the placement of buildings directly at the left boundary line of the parcel, the roles of L and R should be reversed.

Clearly, the regulation outlined above is not applicable, unless parcels are (approximately) rectangular, moreover the exact limits of applicability are undefined. In addition, the application of point 4 requires a precise definition of flag plots, which also creates ambiguity in some cases. It is also noteworthy, that the regulation fails to assign the ownership of a fence to one of the two adjacent land owners if parcels have shifted corners (Fig. 3).

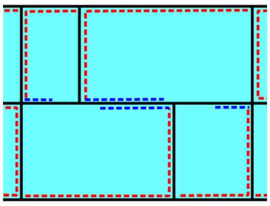


Figure 3. An example of erroneous fence ownership assignment set by RHR of Hungary.

As an additional limitation, it should be noted that private lots, which are not connected to public land (i.e. isolated parcels) cannot be labeled with F,R,B,L. This limitation is however acceptable, as isolated parcels have other fundamental problems due to lack of access, and are not allowed in most countries except for special situation (like transitional states of urban development or historically established parcel boundaries).

The fence ownership laws of Austria, and of the federal states of Saxen, Berlin, and Brandenburg within Germany are similar but simpler. Those laws contain nothing but the first rule of the Hungarian RHR regulation. Fences are jointly erected and owned in all other cases (such as all B edges, and all internal edges of plots with two parallel street fronts). In summary, all versions of the RHR are incomplete, and ambiguous in some situations.

Evaluation of algorithmic laws

Here we propose a list of six properties that a good fence ownership law should possess. A simple scoring system is also proposed. The first property is needed for the general applicability of the law. All others either ensure the sense of justice of those using the rule, or help to minimize the effort of fence erection.

1. Unambiguity: *each boundary point should be assigned to one and only one neighbor regardless of the geometry of parcels.*

Unambiguity is without doubt a crucial requirement. It is not satisfied by the RHR, however all new regulations investigated in the paper meet this requirement.

2. Stability: *an infinitesimally small change in the vertex coordinates of parcels may not induce but infinitesimally small change in the fence assignment.*

Stability implies that inaccurate knowledge or measurements of the parcel geometry enable the owners to make a reasonable estimate of fence ownership. A formal definition of stability would require the mathematical concept of *continuous functions*, however this is omitted for brevity.

3. Locality: *a localized change in the mosaic should not affect the ownership of distant boundaries.*

Localized changes include subdivision of a parcel, merger of two adjacent parcels, or a shift of the boundary between two parcels. We will distinguish between four levels of locality:

3: a change in the network does not affect fence ownership at any of the unchanged boundaries
2: a change in the network does not affect fence ownership anywhere else but at the boundaries of the plot(s) involved in the changes.
1: fence ownership does not change but at the boundaries of the changing plots and their immediate neighbours
0: none of the above

4. Fairness: *the overall length of fences belonging to a parcel is proportional to the size of that parcel.*

Unlike the previous properties, fairness is not evaluated using discrete categories. Instead a continuous scale is proposed. To test fairness, we investigate a sample containing n plots labeled by numbers $i = \{1, 2, ..., n\}$, and evaluate the ratios

$$U_i = O_i/P_i \tag{1}$$

for each plot. Here O_i is the total length of fence belonging to lot i, and P_i is its perimeter, where both lengths exclude the frontal edges. A low level of unfairness means that all U_i values are close to 0.5 with little fluctuation. Accordingly, we will estimate the degree of unfairness of each algorithm by using the *standard deviation* of the dataset formed by the corresponding U_i values:

$$U = \text{std}(\{U_i; i = 1, 2, ..., n\}). \tag{2}$$

High values of U hint towards the lack of fairness of the algorithm. Note however that the exact value of U is specific to the chosen sample. Hence, we will investigate several, relatively big samples with qualitatively different geometric structures.

5. Economy: *each owner is responsible for a small number of connected sections within the plot boundary.*

An economical fence assignment simplifies the organization of erection and also contributes to the uniform appearance of fences around a plot.

Economy will also be evaluated with the aid of a continuous scale, using a similar methodology to the case of fairness. Specific samples are considered and the ratios

$$E_i = S_i/N_i \tag{3}$$

are evaluated for each plot, where S_i is the number of disconnected fence sections belonging to a plot, and N_i is the number of its private neighbors (with a shared edge). The proposed division by N_i is motivated by the observation that it is easy to create an algorithm for which $S_i/N_i = 1$ for each plot.

The level of Economy is then characterized by the *average* value of E_i over the sample:

$$E = \text{avg}(\{E_i; i = 1, 2, ..., n\}).$$

A good level of economy corresponds to a value of E significantly below 1.

6. Similarity: *a new regulation is similar to established rules (if they exist).*

As an example, we will consider similarity to the RHR of Hungary. Similarity implies that a newly introduced law respects existing rights and obligations as much as possible. We will distinguish between four levels of similarity:

- 3: the fence assignment created using the new method is identical to the one created with the RHR whenever the RHR is applicable
- 2: the new method is identical to the RHR when applied to rectangular grids composed of two rows as in Fig. 2A-B (where plots of different widths are allowed) including the assignment of the corner plots.
- 1 point: the same as before, but differences are acceptable in the case of corner plots.
- 0 points: none of the above

New algorithms

A general framework using fencing functions

We now aim to define a general framework to fence ownership regulations in several steps. We allow polygonal parcels of arbitrary shape with the only restriction being that each private parcels has at least one edge adjacent to public land. Such edges will be called *frontal*, and other edges will be referred to as *internal*. An extension to mosaics including private parcels isolated from public land will also be discussed later.

As a preliminary step, private parcels are labeled by integer numbers $i = 1, 2, ...,$ and we choose a reference point O_i on the boundary of each parcel. We always choose O_i to be an interior point of a frontal edge. We then define for each parcel a parameterization of boundary points. Namely, parameter x is defined as the arclength along the boundary measured from point O_i in counter-clockwise direction. Hence the boundary points of parcel i correspond to $0 \le x < p_i$ where p_i is the perimeter of that parcel including frontal edges. Parameter x is used to easily identify individual points of the boundary. Then, a fence ownership regulation will be encoded by an appropriately chosen *fence function* $f(x, ...)$, which assigns a numerical value to each boundary point of each private plot. f may depend on any geometric property of that plot in addition to the value of x. Fence ownership will be determined according to the following rules:

R1: each internal boundary point belongs to the adjacent private plot in which a lower value of the fence function is assigned to that point.
R2: Frontal points belong to the only adjacent private plot.
R3: if the fence functions of two adjacent lots are equal over a finite section of their common boundary, then there are two possible rules for the corresponding boundary section:

(a) it is jointly owned or
(b) it is divided to two equal parts similarly to the regulation of Hungary

We will discuss the consequences of both versions for those fence function where this situation may occur.

Reformulation of the Right Hand Rule

The Right Hand Rule fits into the general framework introduced above. Assume that the edges of all private parcels in a mosaic are labeled by $F, R, B, L,$ and F^* as introduced previously. Let $f(x) = \{0, 1, 2, 3, 4\}$ for all points along $\{F, R, B, L, F^*\}$ edges of a parcel, respectively. The imposed regulation becomes equivalent to Hungarian Right Hand Rule law on any rectangular grid (potentially with flag plots). However recall that the Right Hand Rule relies on an initial labeling, which is in general not available for irregular plots.

In order to satisfy the requirement of unambiguity, we need to define fence functions that do not rely on the $\{F, R, B, L, F^*\}$ labeling. In what follows, several possible functions are introduced.

Auxiliary functions

As a preliminary step, we need to define several auxiliary functions. The first one is the tangent angle $t_i(x)$ of the boundary of plot i with respect to the tangent at the reference point O_i, measured in radian units. Note that this function is constant over straight edges, and undefined in corners. In corners, the tangent angle jumps, i.e. $t_i(x)$ has different left and right limits. We will denote the left limit by $t_i(x^-)$.

An additional pair of auxiliary functions will assign to each point x the closest frontal points if the boundary is traversed in anticlockwise or clockwise direction and will be denoted by $c_i^+(x)$ and $c_i^-(x)$. Note that $c_i^+(x) = c_i^-(x) = x$ if x is a frontal point, and both function values correspond to endpoints of a frontal edge if x belongs to an internal edge.

In order to define the last auxiliary function, the edges of plot i are enumerated by consecutive integers in counterclockwise direction as $1, 2, ..., n_i$ where n_i is the number of edges. Then, the auxiliary function $C(x)$ returns the index of the edge that point x belongs to. Similarly to the tangent angle function, C is undefined at corners, however it has well-defined left and right limits (which are the indices of the two adjacent edges).

New fencing functions

Now, we are ready to define some fence functions all of which satisfy the crucial requirement of unambiguity. All rules are evaluated and compared from the point of view of stability, locality, and similarity, and the results are summarized in Table 2. The investigation of fairness and economy require quantitative analysis on specific samples (except for a few simple cases), which will be done later.

1. Constant function: consider first a constant fence function $f_i(x) = 0$ along all boundaries. If algorithmic laws are defined as per Rule R3(a), then this function dictates joint ownership at all boundaries, i.e. it is the same as having no regulation at all. If, in contrast Rule R3(b) is adopted, the constant function means that each boundary between two adjacent lots is divided to two equal parts. Stability is obviously satisfied. This algorithm is ideal from the point of view of Fairness, and also enjoys the highest degree of Locality. In contrast, similarity is violated, as it is not at all similar to the Right Hand Rule. As we will see, the main weakness of this regulation is lack of Economy: each owner needs to build as many fence sections as the number of its private neighbors (N_i). For example, in a regular rectangular grid of parcels, each owner builds three disconnected section of the fence in contrast to the Right Hand Rule, which assigns one connected section to each parcel.

Tangent angle function: The next function is defined as

$$f_i(x) = \left[t_i(x) - t_i(c_i^-(x^-))\right] \mod 2\pi.$$

In everyday words, we consider the difference between the tangent angle $t_i(x)$ at the point in question, and the tangent angle at the closest frontal edge. If the difference is not within the interval $(0, 2\pi)$ then the modulo 2π operation shifts it by an integer multiple of 2π into that interval. This function assigns constant values to each edge, and constant 0 value to frontal edges. f_i is undefined at the corner points, however in practice that does not make any problem in fence assignment since the vertices are just a finite set of isolated points. In the case of rectangular mesh mosaics similar to Fig. 2, this function assigns values $\{0, \pi/2, \pi, 3\pi/2\}$, to $\{F, R, B, L\}$ edges. The imposed regulation is indeed equivalent to the Right Hand Rule, and the same goes for corner plots, but not for flag plots. Hence, this algorithm has a similarity score 2. At the same time, the requirement of stability is clearly violated. For B edges of rectangular grids, both neighbors have $f(x) = \pi$. Even the slightest perturbation of the corner angles affects the owner of a whole edge, which means lack of stability. In general, one can observe that a fencing function with constant sections violates stability, whereas strictly increasing functions do not. The locality score is 2. Note that the requirements of level 3 may be violated because the $c_i^-(x^-)$ values along a boundary section may change even if that section itself is left unchanged.

Arclength function: The next candidate function

$$f_i(x) = \left[x - c_i^-(x)\right] \mod p_i$$

simply measures distance from the frontal point closest in clockwise direction. p_i is the perimeter of plot i, and the $\mod p_i$ operation shifts the function values into the interval $(0, p_i)$. The function assigns 0 value to frontal points, and positive values to internal ones. The imposed rule again enjoys a locality score 2. Unlike the tangent function, this rule is stable. Furthermore, on a grid of identical rectangles, it is equivalent to the Right Hand Rule. However similarity to the right hand rule is not satisfied on a grid consisting of rectangles of different width, where the owners of narrower plots build more fences than according to the right hand rule, where as wider plots have less fence. Hence, the similarity score is 0.

Another fundamental problem with the arclength function is having a dimension of length, which means that the imposed regulation may discriminate based on the size of a plot. In particular the rule imposed by the arclength function tends to assign fence ownership to smaller plots (in terms of perimeter) as the average value of the arclength function is smaller along their boundaries (Fig. 5c). This property is likely to reduce Fairness, which will be demonstrated by examples in the sequel.

The size effect can be eliminated by nondimensionalization. For example we can define the dimensionless arclength function

$$f_i^*(x) = \frac{[x - c_i^-(x)] \mod p_i}{[c_i^+(x) - c_i^-(x)] \mod p_i}$$

which yields 0 at frontal points, and continuously grows from 0 to 1 along the connected internal section(s) of plot boundaries. The dimensionless arclength function is equivalent to RHR if plots of different width are arranged in a rectangular grid. Similarity is still not perfect though as the RHR is not observed at corner plots and in the case of double plots with two street fronts (Fig. 2d), i.e. the similarity score is 1. Stability, and the locality score are preserved.

Counting function: The counting function is defined by a textual description as follows. Starting from the reference point, we traverse the boundary of parcel i in counterclockwise direction. If we are on a frontal edge, then we assign $f_i(x) = 0$. On internal points, f_i is a positive integer, which is constant for the entire boundary section common with one neighbor. Whenever the boundary section with a new neighbor is reached, f_i increases by 1. Hence, the counting function counts neighbors in counter-clockwise direction along the boundary. This function has a similarity score 2. It is stable, as fence assignment is not at all affected by the geometry of the mosaic but only by its topological structure. The locality score is only 1. The requirements of level 2 are violated for example of the subdivision of a plot affects the number of neighbors of the adjacent plots.

'012' function: this function is similar to the counting function with the only difference being that all values larger than 2 are replaced by the value 2. Interestingly, the similarity, and stability properties are largely preserved, but the locality score is increased to 2.

Extension of the algorithms to isolated lots

At this point it is worth mentioning that all proposed algorithms exclude isolated lots, where the lack of frontal edges leave all functions undefined. Isolated plots may be present in a plot structure due to historical reasons, or as temporary state in an urban development process, such as isolated sections of a planned street. The main problem of isolated plots is the lack of physical access from public land, which is usually solved by an easement agreement or by customary law. If physical access of the plot is clarified, it is possible to extend every algorithm to isolated parcels by introducing an additional rule such as:

R4(a): an edge of an isolated parcel through which physical access to that parcel is assured should be considered as frontal. The corresponding fence belongs to the isolated parcel.

Alternatively, isolated parcel can also be handled by prescribing that

R4(b): fencing functions associated with isolated parcels have constant value of $+\infty$ (infinity).

which in practice means that edges between isolated and non-isolated parcels are assigned to the non-isolated parcel. In what follows, we well use R4(b) in the case studies.

Table 2. Preliminary evaluation results

Algorithm	Unambiguity	Stability	Locality	Similarity
Hungarian RHR	✗	✓	1	(3)
Constant function	✓	✓	3	0
Tangent angle function	✓	✗	2	2
Arclength function	✓	✓	2	0
Dimensionless arclength f.	✓	✓	2	1
Counting function	✓	✓	1	2
012 function	✓	✓	2	2

A case study: Solymár, Hungary

Next, specific samples are chosen, and used to evaluate fairness and economy on different types of plot mosaics. All samples belong to Solymár, a township with approximately 10.000 inhabitants in the suburban area of Budapest, in central Hungary (Fig. 4a).

Historical background and development of Solymár

The area of Solymár was already inhabited in the Bronze Age and during the Roman period. The first written record of the settlement dates back to 1266. It has been inhabited continuously since the early 18th century with a dominantly German population before World War 2.

The settlement is divided into several neighborhoods. The names of these are derived from the previously used German names, although some parts (Krautgarten, Hutweide) still bear their original names (Table 3). The parcel structure of each neighborhood has characteristic features reflecting its historical development. The following morphological types are distinguished:

- Structures that developed organically, in parallel with the gradual population growth of the settlement
- Narrow strips of agricultural parcels converted to residential use
- Regular, pre-planned mosaics developed as part of urban development projects
- Floating plots

Table 3. The types of parcel structures and the regions they are typical in

Type of structure	Example(s)
Organic	central area (5), Szél-hegy (1), Kakukkhegy (9)
Converted agricultural parcels	Krautgarten (7), Györgyyhegy (8)
Pre-planned	Kerekhegy (6) , Hutweide (2), Barackos (3)
Floating plots	housing estate (4)

Sample areas

We will evaluate the E and U values for each algorithm using three neighborhoods within Solymár with 195, 197 and 310 parcels (Fig. 4) representing pre-planned, organic, and converted agricultural areas. The housing estate is not part of the study, as there are no fences built in the area.

For pre-planned structures, most parcels are rectangular and organized in systematic rows, typically with two plots aligned back-to-back along their rear edges. This configuration reflects deliberate, engineered planning.

In agricultural areas, parcels are usually long and narrow, each having two front edges and two neighboring plots. One of the front edges frequently borders a topographical obstacle, such as a stream, a steep hillside, or a forested zone. When these agricultural plots are converted for residential use, they are often subdivided into smaller parcels, accompanied by the introduction of new streets. The selected sample represents such a transitional state, where fragments of future streets begin to appear in the mosaic as isolated parcels.

(a) **(b)**

(c) **(d)**

Figure 4. (a) location of Solymár withing Hungary, and overview the parcel structure of Solymár. Regions mentioned in Table 3 are marked by dotted lines and numbers. Grey fill marks the areas chosen for the evaluation, including an area with long, thin plots previously used for agriculture (b), an engineered area with plots mostly of the same size (c), and the historic city center with flag plots and irregularly shaped plots (d).

Finally, organic, historical structures display a much wider variety of parcel sizes and shapes. This irregularity makes them an ideal testbed for evaluating the robustness and adaptability of algorithmic regulations.

Evaluation for fairness and economy.

In a preliminary step, public parcels (such as streets) have been identified manually, and removed from the mosaic, leaving behind disconnected blocks of plots. Then, a custom-made code was developed in MatLab programming

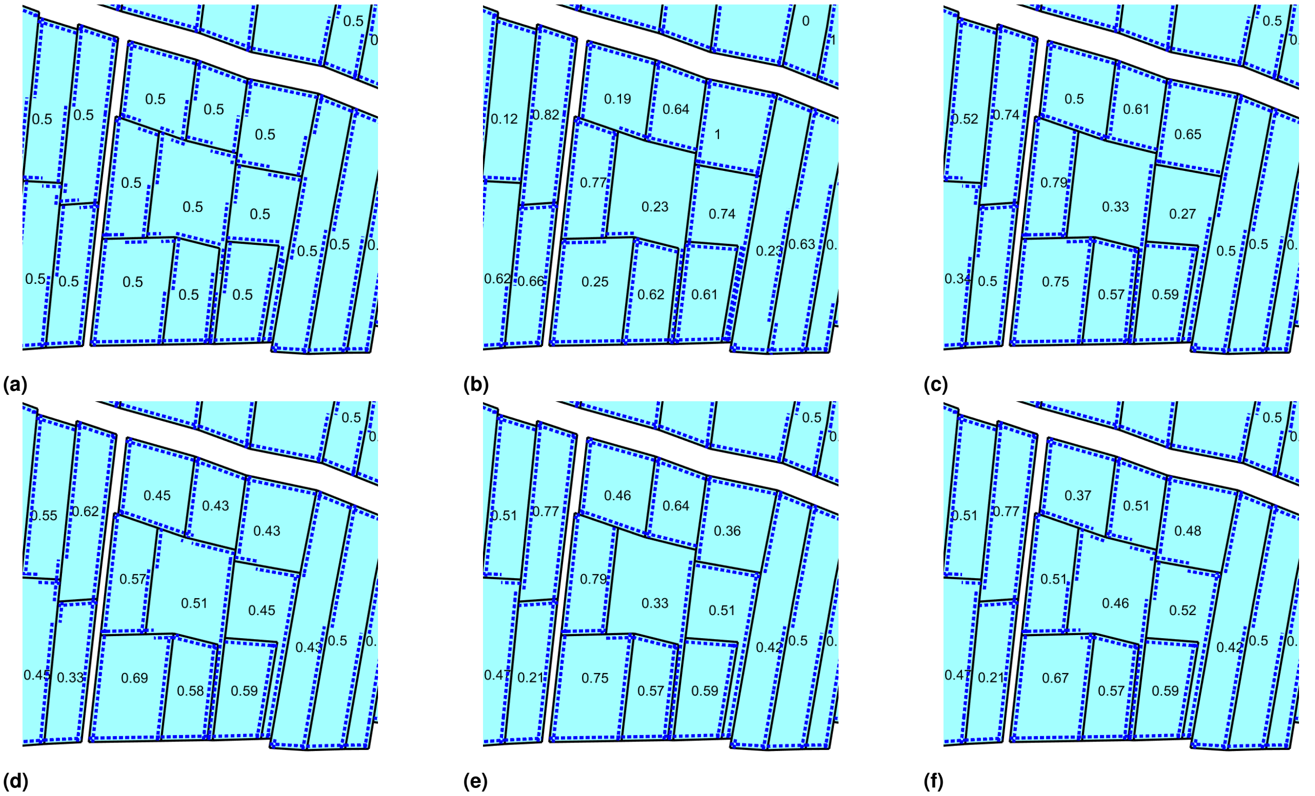


Figure 5. Comparison of the results of the algorithms for an area within the historic city center. Ownership is marked by dashed lines, and U_i values are shown for each plot: (a) constant function (b) tangent angle function (c) arclength function (d) dimensionless arclength function (e) counting function (f) 012 function.

environment, in order to identify individual parcels from the line drawing, with automated correction of small geometric inaccuracy of the original drawing (e.g. minor gaps between segments of boundary lines). Furthermore, the code also identified internal and frontal segments of boundaries, and isolated plots.

Then, all proposed fence ownership algorithms have been implemented, and fence ownership has been determined for the entire areas (Fig. 5). The results can be used to compare algorithms from the point of view of Fairness and Economy. Statistical results are summarized in Table 4.

Table 4 shows that none of the algorithms is perfect or superior to all others in terms of fairness and economy simultaneously. The constant function has trivially optimal U value, and simultaneously the worst E value, because in this case one fence section must be assigned for each neighbor. It is remarkable from the table that the tangent function features, remarkably advantageous Economy value, however high degree of unfairness. All other functions represent tradeoffs between these two extrema.

The comparison of the three sample areas reveals generally better algorithmic performance in the engineered area compared to the historic center. The former agricultural area typically exhibits intermediate results. These differences can be attributed to the varying degrees of geometric complexity inherent to each type of parcel mosaic.

Fig. 6 and Fig. 7 show more detailed statistics for each algorithm, and support further observations. Fig. 6a and Fig. 7a confirm expectations for the constant function. Fig. 6b reflects high variability of U_i values for the tangent angle function, i.e. the lack of fairness. Fig. 6c features a regression line with a relatively high slope, which marks the size-dependence of the algorithm induced by the arclength function, and explains why unfairness values are higher than for the dimensionless arclength function. Finally, Fig. 7f features a sharp increase in the number of fence sections for plots with 8 or more neighbors, which can be understood by noting, that this algorithm becomes similar to the constant function in the case of plots with many neighbors.

Discussion

The proposed set of requirements and the analysis of various algorithms highlight the inherent difficulty of designing a perfect regulation in the context of complex parcel mosaics. Specifically, it appears infeasible to construct a rule with level 3 locality—except for the trivial case of the constant function. Furthermore, achieving the highest level of similarity (level 3) using a simple algorithmic approach is nearly impossible, primarily due to the ad hoc characteristics embedded in the Hungarian RHR regulation. Nonetheless, the fundamental requirement of unambiguity was successfully met by all the algorithms considered.

Additional findings from the investigation enable us to identify the most promising candidates for future application. The constant function can be dismissed, as it merely represents the absence of regulation. Similarly, the tangent function proves disadvantageous due to its lack of robustness and its very high U values. The arclength function is also unsuitable, given its dependency on parcel size. In contrast, the remaining three algorithms emerge as viable candidates.

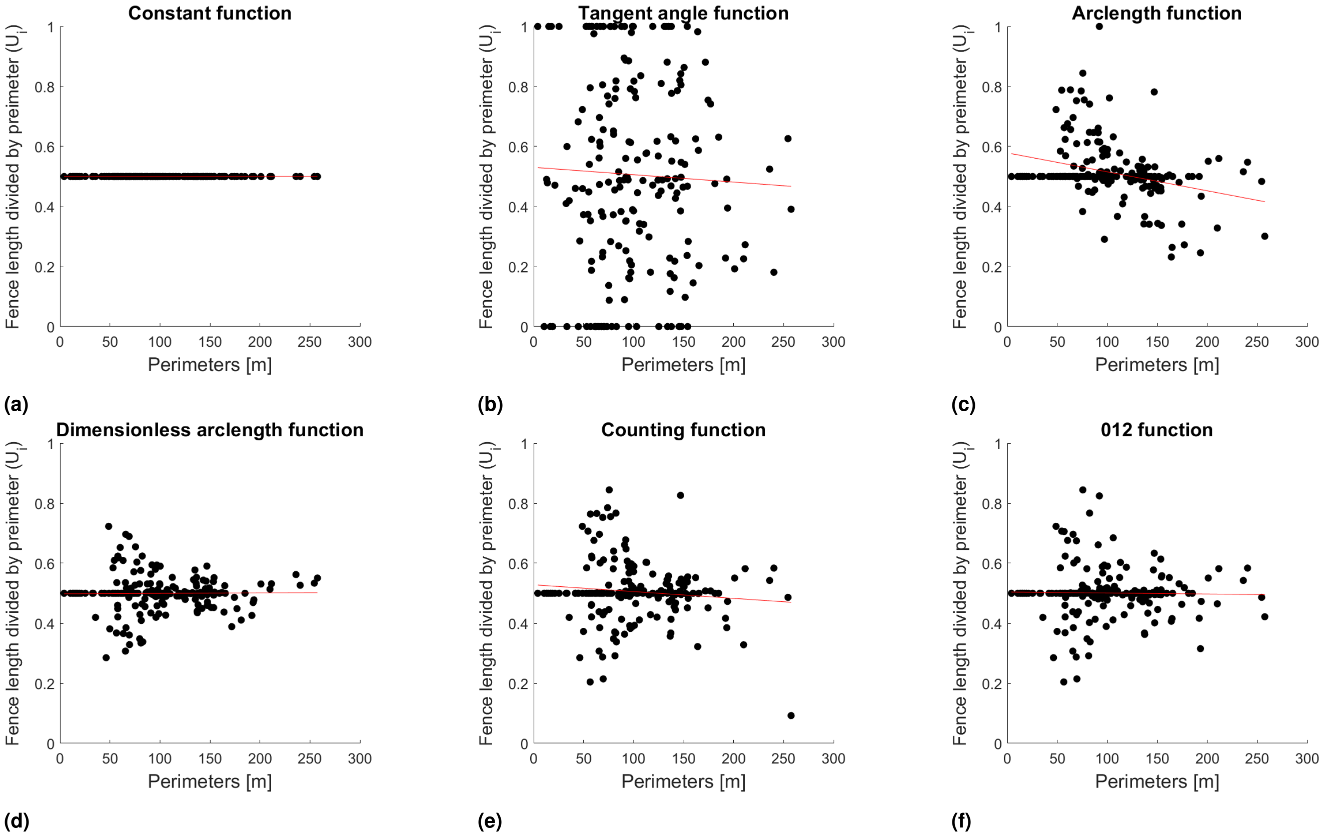


Figure 6. Diagrams showing U_i values of individual plots of the historic city center (dot markers) against plot perimeters for each algorithm. The red lines mark the results of linear regression.

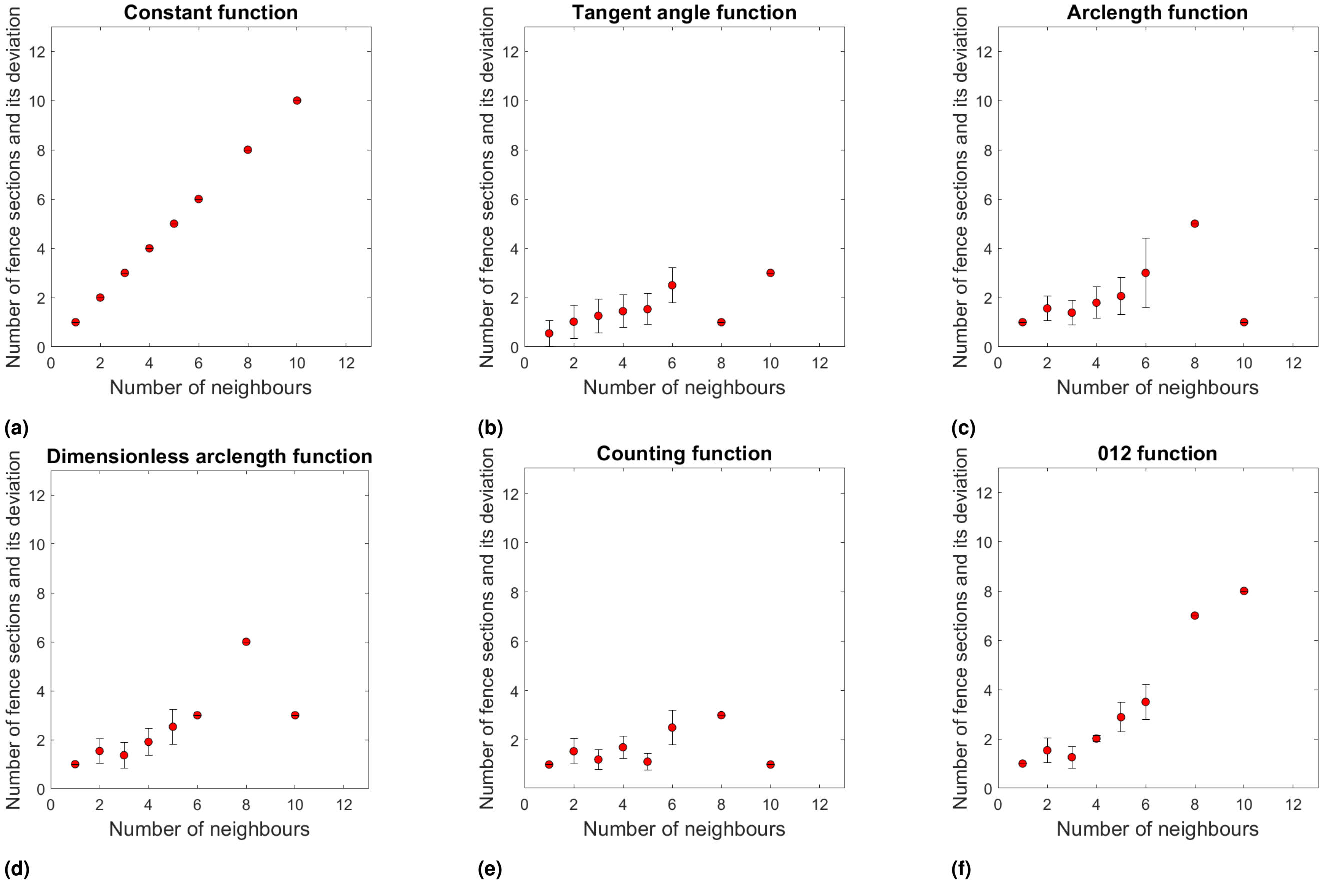


Figure 7. Diagrams showing the average (circle) and standard deviation (error bar) of the number of fence sections against the number of neighbours for all plots the agricultural area.

Table 4. Economy (E) and unfairness (U) scores for each algorithm evaluated over three sample areas within Solymár, Hungary.

	Algorithm	Engineered area	Historic city center	Agricultural area	Average
Economy (E)	Constant	1	1	1	1
	Tangent	0.3897	0.4388	0.4344	0.4240
	Arclength	0.5585	0.6164	0.5858	0.5869
	Dimensionless arclength	0.6049	0.6319	0.5935	0.6101
	Counting	0.5854	0.5845	0.5457	0.5719
	012	0.6123	0.6375	0.5949	0.6149
Unfairness (U)	Constant	0	0	0	0
	Tangent	0.3245	0.3219	0.3220	0.3228
	Arclength	0.0685	0.0988	0.0836	0.0836
	Dimensionless arclength	0.0438	0.0621	0.0604	0.0554
	Counting	0.0744	0.1010	0.1004	0.0919
	012	0.0692	0.0845	0.0808	0.0782

When similarity to the RHR is prioritized among the requirements, the 012 function appears to offer the best performance, achieving level 2 locality and similarity. However, if similarity is not a requisite, the dimensionless arclength function demonstrates slightly superior overall performance, combining level 2 locality with the lowest unfairness values among the tested algorithms.

We emphasize that, while fence ownership plays a moderate role in the economic aspects of estate development and land use, it serves as a representative example of legal regulations applicable to systems involving complex geometric structures. Hence it is an appropriate model problem in this respect. Similar challenges arise in other areas of building law, notably in the regulation of building positions and three-dimensional building geometries, both of which are of significant practical importance.

Placing our results in a broader context, it is worth noting that many fields of law require the use and formal codification of mathematical algorithms to address complex problems. Prominent examples include the regulation of elections (Norris 2004), voting mechanisms (Kóczy 2012) and educational admission systems (Biró et al. 2010), all of which rely heavily on concepts from mathematical game theory (Gertner and Picker 1994). We believe that the application of geometry and geometric algorithms in legal contexts likewise deserve greater attention in future research.

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