State-Dependent Gaussian Broadcast Channel with State Amplification

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***Abstract*—A state amplification problem, where the common additive state to a Gaussian broadcast channel (BC) is to be estimated at both receivers, is considered. The state process, known non-causally at the encoder, is assumed to be independent and identically distributed (i.i.d.) Gaussian. Both receivers must estimate the state process to within acceptable squared error distortion limits. In addition to the state estimation, our setting also requires message transmission to the stronger receiver at a given rate. We are interested in the optimal trade-offs between the distortions incurred at the receivers when a message at a given rate is to be delivered from the encoder to the strong receiver. A complete characterization of the rate-distortion trade-off region is presented. Our result differs from a recent result where an additional common reconstruction constraint was imposed on the state estimates in the same setting, and it was observed that allowing the weak user to decode part of the private message to the stronger user helps the distortion trade-offs.**

I. Introduction

Channels with state are used to model communication settings in which the channel conditional law is controlled by an external process known as *state*. There has been significant interest in the setting where the state information is available non-causally at the transmitter only, starting with the seminal paper by Gelfand & Pinsker [1]. The non-causal setting is highly relevant in the context of coding for memory with defective cells [2] and digital watermarking [3]. For the additive Gaussian state-dependent channel, Costa [4] arrived at the surprising conclusion that the capacity remains the same as that of a channel without state.

Certain state-dependent settings might require the transmitter to communicate the channel state in addition to message transmission. For instance, for the additive white Gaussian noise channel, Sutivong *et. al.* [5] characterized the optimal trade-off region for simultaneous message and state communication. The corresponding discrete memoryless model with state communication was investigated by Kim *et. al.* [6] using a list size reduction metric instead of distortion for state estimation.

The single-user result of Sutivong [5] has been extended in several works. Bross [7] showed that for joint message and state transmission, feedback can improve the optimal trade-off region in the case of causal side information. The dual problem where the sender wishes to mask the state instead of amplifying it, was addressed in [8]. Zhao *et al.* [9] studied source and message communication over a Gaussian BC without state dependence. In [10], a state-dependent Gaussian BC was considered with the goal of amplifying the channel state at one of the receivers while masking it from the other receiver, with no message transmissions. Simultaneous message and state communications was also studied in the case of action-dependent state channels [11] in [12], and in the case of multiple access channels in [13] – see also [14]. The work which is closest to what we study in this paper is [15], where inner and outer bounds were derived for simultaneous message transmission and state estimation over a state-dependent Gaussian BC. Unfortunately, the outer bound in [15] has an error as we point out in Section IV-A. Our converse proof in Section IV overcomes this for the special case where there is no message to the weak user.

Specifically, we study a state estimation problem over the Gaussian BC model shown in Fig. 1, where an additive i.i.d. Gaussian state process ($S \sim \mathcal{N}(0, Q)$) corrupts the transmissions. The channel state is known non-causally to the transmitter. Independent Gaussian noise processes, $Z_1 \sim \mathcal{N}(0, N_1)$ and $Z_2 \sim \mathcal{N}(0, N_2)$, are encountered in the respective links to the two receivers. Without loss of generality, we assume that receiver one encounters a smaller noise variance ($N_1 \leq N_2$), and is thus the strong receiver, while the second receiver is the weak receiver. The goal of our scheme is to reconstruct the state at each receiver to within some prescribed mean squared error distortion limit. Moreover, we are also interested in sending an independent message to the strong receiver. The transmitter is subject to an average power constraint. We wish to characterize the trade-off between the rate of message transmission and the distortions incurred in state estimation at the receivers. In Theorem 1, we derive the optimal rate versus distortions trade-off, with the main contribution being the converse proof.

Notably, our work differs from the one in [16] where the same Gaussian BC setup with a single rate to the strong user was studied, but with an additional common reconstruction [17] constraint on the state estimates. It was observed in [16] that sending a rate to the weak user might improve the distortion trade-offs even though only a private rate is intended to the strong receiver. However, it will be clear from our characterization in Theorem 1 that such a phenomenon does not occur in the absence of common reconstruction constraints.

The paper is organized as follows: we introduce the system model and main results in Section II. Sections III and IV provide the proofs of achievability and converse respectively. We conclude the paper in Section V.

II. System Model and Results

We start with some notational conventions. While denoting $U^n := U_1, \cdots, U_n$ is standard, we will also write U^n as $\mathbf{U}$. Thus capital letters in bold signify multi-letter variables, whereas the corresponding normal font represents single-letter variables. The Gaussian channel capacity function namely $C = \frac{1}{2}\log(1+SNR)$ will be denoted as $g(x) = \frac{1}{2}\log(1+x)$.

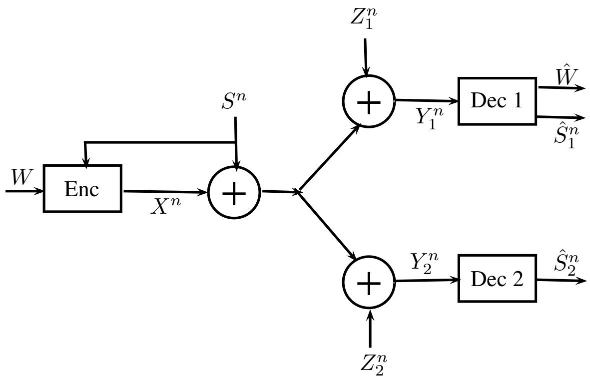

Fig. 1: Dirty Paper BC with State Reconstructions

The dirty paper BC with state is shown in Figure 1, with the receiver k observing,

$$Y_k = X + S + Z_k,$$

with $Z_k \sim \mathcal{N}(0, N_k)$, $S \sim \mathcal{N}(0, Q)$ and $S \perp\!\!\!\perp Z_k, k = 1, 2$. After n observations, decoder k estimates $\hat{\mathbf{S}}_k$, while the strong receiver also decodes the message W, which is assumed be uniform in $\{1, \cdots, 2^{nR}\}$.

Definition 1. *An $(n, R, D_1, D_2, \epsilon)$ state amplification scheme consists of an encoder map $\mathcal{E} : \{1, \cdots, 2^{nR}\} \times \mathcal{S}^n \to \mathcal{X}^n$, a decoding map $\psi : \mathcal{Y}_1^n \to \{1, \cdots, 2^{nR}\}$, and two receiver reconstruction maps $\phi_k : \mathcal{Y}_k^n \to \mathbb{R}^n$ such that for W uniformly distributed over $\{1, \cdots, 2^{nR}\}$, and $\mathbf{X} = \mathcal{E}(W, \mathbf{S})$, $k = 1, 2$*

$$\frac{\mathbb{E}[\|\mathbf{S} - \phi_k(\mathbf{Y}_k)\|^2]}{n} \leq D_k + \epsilon, \quad k = 1, 2, \tag{1}$$

$$\mathbb{P}(\psi(\mathbf{Y}_1) \neq W) \leq \epsilon, \quad k = 1, 2, \tag{2}$$

under an average power constraint $\mathbb{E}\|\mathbf{X}\|^2 \leq nP$.

We say that a triple (R, D_1, D_2) is achievable if a $(n, R, D_1, D_2, \epsilon)$ state amplification scheme exists for every $\epsilon > 0$, possibly by taking n large enough. Let $\mathcal{C}_{\text{est}}^{\text{bc}}(P)$ be the collection of all achievable (R, D_1, D_2) tuples, with $0 \leq D_k \leq Q$, $k = 1, 2$.

Our main result is stated now.

Theorem 1. *For the dirty-paper BC with state estimation requirement, the optimal trade-off region $\mathcal{C}_{est}^{bc}(P)$ between the rate and state estimation distortions is given by the convex closure of the set of $(R, D_1, D_2) \in \mathbb{R}_+^3$ such that*

$$R \leq \frac{1}{2}\log\left(1 + \frac{\gamma P}{N_1}\right), \tag{3}$$

$$D_1 \geq \frac{Q(N_1 + \gamma P)}{P + Q + N_1 + 2\sqrt{(1-\gamma)PQ}}, \tag{4}$$

$$D_2 \geq \frac{Q(N_2 + \gamma P)}{P + Q + N_2 + 2\sqrt{(1-\gamma)PQ}}, \tag{5}$$

for some $\gamma \in [0, 1]$.

Proof. The achievability is given in Section III. The converse is proved in Section IV. ■

III. Proof of Achievability

The achievability can be proved by power-sharing. This is similar to the inner bound in [15, Theorem 1], but we detail it for the sake of completeness. The strategy we employ to prove achievability is to split the available power at the encoder between message transmission and state amplification. The available power at the encoder i.e. P is split into two parts: namely γP for message transmission to the strong user and $(1-\gamma)P$ for state amplification, for some $\gamma \in [0, 1]$. Generate the state amplification signal:

$$X_S^n = \sqrt{\frac{(1-\gamma)P}{Q}} S^n. \tag{6}$$

Hence we can rewrite the channel model as follows:

$$Y_1 = X_M + X_S + S + Z_1 \tag{7}$$

$$Y_2 = X_M + X_S + S + Z_2, \tag{8}$$

where $\mathbb{E}[X_M]^2 \leq \gamma P$, with $X_M \perp\!\!\!\perp S$. Note that the subscript M in X_M denotes that the corresponding signal is intended for message transmission, while the subscript S in X_S is used to denote state amplification signals. Now in order to communicate the message across to the strong receiver, we employ the writing on dirty paper result for a Gaussian BC [18], which states that the capacity region of a state dependent Gaussian BC with non-causal state information at the encoder is the same as the capacity region of the Gaussian BC without state.

The scheme for achieving the rate in (3) can be described as follows: For receiver 1, consider the dirty paper channel with input X_M, known state $(X_S + S)$ and unknown noise Z_1. We choose $U_1 = X_M + \alpha_1(X_S + S)$, $X_M \perp\!\!\!\perp S$ with $X_M \sim \mathcal{N}(0, \gamma P)$ and $\alpha_1 = \frac{\gamma P}{\gamma P + N_1}$. The achievable rate is

$$R = \frac{1}{2}\log\left(1 + \frac{\gamma P}{N_1}\right). \tag{9}$$

Now we turn to the proof of the achievable distortions. Based on the observation Y_1^n, the first receiver forms the (linear) MMSE estimate

$$\hat{S}_1^n = \frac{(Q + \sqrt{(1-\gamma)PQ})Y_1^n}{P + Q + N_1 + 2\sqrt{(1-\gamma)PQ}}. \tag{10}$$

The MMSE can be readily calculated to be

$$D_1 = \frac{Q(N_1+\gamma P)}{P+Q+N_1+2\sqrt{(1-\gamma)PQ}}. \tag{11}$$

Based on the observation Y_2^n, the second receiver forms the (linear) MMSE estimate

$$\hat{S}_2^n = \frac{(Q+\sqrt{(1-\gamma)PQ})Y_2^n}{P+Q+N_2+2\sqrt{(1-\gamma)PQ}}. \tag{12}$$

The MMSE can be readily calculated to be

$$D_2 = \frac{Q(N_2+\gamma P)}{P+Q+N_2+2\sqrt{(1-\gamma)PQ}}. \tag{13}$$

This completes the proof of achievability.

IV. Proof of Converse

We first define some parameters that will be used in the sequel. Assume that (R, D_1, D_2) is achievable. Let n denote the blocklength. The following lemma from the single user result of [5] will be very useful towards the converse, its proof can be found in [5].

Lemma 1. *Any communication scheme achieving distortions* $D_{kn} \triangleq \frac{1}{n}\mathbb{E}[||S^n - \hat{S}_k^n||^2], k=1,2$ *over block length* n *will have*

$$\frac{n}{2}\log\left(\frac{Q}{D_{kn}}\right) \le I(S^n; Y_k^n), k=1,2. \tag{14}$$

The following notation is handy for any three random variables (F, G, H).

$$\sigma^2_{F|G,H} \triangleq \min_{\alpha,\beta} \mathbb{E}[F-\alpha G-\beta H]^2. \tag{15}$$

Let the instantaneous power of X_i be P_i for $i=1,2,\cdots,n$.

We now outline as to why setting $\beta=0$ in the outer bound of Liu *et al.* [15, Theorem 2] does not prove the converse.

A. Errors in the proof of Liu et al. [15]

Since only the strong user has a message in our setup, we can set $W_2=\varnothing$ in [15]. Thus only one parameter would be necessary to characterize the region. Let $\beta_i = 0$ (βP was allocated to the weak user's message in [15]). So as per the definition of γ in equation (53) of [15],

$$h(Y_2^n|X^n,S^n) \le h(Y_2^n|S^n) \tag{16}$$

$$\implies \frac{n}{2}\log(2\pi e N_2) \le h(Y_2^n|S^n) \le \frac{n}{2}\log(2\pi e(P+N_2)). \tag{17}$$

Hence there exists $\gamma \in [0,1]$ such that

$$h(Y_2^n|S^n) = \frac{n}{2}\log(2\pi e(\gamma P+N_2)). \tag{18}$$

By the method of [15], one would then proceed to select a set of parameters $\gamma_i, i=1,2,\cdots,n$ such that

$$\frac{1}{n}\sum_{i=1}^n \gamma_i P_i = \gamma P, \tag{19}$$

$$\mathbb{E}[X_i S_i] = \sqrt{(1-\gamma_i)P_i Q}, \tag{20}$$

which gives $\sigma^2_{X_i|S_i} = \gamma_i P_i$.

But we have

$$\begin{aligned}
h(Y_2^n|S^n) &\overset{(a)}{\le} \sum_{i=1}^n h(Y_{2i}|S_i) = \sum_{i=1}^n h(X_i+Z_{2i}|S_i)\\
&\overset{(b)}{\le} \sum_{i=1}^n \frac{1}{2}\log(2\pi e(N_2+\sigma^2_{X_i|S_i}))\\
&= \sum_{i=1}^n \frac{1}{2}\log(2\pi e(N_2+\gamma_i P_i))\\
&\overset{(c)}{\le} \frac{n}{2}\log\left((2\pi e)\left(N_2+\frac{1}{n}\sum_{i=1}^n \gamma_i P_i\right)\right)\\
&= \frac{n}{2}\log(2\pi e(N_2+\gamma P)),
\end{aligned} \tag{21}$$

where (a) follows since conditioning cannot increase differential entropy, (b) follows from the differential entropy maximizing property of Gaussian random variables for a fixed variance and the independence $Z_{2i} \perp\!\!\!\perp (X_i, S_i)$, while (c) follows from Jensen's inequality.

Now since $h(Y_2^n|S^n) = \frac{n}{2}\log(2\pi e(\gamma P+N_2))$ from (18), all inequalities leading to (21) above must be equalities. In particular, this would force

$$\gamma_i = \gamma \ \forall i \in [1:n], \tag{22}$$

which is overly restrictive.

This renders the outer bound on distortion D_2 in equations (81), (82) of [15] given by

$$\begin{aligned}
\frac{n}{2}\log\frac{Q}{D_{2n}} &\le I(S^n;Y_2^n)\\
&= h(Y_2^n) - h(Y_2^n|S^n)\\
&\overset{(a)}{\le} \sum_{i=1}^n h(Y_{2i}) - \frac{n}{2}\log(2\pi e(\gamma P+N_2))
\end{aligned} \tag{23}$$

incorrect, because the step (a) above would explicitly require the equality in (18). This motivates our solution, which is detailed next.

B. Solution

Define parameters $\gamma_i \in [0,1], i=1,\cdots,n$ such that

$$\mathbb{E}[X_i S_i] = \sqrt{(1-\gamma_i)P_i Q}. \tag{24}$$

Expression (24) also implies that

$$\sigma^2_{X_i|S_i} = \gamma_i P_i. \tag{25}$$

Based on the parameters above, select $\gamma \in [0,1]$ such that

$$n\gamma P = \sum_{i=1}^n \gamma_i P_i. \tag{26}$$

Now using (24) and (15), we further define

$$\sigma^2_{Y_1} \triangleq \frac{1}{n}\sum_{i=1}^n \sigma^2_{Y_{1i}} = P+Q+N_1+2\sqrt{(1-\gamma)PQ}, \tag{27}$$

$$\sigma^2_{Y_2} \triangleq \frac{1}{n}\sum_{i=1}^n \sigma^2_{Y_{2i}} = P+Q+N_2+2\sqrt{(1-\gamma)PQ}, \tag{28}$$

$$\sigma^2_{\hat{Z}|Z_2} \triangleq \frac{1}{n}\sum_{i=1}^{n}\sigma^2_{\hat{Z}_i|Z_{2i}} = \frac{N_1(N_2-N_1)}{N_2} \quad (29)$$

$$\sigma^2_{\hat{Z}|Y_2,S} \triangleq \frac{1}{n}\sum_{i=1}^{n}\sigma^2_{\hat{Z}_i|Y_{2i},S_i} = (N_2-N_1)\frac{(N_1+\gamma P)}{(N_2+\gamma P)}. \quad (30)$$

By Fano's Inequality, we can write:

$$H(W|Y_1^n) \leq n\epsilon_n, \quad (31)$$

where $\epsilon_n \to 0$ as $n \to \infty$. We shall occasionally suppress the $n\epsilon_n$ terms in the rest of the paper.

The rate constraint can now be proved as follows.

$$\begin{aligned}
nR &= H(W) \\
&\overset{(a)}{=} H(W|S^n) \\
&\overset{(b)}{\leq} I(W;Y_1^n|S^n) + n\epsilon_n \\
&= h(Y_1^n|S^n) - h(Y_1^n|W,S^n) + n\epsilon_n \\
&\overset{(c)}{=} h(X^n+Z_1^n|S^n) - h(Y_1^n|W,S^n,X^n) + n\epsilon_n \\
&= h(X^n+Z_1^n|S^n) - h(Z_1^n) + n\epsilon_n \\
&\overset{(d)}{\leq} \sum_{i=1}^{n}[h(X_i+Z_{1i}|S_i) - h(Z_{1i})] + n\epsilon_n \\
&\overset{(e)}{\leq} \sum_{i=1}^{n}\frac{1}{2}\log\left((2\pi e)\sigma^2_{(X_i+Z_{1i})|S_i}\right) \\
&\qquad - \frac{n}{2}\log(2\pi e N_1) + n\epsilon_n \\
&\overset{(f)}{=} \sum_{i=1}^{n}\frac{1}{2}\log\left((2\pi e)(N_1+\sigma^2_{X_i|S_i})\right) \\
&\qquad - \frac{n}{2}\log(2\pi e N_1) + n\epsilon_n \\
&= \sum_{i=1}^{n}\frac{1}{2}\log\left(\frac{N_1+\sigma^2_{X_i|S_i}}{N_1}\right) + n\epsilon_n \\
&= \sum_{i=1}^{n}\frac{1}{2}\log\left(1+\frac{\gamma_i P_i}{N_1}\right) + n\epsilon_n \\
&\overset{(g)}{\leq} \frac{n}{2}\log\left(1+\frac{1}{n}\sum_{i=1}^{n}\frac{\gamma_i P_i}{N_1}\right) + n\epsilon_n \\
&\overset{(h)}{=} \frac{n}{2}\log\left(1+\frac{\gamma P}{N_1}\right) + n\epsilon_n, \quad (32)
\end{aligned}$$

where (a) follows since $W \perp\!\!\!\perp S^n$, (b) follows from Fano's inequality, (c) follows since (W,S^n) determines X^n, (d) follows since conditioning cannot increase differential entropy, (e) follows from the differential entropy maximizing property of Gaussian random variables for a given variance, (f) follows since $Z_{1i} \perp\!\!\!\perp (X_i,S_i)$, (g) follows from Jensen's inequality and (h) follows from (26). Now the desired rate bound results by taking limits $n \to \infty$ in (32) and noting that $\epsilon_n \to 0$ as $n \to \infty$.

For the strong user's distortion, we proceed as follows:

$$\begin{aligned}
nR &+ \frac{n}{2}\log\frac{Q}{D_{1n}} \\
&\overset{(a)}{\leq} I(W;Y_1^n|S^n) + I(S^n;Y_1^n) + n\epsilon_n \\
&= I(W,S^n;Y_1^n) + n\epsilon_n \\
&= h(Y_1^n) - h(Y_1^n|W,S^n) + n\epsilon_n \\
&= h(Y_1^n) - h(Z_1^n) + n\epsilon_n \\
&\overset{(b)}{\leq} \frac{n}{2}\log\left(\frac{\sigma^2_{Y_1}}{N_1}\right) + n\epsilon_n \\
&\overset{(c)}{\leq} \frac{n}{2}\log\left(\frac{P+Q+N_1+2\sqrt{(1-\gamma)PQ}}{N_1}\right) + n\epsilon_n, \quad (33)
\end{aligned}$$

where (a) follows from Fano's inequality and Lemma 1, (b) follows from the differential entropy maximizing property of Gaussian random variables for a given variance and (c) follows from the definitions (27) through (30). Taking limits $n \to \infty$ in (33) and noting that $\epsilon_n \to 0$ as $n \to \infty$ gives the following outer bound:

$$D_1 \geq \frac{QN_1 2^{2R}}{P+Q+N_1+2\sqrt{(1-\gamma)PQ}}. \quad (34)$$

On evaluating the outer bound with $R = \frac{1}{2}\log\left(1+\frac{\gamma P}{N_1}\right)$, the desired bound on D_1 (as in expression (4)) results. Hence the outer bound in expression (34) is achievable.

For the weak user's distortion, we proceed as follows (note that $Z_2^n = Z_1^n + \hat{Z}^n$, where $\hat{Z} \sim \mathcal{N}(0, N_2-N_1)$ and $\hat{Z} \perp\!\!\!\perp Z_1$ without loss of generality):

$$\begin{aligned}
nR &+ \frac{n}{2}\log\frac{Q}{D_{2n}} \\
&\overset{(a)}{\leq} I(W;Y_1^n,Y_2^n|S^n) + I(S^n;Y_2^n) + n\epsilon_n \\
&= I(W;Y_1^n|Y_2^n,S^n) + I(W;Y_2^n|S^n) + I(S^n;Y_2^n) + n\epsilon_n \\
&= I(W,S^n;Y_2^n) + I(W;Y_1^n|Y_2^n,S^n) + n\epsilon_n \\
&\overset{(b)}{=} I(W,S^n;Y_2^n) + I(W;\hat{Z}^n|Y_2^n,S^n) + n\epsilon_n \\
&= h(Y_2^n) - h(Y_2^n|W,S^n) \\
&\qquad + h(\hat{Z}^n|Y_2^n,S^n) - h(\hat{Z}^n|Y_2^n,S^n,W) + n\epsilon_n \\
&\overset{(c)}{=} h(Y_2^n) - h(Z_2^n) + h(\hat{Z}^n|Y_2^n,S^n) - h(\hat{Z}^n|Z_2^n) + n\epsilon_n \\
&\overset{(d)}{\leq} \frac{n}{2}\log\left(\frac{\sigma^2_{Y_2}\sigma^2_{\hat{Z}|Y_2,S}}{N_2\sigma^2_{\hat{Z}|Z_2}}\right) + n\epsilon_n \\
&\overset{(e)}{\leq} \frac{n}{2}\log\left(\frac{(P+Q+N_2+2\sqrt{(1-\gamma)PQ})(N_1+\gamma P)}{N_1(N_2+\gamma P)}\right) \\
&\qquad + n\epsilon_n, \quad (35)
\end{aligned}$$

where (a) follows from Fano's inequality, since Y_2 is degraded w.r.t. Y_1 and from Lemma 1, (b) follows since $Y_2^n = Y_1^n + \hat{Z}^n$, (c) follows since (W,S^n) determines X^n and since $\hat{Z}^n \to Z_2^n \to (W,S^n,X^n,Y_2^n)$ forms a Markov chain, (d) follows from the differential entropy maximizing property of Gaussian random variables for a given variance and (e) follows from the definitions (27) through (30). Taking limits $n \to \infty$ in (35) and noting that $\epsilon_n \to 0$ as $n \to \infty$ gives the following outer bound:

$$D_2 \geq \frac{QN_1(N_2 + \gamma P)2^{2R}}{(P + Q + N_2 + 2\sqrt{(1-\gamma)PQ})(N_1 + \gamma P)}. \quad (36)$$

On evaluating the outer bound with $R = \frac{1}{2}\log\left(1 + \frac{\gamma P}{N_1}\right)$, the desired bound on D_2 (as in expression (5)) results. Hence the outer bound in expression (36) is achievable.

The trade-off between D_1 and D_2 for various values of R is shown in Fig. 2 for an example where $P = Q = N_1 = 1, N_2 = 2$. Clearly it is observed that the state estimation distortions increase as higher rates are demanded.

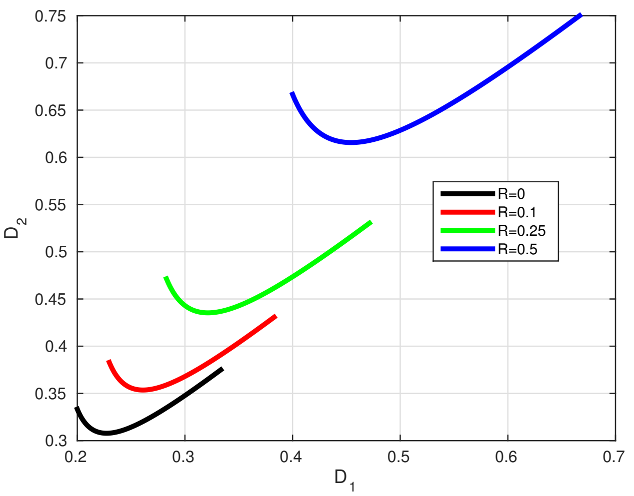

Fig. 2: D_1 versus D_2 for various R

V. Conclusion

In this paper, a complete characterization was derived for message and state communication over a state dependent Gaussian broadcast channel with a single message to the strong user. The differences with existing results for common reconstructions over the same model were pointed out. The general case with two independent messages remains open. The discrete memoryless broadcast channel (DM-BC) with two rates and state-estimation also remains open. While it is possible to obtain achievable regions for the same using standard techniques, obtaining matching converses seem difficult even for the degraded case. An interesting scenario would be to solve the general case with a distortion criterion for state estimation. This will be part of our future investigations.

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