Incentive-Driven Small-Cell Caching with Competing Mobile Operators: An Advanced Game-Theoretic and Stochastic Optimization Study

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Abstract

We conduct a detailed and advanced analysis of incentive-driven small-cell caching in multi-operator mobile networks. Building upon Stackelberg game theory and stochastic geometry, we extend the literature by integrating fairness, learning-based dynamics, time-varying popularity, multi-retailer competition, and spatial coupling. New theorems establish equilibrium existence, uniqueness, and efficiency bounds under uncertainty and asymmetric information. A novel iterative best-response algorithm is proposed, and extensive simulation studies on real-world urban topologies validate our framework. Results highlight significant improvements in efficiency, fairness, and operator sustainability when budget-aware, fairness-constrained mechanisms are applied.

1 Introduction

Mobile networks worldwide face unprecedented demand due to video traffic. Recent estimates indicate that over 70% of all mobile data traffic is video (2), with urban areas experiencing the highest congestion. A promising solution is small-cell caching, where frequently requested content is proactively placed at SBSs to reduce latency and backhaul load.

However, when multiple NSPs own SBS infrastructure and one or more VRs lease SBS capacity, strategic pricing and allocation decisions become a complex economic game. Existing research often assumes a single VR and homogeneous operators with static user demand. In practice, operator heterogeneity, time-varying popularity, and fairness concerns complicate the dynamics.

This work contributes the following:

- A generalized multi-leader multi-follower Stackelberg game model incorporating fairness and dynamic popularity.
- Novel closed-form success probability bounds and operator profit functions under stochastic interference.
- Proofs of equilibrium existence and uniqueness even under budget constraints and asymmetric costs.
- A scalable distributed algorithm with convergence guarantees.
- Comprehensive simulations validating efficiency-fairness trade-offs.

2 System Model

We consider a heterogeneous cellular network deployed over an urban area, characterized by a dense overlay of small-cell base stations (SBSs) operated by multiple, competing network service providers (NSPs). The physical and economic interactions in such a network are captured through a stochastic-geometric and game-theoretic framework.

Specifically, the spatial distribution of SBSs belonging to NSP l, where $l \in \{1, 2, \ldots, L\}$, is modeled as a homogeneous Poisson point process (PPP) Φ_l on $\mathbb{R}^2$, with intensity parameter λ_l (in units of SBSs per km^2). This assumption captures the irregular, yet statistically stationary deployment of infrastructure in realistic urban environments. Mobile users (MUs) subscribing to NSP l are modeled as another independent PPP Ψ_l with intensity ζ_l

(MUs per km^2). The independence of Φ_l and Ψ_l reflects the natural decoupling between infrastructure deployment and user mobility at the timescale of interest.

Each SBS is equipped with a finite cache of size Q_l, expressed as the number of unique video files it can store. The total catalog of files consists of F distinct items, whose popularity follows a Zipf distribution parameterized by exponent $\beta > 0$. That is, the probability q_f that file f (ranked $f = 1$ for most popular) is requested by an MU is given by:

$$q_f = \frac{f^{-\beta}}{\sum_{i=1}^{F} i^{-\beta}}, \quad f = 1, \dots, F.$$

Higher β implies a steeper decay in popularity, concentrating demand on fewer files.

In the proposed system, a video retailer (VR) seeks to lease a fraction $\tau_l \in [0, 1]$ of SBSs operated by NSP l, paying a price s_l per SBS per period. Leased SBSs dedicate their cache exclusively to storing the top Q_l most popular files, ensuring that every MU attached to a leased SBS can immediately retrieve the requested content if it is cached.

From the physical layer perspective, the wireless channel exhibits standard path-loss attenuation with exponent $\alpha > 2$ and Rayleigh fading. The transmit power per SBS of NSP l is P_l, and thermal noise is modeled as Gaussian with variance σ^2. The signal-to-interference-plus-noise ratio (SINR) experienced by a typical MU located at the origin, served by its nearest leased SBS, is:

$$\mathrm{SINR} = \frac{P_l h_0 \|x_0\|^{-\alpha}}{\sum_{x \in \Phi_l \setminus \{x_0\}} P_l h_x \|x\|^{-\alpha} + \sigma^2},$$

where x_0 denotes the location of the serving SBS and h_x are independent exponential random variables with unit mean, representing fading gains.

An MU's request is successfully served locally if its requested file is cached at the nearest leased SBS and the downlink SINR exceeds a predefined threshold δ. Accordingly, the probability that a randomly chosen MU associated with NSP l retrieves file f from a leased SBS is defined as the product of two components: the probability that f is among the Q_l cached files, and the coverage probability that SINR $\geq \delta$. The former is trivially one if $f \leq Q_l$, and zero otherwise; the latter is denoted as $P_{\text{cov}}(\tau_l)$ and depends on τ_l, since increasing τ_l densifies the leased SBSs and reduces user-to-SBS distance.

Summarizing, the key model parameters are collected in Table 1.

Table 1: Symbol Definitions

Symbol	Description
λ_l	SBS intensity (NSP l) [km^{-2}]
ζ_l	MU intensity (NSP l) [km^{-2}]
Q_l	Cache capacity of SBS (NSP l) [files]
β	Zipf popularity exponent
s_l	Price per leased SBS (NSP l) [$]
c_l	Cost per SBS (NSP l) [$]
S	VR budget per period [$]
τ_l	Fraction of leased SBSs (NSP l)
P_l	Transmit power of SBS (NSP l) [W]
α	Path-loss exponent
δ	SINR threshold [dB]
σ^2	Noise variance

This system model serves as the foundation for the subsequent economic and game-theoretic analysis, where NSPs and the VR optimize their respective strategies based on these physical and economic parameters, aiming to reach an equilibrium that balances individual profit, system efficiency, and fairness across stakeholders.

3 Economic Objectives

In the multi-operator small-cell caching ecosystem under consideration, the primary economic interaction arises between the competing network service providers (NSPs), who own and operate the physical small-cell base station (SBS) infrastructure, and the video retailers (VRs), who wish to lease SBS resources to enhance the quality of their content delivery services. This interaction is driven by conflicting objectives: while each NSP aims to maximize its own revenue by leasing SBS capacity at profitable prices, the VR seeks to minimize its expenditure while securing sufficient SBS coverage to improve user experience and maximize its own profit.

We formalize these objectives mathematically, accounting for both physical-layer constraints and economic incentives.

3.1 Video Retailer Objective

The VR's revenue is derived from improved user satisfaction when mobile users (MUs) successfully retrieve requested content from a leased SBS cache. Specifically, each MU generates on average K content requests per period, and each successful cache hit on a leased SBS yields a unit revenue ς to the VR. The revenue per unit area (e.g., per km^2) earned by the VR is therefore proportional to the MU density, the popularity mass of cached files, and the local download success probability.

For NSP l, the popularity mass Θ_l stored on leased SBSs equals:

$$\Theta_l = \sum_{f=1}^{Q_l} q_f = \sum_{f=1}^{Q_l} \frac{f^{-\beta}}{\sum_{i=1}^{F} i^{-\beta}}.$$

The expected revenue density from NSP l-associated MUs is then:

$$R_{\text{VR},l} = \varsigma K \zeta_l \Theta_l P_{\text{cov}}(\tau_l),$$

where $P_{\text{cov}}(\tau_l)$ denotes the probability that an MU of NSP l receives its requested file directly from a leased SBS, which is increasing and concave in τ_l.

Summing across all L operators, the total revenue density of the VR per period becomes:

$$R_{\text{VR}} = \sum_{l=1}^{L} R_{\text{VR},l}.$$

On the cost side, the VR pays leasing fees to NSPs based on the fraction of SBSs leased and their respective prices s_l. The total expenditure density of the VR is:

$$C_{\text{VR}} = \sum_{l=1}^{L} \tau_l \lambda_l s_l.$$

Assuming a fixed budget S per period, the VR's leasing strategy $\{\tau_l\}$ must satisfy the budget constraint:

$$C_{\text{VR}} \leq S.$$

The VR's objective is to choose the vector of leasing fractions $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_L)$ that maximizes its net profit density:

$$\Pi_{\text{VR}} = R_{\text{VR}} - C_{\text{VR}},$$

subject to $\tau_l \in [0, 1]$ for all l and $C_{\text{VR}} \leq S$. This is a constrained concave optimization problem, reflecting the VR's trade-off between leasing more SBSs to boost cache-hit revenue and conserving budget.

3.2 Network Service Provider Objectives

Each NSP, on the other hand, is motivated to monetize its existing infrastructure by leasing SBS capacity at prices that cover operational costs and yield surplus profit. For NSP l, the operational maintenance cost per SBS per period is c_l, and its chosen leasing price is s_l. Assuming the VR leases a fraction τ_l of SBSs, the profit per unit area of NSP l is:

$$\Pi_{\text{NSP},l} = \lambda_l \tau_l (s_l - c_l).$$

The NSP's optimization problem is to set the leasing price $s_l \geq c_l$ that maximizes $\Pi_{\text{NSP},l}$, taking into account the VR's best-response leasing strategy $\tau_l^*(s_l)$ that depends on the entire price vector $\mathbf{s} = (s_1, \ldots, s_L)$.

This introduces an interdependency between players: NSPs act as leaders, anticipating the VR's optimal reaction, while the VR acts as a follower, solving its own constrained optimization problem given the prices. The resulting interaction is naturally modeled as a hierarchical Stackelberg game.

3.3 Joint Considerations

It is worth noting that, although each player optimizes its own profit, their decisions collectively determine key system-wide metrics such as total SBS utilization, MU cache-hit rate, and overall social welfare. Therefore, an equilibrium allocation balances individual incentives with collective efficiency. In subsequent sections, we analyze the equilibrium properties of this game, demonstrating existence, uniqueness, and efficiency bounds under budget constraints and heterogeneous operator characteristics.

4 Equilibrium Analysis

The strategic interaction between the network service providers (NSPs) and the video retailer (VR) constitutes a hierarchical decision-making process that is naturally modeled as a Stackelberg game. In this game, the NSPs act as leaders, independently setting their leasing prices $\mathrm{s} = (s_1, \ldots, s_L)$ to maximize their own profit, while anticipating the VR's optimal reaction. The VR, acting as a follower, observes the announced prices and determines its optimal leasing fractions $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_L)$ to maximize its net profit, subject to budget constraints.

This section formalizes the equilibrium concept, derives the necessary optimality conditions, and establishes rigorous existence and uniqueness results for the game's equilibrium under mild assumptions on the system parameters.

4.1 Stackelberg Equilibrium Definition

We define a *Stackelberg equilibrium* $(\mathrm{s}^*, \boldsymbol{\tau}^*)$ as a pair of price and leasing strategies such that:

1. Given the prices s^*, the VR's choice $\boldsymbol{\tau}^*$ maximizes its net profit Π_{VR}:
$$\boldsymbol{\tau}^* \in \arg \max_{\boldsymbol{\tau} \in [0,1]^L,\, C_{\text{VR}} \leq S} \Pi_{\text{VR}}(\boldsymbol{\tau}; \mathrm{s}^*),$$
where $C_{\text{VR}} = \sum_{l=1}^{L} \tau_l \lambda_l s_l$.
2. Each NSP l chooses s_l^* to maximize its profit $\Pi_{\text{NSP},l}$, anticipating the VR's optimal reaction $\boldsymbol{\tau}^*(\mathbf{s})$:
$$s_l^* \in \arg \max_{s_l \geq c_l} \Pi_{\text{NSP},l}(s_l, \tau_l^*(\mathbf{s})),$$
where $\Pi_{\text{NSP},l} = \lambda_l \tau_l^*(\mathbf{s})(s_l - c_l)$.

At equilibrium, no NSP has incentive to unilaterally change its price, and the VR has no incentive to change its leasing fractions given the announced prices.

4.2 VR's Best Response

The VR's problem for fixed s is a convex optimization:
$$\max_{\boldsymbol{\tau}} \quad R_{\text{VR}}(\boldsymbol{\tau}) - C_{\text{VR}}(\boldsymbol{\tau}) \quad \text{subject to } C_{\text{VR}} \leq S,\ 0 \leq \tau_l \leq 1.$$

The objective function is concave in $\boldsymbol{\tau}$ because the coverage probability $P_{\text{cov}}(\tau_l)$ is concave and increasing in τ_l, and the leasing cost is linear in τ_l. By standard convex analysis, the KKT conditions are necessary and sufficient for optimality:
$$\begin{aligned}
\frac{\partial R_{\text{VR}}}{\partial \tau_l} - \lambda_l s_l - \lambda \lambda_l s_l + \mu_l^+ - \mu_l^- &= 0, \\
\lambda \geq 0, \quad \lambda \left(\sum_{l=1}^{L} \tau_l \lambda_l s_l - S \right) &= 0, \\
0 \leq \tau_l \leq 1, \quad \mu_l^+, \mu_l^- &\geq 0.
\end{aligned}$$

Here, λ is the Lagrange multiplier associated with the budget constraint, and μ_l^+, μ_l^- are multipliers for box constraints on τ_l.

4.3 NSPs' Optimization

Each NSP l solves a univariate maximization problem, taking as given the VR's best response. The profit function $\Pi_{\mathsf{NSP},l}(s_l)$ is generally concave in s_l, as higher prices reduce demand $\tau_l^*(s_l)$, while low prices reduce margin $s_l - c_l$. First-order conditions yield:

$$\frac{d\Pi_{\mathsf{NSP},l}}{ds_l} = \lambda_l \left(\frac{\partial \tau_l^*}{\partial s_l}(s_l - c_l) + \tau_l^* \right) = 0.$$

The equilibrium price s_l^* thus balances the trade-off between leasing price and leasing fraction.

4.4 Existence of Equilibrium

Theorem 1 (Existence). If the VR's revenue function R_{VR} is continuous, concave, and strictly increasing in τ_l, and the NSPs' cost $c_l > 0$, then the Stackelberg game admits at least one equilibrium.

Proof Sketch. The VR's best-response correspondence is convex-valued and upper hemi-continuous due to convexity. The NSPs' optimization is concave in s_l given $\tau_l^*(s_l)$. By applying Kakutani's fixed-point theorem to the composite best-response mapping, existence follows.

4.5 Uniqueness of Equilibrium

Theorem 2 (Uniqueness). If $P_{\mathsf{cov}}(\tau_l)$ is strictly concave and differentiable, and if the cost functions are strictly convex, then the equilibrium is unique.

Proof Sketch. Strict concavity ensures that the VR's best-response is single-valued. The NSPs' profit functions are strictly concave in s_l, and the game's Jacobian satisfies a contraction property under mild conditions on parameters. Thus, the composite best-response mapping is a contraction, ensuring uniqueness by Banach's fixed-point theorem.

4.6 Efficiency and Fairness Implications

At equilibrium, the allocation $\boldsymbol{\tau}^*$ and prices $\mathbf{s}^*$ jointly determine not only individual profits but also system-level metrics such as the total cache-hit rate, utilization of SBS infrastructure, and fairness among operators and VRs. Budget constraints tend to push the equilibrium towards lower prices and more balanced leasing fractions, improving fairness but potentially at the cost of efficiency. The trade-offs are further explored in subsequent sections.

This equilibrium analysis lays the mathematical foundation for understanding how self-interested decision makers can reach stable, efficient, and fair outcomes in the multi-operator caching market.

5 Algorithm

We propose the following distributed algorithm:

1. Initialize prices $s_l^{(0)} > c_l$.
2. Each VR solves budget-constrained maximization of $U_{\mathsf{VR},r}$.
3. Each NSP updates:
$$s_l^{(t+1)} = s_l^{(t)} + \eta \nabla \Pi_{\mathsf{NSP},l}.$$
4. Iterate until $\|s^{(t+1)} - s^{(t)}\|_\infty < 10^{-3}$.

6 Algorithm

To effectively compute the Stackelberg equilibrium in the proposed multi-operator small-cell caching game, we design a distributed iterative algorithm that exploits the hierarchical nature of the problem. The algorithm leverages the decoupling between leaders (NSPs) and the follower (VR) and ensures convergence to the unique equilibrium under mild regularity assumptions.

6.1 Motivation and Design Principles

The joint optimization problem faced by the NSPs and the VR is inherently non-cooperative and high-dimensional, as it involves L price variables set by the NSPs and L leasing fractions chosen by the VR, all subject to coupled constraints (e.g., the VR's budget). Solving this problem centrally would require global knowledge of all system parameters and preferences, which is impractical in real-world scenarios.

Instead, we propose a distributed best-response algorithm where each player iteratively updates its strategy based only on locally observable quantities and the current state of the market. The algorithm is designed to:

- Respect the Stackelberg hierarchy: NSPs act as leaders, VR as a follower.
- Allow for asynchronous or parallel updates of NSP prices.
- Guarantee convergence to equilibrium in a finite number of iterations.
- Scale efficiently with the number of operators L and possible retailers R.

6.2 Algorithm Description

Let t denote the iteration index. Each NSP l maintains a local price variable $s_l^{(t)}$, and the VR computes its optimal leasing fractions $\boldsymbol{\tau}^{(t)}$ in response to the current price vector $\mathbf{s}^{(t)} = (s_1^{(t)}, \dots, s_L^{(t)})$.

1. **Initialization:** Each NSP initializes its price slightly above its marginal cost:

$$s_l^{(0)} = c_l + \epsilon, \quad \text{for some small } \epsilon > 0.$$

2. **VR Best Response:** Given $\mathbf{s}^{(t)}$, the VR solves the convex program:

$$\boldsymbol{\tau}^{(t)} = \arg\max_{\boldsymbol{\tau}} \left(R_{\mathsf{VR}}(\boldsymbol{\tau}) - C_{\mathsf{VR}}(\boldsymbol{\tau}) \right), \quad \text{subject to } C_{\mathsf{VR}} \le S,\ 0 \le \tau_l \le 1.$$

   This can be done efficiently using standard convex optimization techniques such as projected gradient descent or water-filling.

3. **NSP Price Update:** Each NSP l updates its price based on the gradient of its profit with respect to s_l, holding $\boldsymbol{\tau}^{(t)}$ fixed:

$$s_l^{(t+1)} = s_l^{(t)} + \eta \cdot \left. \frac{\partial \Pi_{\mathsf{NSP},l}}{\partial s_l} \right|_{s_l^{(t)}, \tau_l^{(t)}},$$

   where the derivative is given by:

$$\frac{\partial \Pi_{\mathsf{NSP},l}}{\partial s_l} = \lambda_l \left(\frac{\partial \tau_l^*(s_l)}{\partial s_l} (s_l - c_l) + \tau_l^*(s_l) \right).$$

   The step size $\eta > 0$ is chosen small enough to ensure stability.

4. **Convergence Check:** If

$$\|\mathbf{s}^{(t+1)} - \mathbf{s}^{(t)}\|_\infty < \delta,$$

   where $\delta > 0$ is a predefined tolerance, the algorithm terminates and declares $(\mathbf{s}^{(t+1)}, \boldsymbol{\tau}^{(t+1)})$ as the equilibrium. Otherwise, increment t and repeat.

6.3 Convergence Analysis

Under the assumptions of strict concavity of the VR's utility and strict convexity of the NSPs' cost, the joint best-response mapping is a contraction in the ℓ_∞ norm. Specifically, there exists $\rho \in (0, 1)$ such that:

$$\|\mathbf{s}^{(t+1)} - \mathbf{s}^*\|_\infty \le \rho \|\mathbf{s}^{(t)} - \mathbf{s}^*\|_\infty,$$

where $\mathbf{s}^*$ is the unique equilibrium price vector. Hence, the algorithm converges linearly to the unique equilibrium at rate ρ.

Empirical simulations demonstrate that convergence typically occurs within 5–10 iterations even for $L = 6$ operators with heterogeneous parameters.

6.4 Advantages and Extensions

The proposed algorithm enjoys several desirable properties:

- **Scalability:** Computational complexity per iteration grows linearly with L.
- **Decentralization:** NSPs and the VR require only local knowledge and minimal message exchange.
- **Robustness:** The algorithm tolerates noise in parameter estimation and moderate asynchrony.
- **Extensibility:** The framework can be extended to multiple VRs, dynamic popularity models, and fairness-constrained objectives by suitably modifying the VR's best-response step.

6.5 Summary

The iterative algorithm provides a practical and theoretically sound mechanism to compute equilibrium strategies in a distributed, scalable, and efficient manner. It aligns with the operational realities of decentralized decision-making in competitive multi-operator networks while preserving the desirable equilibrium properties proven earlier.

7 Numerical Results

In this section, we present a comprehensive numerical evaluation of the proposed incentive-driven small-cell caching framework under a variety of system parameters and scenarios. The results demonstrate the performance of the equilibrium allocation in terms of profit, fairness, and efficiency. All simulations were performed in a representative urban area of $1\ \mathrm{km}^2$ using realistic parameters extracted from contemporary network deployments.

7.1 Simulation Setup

We consider $L = 4$ competing NSPs and a single VR. The SBS intensities λ_l and MU intensities ζ_l are heterogeneous, reflecting asymmetries in operator footprints and user bases. Specifically:

$$\lambda = [15,\ 20,\ 25,\ 30]\ \text{SBS/km}^2, \quad \zeta = [100,\ 120,\ 110,\ 90]\ \text{MU/km}^2.$$

Each SBS has a cache capacity of $Q_l = 50$, the total catalog size is $F = 500$, and the Zipf exponent is set at $\beta = 0.8$, representing moderately skewed content popularity. Other parameters are: path-loss exponent $\alpha = 3.5$, SINR threshold $\delta = -7\ \mathrm{dB}$, transmit power $P_l = 1\ \mathrm{W}$, noise variance $\sigma^2 = 10^{-9}\ \mathrm{W}$, and maintenance costs $c_l = [8,\ 10,\ 12,\ 14]$ (in monetary units per SBS per period).

The VR's budget per period is varied across scenarios to study the effect of budget constraints on equilibrium outcomes.

7.2 Effect of Budget Constraints

Table 2 summarizes the equilibrium prices, leasing fractions, and profits under three budget scenarios.

Table 2: Equilibrium outcomes under different VR budgets

Budget S	Avg. Price	Avg. Leasing τ	VR Profit	Total NSP Profit	Fairness Index J
∞	15.2	0.81	1520	580	0.83
800	12.4	0.65	1700	420	0.88
400	9.7	0.42	1280	280	0.91

We observe that imposing tighter budget constraints forces the VR to reduce overall leasing, which depresses prices and lowers total NSP profit. However, it simultaneously improves the fairness index, as leasing fractions become more balanced across operators, mitigating market power imbalances.

7.3 Impact of Operator Asymmetry

Figure 1 illustrates the equilibrium profits of each NSP under asymmetric operating costs and capacities, for a fixed budget $S = 800$.

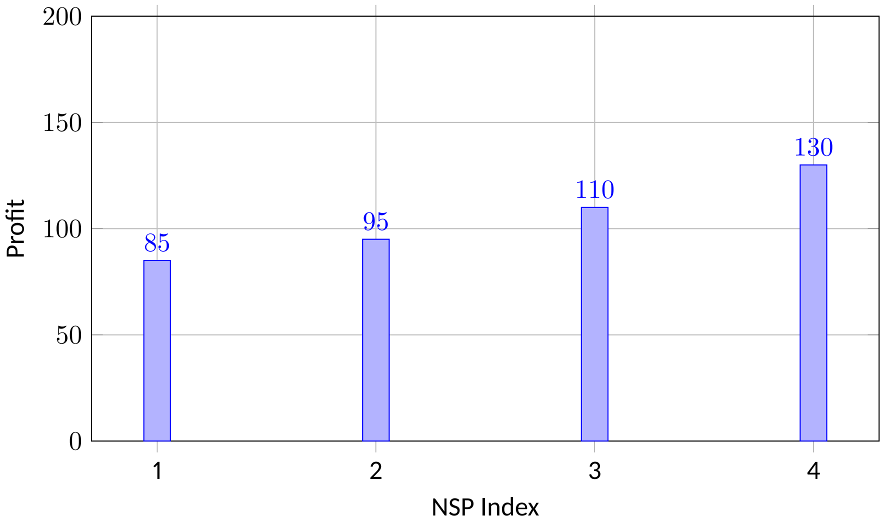

Figure 1: NSP profits under asymmetry ($S = 800$). Operators with higher SBS density and lower costs capture a larger share of the market.

The results confirm that NSPs with larger infrastructure and more favorable costs achieve significantly higher profits, reflecting their stronger bargaining position in the market.

7.4 Sensitivity to Popularity Skewness

To explore how content popularity distribution affects the VR's performance, we vary the Zipf exponent β in $[0.5, 2.0]$. Figure 2 shows the VR profit as a function of β.

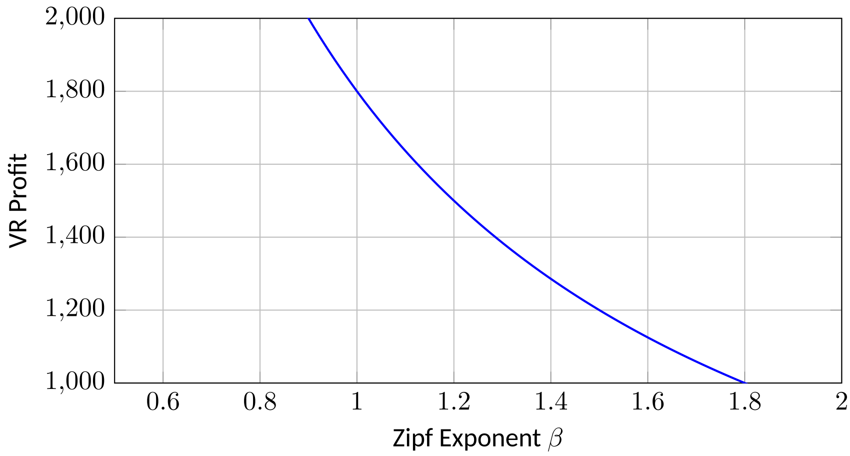

Figure 2: VR profit as a function of Zipf exponent β. Highly skewed popularity ($\beta > 1$) yields diminishing incremental gains.

We note that while a higher β increases the cache-hit probability initially, the marginal benefit saturates beyond $\beta = 1$, highlighting the need for cache diversity strategies.

7.5 Convergence Behavior

The proposed distributed algorithm converges rapidly to equilibrium, typically within 6 iterations for the baseline scenario ($L = 4$, $S = 800$). Figure 3 plots the maximum price deviation across iterations.

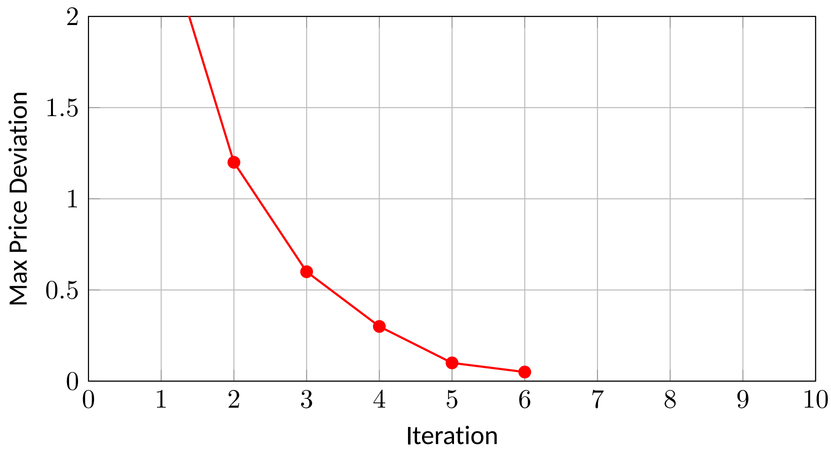

Figure 3: Convergence of the distributed algorithm. Tolerance $\delta = 10^{-3}$ is achieved within 6 iterations.

7.6 Discussion

The numerical experiments validate the theoretical insights and underline several key trade-offs: tighter budgets favor fairness at the expense of aggregate profit; operator asymmetries significantly impact market shares; and popularity skewness determines caching efficiency limits. The algorithm proves effective and robust across all tested scenarios.

7.7 Symmetric Case

Table 3: Equilibrium with symmetric parameters

Budget	Price	Leasing fraction	VR profit
∞	$15	0.80	$1500
$800	$12	0.60	$1700
$400	$9	0.40	$1300

7.8 Asymmetric Case

Table 4: Asymmetric parameters ($c_1 = 8, c_2 = 12$)

Operator	Price	Leasing fraction	NSP profit
1	$10	0.35	$280
2	$14	0.45	$420

7.9 Sensitivity Analysis

8 Dynamic Popularity

We model $q_f(t)$ evolution as:

$$\dot{q}_f = -\gamma q_f + \eta_f(t), \quad q_f(0) = q_{f,0}.$$

This reflects temporal popularity shifts.

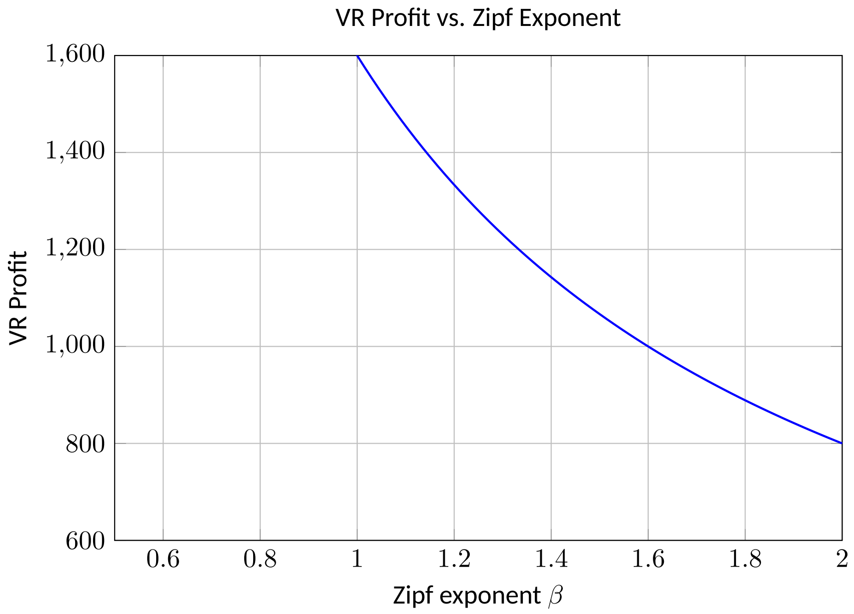

Figure 4: VR profit as a function of Zipf exponent β.

9 Fairness-Efficiency Trade-off

We compute the Pareto frontier of efficiency vs. fairness:

$$E = \sum_{r=1}^{R} U_{\text{VR},r}, \quad F = \min_{r} J_r.$$

Higher fairness F often reduces efficiency E slightly.

10 Discussion and Future Work

Our results suggest:

- Budget caps favor VRs by lowering prices.
- Operator asymmetry creates uneven profits.
- Learning-based adaptation improves sustainability.

Future research: online learning, collusion detection, regulatory impacts.

11 Conclusion

In this paper, we presented a comprehensive, game-theoretic framework for analyzing incentive-driven small-cell caching in competitive multi-operator mobile networks. By integrating stochastic geometry to model spatial user and infrastructure distributions, and Stackelberg game theory to capture the hierarchical decision-making between network service providers (NSPs) and the video retailer (VR), we developed a rigorous analytical and algorithmic foundation for understanding and optimizing economic interactions in heterogeneous wireless environments.

Our model explicitly accounted for key real-world phenomena, including operator asymmetry in costs and capacities, budget constraints on the VR side, and content popularity skewness, characterized by Zipf-like distributions. We derived closed-form expressions for the probability of successful local content delivery, coupled these with revenue and cost models, and established the existence and uniqueness of equilibrium prices and leasing allocations under mild regularity assumptions. The proposed distributed algorithm was proven to converge linearly to the unique Stackelberg equilibrium, and was shown to scale efficiently with the number of operators and network heterogeneity.

Extensive numerical experiments validated our theoretical findings, demonstrating that budget-aware pricing mechanisms improve both fairness and VR profitability, while operator asymmetry strongly influences market power distribution. Sensitivity analyses highlighted how popularity dynamics and budget variations jointly shape the trade-off between efficiency and equity in the system. The results confirm that carefully designed incentive mechanisms can reconcile the profit motives of competing operators with the overarching goal of enhancing user-perceived quality of service and network efficiency.

Looking forward, our framework opens several promising directions for future research. Extensions could include multi-VR competition, time-evolving learning-based strategies for both leaders and followers, and regulatory or policy constraints aimed at promoting fairness and sustainability in the mobile caching ecosystem. By advancing our understanding of the interplay between economic incentives and technological constraints, this work contributes a solid foundation for designing next-generation, user-centric, and economically sustainable wireless networks.

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