Thermax
1 2 3 Excel
Research
Design
Fundamentals

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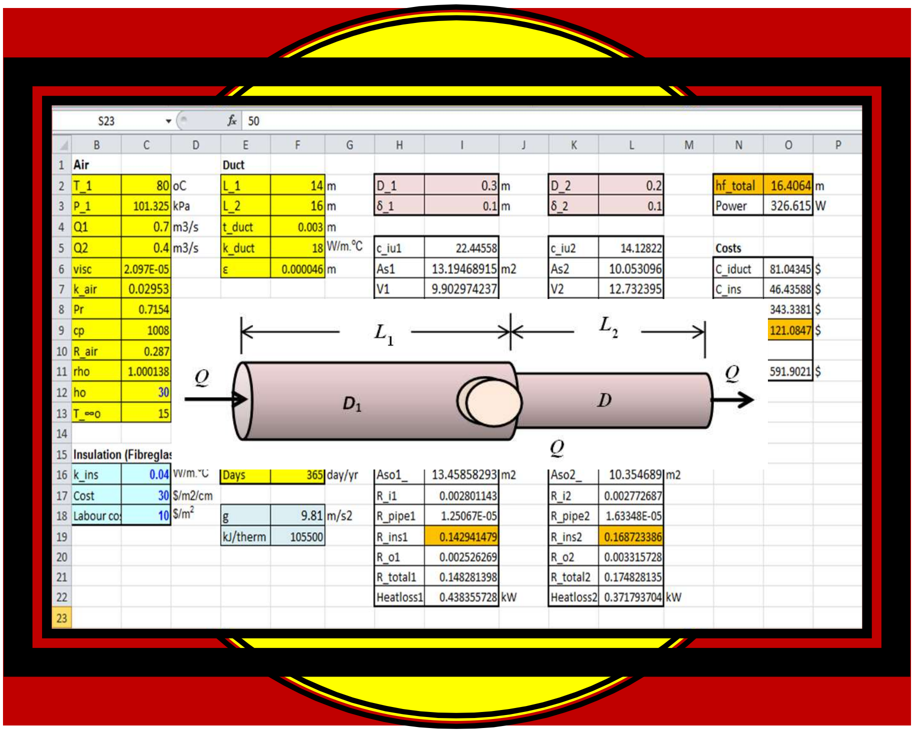
Computer Applications for Engineers using Excel



S23 | 50

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Air**			**Duct**											
2	T_1	80	oC	L_1	14	m	D_1	0.3	m	D_2	0.2		hf_total	16.4064	m
3	P_1	101.325	kPa	L_2	16	m	δ_1	0.1	m	δ_2	0.1		Power	326.615	W
4	Q1	0.7	m3/s	t_duct	0.003	m									
5	Q2	0.4	m3/s	k_duct	18	W/m.°C	c_iu1	22.44558		c_iu2	14.12822		**Costs**		
6	visc	2.097E-05		ε	0.000046	m	As1	13.19468915	m2	As2	10.053096		C_iduct	81.04345	$
7	k_air	0.02953					V1	9.902974237		V2	12.732395		C_ins	46.43588	$
8	Pr	0.7154												343.3381	$
9	cp	1008												121.0847	$
10	R_air	0.287													
11	rho	1.000138												591.9021	$
12	ho	30													
13	T_∞o	15													
14															
15	**Insulation (Fibreglas**														
16	k_ins	0.04	W/m.°C	Days	365	day/yr	Aso1_	13.45858293	m2	Aso2_	10.354689	m2			
17	Cost	30	$/m2/cm				R_i1	0.002801143		R_i2	0.002772687				
18	Labour co	10	$\$/m^2$	g	9.81	m/s2	R_pipe1	1.25067E-05		R_pipe2	1.63348E-05				
19				kJ/therm	105500		R_ins1	0.142941479		R_ins2	0.168723386				
20							R_o1	0.002526269		R_o2	0.003315728				
21							R_total1	0.148281398		R_total2	0.174828135				
22							Heatloss1	0.438355728	kW	Heatloss2	0.371793704	kW			
23															

Mohamed M. El-Awad

Computer Applications for Engineers using Excel

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Mohamed M. El-Awad

April, 2026

This book is dedicated to by beloved family

Ghada, Ula, and Ahmed

Thermax (GAS and AIR groups) can be downloaded from:

https://docs.google.com/document/d/15mOzV33K3S8_VJjnVQVN9-ND0mO5VYYqxbV9If3ot8A/edit?usp=drive_link

Preface

Engineering students usually take at an early stage of their study a preliminary course on computer applications that introduces them to the world of computers and equips them with the basic skills needed for word-processing and data analyses and presentation. After studying the basic engineering subjects, they take an intermediate-level course on computer applications the aim of which is to train the students on the essential computational methods needed for engineering design analyses. Although the various engineering specialisations involve different types of design analyses, there are commonly-shared methods such as those for the solution of linear systems of equations, solution of non-linear equations, solution of ordinary and partial differential equations, and the methods for iterative solutions and optimisation analyses. All these types of computer-oriented methods are frequently encountered in the three thermofluid subjects, thermodynamics, fluid mechanics, and heat-transfer, which are also fundamental subjects for most engineering specialisations including civil, chemical, and mechanical engineering. Therefore, this book illustrates the use of computational methods for engineering design analyses by focussing on thermofluid analyses. This approach also enables the various examples given in the book to be related to practical engineering analyses rather than purely mathematical ones.

Various computational modelling platforms have been used for the intended purpose of this book that include computer-programming languages, such as C++ and Python, and specialised computer applications such as MATLAB and EES. This uses a general-purpose spreadsheet application, which is Microsoft Excel, as the base for its modelling platform. Besides being widely available and easy to learn, Excel is one of the mostly used software by practicing engineers. The Excel-based modelling platform used in the book has four elements; (i) Excel with its user-interface and built-in functions, (ii) the Solver add-in that comes with Excel, (iii), the integrated programming language Visual Basic for Applications (VBA) and (iv) an Excel add-in for fluid properties called Thermax. While the main two components of the platform, Excel and Solver, are adequate for most fluid mechanics and heat-transfer analyses, Thermax and VBA are needed for thermodynamic analyses and for the development of custom functions when the analytical model cannot be applied by only using Excel's built-in functions and Thermax functions. Appropriately used, this Excel-based modelling platform minimises the effort of developing the analytical models so that more attention can be paid to the application of the relevant engineering principles without using black-box models.

This book includes nine chapters the first of which reviews the basic principles of thermofluids, highlights the advantages of computer-aided thermofluid analyses, and briefly describes the Excel-based modelling platform. Chapter 2 shows how Excel's built-in functions and tools can be used for the basic computer-based analyses like the solution of linear systems of equations and the solution of non-linear equations, while Chapter 3 illustrates the use of the other three components of the platform by means of simple examples. Chapter 4 deals with iterative solutions met in the three thermofluid courses, while Chapters 5, 6, and 7 deal with selected computer-aided analyses specifically related to heat-transfer, fluid-dynamics, and thermodynamics, respectively. Chapter 8 deals with single-objective and multi-objective design optimisation analyses. For multi-objective design optimisation, this chapter considers the problem of an end-loaded concrete beam and uses the free version of the MIDACO solver to solve it. Chapter 9 presents seven additional cases which are relatively more challenging than the related cases described in previous chapters of the book. These cases are meant for stimulating ideas for relevant mini projects.

The material covered in this book is mainly compiled from three more elaborative books written by the author that use the Excel-based modelling platform for computer-aided analyses and optimisation of thermal-fluid systems by adopting a learning-by-example approach. The book is intended to suit a one-credit-hour course that is taken after completing the basic engineering courses and builds on the students' theoretical background at that level. Most of the examples given in the book are based on relevant examples given in standard engineering text books or cases obtained from the published literature so that the students can refer to these sources for any additional information needed for developing their analytical models or verifying their results. Exercises are given at the end of Chapter 2 through Chapter 8 to help the students sharpen their skills related to the particular topic and more challenging exercises are provided as mini projects at the end of Chapter 9.

Acknowledgements

The writing of this book would not have been possible without benefitting from the efforts of many colleagues who have made their publications, data, and software available in the open literature or on their websites. A special gratitude goes to the Mechanical Engineering Department at the University of Alabama (USA) whose initiative "*Excel for Mechanical Engineering*" both inspired and helped me throughout this work. I am also indebted to Universiti Putra Malaysia and Universiti Tenaga Nasional (Malaysia), the University of Khartoum (Sudan), and the University of Technology and Applied Sciences (Oman) for their generous support at different periods of my academic career and hope that they find the book a worthy token of appreciation and gratitude.

CONTENTS

1

Introduction

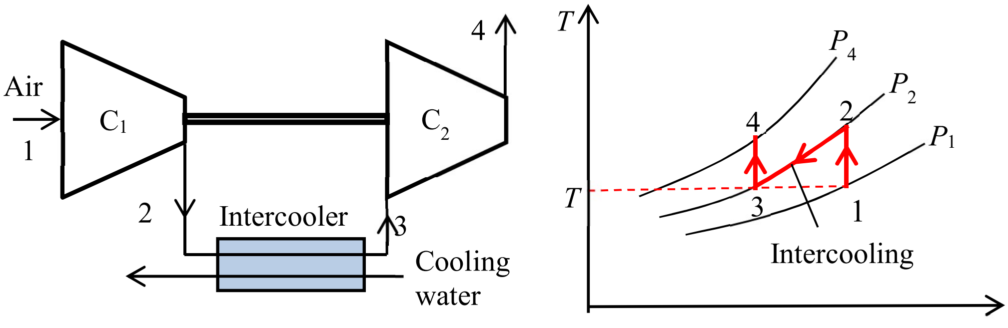
Cars, refrigerators, air-conditioners, computers and mobile phones have become indispensable household items in both developed and developing countries. The energy required to operate these devices mainly comes from burning fossil fuels either directly or indirectly by producing electricity in power-generation plants. Apart from being non-renewable energy sources, large-scale combustion of fossil fuels is the main cause of global warming that caused devastating effects at different parts of the world. Therefore, proper design and operation of the systems that depend on fossil fuels as the source of energy is required. The design methods for the systems that use fossil fuels directly are mainly based on the principles of *thermodynamics*, *fluid mechanics*, and *heat transfer*. This chapter reviews the main principles of these three thermofluid subjects with the view of showing how they can be used to minimise the losses and energy consumption of such systems. For a number of reasons, the equations involved in thermofluid analyses are difficult to solve by using conventional analytical methods and, therefore, these methods introduce many simplifications that reduce their accuracy. In this respect, the chapter highlights the advantages of computer-aided methods for thermofluid analyses and describes the Excel-based modelling platform used in this book for these analyses.

1.1. A review of thermofluid principles

The two main principles that form the framework for thermofluid analyses are the conservation of mass (the continuity equation) and the conservation of energy (the first-law of thermodynamics). These principles take different mathematical forms depending on whether the system under consideration is open or closed and whether the flow is steady or unsteady, compressible or incompressible, laminar or turbulent, etc. Numerous auxiliary relationships are needed in order to quantify the various parameters involved in the resulting equations such as pressure-variations, friction losses, and rates of heat-transfer. In what follows, the main concepts of thermodynamics, fluid dynamics, and heat-transfer are reviewed by considering typical applications.

1.1.1. Thermodynamics

E*ngineering thermodynamics* enables us to determine the amount of energy transfer in the form of work or heat between any system and its surroundings and the efficiency and effectiveness of the energy transfer process. It has four basic laws the most important of which are the first and the second laws of thermodynamics. While the first law accounts for the *quantity* of energy interaction in a process, the second law accounts for the *quality* of this interaction. To apply these two basic laws, property relationships, tables or charts are used to determine the properties of the particular fluid involved and its phase (a liquid, a liquid-vapour mixture, a gas, or a gaseous mixture). To illustrate the application of thermodynamic laws and relationships in a typical analysis, consider the air-compression system shown in Figure 1.1.a that has two stages of compression separated by an intercooler. Air enters the system at a temperature T_1 and pressure P_1. The first-stage compressor, C_1, compresses the air adiabatically to state 2, after which it enters the intercooler where its temperature is reduced to T_3. The second-stage compressor, C_2, then increases the air pressure to P_4 at which the temperature increases to T_4. Figure 1.1.b shows the compression process on a temperature-entropy diagram.



(a) (b)

Figure 1.1. Schematic and *T-s* diagrams of a two-stage air compressor with intercooling

How the total compression work is divided between the two compressor stages depends on their compression ratios and there is a certain value of the intermediate pressure (P_i) that minimises this work. The principles of thermodynamics help us to determine this optimum pressure as shown below.

Treating the two compressor stages as steady-flow processes and neglecting changes in kinetic and potential energy, the first-law of thermodynamics states that [1]:

$$q - w = (h_{out} - h_{in}) \tag{1.1}$$

Where q and w are the amount of heat transfer and work transfer per unit mass flow of air, respectively, and ($h_{out} - h_{in}$) is the resulting enthalpy change over the stage. Equation (1.1) adopts the sign convention that heat into the system is positive, while work into the system is negative. Assuming the compression processes in both stages to be isentropic as shown in Figure 1.1.b could mean that they are adiabatic (q=0) and reversible. Therefore, using an average specific heat for air at constant pressure ($\bar{c}_p$), the compression work per unit mass flow of air in stage 1 (w_1) and in stage 2 (w_2) can be determined from Equation (1.1) as follows:

$$w_1 = -(h_2 - h_1) = -\bar{c}_p (T_2 - T_1) \tag{1.2}$$

$$w_2 = -(h_4 - h_3) = -\bar{c}_p (T_4 - T_3) \tag{1.3}$$

The total compression work (w_{total}) is then given by:

$$w_{total} = w_1 + w_2 = -\bar{c}_p [(T_2 - T_1) + (T_4 - T_3)] \tag{1.4}$$

Assuming perfect intercooling, i.e., $T_3 = T_1$, Equation (1.4) can be rearranged as:

$$w_{total} = \bar{c}_p T_1 \left[\left(1 - \frac{T_2}{T_1}\right) + \left(1 - \frac{T_4}{T_3}\right) \right] = \bar{c}_p T_1 \left[2 - \left(\frac{T_2}{T_1}\right) - \left(\frac{T_4}{T_3}\right) \right] \tag{1.5}$$

Since the two compression processes are assumed to be isentropic and the specific heat for air to be constant, the temperature ratios in Equation (1.5) can be converted into pressure ratios by using the following approximate relationships:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \tag{1.6}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \tag{1.7}$$

Where k is the ratio of specific heats (k=c_p/c_v; c_v is the specific heat for air at constant volume). With another assumption that there is no pressure loss in the intercooler, $P_3 = P_2 = P_i$. Substituting from Equations (1.6) and (1.7), Equation (1.5) becomes:

$$w_{total} = c_p T_1 \left[2 - \left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_4}{p_i}\right)^{\frac{k-1}{k}} \right] \tag{1.8}$$

The variation of the total compression work with the intermediate pressure P_i, can be seen by considering the specific case in which T_1= 300K, P_1=100 kPa, and P_4 = 900 kPa. Using Equation (1.8), the total compression work in the system was calculated for different values of P_i and Figure 1.2 shows the result. The figure shows that the value of P_i at which the total compression work is minimal is around 300 kPa. Increasing or decreasing P_i from this value will increase the compression work.

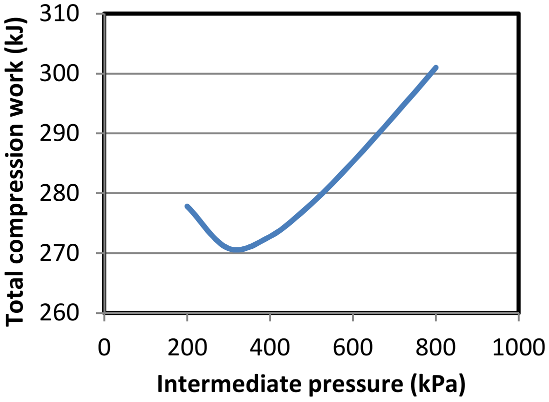


Figure 1.2. Variation of the total compression work with the intermediate pressure

The principles of thermodynamics are also useful for performance evaluation and design optimisation of power-generation and refrigeration systems. For example, consider the regenerative steam-turbine power plant shown in Figure 1.3. This plant consists of a boiler house for producing superheated steam, a high-pressure steam turbine (HPT), a low-pressure steam turbine (LPT), a condenser, an open feed-water heater (FWH) and two feed-water pumps. A fraction of the steam (y) is extracted after the HPT for preheating the feed-water before going back to the boiler house.

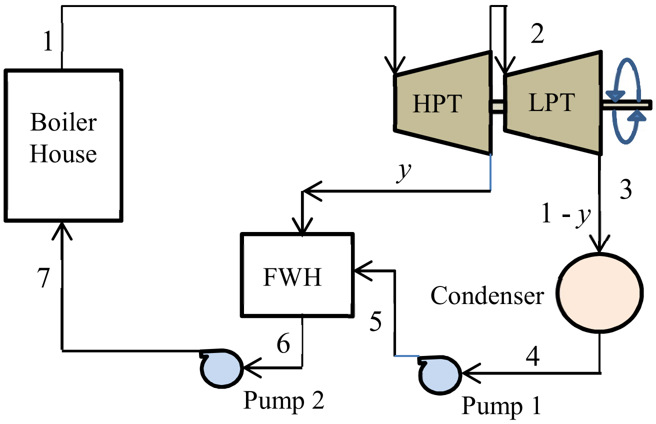


Figure 1.3. Schematic diagram of a regenerative steam-turbine power plant

Although the extracted steam reduces the work output of the LPT, it reduces the amount of heat added in the boiler and its net effect is to increase the thermal efficiency of the plant. There is also a certain steam-extraction pressure at which the plant's thermal efficiency attains a maximum value. As shown below, the principles of thermodynamics can be used to determine this optimum pressure.

The total specific work output from the two turbines (w_{out}) and the total specific work input to the two pumps (w_{in}) are given by:

$$w_{out} = w_{HPT} + w_{LPT} \tag{1.9}$$

$$w_{in} = w_{P1} + w_{P2} \tag{1.10}$$

Where w_{HPT} and w_{LPT} are the specific work output from the high-pressure turbine and the low-pressure turbine, respectively, and w_{P1} and w_{P2} are the specific work inputs in pump 1 and pump 2, respectively. Assuming the two turbines and the two pumps to be adiabatic and neglecting the changes in kinetic and potential energies, the work output or input for these devices per each kg of steam generated in the boiler are given by:

$$w_{HPT} = (h_1 - h_2) \tag{1.11}$$

$$w_{LPT} = (1-y)(h_2 - h_3) \tag{1.12}$$

$$w_{P1} = (1-y)(h_5 - h_4) \tag{1.13}$$

$$w_{P2} = (h_7 - h_6) \tag{1.14}$$

Mass and energy balance over the feed-water heater gives:

$$yh_2 + (1-y)h_5 = 1 \times h_6 \tag{1.15}$$

The specific heat input to the boiler (q_{in}) is determined by the relevant enthalpy change as follows:

$$q_{in} = (h_1 - h_7) \tag{1.16}$$

Finally, the net specific work output from the plant (w_{net}) and the thermal efficiency of the plant (η) can be calculated from:

$$w_{net} = w_{out} - w_{in} \tag{1.17}$$

$$\eta = w_{net} / q_{in} \tag{1.18}$$

Both w_{net} and η depend on the fraction of steam extracted for regeneration (y); which in turn depends on the extraction pressure (P_2). Figure 1.4 shows the variation of y and η with P_2 for an ideal cycle in which P_1 = 15 MPa, T_1 = 600°C, and P_4 = 10 kPa. The figure shows that η attains a maximum value of 45.55% when P_2 is in the range of 1000 kPa.

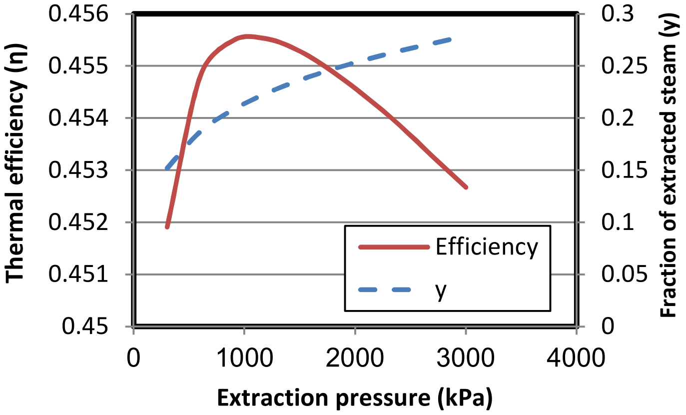


Figure 1.4. The effect of intermediate pressure (P_2) on the fraction of extracted steam (y) and thermal efficiency (η) of an ideal regenerative steam-turbine power plant

It should be mentioned that the working fluid in the above power plant, which is water, changes its phase from liquid to superheated steam in the boiler, to saturated mixture of water and steam in the low-pressure turbine, and returns to liquid water in the condenser. Therefore, appropriate property tables or charts are needed for determining the thermodynamic properties of water at these different states. In general, thermodynamic analyses require many tables and charts for various working fluids. The principles of thermodynamics are also needed for the analyses of air-conditioning systems and processes and for the analyses of the processes that involve combustion and other chemical reactions. For such analyses, thermodynamics provides the basic relationships needed to quantify the effects of fluid mixing and chemical reactions on the properties of the working fluids and to determine the transfer of energy and effluents to or from the system under consideration.

1.1.2. Fluid dynamics

In addition to pipes and ducts, fluid-transporting systems require various equipment such as pumps and compressors, control valves, flow-diversion devices and flow-measuring devices. The principles of *fluid dynamics* help us to estimate the power needed for overcoming friction in these equipment and to determine suitable types and sizes for them. To illustrate the application of these principles, consider the pump-pipe system shown in Figure 1.5 that conveys a liquid between two non-pressurised tanks A and B through a pipe of known length L, diameter D, and roughness ε. Suppose that we want to determine the needed pump power for transporting the liquid at a certain flow rate Q.

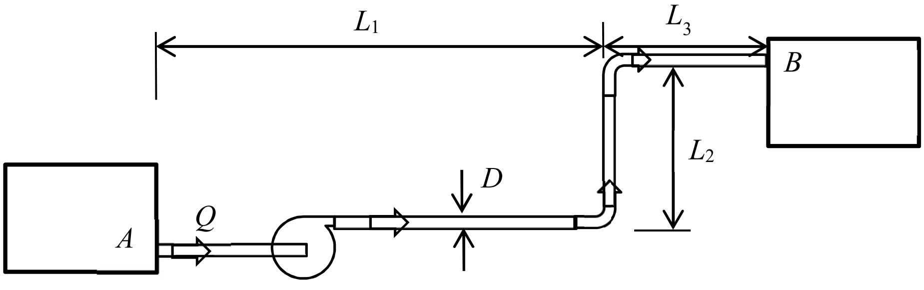


Figure 1.5. Schematic diagram of a simple pump-pipe system

The required pump power ($\dot{W}$) is determined from the following "power equation" [2]:

$$\dot{W} = \frac{\gamma \times Q \times h_p}{\eta} \tag{1.19}$$

Where γ is the specific weight of the transported liquid, Q is the volume flow rate of the liquid, h_p is the pump head needed to circulate the fluid through the pipe from A to B, and η is the combined efficiency of the pump and the electric motor. For a steady flow of an incompressible fluid, h_p can be determined from the following "energy equation":

$$h_p = h_{f,total} + (Z_B - Z_A) + \frac{V_B^2 - V_A^2}{2g} \tag{1.20}$$

Where $h_{f,total}$ is the total head loss through the system due to friction, Z_A and Z_B are the elevations at points A and B, respectively, and V_A and V_B are the corresponding fluid velocities. If the two tanks are not open to the atmosphere, the energy equation should include another term for the pressure difference between the tanks.

The total friction head loss $h_{f,total}$ consists of two parts: the *major friction loss* (h_f), which is the part lost in the pipe itself, and the *minor friction head loss* (h_c), which is the part lost in other components of the system like nozzles, elbows, valves, etc. The major friction loss can be determined from the following Darcy-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{1.21}$$

Where f is the dimensionless Darcy friction factor, V the fluid velocity, L the total length of the pipe, and D the internal diameter of the pipe. The value of the friction factor, which depends on the roughness of the pipe surface and on whether the flow is laminar or turbulent, can be obtained from a Moody diagram or calculated from a relevant formula. For laminar flows, it can be calculated from:

$$f = 64/\mathrm{Re} \qquad \mathrm{Re} < 2300 \tag{1.22}$$

Where Re is the Reynolds number defined as:

$$\mathrm{Re} = VD/\nu \tag{1.23}$$

Where ν is the kinematic viscosity of the flowing fluid. For a turbulent flow in rough pipes, f can be obtained from the following Swamee-Jain formula:

$$f = 0.25 \Big/ \left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2 \qquad \mathrm{Re} > 4000 \tag{1.24}$$

For more accuracy, the friction factor for a turbulent flow can be determined by using the following Colebrook-White formula (frequently referred to as the Colebrook equation):

$$\sqrt{\frac{1}{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1.25}$$

The Colebrook equation is an example of the implicit equations met in thermofluid analyses that need to be solved iteratively. For turbulent flows in smooth tubes, f can be determined from the first Petukhov formula:

$$f = (0.790 \ln(\mathrm{Re}) - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{1.26}$$

Chemical engineers usually determine the pipe friction by using the following Chezy-Manning equation instead of the Darcy-Weisbach equation:

$$h_f = 2f \frac{L}{D} \frac{V^2}{g} \tag{1.27}$$

Where f is the Fanning friction factor. Comparison with Equation (1.21) reveals that the value of the Fanning friction factor is four times the corresponding value of the Darcy friction factor. Civil engineers determine the friction head loss in water-transporting pipes by using the following Hazen-Williams equation:

$$h_f = \frac{10.67 L Q^{1.852}}{C^{1.852} D^{4.8704}} \tag{1.28}$$

Where C is a coefficient that depends on the roughness of the pipe. Unlike Equations (1.21) and (1.27), Equation (1.28) is applicable for both laminar and turbulent flows.

The minor friction losses, h_c, can be determined from the following equation:

$$h_c = \sum_{1}^{n} K \frac{V^2}{2g} \tag{1.29}$$

Where n is the total number of components in the fluid system and K is a coefficient the value of which can be found for each component in relevant tables.

Given the values of the length and diameter of the pipe and its material or roughness and the flow rate and fluid viscosity, the equations described above can be used to determine the required pump power. In principle, the equations can also be used to determine the maximum flow rate of the fluid to be delivered via a pipe of a certain diameter such that the friction loss in the system or the needed pump power does not exceed a specified limit. Moreover, by taking into consideration the initial cost of the pump-pipe system (which increases with D), and the cost of electrical energy needed by the pump (which decreases with D), the equations can also be used to determine the economic pipe diameter D_{opt} that gives the lowest total owning cost for the system over its entire life-time. The equations are also applicable for analysing and optimising pipe-networks.

The principles of fluid dynamics also enable us to select the appropriate type and size of the pump for a given pump-pipe system by matching the “pump curve” with the “system curve”. This is achieved with the help of pump characteristic curves usually provided by the manufacturers. In many situations a single pump or compressor may not be adequate for the required flow rate or delivery pressure and more than one pump or compressor have to be used. In this situation, the principles of fluid dynamics help us to decide when to arrange the pumps/compressors in parallel or in series.

1.1.3. Heat transfer

The design practices of energy-conversion equipment that deal with the transfer of thermal energy such as boilers, condensers, and heat exchangers are mainly based on the principles of *heat transfer*. Three independent physical laws are used in heat-transfer analyses to quantify the *rate* of heat transfer between an object and its surroundings depending on whether the transfers is by conduction (Fourier’s Law), convection (Newton’s law of cooling), or radiation (Stefan-Boltzmann law). The physical properties that determine the rate of heat transfer by conduction, radiation, and convection are the thermal conductivity (k), the surface emissivity (ε) and absorptivity (α), and the heat-transfer coefficient (h), respectively. While k, ε, and α are material or surface-specific, h depends on both the fluid and the flow. Numerous analytically-obtained relationships and empirical formulae are used for determining h depending on whether the flow is forced or natural and whether the flow is internal or external to the system being considered. These formulae usually give the Nusselt number (Nu) which is related to h as follows:

$$h = \frac{k}{D} Nu \tag{1.30}$$

Where D is the pipe’s diameter. Many analytical or empirical formulae are used for determining the Nusselt number for forced or natural flows over single tubes, bank of tubes, plates, etc. For example, the following Dittus-Boelter equation is used for determining Nu inside a fluid-transporting pipe due to forced convection [3]:

$$Nu = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{n} \tag{1.31}$$

Where Re is the Reynolds number, Pr the Prandtl number, and n is a constant that takes a value of 0.4 when the pipe is being heated and 0.3 when it is being cooled.

The subject also describes the methods that can be used to minimise or maximise the rate of heat-transfer between the system’s components or between the system and its surroundings by means of thermal insulation, fins, heat-pipes, etc. To illustrate the use of heat-transfer concepts in thermal-insulation analyses, consider the metal pipe shown in Figure 1.6 that has an internal radius r_1 and external radius r_2. The pipe carries a fluid at a temperature T_i, while the surrounding air is at a different temperature T_∞.

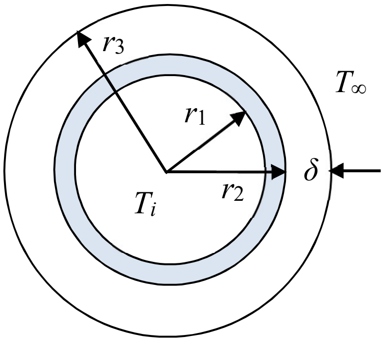


Figure 1.6. Schematic for an insulated metal pipe

The temperature difference between the pipe and the surroundings causes heat gain or heat loss to/from the pipe and, in order to reduce this undesired heat gain or heat loss, the pipe has to be covered by an insulating material. The principles of heat transfer help us to account for the effect of thermal insulation on the rate of heat-transfer ($\dot{Q}$) to/from the pipe which can be calculated from [3]:

$$\dot{Q} = \frac{T_i - T_\infty}{R_{th}} \tag{1.32}$$

Where R_{th} is the combined thermal resistance to heat-transfer by conduction, convection, and radiation, which is given by:

$$R_{th} = \frac{1}{h_i A_1} + \frac{\ln(r_2 / r_1)}{2\pi L k_1} + \frac{\ln(r_3 / r_2)}{2\pi L k_2} + \frac{1}{h_o A_3} \tag{1.33}$$

Where h_i and A_1 are the heat-transfer coefficient and surface area inside the pipe, respectively, h_o and A_3 are the heat-transfer coefficient and surface area outside the insulated pipe, respectively, L is the length of the pipe, and k_1 and k_2 are the thermal conductivities of the pipe and the insulation, respectively. To simplify the analysis, h_o in Equation (1.33) is allowed to take into account the heat-transfer by both convection and radiation to/from the insulation surface. The thickness of the metal pipe is usually small compared to its diameter, while its thermal conductivity is much higher than that of the insulation material. Therefore, the equation can be simplified further by neglecting the term that represents the thermal resistance due to conduction through the pipe.

For other design applications Equations (1.32) and (1.33) are used to determine the required thickness of insulation (δ) for reducing the rate of heat transfer to the required tolerance or for controlling the surface temperature within a range that is dictated by safety or other practical considerations. Although the thicker the insulation the lower will be the rate heat transfer, the cost of insulation increases with its thickness and, therefore, adding more insulation may not be economically profitable beyond a certain thickness. By extending the above heat-transfer model so that the cost of insulation and the value of the saved thermal energy can be calculated and compared, the above equations can also be used to determine the economically optimal thickness of insulation (δ_{opt}).

In many design applications the objective is to enhance the transfer of heat instead of minimising it. Figure 1.7 shows a metal pipe with circular fins attached to its surface so as to boost the rate of heat-transfer between the fluid being transported with the pipe and the surrounding medium, usually air. In this respect, the principles of heat transfer can be used to develop the required mathematical models that determine the the rate of heat transfer from the pipe so as to evaluate the effectiveness and efficiency of the fins.

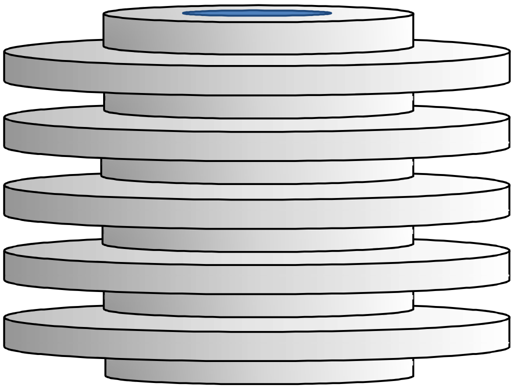


Figure 1.7. Circular fins attached to a metal pipe

Another important application of heat-transfer principles is that related to the design and selection heat-exchangers. A heat-exchanger is any device that allows the transfer of thermal energy between two fluids through a separating surface usually a pipe, a duct, a tube, or a plate. Figures 1.8 and 1.9 show two types of heat-exchangers commonly used in industries and power-plants.

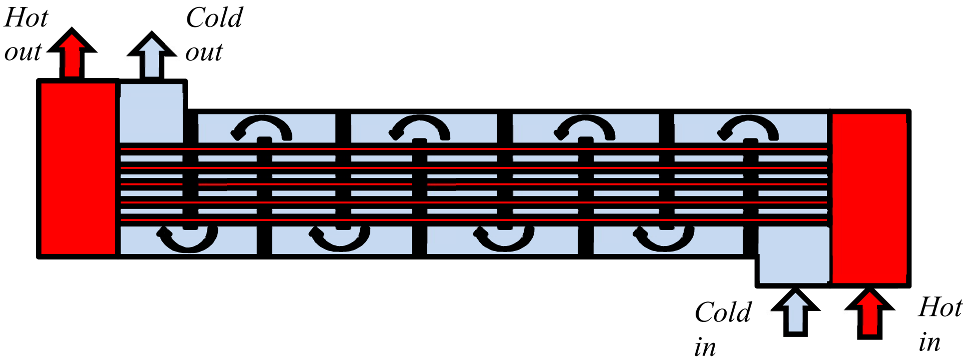


Figure 1.8. A parallel-flow shell-and-tube exchanger

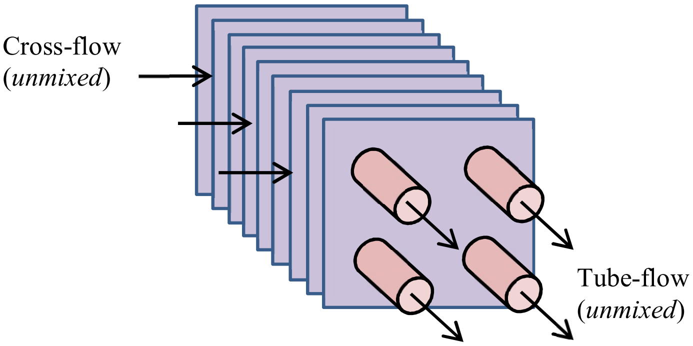


Figure 1.9. A cross-flow exchanger with both streams unmixed (adapted from [3])

Figure 1.8 shows a shell-and-tube heat-exchanger, while Figure 1.9 shows a cross-flow heat-exchanger. Heat-exchanger analyses either aim at determining the required size (i.e., surface area) for a specified heat-transfer duty or determining the exit temperatures of the two streams from a specified heat-exchanger type and size. Two methods are used for these two types of analyses which are the log-mean temperature difference (LMTD) method and the effectiveness-number of transfer units (ε-NTU) method. Complex thermal systems use heat-exchanger networks [HENs] and finding the configuration that minimises the annual cost of the network is also based on the principles of heat transfer.

1.2. Advantages of computer-aided thermofluid analyses

Apart from saving time and eliminating possible human errors, computer-aided thermofluid of analyses offer a number of advantages over traditional analytical methods that use property tables and charts. An important advantage of computer-aided methods is their ability to give more realistic results by avoiding unnecessary simplification of the models and by using more accurate formulae for fluid properties. Moreover, they offer reliable techniques for iterative solutions and optimisation analyses and for the analyses of complex fluid-thermal systems. In what follows, these advantages are illustrated by means of relevant examples.

A. Avoiding excessive simplification of the model

In many situations, traditional analytical methods adopt excessive simplifications of the analytical models which makes their results grossly deviate from the behaviour of real systems. A good example of this situation is given by the models of internal-combustion (IC) engines. Traditional air-standard models of IC engines, such as the Otto cycle and the Diesel cycle, neglect heat-transfer and friction losses, treat the combustion process as heat-addition from an external source, and use constant specific heats of the working fluids. These assumptions enable the engine processes to be represented by simple closed-form relations for calculating the amount of heat added to the engine and net work from it [4]. However, air-standard models usually overestimate the engine's output and thermal efficiency. By comparison, computer-aided models of IC engines closely mimic the behaviour of actual IC engines by taking into consideration the geometrical as well as the thermodynamic characteristics of the engines. Therefore, these models can be used to investigate the effect of important design and operation factors such the ignition or injection timing on the engine performance or the effect of engine' speed on the specific fuel consumption. However, the formulation of these models leads to a set of ordinary differential equations that need to be solved simultaneously by using a numerical method such as the Newton-Raphson method [5].

B. Accurate representation of fluid properties and processes

The ideal-gas law (*Pv*=*RT*) can be used with reasonable accuracy to determine the specific volume of a superheated vapour. However, when the temperature approaches the saturation line, the value of the specific volume thus determined departs significantly from the actual volume. More accurate estimates can be obtained by using more complex models such as the following Soave-Redlich-Kwong (SRK) equation of state [1]:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \tag{1.34}$$

Where P is the absolute pressure of the gas, $\tilde{v}$ is the molar specific volume, R_u is the universal gas constant, T is the absolute temperature of the gas, and the constants a, b and α are fluid-dependent. Figure 1.10 shows the deviations from the tabulated values by those obtained from the ideal-gas law and the SRK equation of state for refrigerant R134a at 0.2 MPa.

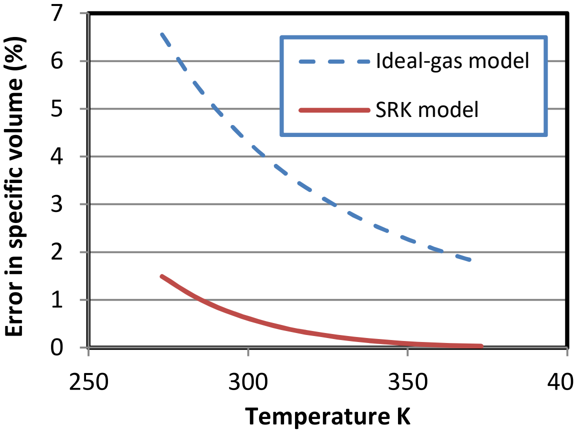


Figure 1.9. Errors in the specific volume of R134a by the ideal-gas law and the SRK equation of state

The figure shows that the error of the ideal-gas law is more that 2% even at high temperatures and increases as the temperature approaches the saturation value, but the accuracy of the SRK equation remained higher than 99% even close to the saturation line. However, the SRK equation is implicit in $\tilde{v}$ and, therefore, it cannot be used directly to determine the specific volume. A number of standard iterative procedures (e.g. Newton-

Raphson method) can be used to solve the equation, but they are more suitable for computer-aided analyses than hand calculations. Another important implicit equation for thermofluid analyses is the Colebrook-White equation, Equation (1.25), that determines the friction factor (*f*) for turbulent pipe-flows. Since the equation involves *f* on both sides and needs to be solved iteratively, the explicit relationships such as the Swamee-Jain formula are preferred in conventional analytical models even though the Colebrook-White equation is more accurate. Many other nonlinear equations like the SRK equation and the Colebrook-White equation give advantage to computer-aided thermofluid analyses by enabling more realistic and accurate estimations.

C. Dealing with iterative solutions and optimisation analyses

Thermofluid analyses that require iterative solutions are very common. A good example of these is given by the pump-pipe analyses discussed in Section 1.1.2. The problems that require the friction head loss to be determined when both the diameter and flow rate are known can be solved in a straightforward manner by using Equation (1.21). However, in design analyses of pump-pipe systems we may need to find the flow rate in a given pipe that gives a specified head loss or to find a suitable pipe diameter for specified head loss, flow rate, and pipe length. In these two cases, the friction factor *f* cannot be determined in advance because it depends on the Reynolds number. Therefore, these two types of pipe-flow problems, referred to as type-2 and type-3 problems, need to be solved by iteration. It is much easier to carry out the iterative process to the required level of accuracy by using a computer-aided method than by doing it manually. Two examples of the analyses that require iterative solutions in heat-transfer and thermodynamics are the rating analyses of heat exchanger and the determination of the adiabatic flame temperature by first-law analysis of the combustion process.

Optimisation analyses are needed for determining the best design for a fluid-thermal system such as the optimum intermediate pressure for the two-stage air-compression system and the optimum steam-extraction pressure for the regenerative Rankine cycle discussed in Section 1.1.1 and the economic insulation thickness for a pipe discussed in Section 1.1.3. While certain simple optimisation analyses that involve a single design parameter can be performed by means of calculus techniques and graphic tools, optimisation analyses of complex systems that involve multiple design variables and multi-objective optimisation analyses require the use of computer-aided techniques.

D. Analyses involving complex models

The complexity of modelling certain fluid-thermal systems makes their analyses only possible with the help of computer-aided methods. The model complexity can be either due to the complexity of the physical structure of the system itself or the complexity of its mathematical representation. An example of the physically complex systems is the pipe network shown in Figure 1.11 that consists of four pipe loops and four consumption points fed by two water tanks; tank A and tank B. Suppose that the flow rates from the two supply tanks are specified together with the pipe diameters and lengths and it is required to determine the discharges at the four consumption points. Although the solution is mainly based on the principles of fluid dynamics discussed in Section 1.1.2, it is difficult to solve the problem by using manual analytical methods especially when a minimum or a maximum pressure level is to be met at the discharge points. In this case, a computer-aided method, such as the Hardy-Cross method, has to be used [6, 7]. The optimisation analyses of heat-exchanger networks give another example of the models that deal with physically complex systems [8].

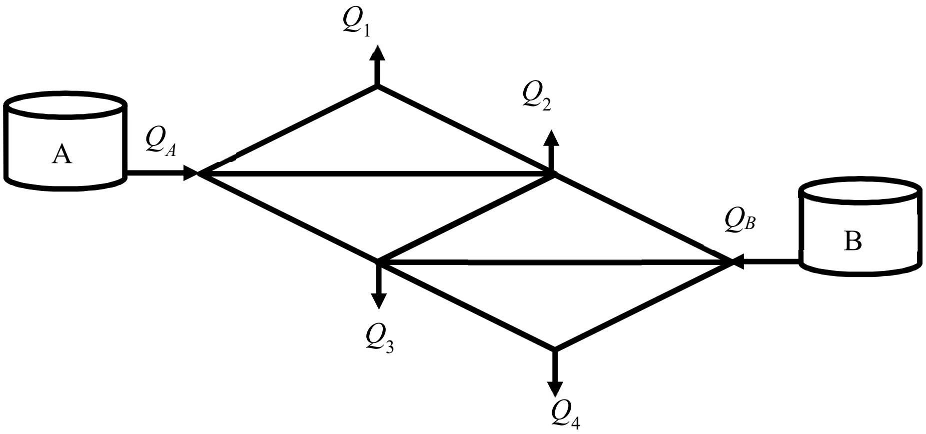


Figure 1.11. A looped pipe network supplied by two tanks

Examples of the mathematically complex thermofluid models that need computer-aided numerical methods are found in multi-dimensional fluid-flow and heat transfer analyses. This type of analyses involves coupled and nonlinear partial differential equations that have to be solved by using computational fluid dynamics (CFD) methods such as the finite-volume method or the finite-difference method. These types of analyses in particular require the use of dedicated software because the same software can be used for many fluids and system configurations. Only a dedicated software can give us the ability to choose the fluid type and system configuration and provide the required information regarding the flow whether it is compressible or incompressible, laminar or turbulent, with or without mixing or chemical reaction, etc., as well as the information required by the numerical method to construct its grid. Luckily, many commercial CFD applications are available nowadays that offer great flexibility and user-friendliness.

1.3. The Excel-based modelling platform for thermofluid analyses

Microsoft Excel is commonly used for data visulation and analyses and has also been used for dealing with simple computer-based operations like matrix inversion and matrix multiplications [3,9]. However, Excel is equipped with numerous features and tools that make it a capable modelling platform for a wide range of engineering analyses including its Goal Seek command and the Solver add-in [10-12]. The “Developer” ribbon in Excel provides a programming language called Visual Basic for Applications (VBA) that can be used for developing customised user-defined functions (UDFs) not provided by Excel. The Developer ribbon also allows the use of macros to remove the tedium of parametric studies and repetitive calculations. The main limitation of Excel for thermofluid analyses, which is the lack of built-in functions for fluid properties, could be resolved by various academic and research institutions via the development of suitable add-ins [13-16].

This book uses an Excel-based modelling platform that includes, in addition to Excel, Solver, and VBA, an educational add-in called Thermax [17-20]. Thermax provides seven groups of property functions for ideal gases, saturated water and superheated steam, synthetic and natural refrigerants, atmospheric humid air for psychrometric analyses, two aqua solutions for vapour-absorption refrigeration, chemically-reacting substances, and air at standard atmospheric pressure for the usual fluid dynamics and heat-transfer analyses. Thermax also provides two interpolation functions and a Newton-Raphson solver for nonlinear equations that enhance the usefulness of the Excel-based modelling platform. Table 1.1 summarises the roles of the four components of the Excel-based modelling platform as used in this book.

Table 1.1. Roles of the four components of the Excel-based modelling platform

Component	Role
Excel	• Provides the basic functions needed for the development of mathematical models including the general mathematical functions and the matrix-operation functions • Provides the Goal Seek command that can be used to perform unconstrained iterative solutions involving a single parameter • Allows circular calculations which can be a convenient method for dealing with iterative solutions of a set of linear or non-linear equations in certain analyses • Provides graphical tools needed for data visualisation and analyses • Allows macros to be recorded for repetitive calculations
Solver	• Offers three solution options that suit different types of analyses • Allows constrained iterative solutions involving multiple parameters • Allows optimisation analyses with single and multiple design variables and can be used for multi-objective optimisation
Thermax	• Provides the physical properties of various fluids • Provides two interpolation functions for tabulated data and a Newton-Raphson solver for non-linear equations such as the Colebrook-White equation and the SRK equation
VBA	• Needed for developing additional fluid property functions or other functions not provided by Excel or Thermax, e.g.: - numerical solvers for large systems of linear equations - custom function for standard pipe dimensions - custom functions for global-warming potential (GWP) of the various refrigerants

1.4. Closure

The following two chapters describe the Excel-based modelling platform in more details. Chapter 2 focuses on the features of Excel that are mostly needed for thermofluid analyses such as its matrix functions and the Goal Seek command, while Chapter 3 introduces the other three components of the modelling platform. Chapter 3 gives examples of using the three solution methods offered by Solver, describes the development of user-defined functions with VBA, and shows how the property functions provided by Thermax can be used in Excel formulae. Chapter 4 uses the Excel-based platform to deal with a common type of thermofluid analyses, which is iterative solutions. The chapter gives examples of using Excel's Goal Seek command and Solver for this type of analyses in the fields of fluid dynamics, heat-transfer, and thermodynamics.

Chapters 5, 6, and 7 use the Excel-based platform for computer-aided analyses associated with the three areas of thermofluids; heat-transfer, fluid dynamics, and thermodynamics, respectively. Chapter 5 deals with the numerical solution of the steady heat-conduction equation by using the finite-difference (FD) method. Chapter 6 focuses on hydraulic analyses of multi-pipe and pump-pipe systems and gives examples of using Excel's Goal Seek command and Solver for this type of analyses. Chapter 7 deals with thermodynamic analyses of five basic power generation and vapour-compression refrigeration (VCR) cycles by using Thermax property functions. With respect to power-generation cycles, the chapter focuses on the Brayton cycle, the Otto cycle, the conventional Rankine cycle, and the organic Rankine cycle (ORC). With respect to refrigeration cycles, it focuses on the simple single-stage VCR cycle.

Chapter 8 uses the Excel-based platform to deal with single-objective and multi-objective design optimisation analyses. The case considered for multi-objective optimisation is that of a tip-loaded circular concrete beam for which the length and diameter are to be selected such that they simultaneously minimise its weight and end-deflection. The dual-objective optimisation problem is solved by using the MIDACO solver [21]. Chapter 9 presents seven additional cases that come from the three thermofluid areas; heat-transfer, fluid-dynamics, and thermodynamics, but their analyses are relatively more challenging than the related cases described in previous chapters of the book. These cases are meant to demonstrate the advantages of computer-aided methods over the traditional analytical methods and to stimulate ideas for relevant mini projects. More project ideas are provided at the end of the chapter.

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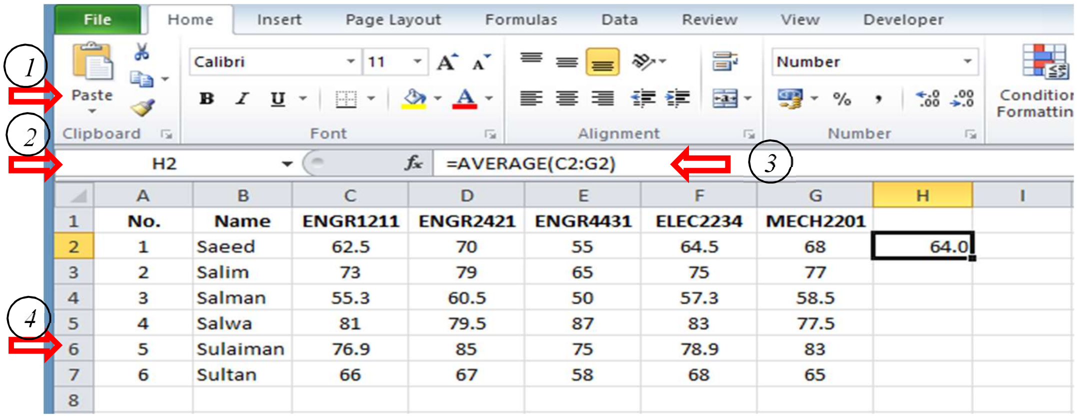
2

Excel

With its graphical tools, iterative tools, and the "Developer" options, Excel forms the backbone of the modelling platform for the various analyses considered in this book. Excel allows the manipulation of stored data by providing a large set of built-in functions and its user-interface provides many tools for general data analyses, but this chapter focuses on those needed for building analytical models relevant to the book's methodology. In this respect, the chapter highlights the use of "cell-labelling" for writing Excel's formulae instead of the usual referencing by location and illustrates the use of Excel's matrix functions for the solution of linear systems of equations. The chapter also illustrates the use of the "Goal Seek" command and the solution of nonlinear equations by "circular calculations" and its final section on Excel's graphical tools illustrates the use of the "trendline" feature for curve-fitting of tabulated data. More detailed information about Excel can be found in more specialised books, e.g., Walkenbach [1,2].

2.1. Elements of Excel's user-interface

Excel's user-interface allows the user to adjust the appearance of the workspace and present his/her primary data and analysis results in various forms. To allow easy access to the large number of functions, tools and commands provided by the user-interface, it is divided into a number of elements with different purposes. For example, Figure 2.1 shows a screenshot of an Excel sheet that stores the scores obtained by a group of students in one semester.



H2

=AVERAGE(C2:G2)

	A	B	C	D	E	F	G	H	I
1	No.	Name	ENGR1211	ENGR2421	ENGR4431	ELEC2234	MECH2201		
2	1	Saeed	62.5	70	55	64.5	68	64.0	
3	2	Salim	73	79	65	75	77		
4	3	Salman	55.3	60.5	50	57.3	58.5		
5	4	Salwa	81	79.5	87	83	77.5		
6	5	Sulaiman	76.9	85	75	78.9	83		
7	6	Sultan	66	67	58	68	65		
8									

Figure 2.1. The main elements of Excel's user-interface

Figure 2.1 shows four elements of Excel's user-interface, which are:

1. The ribbon
2. The name box
3. The formula bar
4. The workspace

The **Ribbon**, which occupies the top part of the sheet, organises the numerous commands provided by Excel into nine "tabs", e.g., the **File**, **Home**, and **Insert** tabs. Each tab consists of a number of command-groups that have a common purpose. For example, the **Developer** tab shown in Figure 2.2 allows the user to write customised functions using the Visual Basic for Applications (VBA) language and to record **Macros** together with other useful development tools.

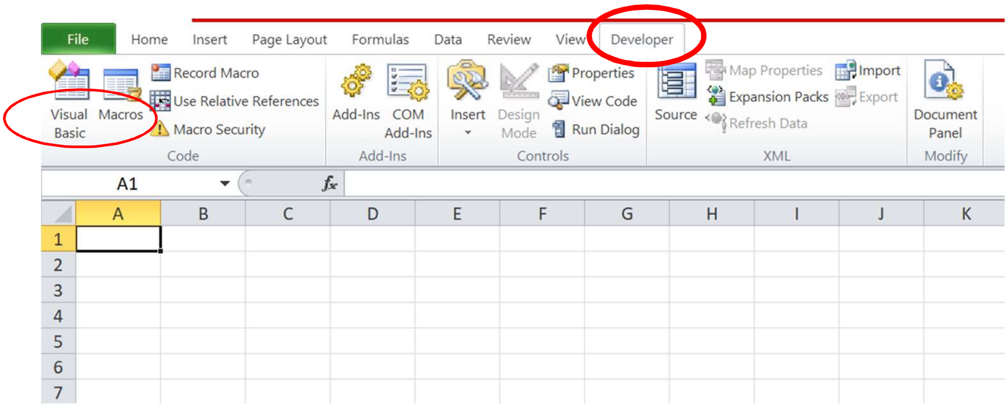


Figure 2.2. Elements of the Developer tab

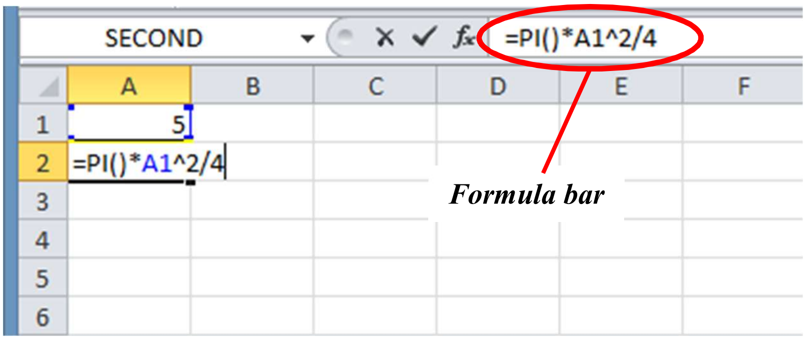
The **Workspace**, which is the main part of the sheet, is divided into a grid of columns and rows that form separate "**cells**" at their intersections. A cell is referred to by a letter and a number, e.g., A1, B3, H2, etc. The letter represents the cell's column, while the number represents its row. The **Name box** shows the name of the cell where the pointer is positioned which is H2 in Figure 2.1. As Figure 2.1 shows, a cell can simply contain a character data, such as "Saeed" and "Salim", or a numerical data, such as 62.5 and 70. A cell can also contain a formula for data manipulation using the numerous built-in functions provided by Excel. The formula bar in Figure 2.1 reveals the formula typed in cell H2 that uses the built-in function "**AVERAGE**" to determine the average score of the first student in the list (Saeed) in the five subjects as 64.0. Note that, unlike the number or character cells, a cell that includes a formula must start with the equal sign "=". The writing of Excel's formulae will be explained in more details in the following section.

2.2. Excel's formulae

In general, Excel's formulae include mathematical or logical operators, built-in functions, and cell references. Excel provides two ways to refer to a particular cell in a formula; either by its location in the sheet, e.g., A2, C10, etc., or by giving it a relevant name, e.g., efficiency, diameter, etc. The two methods will be illustrated below.

2.2.1. Cell reference by location

To illustrate this method, let us write a formula to calculate the area of a circle that has a radius of 5 m. Open a new Excel sheet and type the number 5, which is the radius of the circle, in cell A1 as shown in Figure 2.3. Now, go to cell A2 and type the following formula:


SECOND

=PI()*A1^2/4

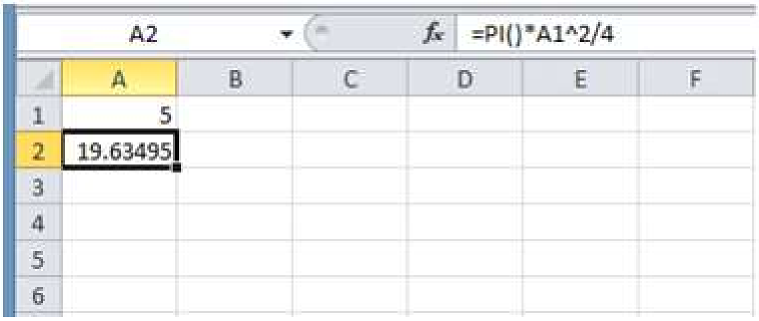
5

=PI()*A1^2/4

Figure 2.3. Writing an Excel formula to determine the area of a circle

=PI()*A1^2/4

The formula starts with the equal sign as mentioned above. The function "**PI()**" is a built-in function that returns the value of Archimedes' constant π. The formula also contains a reference to cell A1 that stores the value of the circle's radius, the multiplication operator *, the power operator ^, the division operator /, and the constants 2 and 4. Note that the formula is shown in the formula bar which can also be used to edit the formula. Pressing the **Enter** key after typing the formula, the result will be as shown in Figure 2.4; which is 19.63495 square metres. The following example shows how Excel's formulae and built-in functions can be used in a simple thermodynamic analysis.


A2

=PI()*A1^2/4

5

19.63495

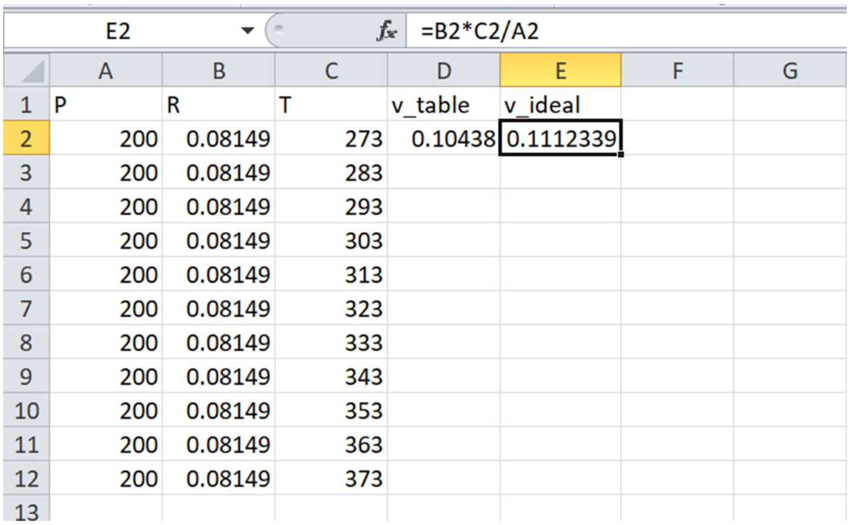
Figure 2.4. The Excel sheet with a formula that determines the area of a circle

Example 2-1. Determining the error in the specific volume of R134a calculated by using the ideal-gas law

It is required to develop an Excel sheet that calculates the error in the specific volume (v) of superheated refrigerant R134a that results from applying the ideal-gas law at a pressure of 200 kPa (T_{sat} = -9.09°C) and temperatures in the range 0°C to 100°C (273 to 373 K).

Solution

Figure 2.5 shows the Excel sheet prepared for this example. The pressure (P), the gas constant (R), and the temperature (T) are stored in columns A, B, and C, respectively. Since the pressure (P) and the gas constant (R) do not change in this example, Excel allows for these parameters to be stored in single cells instead of being repeated as shown in Figure 2.5 (see Section 2.4.1).



E2 f_x =B2*C2/A2

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal		
2	200	0.08149	273	0.10438	0.1112339		
3	200	0.08149	283				
4	200	0.08149	293				
5	200	0.08149	303				
6	200	0.08149	313				
7	200	0.08149	323				
8	200	0.08149	333				
9	200	0.08149	343				
10	200	0.08149	353				
11	200	0.08149	363				
12	200	0.08149	373				
13							

Figure 2.5. The sheet developed for Example 2-1

Column D stores the values of *v* obtained from relevant property tables and column E stores the corresponding values obtained by using the ideal-gas law, i.e.:

$$v = RT / P \qquad (2.1)$$

Where, *P* and *T* are the absolute pressure and temperature, respectively, and *R* is the gas constant (for R134a, $R = 0.08149$ kJ/kg.K). Note that the formula bar in Figure 2.5 reveals the formula in cell E2 that applies Equation (2.1).

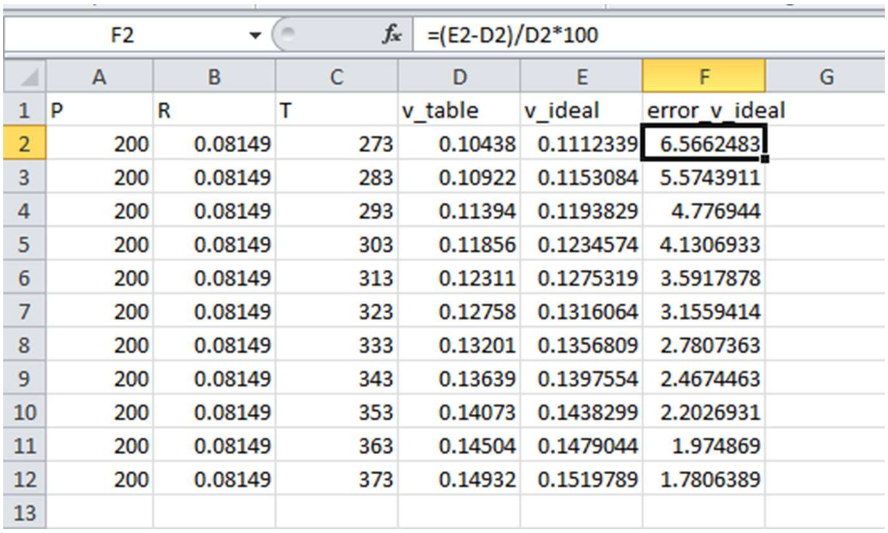
Figure 2.5 shows the tabulated value of the specific volume (v_table) and that determined by Equation (2.1) (v_ideal) at 273K. The percentage error of the ideal-gas law in estimating the specific volume is given by:

$$Error = \frac{v_{Ideal} - v_{Table}}{v_{Table}} \times 100 \qquad (2.2)$$

To determine the percentage error at 273K, go to cell F2 as shown in Figure 2.6 and type the following formula, which is equivalent to Equation (2.2):

=(E2 – D2)/D2*100

Note that the formula bar in Figure 2.6 reveals the above formula for 273K. When you press the **Enter** key, the number **6.566** will appear in cell F2 as shown in the figure.



=(E2-D2)/D2*100

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal	error_v_ideal	
2	200	0.08149	273	0.10438	0.1112339	6.5662483	
3	200	0.08149	283	0.10922	0.1153084	5.5743911	
4	200	0.08149	293	0.11394	0.1193829	4.776944	
5	200	0.08149	303	0.11856	0.1234574	4.1306933	
6	200	0.08149	313	0.12311	0.1275319	3.5917878	
7	200	0.08149	323	0.12758	0.1316064	3.1559414	
8	200	0.08149	333	0.13201	0.1356809	2.7807363	
9	200	0.08149	343	0.13639	0.1397554	2.4674463	
10	200	0.08149	353	0.14073	0.1438299	2.2026931	
11	200	0.08149	363	0.14504	0.1479044	1.974869	
12	200	0.08149	373	0.14932	0.1519789	1.7806389	
13							

Figure 2.6.The completed sheet developed for Example 2-1

To find the percentage errors at other temperatures you can simply copy the formula in cell F2 and paste it on cells F3 to F12. Figure 2.6 shows the completed Excel sheet for the required temperature range. The calculated values of the errors show that the maximum error occurs at the lowest temperature, which is 273K. Note that the error decreases gradually as the temperature increases (refer to Figure 1.10).

2.2.2. Use of cell labels

The usual reference to cells by their columns and rows suits perfectly statistical analyses in which the same formula is applied to a large body of data that is stored column-wise or row-wise. For example, we may want to determine the average value, maximum value, or minimum value of the tabulated data. However, thermofluid analyses usually involve the application of numerous formulae to a small set of data, e.g. the diameter or length of a pipe, the fluid-flow rate, the density or viscosity of the fluid, etc. For such analyses, it is more convenient to give the cell a meaningful name or "label" that matches its content. The cell can then be referred to by its label instead of its relative location. This method makes it easier to recognise the quantities involved in the Excel formulae.

For the purpose of illustration, let us develop an Excel sheet to compare the density of air before and after an isentropic compression process from an initial condition of P_1 = 100 kPa and T_1 =300K to a final pressure of P_2 = 800 kPa. The two air densities involved can be calculated from the ideal-gas law as follows:

$$\rho_1 = P_1 / RT_1 \tag{2.3}$$

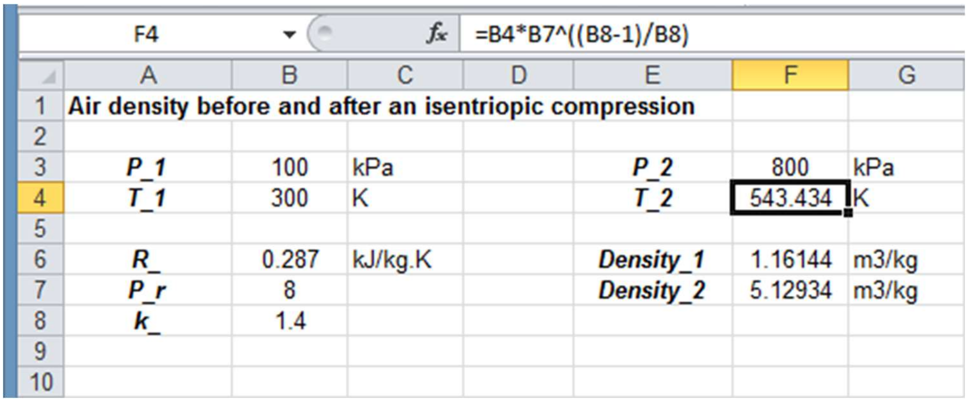
$$\rho_2 = P_2 / RT_2 \tag{2.4}$$

Where R is the gas constant (for air, $R = 0.287$ kJ/kg.K). For an isentropic process, T_2 is related to T_1 according to the following approximate relationship:

$$T_2 = T_1 \times \left(P_2 / P_1\right)^{\frac{k-1}{k}} \tag{2.5}$$

Where k is the ratio of specific heat at constant pressure (c_p) and at constant volume (c_v). At the given temperature range, k for air can be taken as 1.4. Note that the symbol k is also used for the thermal conductivity in Appendix A, Table A.1.

Figure 2.7 shows the sheet prepared for this analysis in which the respective cell labels are typed in the column to the left of the different pressures and temperatures, while the corresponding units are written in the column to the right of each quantity. This is also done to the other quantities in the calculations. The sheet also shows the units of the different properties involved for more clarification.



F4 =B4*B7^((B8-1)/B8)

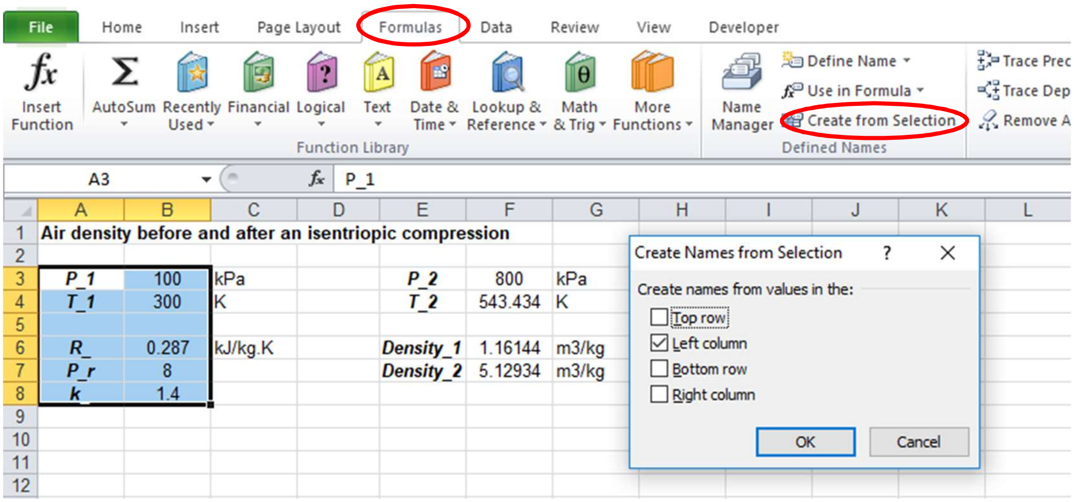
	A	B	C	D	E	F	G
1	**Air density before and after an isentriopic compression**						
2							
3	***P_1***	100	kPa		***P_2***	800	kPa
4	***T_1***	300	K		***T_2***	543.434	K
5							
6	***R_***	0.287	kJ/kg.K		***Density_1***	1.16144	m3/kg
7	***P_r***	8			***Density_2***	5.12934	m3/kg
8	***k_***	1.4					
9							
10							

Figure 2.7. Excel sheet for calculating the air densities before and after compression

Placing the cursor on cell F4 makes the formula bar reveal the formula used for the calculation of the temperature T_2 according to Equation (2.5) which is:

=B4*B7^((B8-1)/B8)

The problem with the above formula is that it is not self-explanatory and one has to go to each cell in the formula in order to figure out what the formula represents; which is very difficult when there are many such formula in the model. The formula can be made easily understandable by using familiar labels to refer to the different cells involved. To do that, select the cells in columns A and B as shown in Figure 2.8, then go to **Formulas** and, at the **Name Manager**, select **Create** from **Selection**. When the form shown in Figure 2.8 appears to you, tick the “Left column” option. Pressing the “**OK**” button will make Excel create names for the different values in the selection box according to the labels written on the left column. The cell F3 that stores the value of P_2 can also be associated with its corresponding label in cell E3.



A3 | f_x P_1

	A	B	C	D	E	F	G
1	**Air density before and after an isentriopic compression**						
2							
3	***P_1***	100	kPa		***P_2***	800	kPa
4	***T_1***	300	K		***T_2***	543.434	K
5							
6	***R_***	0.287	kJ/kg.K		***Density_1***	1.16144	m3/kg
7	***P_r***	8			***Density_2***	5.12934	m3/kg
8	***k_***	1.4					

Create Names from Selection

Create names from values in the:

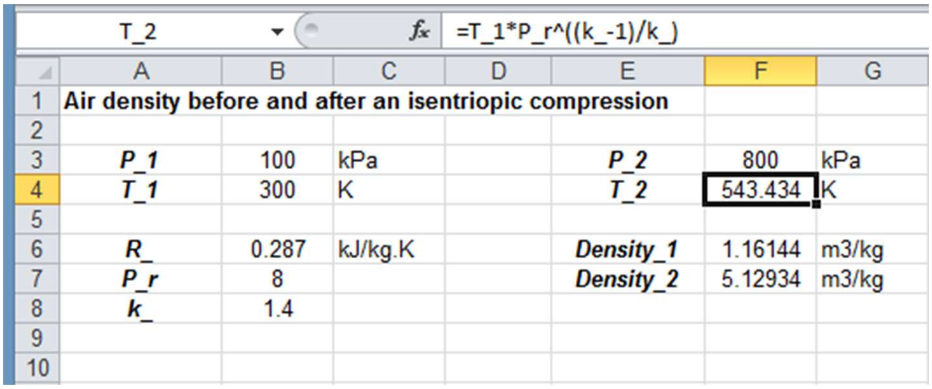
☐ Top row
☑ Left column
☐ Bottom row
☐ Right column

Figure 2.8. Creating names for a selected group of cells

Now, retype the formula in cell F4 that determines T_2 as follows:

=T_1*P_r^((k_-1)/k_)

The formula bar in the sheet shown in Figure 2.9 reveals the formula with the corresponding labels instead of the location references. The procedure can also be applied to the formulae that determine the two densities.



T_2 | f_x =T_1*P_r^((k_-1)/k_)

	A	B	C	D	E	F	G
1	**Air density before and after an isentriopic compression**						
2							
3	***P_1***	100	kPa		***P_2***	800	kPa
4	***T_1***	300	K		***T_2***	543.434	K
5							
6	***R_***	0.287	kJ/kg.K		***Density_1***	1.16144	m3/kg
7	***P_r***	8			***Density_2***	5.12934	m3/kg
8	***k_***	1.4					
9							
10							

Figure 2.9. Formulae using cells labels instead of locations

Labelled formulae are easier to edit than those using location referencing particularly when intricate formulas are involved. Another advantage of cell-labelling is that if you copy a labelled formula and paste it in any other cell you will get the same formula and the same answer, which is not the case if you copy a formula that uses the usual referencing by location in another cell. When naming your cells, choose suitable representative names for the variables involved, e.g. P_1 and T_1 for P_1 and T_1. Note that Excel does not accept “P1” or “T1” as labels since these can be confused with usual cell references by locations. To avoid this, Excel automatically changes these labels to “P1_” and “T1_”. To reveal all the formulae in the sheet, press the control key (ctrl) with the tilde key (~). More information about Excel’s formulae can be found in Walkenbach [1].

2.3. Excel’s built-in functions

Excel provides a large library of built-in functions for data manipulation, like the **AVERAGE** function, and other mathematical functions commonly used in engineering analyses like the **PI**, **SIN**, and **COS** functions. To view the full range of Excel’s functions, type “=” in any Excel cell as shown in Figure 2.10 and then place the cursor on the **Insert Function** “*fx*” button in the formula bar and click it. The dialog box shown in Figure 2.11 will appear to you. You can list all the categories via the “**Select a category**” slot.

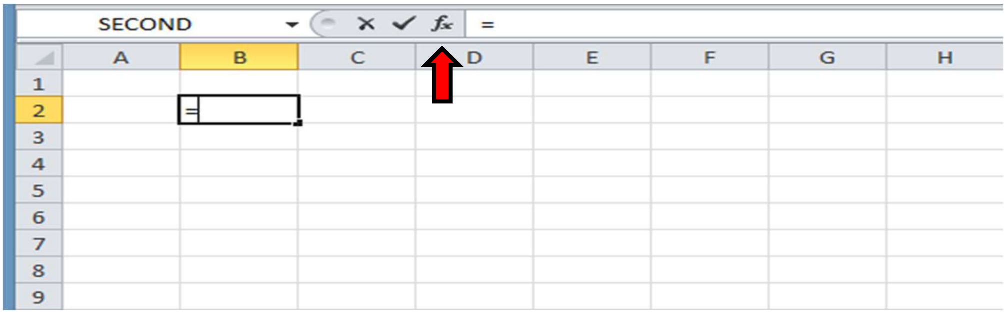
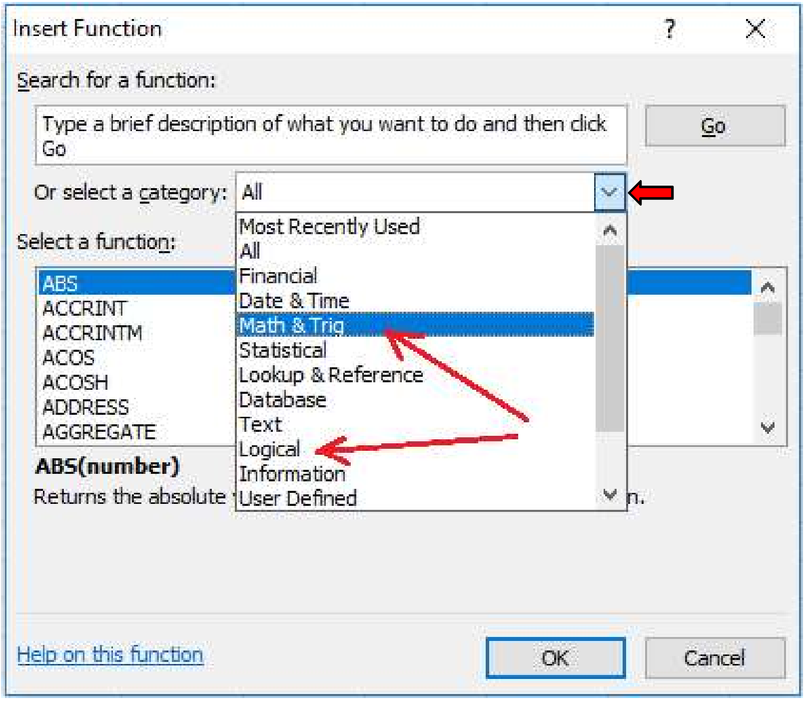


Figure 2.9. Exploring Excel’s built-in functions



Type a brief description of what you want to do and then click Go

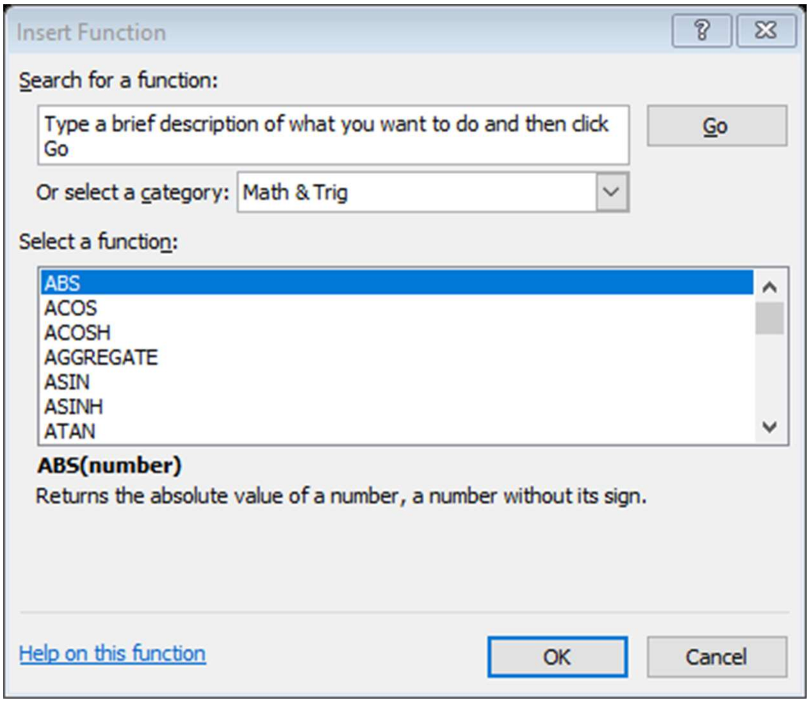
Or select a category: All

ABS(number)

Returns the absolute n.

Figure 2.11. The various categories of Excel’s built-in functions

The **Math & Trig** group includes the mathematical and trigonometric functions used in different types of engineering analyses. Figure 2.12 shows some of the numerous functions in this group. Note that a brief explanation of the function you select automatically appears below the selection window. For example, the explanation given to the **ABS** function is that it returns the absolute value of a number. The functions **ACOS**, **ASIN**, and **ATAN** apply the familiar inverse trigonometric functions: $\cos^{-1}$, $\sin^{-1}$, and $\tan^{-1}$, respectively. By scrolling down the list, you can find many other familiar functions. The following discussion focuses on two types of functions that are needed for the development of analytical models in subsequent chapters of the book, which are (a) the logical functions and (b) the functions for matrix operations.



Type a brief description of what you want to do and then click Go

Or select a category: Math & Trig

ABS(number)

Returns the absolute value of a number, a number without its sign.

Figure 2.12. Common mathematical functions supported by Excel

2.3.1. Logical functions

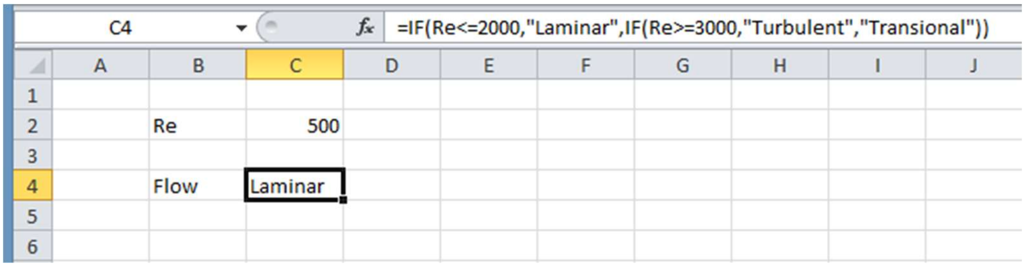
To determine the major friction loss (h_f) in a pipe by using the Darcy-Weisbach equation we have to establish whether the flow is laminar or turbulent so as to select the relevant formula for the friction factor (f). The flow remains laminar before the Reynolds number (Re) reaches a certain value, which can be taken as 2,000, but the flow can only be considered fully turbulent beyond Re = 3000. There is a transitional region between laminar and turbulent flows when $2000 < \mathrm{Re} < 3{,}000$. Suppose that we want to write an Excel formula that determines the type of flow from the given value of the Reynolds number. Using a simple **IF** function, we can write the following formula:

=IF(Re<=2000, “Laminar”, “Turbulent or transitional”)

Note that the above **IF**-formula does not differentiate between turbulent flow and transitional flows. However, we can do that by using a second logical test inside the first logical test according to the following nested IF function:

=IF(Re<=2000, “Laminar”, IF(Re>=3000, “Turbulent”, “Transitional”))

Figure 2.13 shows an Excel sheet containing the above formula (shown in the formula bar) and the response of the formula when the value of Re stored in cell C2 is 500, which is “Laminar”. Depending on the value of Re, the result of the If-formula can be “laminar”, “Turbulent”, or “Mixed”. Excel supports six other logical functions; **AND**, **FALSE**, **IFERROR**, **NOT**, **OR** and **TRUE**. These functions can be combined in the same formula so as to handle more intricate formulae.

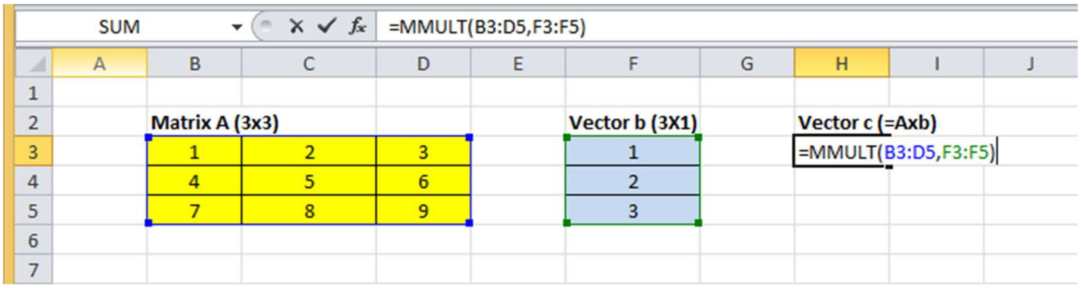

C4 =IF(Re<=2000,"Laminar",IF(Re>=3000,"Turbulent","Transional"))

	A	B	C	D	E	F	G	H	I	J
1										
2		Re	500							
3										
4		Flow	Laminar							
5										
6										

Figure 2.13. A formula using the nested IF function to determine the type of flow

2.3.2. Functions for matrix operations

A group of adjacent cells can be treated by Excel as a matrix or a vector and one of its function groups allow for the addition, subtraction, and multiplication of these matrices and vectors according to the established rules of matrix operations. Figure 2.14 shows a 3x3 matrix (A) stored in the cells B3:D5 and a vector (b) stored in cells F3:F5.


SUM =MMULT(B3:D5,F3:F5)

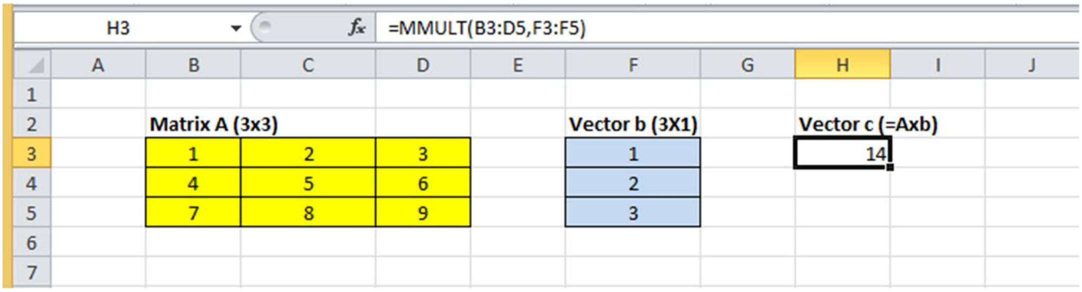
	A	B	C	D	E	F	G	H	I	J
1										
2		Matrix A (3x3)				Vector b (3X1)		Vector c (=Axb)		
3		1	2	3		1		=MMULT(B3:D5,F3:F5)		
4		4	5	6		2				
5		7	8	9		3				
6										
7										

Figure 2.14. Step 1 of using the matrix multiplication function

Matrix (A) and vector (b) can be multiplied and the result stored in a third vector (c) by using the matrix function **MMULT**. The procedure is as follows:

1. After keying in the data of matrix (A) and vector (b) as shown on Figure 2.14, position the cursor at cell H3 and type the formula: =MMULT(B3:D5;F3:F5).

2. Now press ENTER key and cell H3 will take the value 14, which the result of multiplying the first row of the matrix with the vector (b) as Figure 2.15 shows.



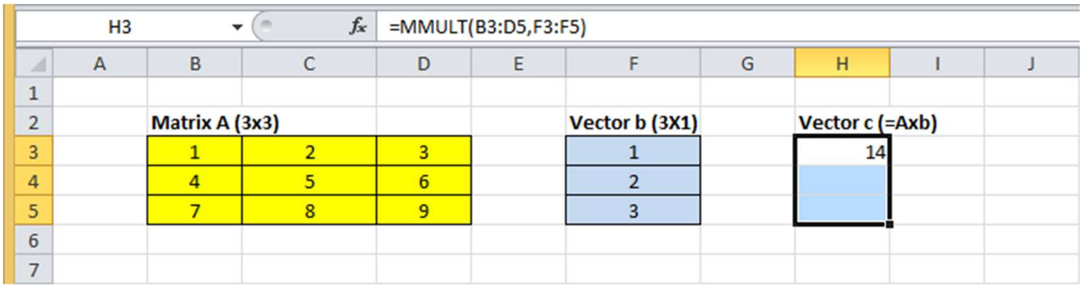
H3 =MMULT(B3:D5,F3:F5)

	A	B	C	D	E	F	G	H	I	J
1										
2		Matrix A (3x3)				Vector b (3X1)		Vector c (=Axb)		
3		1	2	3		1		14		
4		4	5	6		2				
5		7	8	9		3				
6										
7										

Figure 2.15. Step 2 of using the matrix multiplication function

The other two elements of the result vector will not appear automatically. To view the complete solution vector, do what follows:

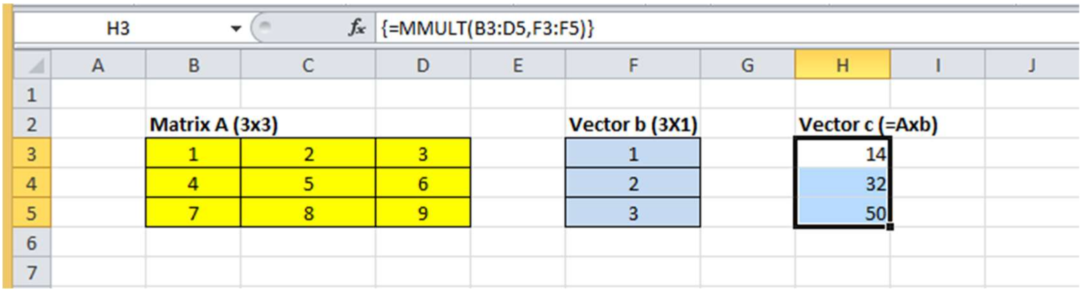
1. Select the cells H3:H5 as shown on Figure 2.16,
2. Press the function key F2 once and then simultaneously hold the (**SHIFT** + **CONTROL**) keys together and press ENTER. The complete solution vector (c) will now appear as shown on Figure 2.17.



H3 =MMULT(B3:D5,F3:F5)

	A	B	C	D	E	F	G	H	I	J
1										
2		Matrix A (3x3)				Vector b (3X1)		Vector c (=Axb)		
3		1	2	3		1		14		
4		4	5	6		2				
5		7	8	9		3				
6										
7										

Figure 2.16. Step 3 of using the matrix multiplication function



H3 {=MMULT(B3:D5,F3:F5)}

	A	B	C	D	E	F	G	H	I	J
1										
2		Matrix A (3x3)				Vector b (3X1)		Vector c (=Axb)		
3		1	2	3		1		14		
4		4	5	6		2		32		
5		7	8	9		3		50		
6										
7										

Figure 2.17. Step 4 of using the matrix multiplication function

Another matrix-operation function provided by Excel is the matrix-inversion function **MINVERSE** which is useful for the solution of linear systems of equations. The following example illustrates the use of this function.

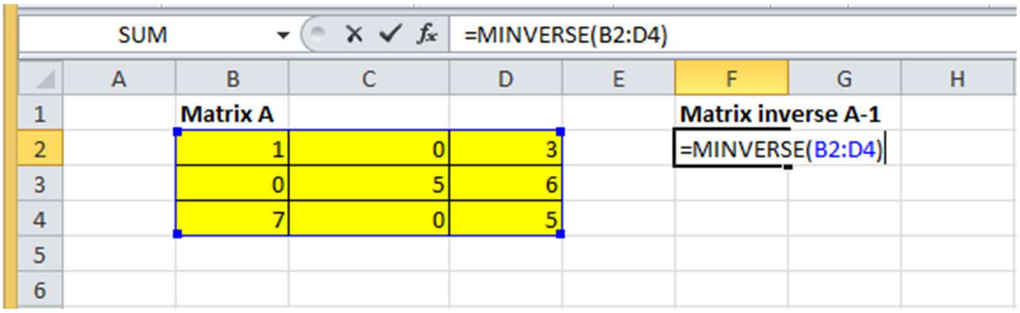
Example 2-2. Using the matrix inversion function

By using the **MINVERSE** function, find the inverse of matrix [A] given by:

$$[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 6 \\ 7 & 0 & 5 \end{bmatrix}$$

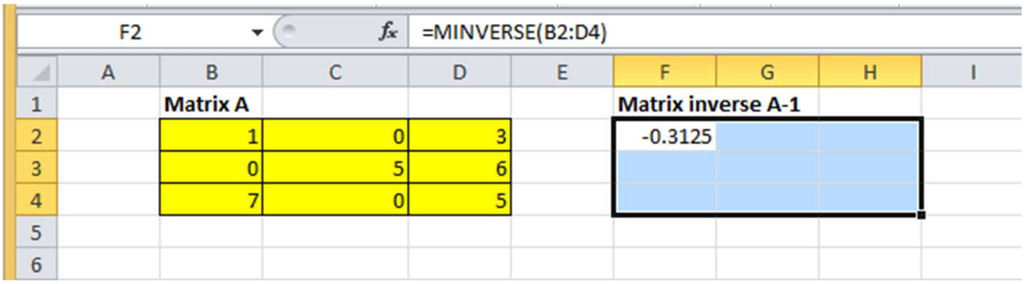
Solution

The first step of the solution is to enter the elements of the matrix as shown on Figure 2.18. After entering the data, go to cell F2 and type the formula "=MINVERSE(B2:D4)". When you press **ENTER**, this cell will have the value -0.3125, which is the first element of the inverse matrix $[A]^{-1}$ shown on Figure 2.19.



	A	B	C	D	E	F	G	H
1		Matrix A				Matrix inverse A-1		
2		1	0	3		=MINVERSE(B2:D4)		
3		0	5	6				
4		7	0	5				
5								
6								

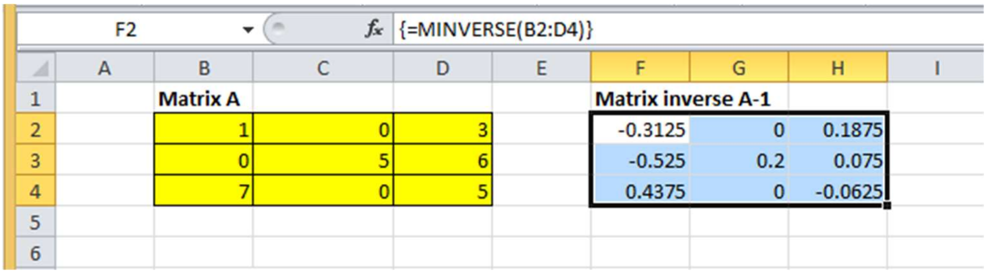
Figure 2.18. Step 1 of using the MINVERSE function



	A	B	C	D	E	F	G	H	I
1		Matrix A				Matrix inverse A-1			
2		1	0	3		-0.3125			
3		0	5	6					
4		7	0	5					
5									
6									

Figure 2.19. Step 2 of using the MINVERSE function

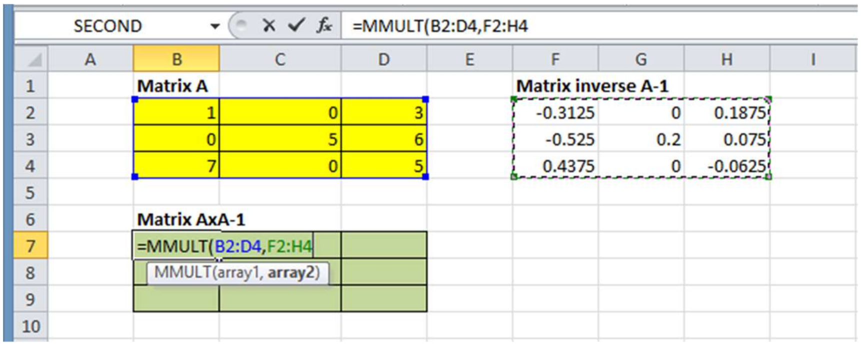
In this case, the result of the matrix operation is another matrix. Starting with the formula in cell F2, select the range F2 to H4 as shown on Figure 2.19. Press and release the function key F2 and then simultaneously hold the **CTRL+SHIFT** keys and press **ENTER**. The other elements of the inverse matrix $[A]^{-1}$ will then appear as shown on Figure 2.20. You can check the solution by multiplying matrix [A] with its inverse by using the **MMULT** functions as explained above. The steps of applying the procedure are shown on Figures 2.21 to 2.23. As should be expected, Figure 2.23 shows that the resultant matrix is the identity matrix.



F2 {=MINVERSE(B2:D4)}

	A	B	C	D	E	F	G	H	I
1		Matrix A				Matrix inverse A-1			
2		1	0	3		-0.3125	0	0.1875	
3		0	5	6		-0.525	0.2	0.075	
4		7	0	5		0.4375	0	-0.0625	
5									
6									

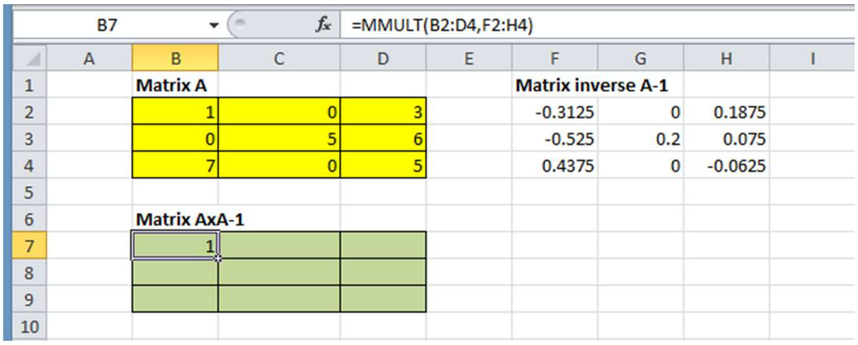
Figure 2.20. The complete inverse matrix $[A]^{-1}$



SECOND =MMULT(B2:D4,F2:H4

	A	B	C	D	E	F	G	H	I
1		Matrix A				Matrix inverse A-1			
2		1	0	3		-0.3125	0	0.1875	
3		0	5	6		-0.525	0.2	0.075	
4		7	0	5		0.4375	0	-0.0625	
5									
6		Matrix AxA-1							
7		=MMULT(B2:D4,F2:H4							
8									
9									
10									

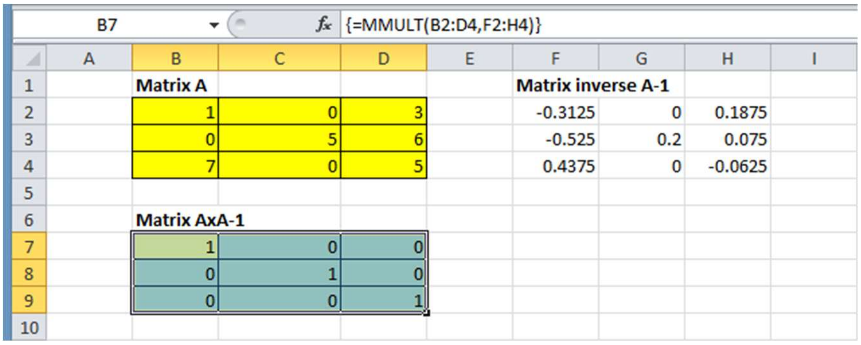
Figure 2.21. Multiplying matrix [A] by its inverse $[A]^{-1}$



B7 =MMULT(B2:D4,F2:H4)

	A	B	C	D	E	F	G	H	I
1		Matrix A				Matrix inverse A-1			
2		1	0	3		-0.3125	0	0.1875	
3		0	5	6		-0.525	0.2	0.075	
4		7	0	5		0.4375	0	-0.0625	
5									
6		Matrix AxA-1							
7		1							
8									
9									
10									

Figure 2.22. The first element of the product (identity) matrix



B7 {=MMULT(B2:D4,F2:H4)}

	A	B	C	D	E	F	G	H	I
1		Matrix A				Matrix inverse A-1			
2		1	0	3		-0.3125	0	0.1875	
3		0	5	6		-0.525	0.2	0.075	
4		7	0	5		0.4375	0	-0.0625	
5									
6		Matrix AxA-1							
7		1	0	0					
8		0	1	0					
9		0	0	1					
10									

Figure 2.23. The complete solution which is the identity matrix

2.3.3. Solution of linear system of equations

The problems that involve systems of linear equations are very common in engineering analyses. In thermofluid analyses an example of such problems is the solution of the heat conduction equation by the finite-difference method. One of the methods offered by Excel for solving linear systems of equations is by applying the matrix-inversion method. For illustration, consider the following linear system written in matrix notation:

$$[A]\{x\}=\{y\} \tag{2.6}$$

Where [A] is the coefficient matrix, $\{x\}$ the vector of unknowns, and $\{y\}$ the right-side or "load" vector. By applying the matrix-inversion method, the solution vector $\{x\}$ can be obtained as follows:

$$\{x\}=[A]^{-1}\{y\} \tag{2.7}$$

Where $[A]^{-1}$ is the inverse of matrix [A]. The method is applied in two steps: (1) inversion of the coefficient matrix and (2) multiplication of the inversed matrix with the load vector. The resulting vector is the solution. The following example illustrates the procedure of applying the method by using Excel's matrix functions.

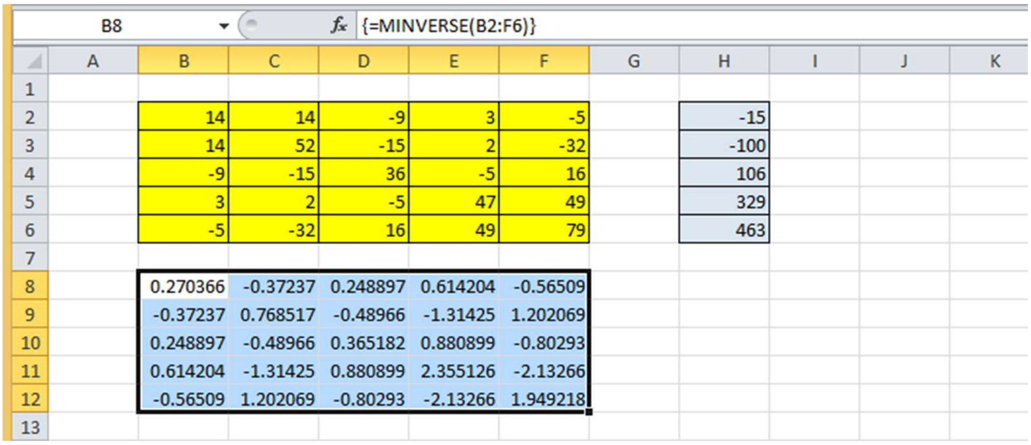
Example 2-3. Solution of a system of linear equations

Find the values of x_i in the following system of linear equations:

$$\begin{bmatrix} 14 & 14 & -9 & 3 & -5 \\ 14 & 52 & -15 & 2 & -32 \\ -9 & -15 & 36 & -5 & 16 \\ 3 & 2 & -5 & 47 & 49 \\ -5 & -32 & 16 & 49 & 79 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} -15 \\ -100 \\ 106 \\ 329 \\ 463 \end{Bmatrix} \tag{2.8}$$

Solution

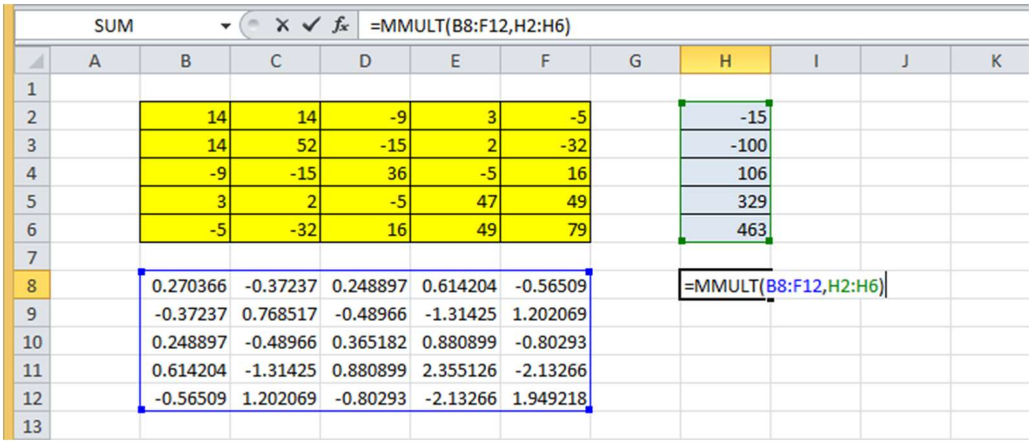
Note that the system is symmetric; which is typically the case with the linear systems that arise in the solution of heat-conduction problems by the finite-difference method. For larger systems of equations, the symmetry of the system can be utilised for reducing the required computer memory by storing only half of the coefficient matrix. However, this requires a complicated computer programming. For small systems like the one considered here, it is more convenient to use Excel's matrix inversion and multiplication functions. Figure 2.24 shows the Excel sheet that stores both the coefficient matrix [A] and the load vector $\{y\}$. The inverse matrix $[A]^{-1}$, which is obtained by following the procedure described in the previous section, is stored below the coefficient matrix as shown in the figure. The inverse matrix $[A]^{-1}$ is then multiplied with the load vector {y} and the result stored below the load vector as shown on Figure 2.25. The complete solution is shown on Figure 2.26. The first element is practically zero and, therefore, the solution vector is{x} = (0, 1, 2, 3, 4).


B8

{=MINVERSE(B2:F6)}

	A	B	C	D	E	F	G	H	I	J	K
1											
2		14	14	-9	3	-5		-15			
3		14	52	-15	2	-32		-100			
4		-9	-15	36	-5	16		106			
5		3	2	-5	47	49		329			
6		-5	-32	16	49	79		463			
7											
8		0.270366	-0.37237	0.248897	0.614204	-0.56509					
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069					
10		0.248897	-0.48966	0.365182	0.880899	-0.80293					
11		0.614204	-1.31425	0.880899	2.355126	-2.13266					
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218					
13											

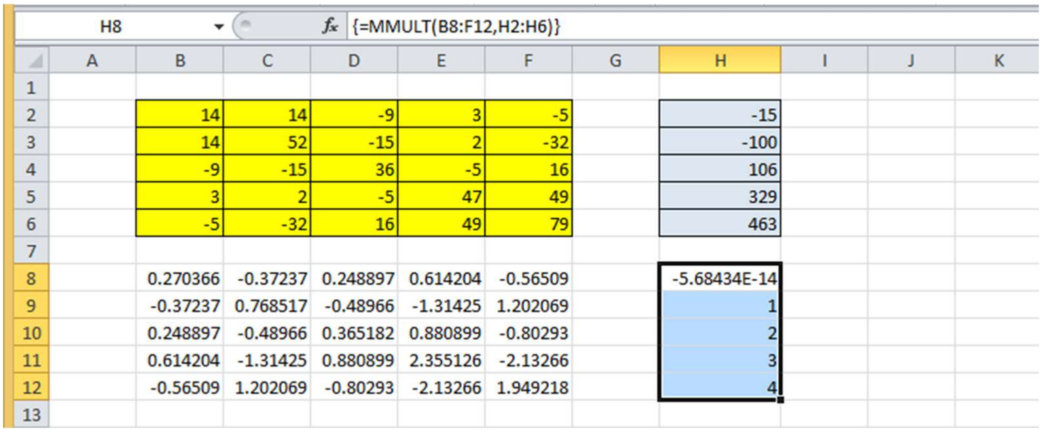
Figure 2.24. The coefficient matrix [A], the load vector {y} and the inverse matrix $[A]^{-1}$


SUM

=MMULT(B8:F12,H2:H6)

	A	B	C	D	E	F	G	H	I	J	K
1											
2		14	14	-9	3	-5		-15			
3		14	52	-15	2	-32		-100			
4		-9	-15	36	-5	16		106			
5		3	2	-5	47	49		329			
6		-5	-32	16	49	79		463			
7											
8		0.270366	-0.37237	0.248897	0.614204	-0.56509		=MMULT(B8:F12,H2:H6)			
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069					
10		0.248897	-0.48966	0.365182	0.880899	-0.80293					
11		0.614204	-1.31425	0.880899	2.355126	-2.13266					
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218					
13											

Figure 2.25. The first step of multiplying the inverse matrix $[A]^{-1}$ with the load vector {y}


H8

{=MMULT(B8:F12,H2:H6)}

	A	B	C	D	E	F	G	H	I	J	K
1											
2		14	14	-9	3	-5		-15			
3		14	52	-15	2	-32		-100			
4		-9	-15	36	-5	16		106			
5		3	2	-5	47	49		329			
6		-5	-32	16	49	79		463			
7											
8		0.270366	-0.37237	0.248897	0.614204	-0.56509		-5.68434E-14			
9		-0.37237	0.768517	-0.48966	-1.31425	1.202069		1			
10		0.248897	-0.48966	0.365182	0.880899	-0.80293		2			
11		0.614204	-1.31425	0.880899	2.355126	-2.13266		3			
12		-0.56509	1.202069	-0.80293	-2.13266	1.949218		4			
13											

Figure 2.26. The complete solution vector {x}

It should be mentioned that the linear systems generated in multi-dimensional heat-transfer and fluid-dynamics analyses are usually too large to be solved efficiently by using the matrix-inversion method described above. As mentioned in Chapter 1, page 16, such analyses require dedicated software for other reasons any way. Small systems of linear equations can also be solved by using Solver as described in Chapter 3.

2.4. Itearative solutions with Excel

Iterative solutions are another type of analyses commonly encountered in engineering analyses. Two methods can be used to perform iterative solutions with Excel: (i) by using the **Goal Seek** command and (ii) by using **circular calculations**. In what follows the two methods are illustrated with the help of simple examples.

2.4.1. Itearative solutions with Goal Seek

The **Goal Seek** command is used for finding the value of an independent variable (x) that yields a specified value of a dependent variable (y). It is a simple, yet very useful tool for "What-if" analyses. The following example illustrates how this command can be used to solve a nonlinear equation.

Example 2-4. Solution of a nonlinear equation by Goal Seek

A centrifugal pump is used for lifting water from the utility network at the ground level to a tank at the top of a building that is 30-m high as shown on Figure 2.27. The pump's characteristic curve can be represented by the following formula:

$$h_p = h_0 - aQ - bQ^2 - cQ^3 \qquad (2.9)$$

Where h_p and Q are the pump's head (m) and discharge (m^3/s), respectively, and h_0, a, b, and c are constants the values of which are 47.22, 2.985x10^3, 1.549x10^5, and 2.348x10^8, respectively. Neglecting friction losses in the pipe, determine the water flow rate (m^3/s) that can be delivered by the pump.

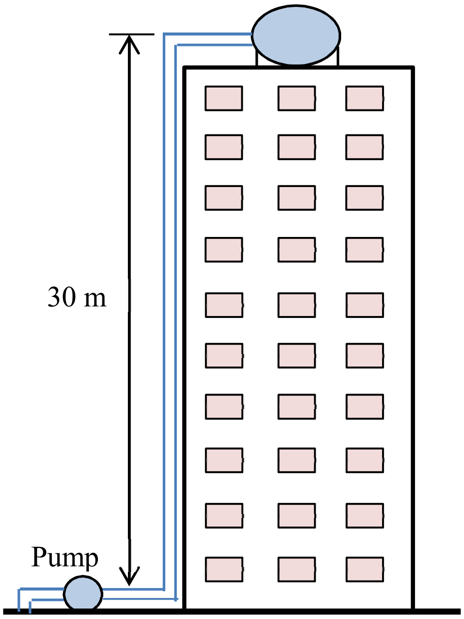
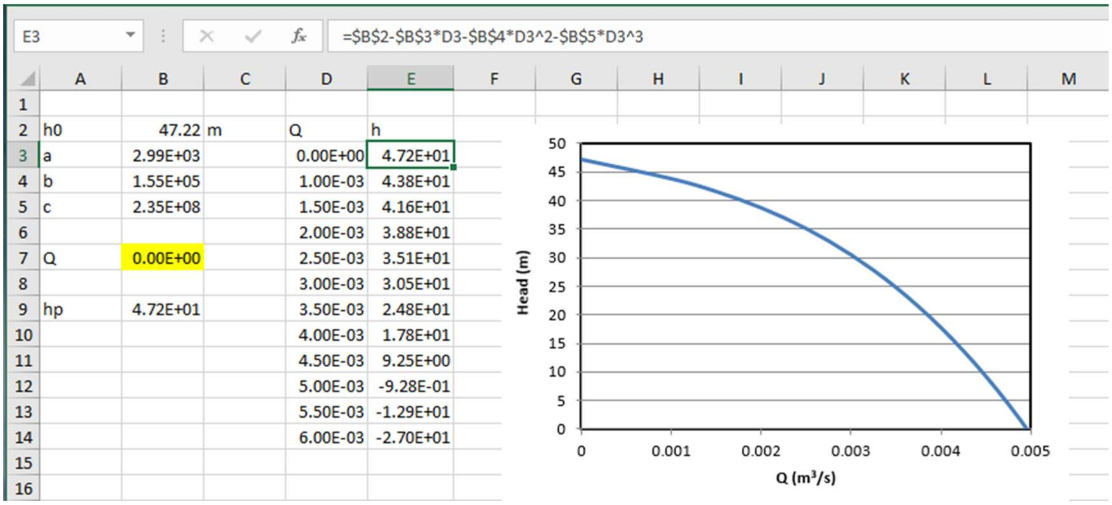


Figure 2.27. Schematic for Example 2-4

Solution

Figure 2.28 shows the Excel sheet prepared for this example in which the values of the four constants in Equation (2.9) are stored on the left side of the sheet. The pump's head is calculated at various values of the discharge and plotted as shown on the figure. We can see from the plot that the value of Q that yield h_p = 30 m is approximately 0.003 m^3/s.



E3 =$B$2-$B$3*D3-$B$4*D3^2-$B$5*D3^3

	A	B	C	D	E
1					
2	h0	47.22	m	Q	h
3	a	2.99E+03		0.00E+00	4.72E+01
4	b	1.55E+05		1.00E-03	4.38E+01
5	c	2.35E+08		1.50E-03	4.16E+01
6				2.00E-03	3.88E+01
7	Q	0.00E+00		2.50E-03	3.51E+01
8				3.00E-03	3.05E+01
9	hp	4.72E+01		3.50E-03	2.48E+01
10				4.00E-03	1.78E+01
11				4.50E-03	9.25E+00
12				5.00E-03	-9.28E-01
13				5.50E-03	-1.29E+01
14				6.00E-03	-2.70E+01
15					
16					

Figure 2.28. Excel sheet for Example 2-4

To solve the problem by using Goal Seek, enter an initial guess for Q in cell B7, say 0, and then enter the following formula that uses Equation (2.9) to calculate h_p in cell B9:

=$B$2-$B$3*B7-$B$4*B7^2-$B$5*B7^3

Note the dollar sign ($) that has been added to the references of the four constants, e.g. B2 has become $B$2. The formula bar in Figure 2.28 reveals a copy of the above formula in cell E3 that uses Equation 2.9 to calculate the head from the discharge. Copying and pasting this formula in the following cells keeps the references to the cells that store the four constants and only changes D3 to D4, D5, etc. To activate the Goal Seek command, go to the **Data** tab, select the **What-If-Analysis** option in the **Data Tools** group and then select **Goal Seek,** as shown on Figure 2.29. The Goal Seek dialog box shown on Figure 2.30.a will then appear to you and asks you to select the "**Set cell**", i.e. the cell that contains the dependent variable, which is B9 in this case. You also have to specify the value sought for this cell and the adjustable cell that stores the parameter to be changed. In this case, we seek the value in the cell B9 to be 30 by changing the value of the cell B7. The completed form is shown on Figure 2.30.b.

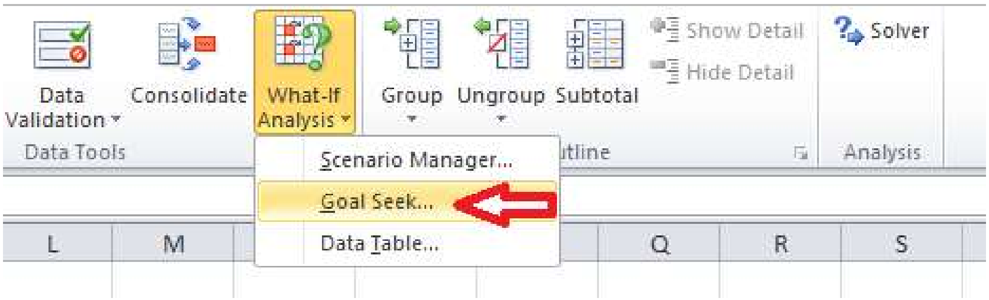
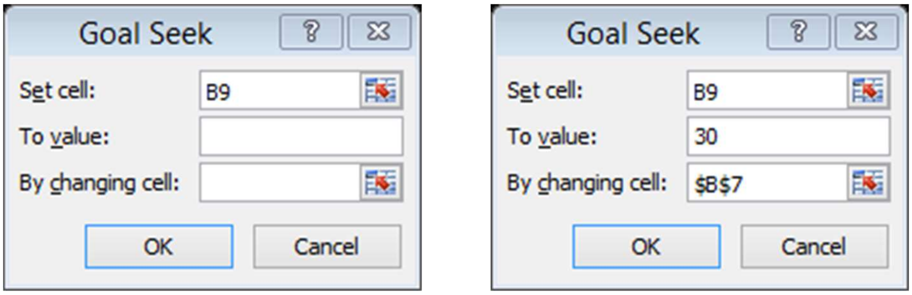


Figure 2.29. Activation of the Goal Seek command



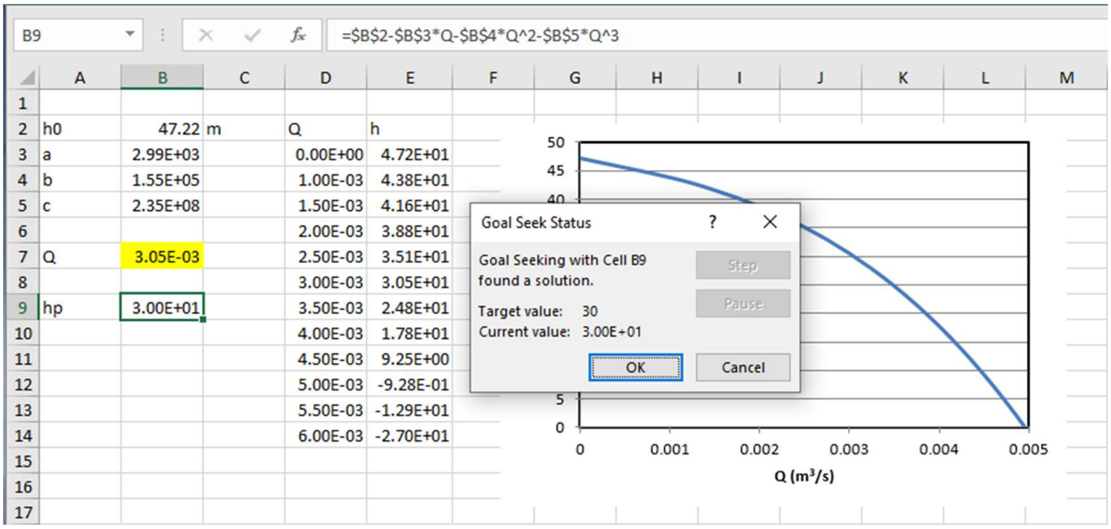
(a) Set cell: B9
To value:
By changing cell:

(b) Set cell: B9
To value: 30
By changing cell: $B$7

(a) (b)

Figure 2.30. Goal Seek Set-up for Example 2-4: (a) before completion (b) the completed box

By pressing the "**OK**" button after completing the Goal Seek form, Excel will change the value in the adjustable cell (B7) until the Set cell (B9) acquires the required value. As shown on Figure 2.31, the answer obtained for Q is 0.003 m^3/s, which agrees with the estimated value from the plot on Figure 2.28.



B9 =$B$2-$B$3*Q-$B$4*Q^2-$B$5*Q^3

	A	B	C	D	E
1					
2	h0	47.22	m	Q	h
3	a	2.99E+03		0.00E+00	4.72E+01
4	b	1.55E+05		1.00E-03	4.38E+01
5	c	2.35E+08		1.50E-03	4.16E+01
6				2.00E-03	3.88E+01
7	Q	3.05E-03		2.50E-03	3.51E+01
8				3.00E-03	3.05E+01
9	hp	3.00E+01		3.50E-03	2.48E+01
10				4.00E-03	1.78E+01
11				4.50E-03	9.25E+00
12				5.00E-03	-9.28E-01
13				5.50E-03	-1.29E+01
14				6.00E-03	-2.70E+01
15					
16					
17					

Goal Seek Status

Goal Seeking with Cell B9 found a solution.

Target value: 30
Current value: 3.00E+01

Figure 2.31. Goal Seek solution for Example 2-4

2.4.2. Iterative solution with circular calculations

A circular reference occurs when an Excel formula refers to its own cell in a direct or indirect manner. This occurs, for example, when solving a system of linear or non-linear equations because the various elements of the solution depend on each other. **Circular calculation** allows Excel to itearate until all the formulae involved are satisfied. The following example illustrates this special feature which is useful for thermofluid analyses.

Example 2-5. Determining the final temperature of heated air

Heat is added to a piston-cylinder device that contains one kg of air initially at 300K. If 100 kJ of heat is added to the air at constant pressure, determine the final temperature of air taking into consideration that its molar specific-heat ($\tilde{c}_p$) varies with temperature according to the following formula:

$$\tilde{c}_p = a + bT + cT^2 + dT^3 \quad \text{[kJ/kmol]} \tag{2.10}$$

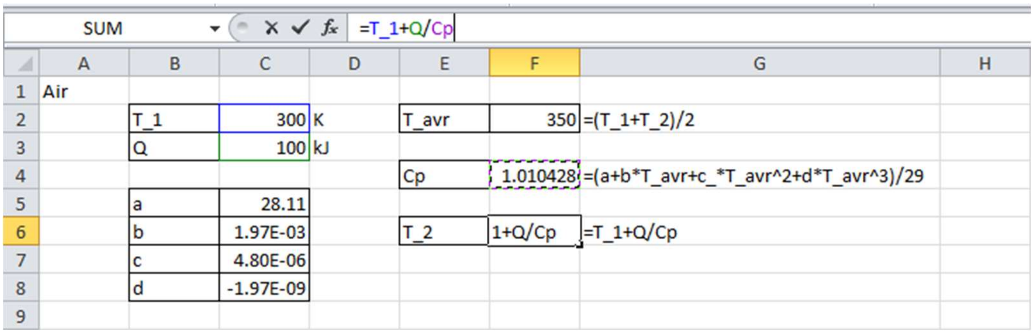
Where $a = 28.11$, $b = 1.97\text{x}10^{-03}$, $c = 4.80\text{x}10^{-06}$, and $d = -1.97\text{x}10^{-09}$.

Solution

From the defenition of specific heat, the final temperature (T_2) is given by:

$$T_2 = T_1 + Q/\left(\tilde{c}_p / M\right) \tag{2.11}$$

Where T_1 is the initial temperature, Q is the amount of heat added, and M is the molar mass (for air, M=29). If the variation of $\tilde{c}_p$ with temperature is ignored and its value at T_1 alone is used, Equation (2.11) determines T_2 as 399.73K. However, the result can be more accurate by using Equation (2.10) to determine $\tilde{c}_p$ at the average temperature, $T_{avr} = (T_1+T_2)/2$. Figure 2.32 shows the Excel sheet developed for this method which reveals the formulae inserted in cells F2, F4, and F6.



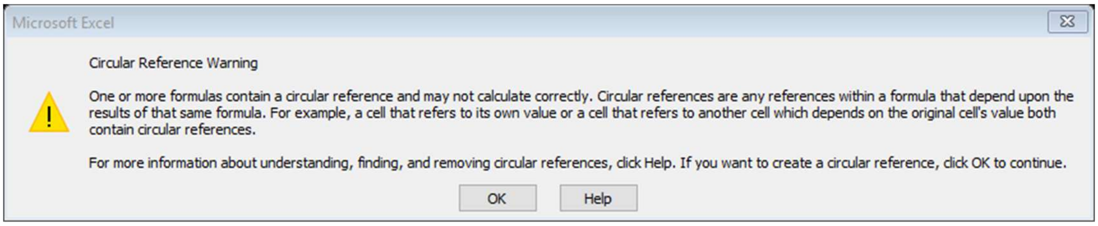
SUM

=T_1+Q/Cp

	A	B	C	D	E	F	G	H
1	Air							
2		T_1	300	K	T_avr	350	=(T_1+T_2)/2	
3		Q	100	kJ				
4					Cp	1.010428	=(a+b*T_avr+c_*T_avr^2+d*T_avr^3)/29	
5		a	28.11					
6		b	1.97E-03		T_2	1+Q/Cp	=T_1+Q/Cp	
7		c	4.80E-06					
8		d	-1.97E-09					
9								

Figure 2.32. Excel sheet developed for Example 2-5

As soon as we type Equation (2.11) in cell F6, Excel will make the warning message that there is a circulare refernce as shown on Figure 2.33.



Microsoft Excel

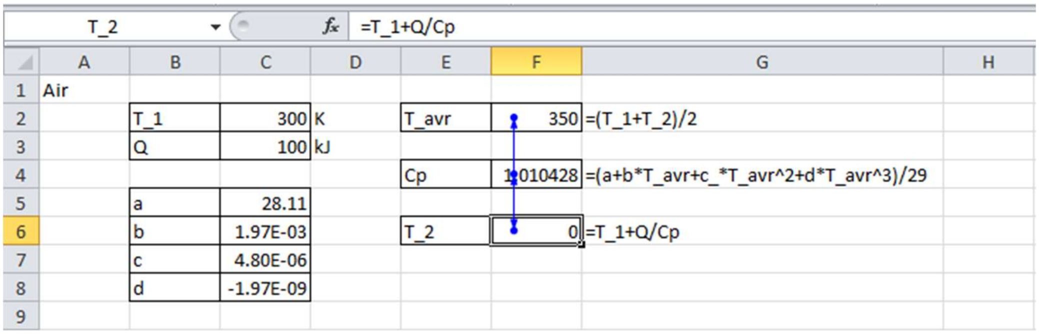
Circular Reference Warning

One or more formulas contain a circular reference and may not calculate correctly. Circular references are any references within a formula that depend upon the results of that same formula. For example, a cell that refers to its own value or a cell that refers to another cell which depends on the original cell's value both contain circular references.

For more information about understanding, finding, and removing circular references, click Help. If you want to create a circular reference, click OK to continue.

Figure 2.33. The circular-reference prompt

The circular reference occurs because T_2 depends on $\tilde{c}_p$ according to Equation (2.11) while $\tilde{c}_p$ itself depends on T_2 according to Equation (2.10). If we press the "OK" button shown on Figure 2.33, the cells involved in the circular reference whill be identified as shown on Figure 2.34. In this case, three cells are involved in the circular reference, which are F2, F4, and F6.

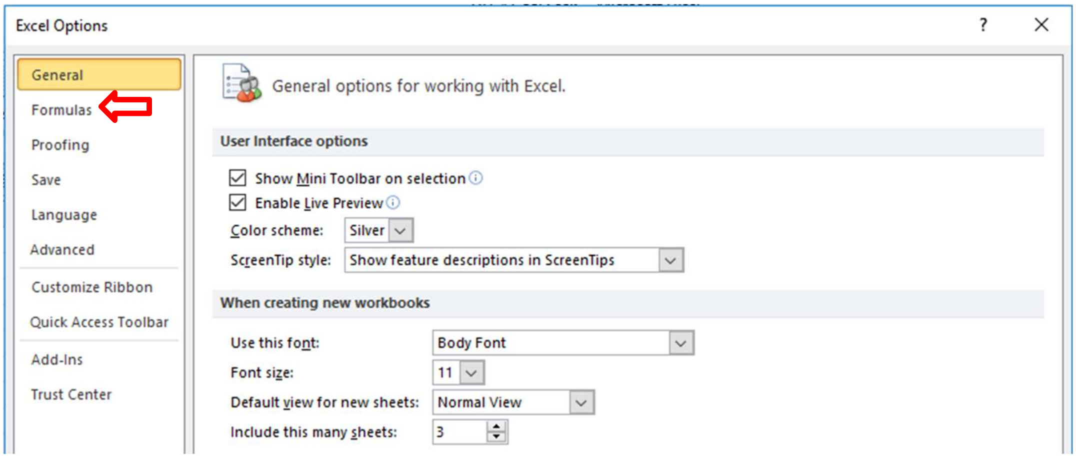


T_2 =T_1+Q/Cp

	A	B	C	D	E	F	G	H
1	Air							
2		T_1	300	K	T_avr	350	=(T_1+T_2)/2	
3		Q	100	kJ				
4					Cp	1.010428	=(a+b*T_avr+c_*T_avr^2+d*T_avr^3)/29	
5		a	28.11					
6		b	1.97E-03		T_2	0	=T_1+Q/Cp	
7		c	4.80E-06					
8		d	-1.97E-09					
9								

Figure 2.34. The cells involved in the circular reference

By allowing circular calculations, Excel will iterate to determine the values of both T_2 and $\tilde{c}_p$ that satisfy the relevant equations. However, the iterative-calculation option is not allowed by default. To allow it, go to **File** and select **Options**. The **Backstage View** form shown on Figure 2.35 will appear to you. Select **Formulas**, then the form will appear as shown on Figure 2.36. Enable iterative calculations by ticking (√) the box indicated in the figure and press the "OK" button. Excel can now iterate to find the values of T_2 and $\tilde{c}_p$ that simultaneously satisfy Equations (2.10) and (2.11). Figure 2.37 shows the solution found by this method, which is T_2 =398.976K. By using a constant value for $\tilde{c}_p$, the value obtained earlier was 399.73K. The difference between the two solutions will increase as more heat is added to the air.



General options for working with Excel.

User Interface options

☑ Show Mini Toolbar on selection

☑ Enable Live Preview

Color scheme: Silver

ScreenTip style: Show feature descriptions in ScreenTips

When creating new workbooks

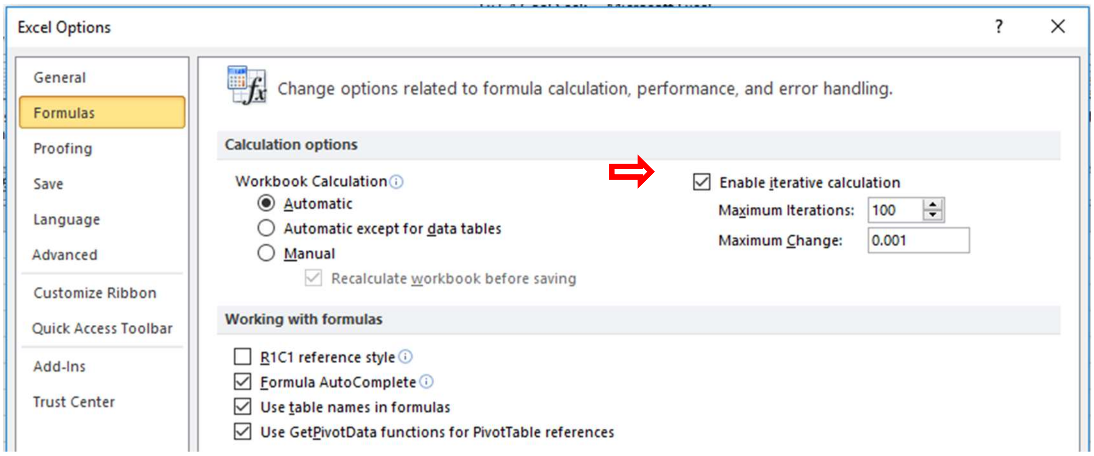
Use this font: Body Font

Font size: 11

Default view for new sheets: Normal View

Include this many sheets: 3

Figure 2.35. Selecting Excel's option (Formulas)



Change options related to formula calculation, performance, and error handling.

Calculation options

Workbook Calculation
- ● Automatic
- ○ Automatic except for data tables
- ○ Manual
  - ☑ Recalculate workbook before saving

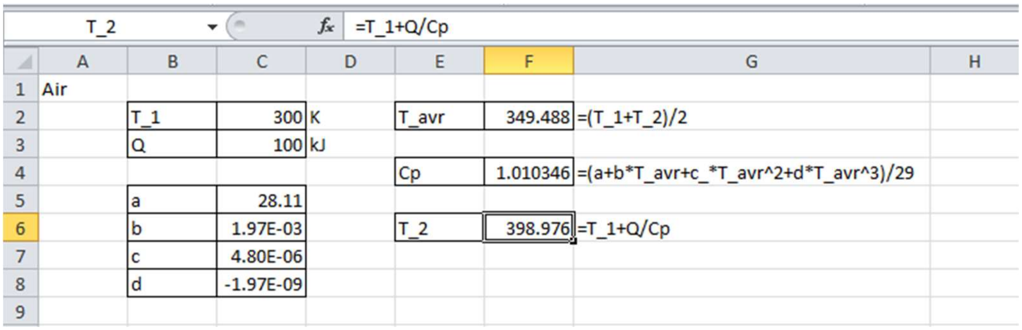
☑ Enable iterative calculation

Maximum Iterations: 100

Maximum Change: 0.001

Working with formulas
- ☐ R1C1 reference style
- ☑ Formula AutoComplete
- ☑ Use table names in formulas
- ☑ Use GetPivotData functions for PivotTable references

Figure 2.36. Enabling iterative calculations from Excel's Formulas option



T_2 =T_1+Q/Cp

	A	B	C	D	E	F	G	H
1	Air							
2		T_1	300	K	T_avr	349.488	=(T_1+T_2)/2	
3		Q	100	kJ				
4					Cp	1.010346	=(a+b*T_avr+c_*T_avr^2+d*T_avr^3)/29	
5		a	28.11					
6		b	1.97E-03		T_2	398.976	=T_1+Q/Cp	
7		c	4.80E-06					
8		d	-1.97E-09					
9								

Figure 2.37. Solution of Example 2-5 by circular calculations

This example can also be solved by using the Goal Seek command. With this method, we start the iterative solution by providing Excel with a guessed value for T_2, call it T_{2o}, based on which a new value for T_2 is calculated, called it T_{2c}. Since the guessed value T_{2c} is unlikely to be correct, it will be different from T_{2o}. Goal Seek is then used to adjust the value of T_{2o} until the difference (Diff = T_{2c} - T_{2o}) vanishes.

Most iterative solutions in this book are obtained by using the Goal Seek command. However, the circular-calculation option is more useful than Goal Seek in certain situations as demonstrated in Chapter 5 that deals with the numerical solution of the heat-conduction equation with the finite-difference method and Chapter 6 that deals with the hydraulic analyses of pump-pipe systems. The option can also be used to solve small systems of linear or non-linear equations. This is left as an exrcise (refer to Exercise 2.3).

2.5. Excel’s graphical tools for data presentation and analysis

Excel has numerous graphical tools that can be used to present the stored data in a variety of charts. Figure 2.38 shows one type of Excel charts that displays the annual variation of temperature and relative humidity at a hypothetical location on a certain day. The figure shows a line chart in which the temperature is scaled on the primary *y*-axis (on the left) while the humidity is scaled on the secondary *y*-axis (on the right). This arrangement is useful for displaying two or more types of data that differ significantly in magnitude, such as the net specific work and thermal efficiency of a power cycle, on the same chart.

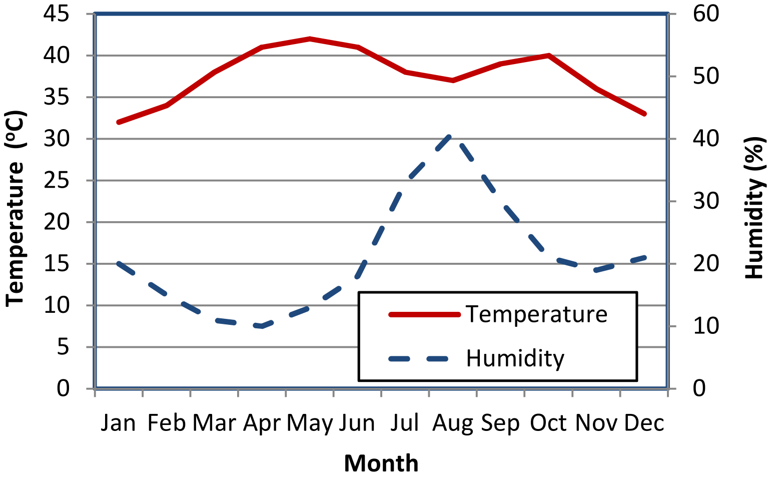


Figure 2.38. An example of line charts

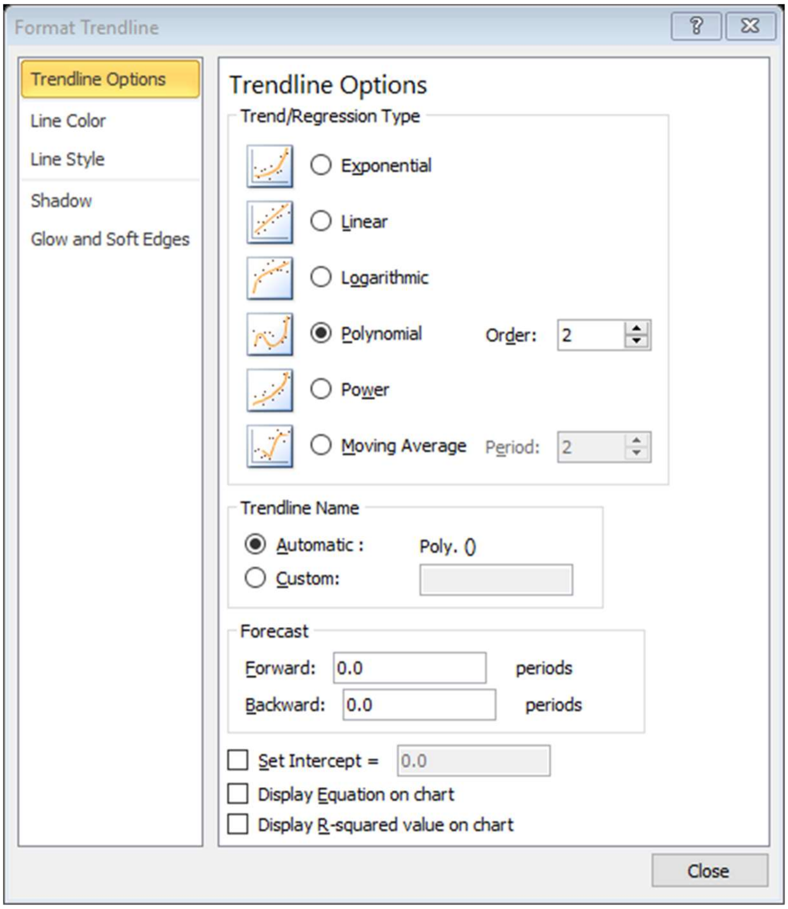
Excel supports other types of charts that allow the user to select the most appropriate way to display his/her data in the form of bar, area, or scatter charts. For more information about the different types of Excel's charts, the reader can refer to specialised references such as Walkenbach [2]. A number of tutorials and videos that show how to create different types of charts can also be found in the internet.

A useful feature of Excel is that its charts provide a curve-fitting capability of numerical data by using the **Trendline** feature. This capability is particularly useful for computer-aided thermofluid analyses because it can be used to convert tabulated fluid-properties and other data into analytical equations that make the data more suitable for iterative solutions and optimisation analyses. To illustrate the use of this feature, consider Table 2.1 that shows properties of saturated water in the range 0.001°C – 60°C. These values of the saturation pressure (P_{sat}) and saturated liquid enthalpy (h_f) are used in psychrometric analyses of air-conditioning applications. For computer-aided analyses, it is useful to convert these data into relevant equations.

The trendline feature provides a number of options, which include exponential, linear, logarithmic, polynomial, and power equations as shown on Figure 2.39. To fit a trendline to the data, we have to create line charts for the two properties as shown on Figures 2.40.a and 2.40.b. Trendlines can then be added on the line charts. Figures 2.40.a and 2.40.b also show the corresponding trendline equations of the tabulated data as determined by using polynomial equations. As Figure 2.40.b shows, a linear equation is adequate for the h_f data since its variation over the given temperature range is mild. However, a third-order polynomial is required to represent the variation of P_{sat} with temperature as shown in Figure 2.40.a.

Table 2.1. Properties of saturated water at temperatures in the range 0ºC- 60ºC taken from Cengel and Boles [3]

TºC	P_{sat} [kPa]	v_f [m^3/kg]	v_g [m^3/kg]	u_f [kJ/kg]	u_g [kJ/kg]	h_f [kJ/kg]	h_g [kJ/kg]	s_f [kJ/kg.K]	s_g [kJ/kg.K]
0.01	0.6117	0.001000	206.00	0.000	2374.9	0.001	2500.9	0.0000	9.1556
5	0.8725	0.001000	147.03	21.019	2381.8	21.020	259.1	0.0763	9.0249
10	1.2281	0.001000	106.32	42.020	2388.7	42.022	2519.2	0.1511	8.8999
15	1.7057	0.001001	77.885	62.980	2395.5	62.982	2528.3	0.2245	8.7803
20	2.3392	0.001002	57.762	83.913	2402.3	83.915	2537.4	0.2965	8.6661
25	3.1698	0.001003	43.340	104.83	2409.1	104.83	2546.5	0.3672	8.5567
30	4.2469	0.001004	32.879	125.73	2415.9	125.74	2555.6	0.4368	8.4520
35	5.6291	0.001006	25.205	146.63	2422.7	146.64	2564.6	0.5051	8.3517
40	7.3851	0.001008	19.515	167.53	2429.4	167.53	2573.5	0.5724	8.2556
45	9.5953	0.001010	15.251	188.43	2436.1	188.44	2582.4	0.6386	8.1633
50	12.352	0.001012	12.026	209.33	2442.7	209.34	2591.3	0.7038	8.0748
55	15.763	0.001015	9.5639	230.24	2449.3	230.26	2600.1	0.7680	7.9898
60	19.947	0.001017	7.6670	251.16	2455.9	251.18	2608.8	0.8313	7.9082



Trendline Options

Trend/Regression Type

○ Exponential
○ Linear
○ Logarithmic
● Polynomial Order: 2
○ Power
○ Moving Average Period: 2

Trendline Name

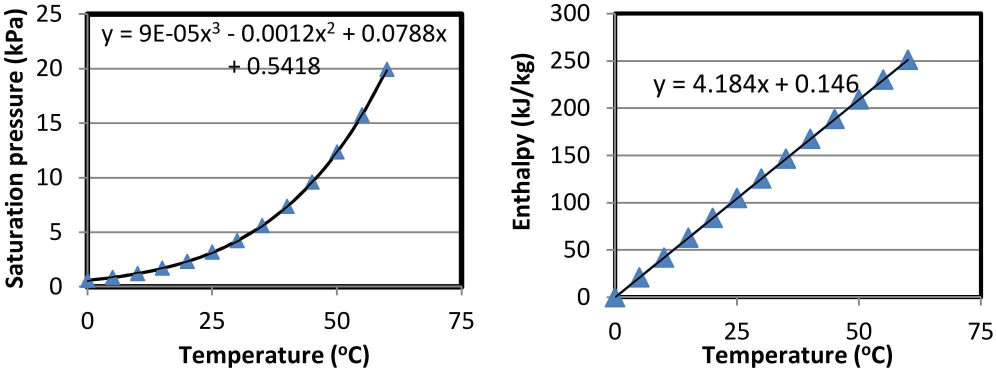
● Automatic : Poly. ()
○ Custom:

Forecast

Forward: 0.0 periods
Backward: 0.0 periods

☐ Set Intercept = 0.0
☐ Display Equation on chart
☐ Display R-squared value on chart

Figure 2.39. The Format Trendline window



(a) (b)

Figure 2.40. Fitting trendlines on tabulated data for water of (a) saturation pressure and (b) saturated liquid enthalpy

2.6. Closure

This chapter focusses on the basic features of Excel that are needed for developing Excel-aided models for thermofluid analyses. The chapter highlights the importance of using cell labelling with Excel's formulae and illustrates the use of Excel's general mathematical functions, logical functions, and the functions for matrix operations. The chapter also demonstrates the use of Excel's two iterative tools: Goal Seek and circular calculations. In spite of its simplicity, the Goal Seek command is very useful for iterative solutions as shown in later chapters of this book. Finally, the chapter illustrates the usefulness of Excel's charting tools for the presentation of tabulated data particularly the trendline feature.

It should be mentioned that the **Developer** tab of Excel's user-interface provides a number of useful features that enhance the effectiveness of Excel as modelling platform for thermofluid analyses, but these features have not discussed in the chapter. Two of these features are the ability to develop user-defined functions with VBA and the ability to record **macros** for conducting repetitive calculations and parametric analyses. VBA will be discussed in Chapter 3, but for more information about the use of macros the reader needs to refer to more specialised books on Excel or the internet.

References

[1] J. Walkenbach, *Excel 2010 Formulas*, Wiley Publishing Inc., 2010

[2] J. Walkenbach, *Excel 2007 Charts*, Wiley Publishing Inc., 2007

[3] Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007

[4] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*, 6th Edition, McGraw Hill, 2010

Exercises

1. The following table shows measured values of the temperature by two different methods compared to the correct corresponding values. Find the average error for each method.

***Correct T* (°C)**	***Method* 1**	***Method* 2**
0	0.1044	0.1112
10	9.1092	9.1153
20	20.1139	20.1194
30	30.1186	30.1235
40	40.1231	40.1275
50	50.1276	50.1316
60	60.1320	60.1357
70	70.1364	70.1397
80	80.1407	80.1438
90	90.1450	90.1479
100	100.1493	100.1520

2. The following table shows the data for the saturation pressure of a certain fluid. Use a nested IF statement to develop an interpolation formula that determines the saturation pressure of the fluid at any temperature in the range 5°C $\le T \le$ 30°C.

T(°C)	P_{sat} (kPa)
5	0.872
10	1.228
15	1.705
20	2.339
25	3.169
30	4.246

3. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations to determine the values of the three unknowns a, b, and c by using:

(a) The matrix inversion method
(b) The iterative solution option.

This exercise is based on Example 9.11 in Chapra and Canale [4]. The answer is: $a = 8.5941$, b=34.4118, and $c = 36.7647$.

4. Figure 2.P4 shows a triangular fin attached to the surface of a wall. Solution of the conduction heat transfer equation with the finite-difference method resulted in the following system of linear equations the solution of which gives the temperatures in °C at different distances from the fin base as shown in the figure:

$$-8.008\,T_1 + 3.5\,T_2 = -900.209$$

$$3.5\,T_1 - 6.008\,T_2 + 2.5\,T_3 = -0.209$$

$$2.5\,T_2 - 4.008\,T_3 + 1.5\,T_4 = -0.209$$

$$1.5\,T_3 - 2.008\,T_4 + 0.5\,T_5 = -0.209$$

$$T_4 - 1.008\,T_5 = -0.209$$

Use Excel functions to solve the above system of linear equations.

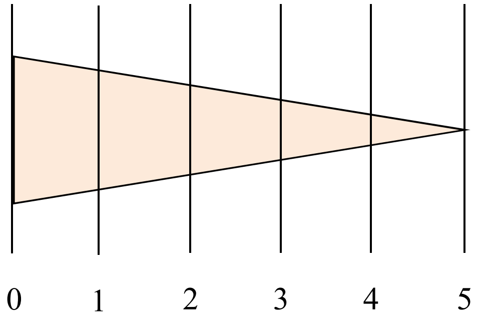


Figure 2.P4. Triangular fin

5. Adopting suitable names in your formulae, prepare an Excel sheet for calculating the frictional loss (h_f) in a circular pipe of diameter D, length L, and roughness k_s. Use your sheet to determine h_f in the following cases:

(a) $D = 25$ cm, $L = 150$ m, $V = 2$ m/s, $k_s = 0.045$ mm, carrying water at 20°C.
(b) $D = 25$ cm, $L = 150$ m, $V = 0.2$ m/s, $k_s = 0.045$ mm, carrying oil at 20°C.
(c) $D = 25$ cm, $L = 150$ m, $V = 7$ m/s, $k_s = 0.045$ mm, carrying air at 20°C.

Use the Darcy-Weisbach equation and determine the values of the kinematic viscosity from relevant property tables.

6. Using a line chart, plot the variation of sine θ for $-180 \leq \theta \leq 180$ in steps of 10°then add cosine θ on the same chart.

7. Using the data shown in Table 2.1, make a line chart for v_f and v_g. Add polynomial trendlines for both properties and comment on the trendlines equations.
8. The table below shows some of the thermo-physical properties of air at atmospheric pressure and different temperatures. Use Excel charts to show the variation of the properties ρ, β, c_p, k, α, μ, v, and Pr with temperature and use trendline to obtain suitable equations for these properties.

T (K)	ρ (kg/m^3)	$\beta\times10^3$ (1/K)	c_p (J/kg.K)	k (W/m.K)	α (m^2/s)	$\mu\times10^6$ (N S/m^2)	$\nu\times10^6$ (m^2/s)	Pr
273	1.252	3.66	1011	0.0237	19.2	17.456	13.9	0.71
293	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71
313	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71
333	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71
353	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71
373	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71
473	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71
573	0.596	1.75	1047	0.0429	68.9	29.322	49.2	0.71
673	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72
773	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72

9. The volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid by:

   $V = \pi h^2(3r - h)/3$

   Using the Goal Seek command, determine the value of h for the tank with r=1 m and V = 0.5 m^3.

   This exercise is based on Problem 8.9 in Chapra and Canale [4]. Answer: h = 0.431 m.