Thermax

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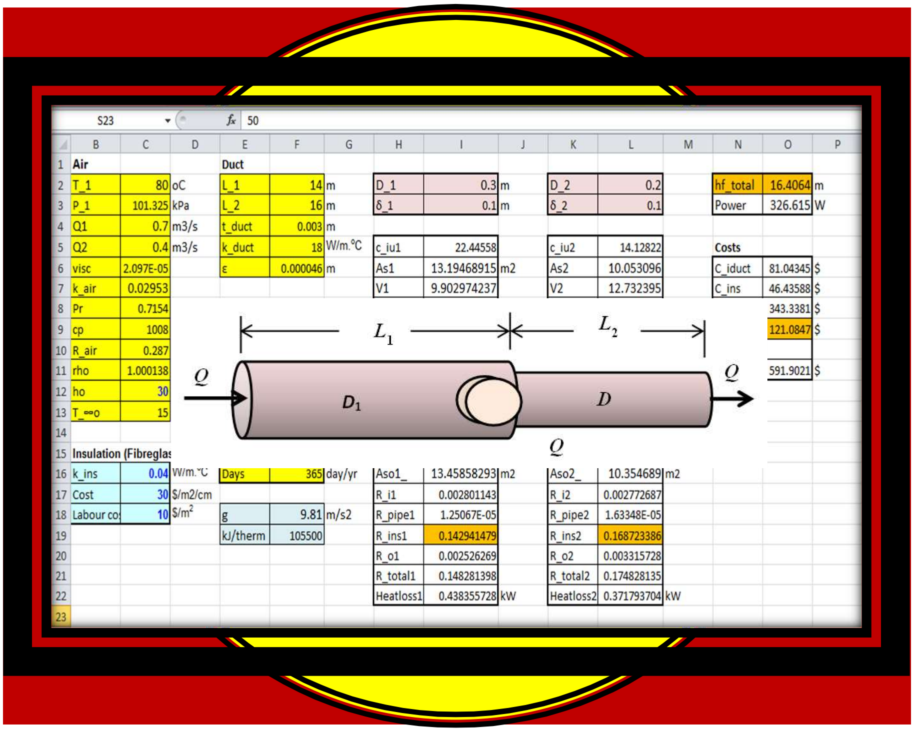
Computer Applications for Engineers using Excel



Mohamed M. El-Awad

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Mohamed M. El-Awad

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This book is dedicated to by beloved family

Ghada, Ula, and Ahmed

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Preface

Engineering students usually take at an early stage of their study a preliminary course on computer applications that introduces them to the use of computers and equips them with the basic skills needed for word-processing and data analyses. After studying the basic engineering subjects, they are taught a more advanced course on computer applications for engineering analyses. The aim of this intermediate-level course is to train the students on the essential computational methods needed for solving problems of engineering design and selection. Although the various engineering specialisations involve different types of design analyses that require the use of computational methods, there are essential methods which are shared by most of them like the solution of linear systems of equations, solution of non-linear equations, solution of ordinary and partial differential equations, iterative solutions, and optimisation analyses. The different types of computer-oriented methods are frequently encountered in the three thermofluid subjects: thermodynamics, fluid mechanics, and heat-transfer, which are fundamental components for most engineering specialisations. Although the use of tables and charts is essential for teaching the basics of these three subjects, computers and computer software become more useful when design analyses are addressed at a later stage.

This book illustrates the use of computational methods in engineering analyses by focussing on thermofluid analyses and by using a general-purpose spreadsheet application which is Microsoft Excel. The Excel-based modelling platform described in the book has four elements; (i) Excel with its user-interface and built-in functions, (ii) the Solver add-in that comes with Excel, (iii), the integrated programming language Visual Basic for Applications (VBA) and (iv) an Excel add-in for fluid properties called Thermax. While the two main components, Excel and Solver, are adequate for most fluid mechanics and heat-transfer analyses, Thermax helps the students to perform thermo-dynamic analyses with Excel. VBA is needed for the development of custom functions when the analytical model cannot be executed by only using Excel's built-in functions and Thermax functions. Properly used, the Excel-based modelling platform minimises the effort of developing the analytical computer model so that more attention can be paid to the application of the relevant thermofluid principles. Apart from the wide availability of Excel on personal computers, the modelling platform allows the students to be more involved in the process of developing their models compared to other applications that are dedicated to thermofluid analyses.

This book has been compiled from the first two books of a set of three books that illustrate the use of the Excel-based modelling platform for various types of computer-aided thermofluid analyses. The selected topics it includes are intended to suit a credit-hour course on computer-applications for engineers that is preferably taken after completing the basic three thermofluid courses. The book builds on the students' theoretical background at an intermediate level and does not discuss advanced related topics such as CFD (Computational Fluid Dynamics), the finite-element method, exergo-economics, or multi-objective optimisation. The last two topics are discussed in the third book of this set and can be added if required. Most of the examples given in the book are based on relevant cases or examples given in standard thermofluid textbooks or obtained from the published literature so that the students can look for any additional information needed to validate their Excel models and verify their results. Exercises are given at the end of each chapter to help students sharpen their skills related to the particular topic.

The book includes nine chapters the first of which mainly reviews the basic principles of thermofluid analyses. Chapter 2 shows how Excel's built-in functions and tools can be used for the basic computer-based analyses like the solution of linear systems of equations and the solution of non-linear equations, while Chapter 3 illustrates the use of the other three components by means of simple examples. Chapter 4 and Chapter 5 deal with the use of the Excel-based platform for iterative solutions and optimisation analyses, respectively, while Chapters 6, 7, and 8 deal with its use for selected topics in heat-transfer, fluid-dynamics, and thermodynamics, respectively. Chapter 9 presents additional cases which are relatively more challenging than the related cases described in previous chapters of the book. The cases are meant for stimulating ideas for relevant mini projects and more project ideas are provided at the end of the chapter.

Acknowledgements

The development of Thermax and the writing of this set of books would not have been possible without benefitting from the efforts of many colleagues who have made their publications, data, and software available in the open literature or on their websites. A special gratitude goes to the Mechanical Engineering Department at the University of Alabama (USA) whose initiative "*Excel for Mechanical Engineering*" both inspired and helped me to develop Thermax. I am also indebted to Universiti Putra Malaysia - UPM and Universiti Tenaga Nasional - UNITEN (Malaysia), the University of Khartoum - U. of K. (Sudan), and the University of Technology and Applied Sciences - UTAS (Oman) for their generous support at different periods of my academic career and hope that they find the books a worthy token of appreciation and gratitude. Last, but not least, I am grateful to my beloved family for the unfailing support I needed desperately to complete this work at a time of conspiracy, betrayal, and war.

CONTENTS

1

Introduction

Cars, refrigerators, and air-conditioners have become indispensable items for families in both developed and developing countries. The energy required to operate these systems mainly comes from burning fossil fuels in power-generation plants. Apart from being non-renewable energy sources, large-scale combustion of fossil fuels is the main cause for global warming and its increasingly devastating effects at different parts of the world. Therefore, proper design and operation of these and other energy-demanding devices are required for minimising these effects. The design methods of these systems are mainly based on the principles of *thermodynamics*, *fluid mechanics*, and *heat transfer*. This chapter reviews the main principles of these three thermofluid subjects with the objective of showing how they can be used to reduce the losses and minimise energy consumption. For a number of reasons, the equations involved in thermofluid analyses are difficult to solve without using computer-aided methods. Therefore, these analyses introduce many simplifications that reduce their accuracy. In this respect, the chapter highlights the advantages of computer-aided methods for thermofluid analyses and describes the Excel-based modelling platform used in this book for these analyses.

1.1. A review of thermofluid principles

The two main principles that form the framework for thermofluid analyses are the conservation of mass (the continuity equation) and the conservation of energy (the first-law of thermodynamics). These principles take different mathematical forms depending on whether the system under consideration is open or closed and on whether the flow is steady or unsteady, compressible or incompressible, laminar or turbulent, etc. Numerous auxiliary relationships are needed in order to quantify the various parameters involved in the resulting equations such as pressure-variations, friction losses, and rates of heat-transfer. In what follows, the main concepts of thermodynamics, fluid dynamics, and heat-transfer are reviewed by considering typical applications.

1.1.1. Thermodynamics

The principles of *engineering thermodynamics* allow us to determine the amount of energy transfer in the form of work or heat between any system and its surroundings. They also allow us to determine the efficiency and effectiveness of the system if energy conversion is involved. There are four basic laws for thermodynamics the most important of which are the first and the second laws of thermodynamics. To apply these two basic laws, thermodynamic analyses use many property relationships, tables, and charts that determine the properties of the particular fluid involved and its phase (a liquid, a liquid-vapour mixture, a gas, or a gaseous mixture). To illustrate the application of thermodynamic laws and relationships in a typical analysis, consider the air-compression system shown in Figure 1.1.a. Air enters the system that has two stages of compression separated by an intercooler at a temperature T_1 and pressure P_1. The first-stage compressor, C_1, compresses the air adiabatically to state 2, after which it enters the intercooler where its temperature is reduced to T_3. The second-stage compressor, C_2, then increases the air pressure to P_4 at which the temperature increases to T_4. Figure 1.1.b shows the compression process on a temperature-entropy diagram.

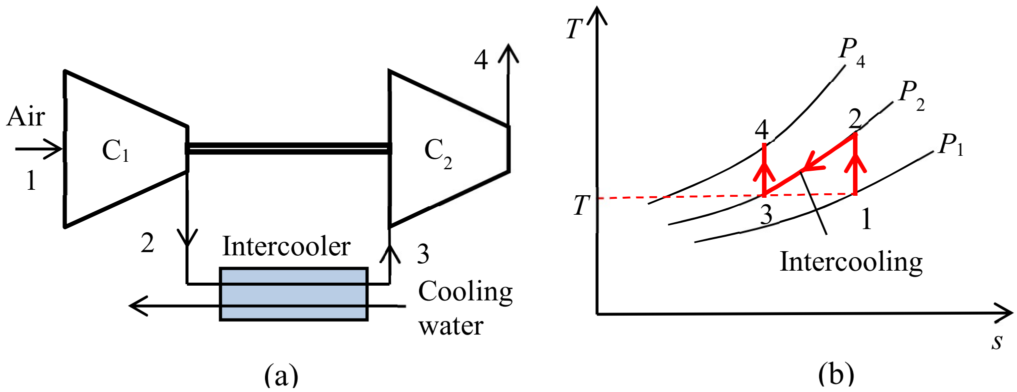

Figure 1.1. Schematic and *T*-*s* diagrams of a two-stage air compressor with intercooling

How the total compression work is divided between the two compressor stages depends on their compression ratios and there is a certain value of the intermediate pressure (P_i) that minimises the total compression work. The principles of thermodynamics help us to determine this optimum pressure as shown below.

Treating the two compressor stages as steady-flow processes and neglecting changes in kinetic and potential energy, the first-law of thermodynamics is expressed as [1]:

$$q - w = (h_{out} - h_{in}) \tag{1.1}$$

Where q and w are the amounts of heat transfer and work transfer per unit mass flow of air, respectively, and ($h_{out} - h_{in}$) is the resulting enthalpy change over the stage. Equation (1.1) adopts the usual sign convention that heat into the system is positive, while work into the system is negative. Assuming the compression processes in both stages to be isentropic as shown in Figure 1.1.b means that they are adiabatic (q=0) and reversible. Therefore, using an average specific heat for air at constant pressure (c_p), the compression work per unit mass flow of air in stage 1 (w_1) and in stage 2 (w_2) can be determined from Equation (1.1) as follows:

$$w_1 = -(h_2 - h_1) = -c_p(T_2 - T_1) \tag{1.2}$$

$$w_2 = -(h_4 - h_3) = -c_p(T_4 - T_3) \tag{1.3}$$

The total compression work (w_{total}) is then given by:

$$w_{total} = w_1 + w_2 = -c_p[(T_2 - T_1) + (T_4 - T_3)] \tag{1.4}$$

Assuming perfect intercooling, i.e., $T_3 = T_1$, Equation (1.4) can be rearranged as:

$$w_{total} = c_p T_1 \left[\left(1 - \frac{T_2}{T_1}\right) + \left(1 - \frac{T_4}{T_3}\right) \right] = c_p T_1 \left[2 - \left(\frac{T_2}{T_1}\right) - \left(\frac{T_4}{T_3}\right) \right] \quad (1.5)$$

Since the two compression processes are assumed to be isentropic and the specific heat c_p for air to be constant, the temperature ratios in Equation (1.5) can be converted into pressure ratios by using the following approximate relationships:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad (1.6)$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \quad (1.7)$$

Where k is the ratio of specific heats (k=c_p/c_v; c_v is the specific heat for air at constant volume). With the assumption that there is no pressure loss in the intercooler, $P_3 = P_2 = P_i$. Substituting from Equations (1.6) and (1.7), Equation (1.5) becomes:

$$w_{total} = c_p T_1 \left[2 - \left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_4}{p_i}\right)^{\frac{k-1}{k}} \right] \quad (1.8)$$

To see how the total compression work varies with the intermediate pressure P_i, a specific case was considered in which T_1= 300K, P_1=100 kPa, and P_4 = 900 kPa. Using Equation (1.8), the total compression work in the system was calculated for different values of P_i and Figure 1.2 shows the result. The figure shows that the value of P_i at which the total compression work is minimal is around 300 kPa. Increasing and decreasing P_i from this value both increase the compression work.

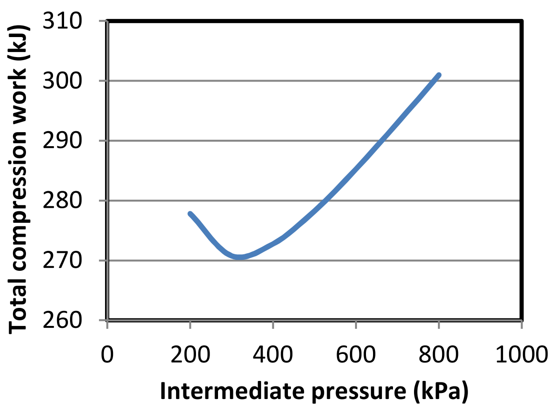

Figure 1.2. Variation of the total compression work with the intermediate pressure

The principles of thermodynamics are particularly useful for performance evaluation and design optimisation of power-generation and refrigeration systems. For example, consider the regenerative steam-turbine power plant shown in Figure 1.3. This plant consists of a boiler house for producing superheated steam, a high-pressure steam turbine (HPT), a low-pressure steam turbine (LPT), a condenser, an open feed-water heater (FWH) and two feed-water pumps. A fraction of the steam (y) is extracted after the HPT for preheating the feed-water before going back to the boiler house.

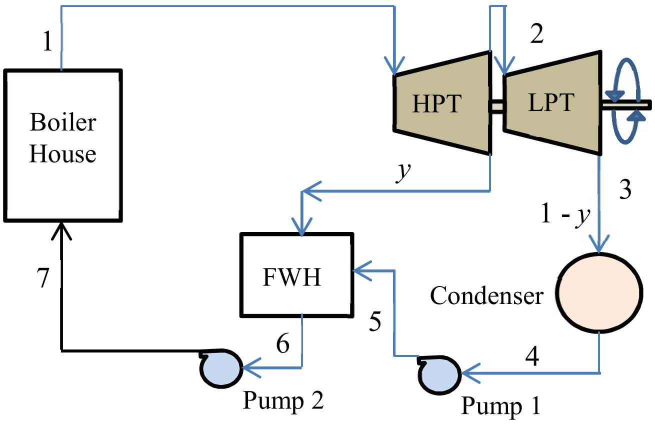

Figure 1.3. Schematic diagram of a regenerative steam-turbine power plant

Although extracted steam reduces the work output from plant, it reduces the amount of heat added in the boiler and its net effect is to increase the thermal efficiency of the plant. There is also a certain extraction pressure for the steam at which the plant's thermal efficiency attains a maximum value. As shown below, the principles of thermodynamics can be used to determine this optimum steam-extraction pressure.

The total specific work output from the two turbines (w_{out}) and the total work input to the two pumps (w_{in}) are given by:

$$w_{out} = w_{HPT} + w_{LPT} \tag{1.9}$$

$$w_{in} = w_{P1} + w_{P2} \tag{1.10}$$

Where w_{HPT} and w_{LPT} are the specific work output from the high-pressure turbine and the low-pressure turbine, respectively, and w_{P1} and w_{P2} are the specific work inputs in pump 1 and pump 2, respectively. Assuming the two turbines and the two pumps to be adiabatic and neglecting the changes in kinetic and potential energies, the work output or input for each device can be determined from the enthalpy difference across the device. Per each kg of steam generated in the boiler, these are given by:

$$w_{HPT} = (h_1 - h_2) \quad (1.11)$$

$$w_{LPT} = (1 - y)(h_2 - h_3) \quad (1.12)$$

$$w_{P1} = (1 - y)(h_5 - h_4) \quad (1.13)$$

$$w_{P2} = (h_7 - h_6) \quad (1.14)$$

Mass and energy balance over the open feed-water heater gives:

$$yh_2 + (1 - y)h_5 = 1 \times h_6 \quad (1.15)$$

The net specific work output from the plant (w_{net}) is then given by:

$$w_{net} = w_{out} - w_{in} \quad (1.16)$$

Similarly, the specific heat input to the boiler (q_{in}) is determined by the relevant enthalpy change as follows:

$$q_{in} = (h_1 - h_7) \quad (1.17)$$

Finally, the thermal efficiency of the plant (η) can be calculated from:

$$\eta = w_{net} / q_{in} \quad (1.18)$$

Both w_{net} and η depend on the fraction of steam extracted for regeneration (y); which in turn depends on the extraction pressure (P_2). Figure 1.4 shows the variation of y and η with P_2 for an ideal cycle in which P_1 = 15 MPa, T_1 = 600ºC, and P_4 = 10 kPa. The figure shows that the cycle's efficiency attains a maximum value of 45.55% when P_2 is in the range of 1000 kPa.

It should be mentioned that the working fluid in the above power plant, which is water, changes its phase from a liquid to superheated steam in the boiler, to a saturated mixture of water and steam in the low-pressure turbine, and returns to liquid water in the condenser. Therefore, appropriate property tables or charts are needed for determining the thermodynamic properties of water at the different states. In general, thermodynamic analyses use many tables and charts for various working fluids. The principles of thermodynamics are also needed for the analyses of air-conditioning systems and processes and for the analyses of the processes that involve combustion and other chemical reactions. For such analyses, thermodynamics provides the basic relationships needed to quantify the effects of fluid mixing and chemical reactions on the properties of the working fluids and to determine the transfer of energy and effluents to or from the system.

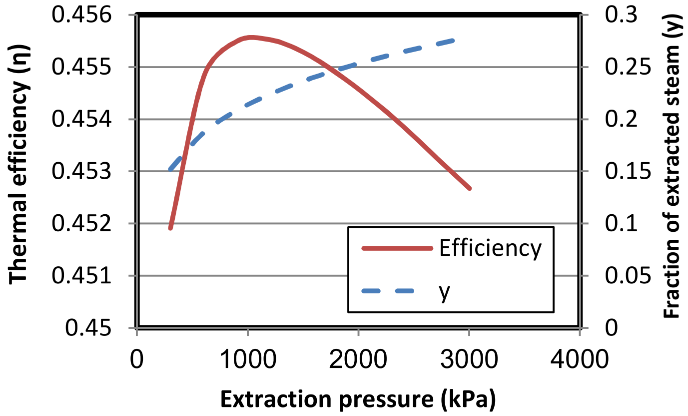

Figure 1.4. The effect of intermediate pressure (P_2) on the fraction of extracted steam (y) and thermal efficiency (η) of a regenerative steam-turbine power plant

1.1.2. Fluid dynamics

In addition to pipes and ducts, fluid-transporting systems require various equipment such as pumps, compressors, control valves, and flow-measuring devices. The principles of *fluid dynamics* help us to estimate the power needed for overcoming friction in these equipment and to determine suitable types and sizes for them. To illustrate the application of these principles, consider the pump-pipe system shown in Figure 1.5 that conveys a liquid between two non-pressurised tanks A and B through a pipe of known length L, diameter D, and roughness ε. Suppose that we want to determine the needed pump power for transporting the liquid at a certain flow rate Q.

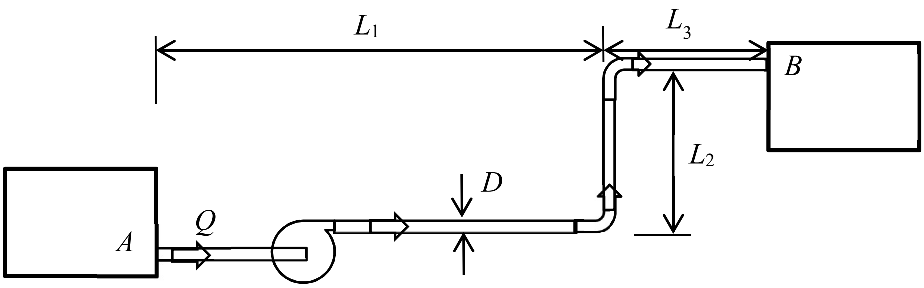

Figure 1.5. Schematic diagram of a simple pump-pipe system

The required pump power ($\dot{W}$) can be determined from the following "power equation":

$$\dot{W} = \gamma \times Q \times h_p / \eta \qquad [\text{W}] \tag{1.19}$$

Where γ is the specific weight of the transported liquid (N/m^3), Q is the volume flow rate of the liquid (m^3/s), h_p is the pump head (m) needed to circulate the fluid through the pipe from A to B, and η is the combined efficiency of the pump and the electric motor. For a steady flow of an incompressible fluid, h_p can be determined from the following "energy equation":

$$h_p = h_{f,total} + \left(Z_B - Z_A\right) + \frac{V_B^2 - V_A^2}{2g} \qquad \text{[m]} \qquad (1.20)$$

Where $h_{f,total}$ is the total head loss through the system due to friction (m), Z_A and Z_B are the elevations (m) at points A and B, respectively, and V_A and V_B are the corresponding fluid velocities (m/s). If the two tanks are not open to the atmosphere, the energy equation should also include a term for the pressure difference between the tanks.

The total friction head loss $h_{f,total}$ consists of two parts: the *major friction loss* (h_f), which is the part lost in the pipe itself, and the *minor friction head loss* (h_c), which is the part lost in other components of the system like nozzles, elbows, valves, etc. The major friction loss can be determined from the following Darcy-Weisbach equation [2]:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad \text{[m]} \qquad (1.21)$$

Where f is the dimensionless Darcy friction factor, V the fluid velocity (m/s), L the total length of the pipe (m), and D the internal diameter of the pipe (m). The value of the friction factor, which depends on the roughness of the pipe surface and on whether the flow is laminar or turbulent, can be obtained from a Moody diagram [2] or calculated from a relevant formula. For laminar flows, f can be calculated from:

$$f = 64/\mathrm{Re} \qquad \mathrm{Re} < 2300 \qquad (1.22)$$

Where Re is the Reynolds number defined as:

$$\mathrm{Re} = VD/\nu \qquad (1.23)$$

Where ν is the kinematic viscosity of the flowing fluid (m^2/s). For a turbulent flow in rough pipes, f can be obtained from the following Swamee-Jain formula [2]:

$$f = 0.25 / \left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2 \qquad \mathrm{Re} > 4000 \qquad (1.24)$$

For more accuracy, the friction factor for a turbulent flow can be determined by using the following Colebrook-White formula (frequently referred to as the Colebrook equation):

$$\sqrt{\frac{1}{f}} = -2.0\log_{10}\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{1.25}$$

The Colebrook equation is an example of the implicit equations met in thermofluid analyses that need to be solved iteratively. For turbulent flows in smooth tubes, *f* can be determined from the first Petukhov formula [2]:

$$f = \left(0.790\ln(\mathrm{Re}) - 1.64\right)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{1.26}$$

Chemical engineers usually determine the pipe friction by using the following Chezy-Manning equation instead of the Darcy-Weisbach equation:

$$h_f = 2f\frac{L}{D}\frac{V^2}{g} \qquad [\mathrm{m}] \tag{1.27}$$

Where *f* is the Fanning friction factor. Comparison with Equation (1.21) reveals that the value of the Fanning friction factor is 4 times the corresponding value of the Darcy friction factor. Civil engineers determine the friction head loss in water-transporting pipes by using the following Hazen-Williams equation:

$$h_f = \frac{10.67 L Q^{1.852}}{C^{1.852} D^{4.8704}} \qquad [\mathrm{m}] \tag{1.28}$$

Where C is a coefficient that depends on the roughness of the pipe. Unlike Equations (1.21) and (1.27), Equation (1.28) is applicable for both laminar and turbulent flows.

The minor friction losses, h_c, can be determined from the following equation:

$$h_c = \sum_{1}^{n} K\frac{V^2}{2g} \qquad [\mathrm{m}] \tag{1.29}$$

Where n is the total number of components in the fluid system and K is a coefficient the value of which can be found for each component in relevant tables.

Given the values of the pipe length and diameter, flow rate, fluid viscosity, and pipe material or roughness, the equations described above can be used to determine the required pump power. The equations can also be used to determine the maximum flow rate of the fluid to be delivered via a pipe of a certain diameter such that the friction loss in the system or the needed pump power does not exceed a specified limit. Moreover, by taking into consideration the initial cost of the pump-pipe system that increases with D, and the cost of electrical energy needed by the pump that decreases with D, the equations can be used to determine the economic pipe diameter D_{opt} that gives the lowest total owning cost for the system over its entire life-time. In general, the equations are also applicable for analysing and optimising pipe-networks.

The principles of fluid dynamics also enable us to select the appropriate type and size of the pump for a given pump-pipe system by matching the "pump curve" with the "system curve". This is achieved with the help of pump characteristic curves usually provided by the manufacturers. In many situations a single pump or compressor may not be adequate for the required flow rate or delivery pressure and more than one pump or compressor have to be used. In this situation, the principles of fluid dynamics help us to decide when to arrange the pumps/compressors in parallel or in series.

1.1.3. Heat transfer

The design practices of energy-conversion equipment that deal with the transfer of thermal energy such as boilers, condensers, and heat exchangers are mainly based on the principles of *heat transfer*. Three independent physical laws are used in heat-transfer analyses to quantify the *rate* of heat transfer between an object and its surroundings depending on whether the heat transfers by conduction (Fourier's Law), convection (Newton's law of cooling), or radiation (Stefan-Boltzmann law). The physical properties that determine the rate of heat transfer by conduction, radiation, and convection are the thermal conductivity (k), the surface emissivity (ε) and absorptivity (α), and the heat-transfer coefficient (h), respectively.

While k, ε, and α are material or surface-specific, h depends on both the fluid and the flow. Numerous analytically-obtained relationships and empirical formulae are used for determining h depending on whether the flow is forced or natural and whether the flow is internal or external to the system being considered. These formulae usually give the Nusselt number (Nu) which is related to h as follows:

$$h = \frac{k}{D} Nu \qquad [\mathrm{W/m^2.K}] \tag{1.30}$$

Where D is the pipe diameter and k is the thermal conductivity of the transported fluid. Many analytical or empirical formulae are used for determining the Nusselt number for forced or natural flows over single tubes, bank of tubes, plates, etc. For example, the following Dittus-Boelter equation is used for determining Nu inside a fluid-transporting pipe due to forced convection [3]:

$$Nu = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{n} \tag{1.31}$$

Where Re is the Reynolds number, Pr the Prandtl number, and n is a constant that takes a value of 0.4 when the pipe is being heated and 0.3 when it is being cooled.

The subject also describes the methods that can be used to minimise or maximise the rate of heat-transfer between the system's components or between the system and its surroundings by means of thermal insulation, fins, heat-pipes, etc. To illustrate the use of heat-transfer concepts in thermal-insulation analyses, consider the metal pipe shown in Figure 1.6 that has an internal radius r_1 and external radius r_2. The pipe carries a fluid at a temperature T_i, while the surrounding air is at a different temperature T_∞.

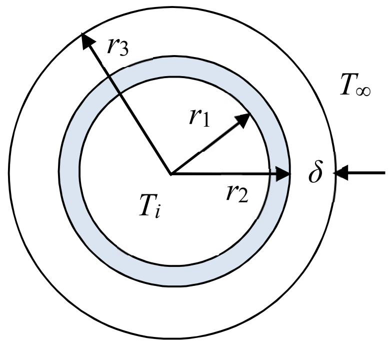

Figure 1.6. Schematic for an insulated metal pipe

The temperature difference between the pipe and the surroundings will cause heat gain or heat loss to/from the pipe and, in order to reduce this heat gain or heat loss, the pipe has to be covered by an insulating material. The principles of heat transfer help us to account for the effect of thermal insulation on the rate of heat-transfer to/from the pipe. The rate of heat transfer ($\dot{Q}$) can be calculated from [3]:

$$\dot{Q} = \left(T_i - T_\infty\right) / R_{th} \qquad \text{[W]} \qquad (1.32)$$

Where R_{th} is the combined thermal resistance to heat-transfer by conduction, convection, and radiation, which is given by [3]:

$$R_{th} = \frac{1}{h_i A_1} + \frac{\ln\left(r_2 / r_1\right)}{2\pi L k_1} + \frac{\ln\left(r_3 / r_2\right)}{2\pi L k_2} + \frac{1}{h_o A_3} \qquad \text{[K/W]} \qquad (1.33)$$

Where h_i and A_1 are the heat-transfer coefficient and surface area inside the pipe, respectively, h_o and A_3 are the heat-transfer coefficient and surface area outside the insulated pipe, respectively, L is the length of the pipe, and k_1 and k_2 are the thermal conductivities of the pipe and the insulation, respectively. To simplify the analysis, h_o in Equation (1.33) takes into account the heat-transfer by both convection and radiation to/from the insulation surface. The thickness of the metal pipe is usually small compared to its diameter, while its thermal conductivity is much higher than that of the insulation material. Therefore, the equation can be simplified further by neglecting the term that represents the thermal resistance due to conduction through the pipe.

Equations (1.32) and (1.33) can be used to determine the required thickness of insulation (δ) for reducing the rate of heat transfer to the required tolerance or for controlling the surface temperature within a range that is dictated by safety or other practical considerations. Although the thicker the insulation the lower will be the rate heat transfer, the cost of insulation increases with its thickness and, therefore, adding more insulation will not be economically profitable beyond a certain thickness. By extending the above heat-transfer model so that the cost of insulation and the value of the saved thermal energy can be calculated and compared, the above equations can also be used to determine the economically optimal thickness of insulation (δ_{opt}).

Figure 1.7 shows a metal pipe with circular fins attached to its surface so as to boost the rate of heat-transfer between the fluid being transported with the pipe and the surrounding medium, usually air. The principles of heat transfer can be used to develop the required mathematical equations that determine the the rate of heat transfer from the pipe and, therefore, to evaluate the effectiveness and efficiency of the fin.

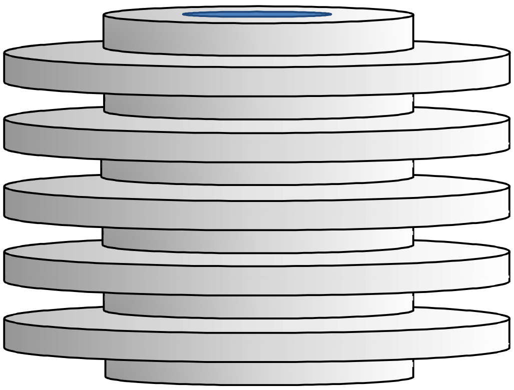

Figure 1.7. Circular fins attached to a metal pipe

Another important application of heat-transfer principles is that related to the design and selection heat-exchangers. A heat-exchanger is any device that allows the transfer of thermal energy between two fluids through a separating surface usually a pipe, a duct, a tube, or a plate. Figures 1.8 and 1.9 show two types of heat-exchangers commonly used in industries and power-plants. Figure 1.8 shows a shell-and-tube heat-exchanger while Figure 1.9 shows a cross-flow heat-exchanger. Heat-exchanger analyses either aim at determining the required size (i.e., surface area) for a specified heat-transfer duty or determining the exit temperatures of the two streams from a specified heat-exchanger type and size. Two methods are used for these two types of analyses which are the log-mean temperature difference (LMTD) method and the effectiveness-number of transfer units (ε-NTU) method. Complex thermal systems use heat-exchanger networks [HENs] and finding the configuration that minimises the annual cost of the network is based on the principles of heat transfer.

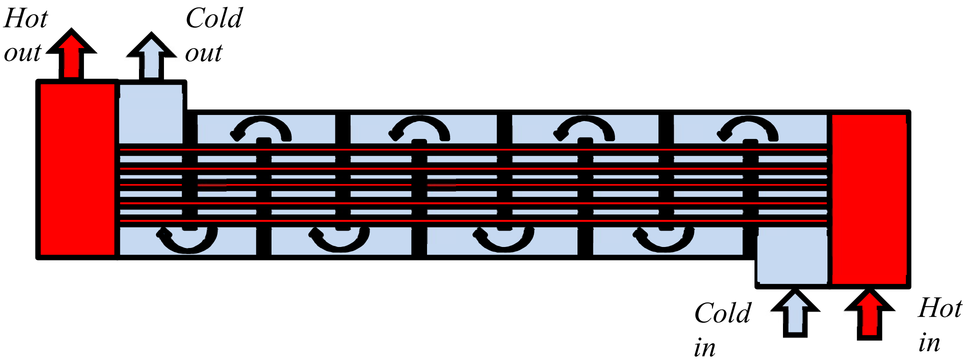

Figure 1.8. A parallel-flow shell-and-tube exchanger

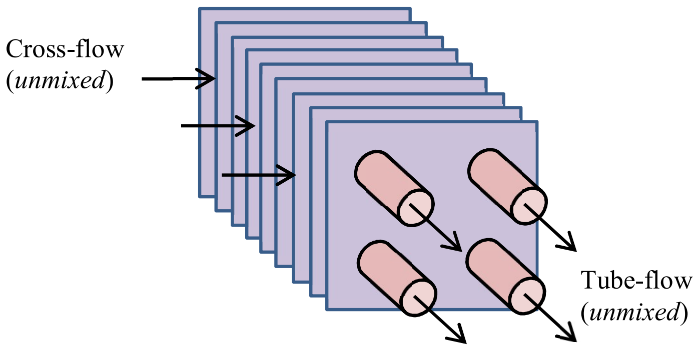

Figure 1.9. A cross-flow exchanger with both streams unmixed

1.2. Advantages of computer-aided thermofluid analyses

Apart from saving time and eliminating possible human errors, computer-aided thermofluid of analyses offer a number of advantages over traditional methods that use property tables and charts. An important advantage of computer-aided methods is their ability to give more realistic results by avoiding unnecessary simplification of the models and by using more accurate estimations of fluid properties. Moreover, they offer reliable techniques for iterative solutions, optimisation analyses, and the analyses of complex fluid-thermal systems. In what follows, these advantages are illustrated by means of relevant examples.

A. Avoiding excessive simplification of the model

In many situations, traditional analytical methods adopt excessive simplifications of the analytical models; which makes their results grossly deviate from the behaviour of real systems. A good example of this situation is given by the models of internal-combustion (IC) engines. Traditional air-standard models of IC engines, such as the Otto cycle and the Diesel cycle, neglect heat-transfer and friction losses, treat the combustion process as heat-addition from an external source, and use constant specific heats for the working fluids. These assumptions enable the engine processes to be represented by simple closed-form relations for calculating the amount of heat added to the engine and net work from the engine [4]. However, air-standard models usually overestimate the engine's output and thermal efficiency.

By comparison, computer-aided models of IC engines closely mimic the behaviour of actual IC engines by taking into consideration the geometrical as well as the thermodynamic characteristics of the engines. Therefore, these models can be used to investigate the effect of important design and operation factors such the ignition or injection timing on the engine performance or the effect of engine speed on the specific fuel consumption. However, the formulation of these models leads to a set of ordinary differential equations that need to be solved simultaneously by using a numerical method such as the Newton-Raphson method [5].

B. Accurate representation of fluid properties and processes

The ideal-gas law can be used with reasonable accuracy for determining the specific volume of a superheated vapour, but when the temperature approaches the saturation line, the value of the specific volume determined by the ideal-gas law departs significantly from the actual volume. More accurate estimates can be obtained by using the following Soave-Redlich-Kwong (SRK) equation of state [1]:

$$P = \frac{R_u T}{\tilde{v} - b} - \frac{a\alpha}{\tilde{v}(\tilde{v} + b)} \tag{1.34}$$

Where P is the absolute pressure of the gas, $\tilde{v}$ is the molar specific volume, R_u is the universal gas constant, T is the absolute temperature of the gas, and the constants a, b and α are fluid-dependent. Figure 1.10 shows the deviations from the tabulated values by those obtained from the ideal-gas law and the SRK equation of state for refrigerant R134a at 0.2 MPa. The figure shows that the error of the ideal-gas law is more that 2% even at high temperatures and increases as the temperature approaches the saturation value, but the accuracy of the SRK equation remained higher than 99% even close to the saturation line. However, since the SRK equation is implicit in $\tilde{v}$ it cannot be used directly to determine the specific volume. A number of standard iterative procedures (e.g. Newton-Raphson method) can be used to solve the equation, but they are more suitable for computer-aided analyses than hand calculations.

Another important implicit equation for thermofluid analyses is the Colebrook-White equation, Equation (1.25), that determines the friction factor (f) for turbulent pipe-flows. Since the equation involves f on both sides and needs to be solved iteratively, the explicit relationships such as the Swamee-Jain formula are preferred even though the Colebrook-White equation is more accurate. Many other nonlinear equations like the SRK equation and the Colebrook-White equation give advantage to computer-aided thermofluid analyses by enabling more realistic and accurate estimations.

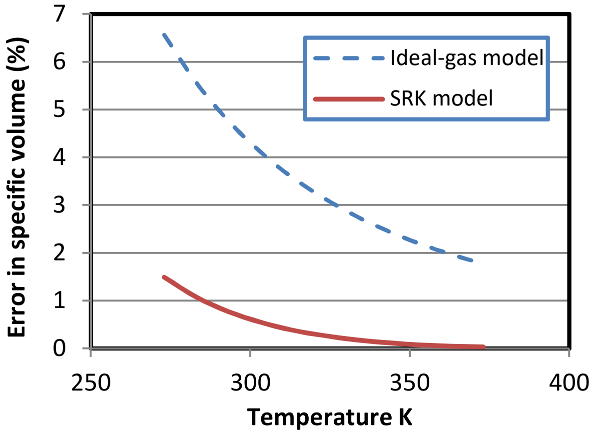

Figure 1.10. Errors in the specific volume of R134a by the ideal-gas law and the SRK equation of state

C. Dealing with iterative solutions and optimisation analyses

A good example of thermofluid analyses that require iterative solutions is found in pipe-flow analyses. Pipe flow problems that require the friction head loss to be determined when both the diameter and flow rate are known can be solved in a straightforward manner by using Equation (1.21). However, in design analyses of pump-pipe systems we may need to find the flow rate in a given pipe that gives a specified head loss or to find a suitable pipe diameter for specified head loss, flow rate, and pipe length. In these two cases, the friction factor f cannot be determined in advance because it depends on the Reynolds number. Therefore, these two types of pipe-flow problems, referred to as type-2 and type-3 problems, need to be solved by iteration. It is much easier to carry out the iterative process to the required level of accuracy by using a computer-aided method than by doing it manually. Other types of thermofluid analyses that also require iterative solutions include rating analyses of heat exchanger and the determination of the adiabatic flame temperature by first-law analysis of the combustion process.

Optimisation analyses are needed for determining the best design for a fluid-thermal system such as the optimum intermediate pressure for a two-stage air-compression system, the optimum steam-extraction pressure for a regenerative Rankine cycle, and the economic thickness of insulation for a pipe. While simple optimisation analyses that involve a single design parameter can be performed by means of calculus techniques and graphic tools, optimisation analyses of complex systems that involve multiple design variables require the use of computer-aided techniques.

D. Analyses involving complex models

The complexity of modelling certain fluid-thermal systems makes their analyses only possible with the help of computer-aided methods. The model complexity can be either due to the complexity of the physical structure of the system itself or the complexity of its mathematical representation. An example of physically complex systems is the pipe network shown in Figure 1.11 that consists of four pipe loops and four consumption points and fed by two water tanks; tank A and tank B. Suppose that the flow rates from the two supply tanks are specified together with the pipe diameters and lengths and it is required to determine the discharges at the four consumption points.

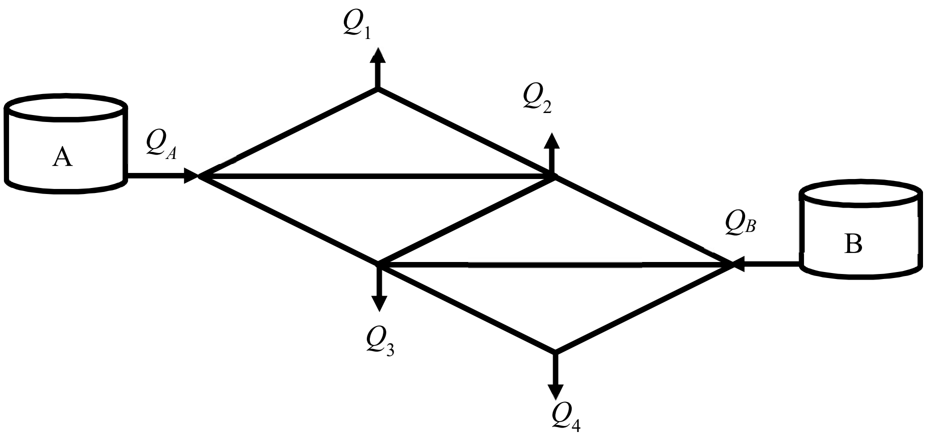

Figure 1.11. A looped pipe network supplied by two tanks

Although the solution is mainly based on the two well-known principles of conservation of mass and conservation of energy, it is difficult to solve the problem by using manual analytical methods especially when a minimum or a maximum pressure level is to be met at the discharge points. In this case, a computer-aided method, such as the Hardy-Cross method, has to be used [6, 7]. The optimisation analyses of heat-exchanger networks give another example of the models that deal with physically complex systems [8].

Mathematically complex thermofluid models that need computer-aided numerical methods are found in multi-dimensional fluid-flow and heat transfer analyses. This type of analyses involves coupled and nonlinear partial differential equations that have to be solved by using computational fluid dynamics (CFD) methods such as the finite-volume method or the finite-difference method. Many commercial CFD applications are available nowadays that offer great flexibility and user-friendliness.

1.3. The Excel-based modelling platform for thermofluid analyses

Microsoft Excel is commonly used for data visulation and for dealing with simple computer-based operations like matrix inversion and matrix multiplications [3,9]. However, Excel is equipped with other features that make it a capable modelling platform for a wide range of engineering "What-if" analyses [10-12]. In addition to its Goal Seek command and the Solver add-in, the "Developer" ribbon in Excel provides a programming language called Visual Basic for Applications (VBA) that can be used for developing customised user-defined functions (UDFs) not provided by Excel. The Developer ribbon also allows the use of macros to remove the tedium of parametric studies and repetitive calculations.

The lack of built-in functions for fluid properties that limits the usefulness of Excel for thermofluid analyses could be solved by developing suitable add-ins by various academic and research institutions [13-16]. This book uses an Excel-based modelling platform for thermofluid analyses that includes in addition to Excel, Solver, and VBA, an educational Excel add-in called Thermax [17]. Thermax provides seven groups of property functions for ideal gases, saturated water and superheated steam, synthetic and natural refrigerants, atmospheric humid air for psychrometric analyses, two aqua solutions for vapour-absorption refrigeration, chemically-reacting substances, and air at standard atmospheric pressure. Thermax also provides two interpolation functions and a Newton-Raphson solver for nonlinear equations that enhance the usefulness of the Excel-based modelling platform. Table 1.1 summarises the roles of the four components of the Excel-based modelling platform as used in this book.

Table 1.1. Roles of the four components of the Excel-based modelling platform

Component	Role
Excel	• Provides the basic functions needed for thermofluid analyses including the general mathematical functions and the matrix-operation functions • Provides the Goal Seek command needed for performing unconstrained iterative solutions involving a single parameter • Allows circular calculations which can be a convenient method for dealing with parameter dependency in certain analyses • Provides graphical tools needed for data visualisation and analyses • Allows macros to be recorded for repetitive calculations
Solver	• Allows constrained iterative solutions involving multiple parameters • Allows optimisation analyses with single and multiple design variables • Offers three solution options that suit different types of analyses
Thermax	• Provides the physical properties of various fluids • Provides two interpolation functions for tabulated data and a Newton-Raphson solver for non-linear equations such as the Colebrook-White equation and the SRK equation
VBA	• Needed for developing custom functions for the non-linear equations involved in iterative solutions and optimisation analyses • Needed for developing numerical solvers for large systems of linear equations • Can be used to develop additional fluid property functions or other functions not provided by Excel or the Thermax add-in

1.4. Closure

The following two chapters describe the Excel-based modelling platform in more details. While Chapter 2 focuses on the features of Excel that are mostly needed for thermofluid analyses such as its matrix functions and its Goal Seek command, Chapter 3 introduces the other three components of the modelling platform. Chapter 3 gives examples of using the three solution methods offered by Solver, describes the development of user-defined functions with VBA, and shows how the property functions provided by Thermax can be used in Excel formulae. Chapter 4 and Chapter 5 use the Excel-based platform to deal with two common types of thermofluid analyses; iterative solutions and optimisation analyses. Chapter 4 that deals with iterative solutions gives examples of using Excel's Goal Seek command and Solver for this type of analyses in the fields of fluid dynamics, heat-transfer, and thermodynamics. Chapter 5 that deals with optimisation analyses of fluid-thermal systems shows how Solver can be used for the analyses that involve a single design variable and multiple design variables.

Chpaters 6, 7, and 8 show how the Excel-based platform can be used for computer-aided analyses associated with the three areas of thermofluids; heat-ransfer, fluid dynamics, and thermodynamics, respectively. Chapter 6 deals with the numerical solution of the steady heat-conduction equation by using the finite-difference (FD) method. The chapter presents two approaches for applying the FD method with Excel the first of which assembles the system of linear equations and uses Excel's matrix functions to solve it, while the second uses circular calculations and, therefore, does not require the system of linear equations to be explicitly assembled. Chapter 7 focuses on hydraulic analyses of multi-pipe and pump-pipe systems and gives examples of using Excel's Goal Seek command and Solver for this type of analyses. Different pipe and pump arrangements are analysed in the chapter to determine the system's friction losses, power requirement, or operating point. Chapter 8 deals with thermodynamic analyses of the basic power generation and vapour-compression refrigeration (VCR) cycles by using Thermax property functions. With respect to the power-generation cycles, the chapter focuses on the Brayton cycle, the Otto cycle, and the Rankine cycle. With respect to the refrigeration cycles, the chapter focuses on the simple single-stage VCR cycle.

Chapter 9 presents six cases which are relatively more challenging than the related cases described in previous chapters of the book. The first of these cases shows how the Excel-based platform can be used to deal with the analyses of two-dimensional heat conduction in a complicated geometry. The second, third, and fourth cases deal with practical optimisation analyses of thermofluid systems that include a hot-water generation system with two steam heaters, an air-conditioning duct with two sections, and a pump-pipe system. The last two cases deal with thermodynamic analyses of the combined Brayton-Rankine power generation cycle and the cascade VCR cycle. The six cases that come from the three thermofluid areas; heat-transfer, fluid-dynamics, and thermodynamics, are meant for stimulating ideas for relevant mini projects. More project ideas are provided at the end of the chapter.

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2

Excel

Excel allows the manipulation of stored data by providing a large set of built-in functions and a number of analytical tools. With its graphical tools, iterative tools, and the “Developer” options, Excel forms the backbone of the modelling platform used in this book for thermofluid analyses. Excel’s user-interface provides many tools for general data analyses, but this chapter focuses on those needed for building analytical models for thermofluid analyses. The chapter highlights the use of “cell-labelling” for writing Excel’s formulae instead of the usual referencing by location and illustrates the use of Excel’s matrix functions for the solution of linear systems of equations. The chapter also illustrates the use of “Goal Seek” and “circular calculations” for the solution of nonlinear equations and its final section on Excel’s graphical tools illustrates the use of the “trendline” feature for curve-fitting of tabulated data. More detailed information about Excel can be found in, e.g., Walkenbach [1,2].

2.1. Elements of Excel’s user-interface

Excel’s user-interface allows the user to adjust the appearance of the workspace and present his/her primary data and analysis results in various forms. To allow easy access to the large number of functions, tools, and commands provided by Excel, the user-interface is divided into a number of elements with different purposes. For example, Figure 2.1 shows a screenshot of an Excel sheet that stores the scores obtained by a group of students in one semester.

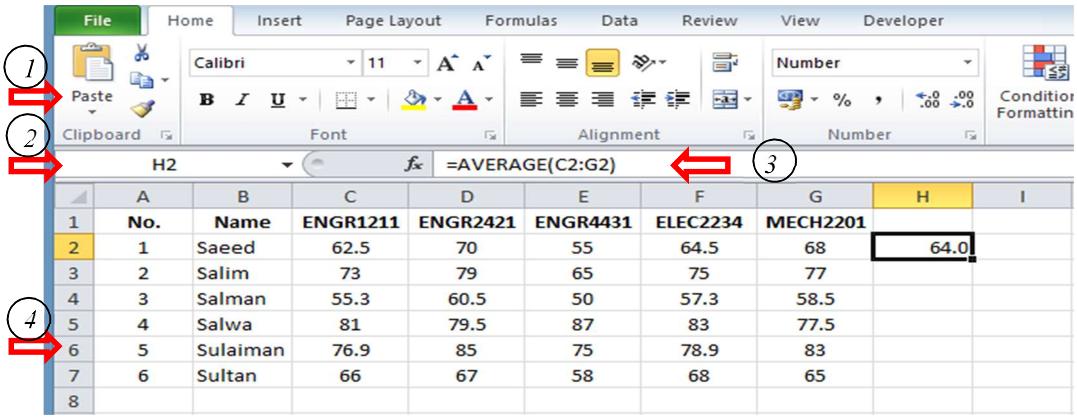


Figure 2.1. The main elements of Excel’s user-interface

Figure 2.1 shows four elements of Excel’s user-interface, which are:

1. The ribbon
2. The name box
3. The formula bar
4. The workspace

The **Ribbon**, which occupies the upper part of the sheet, organises the numerous commands provided by Excel into nine “tabs”, e.g., the **File**, **Home**, and **Insert** tabs. Each tab consists of a number of command-groups that have a common purpose. For example, the **Developer** tab shown in Figure 2.2 allows the user to write customised functions using the Visual Basic for Applications (VBA) language and to record **Macros** together with other useful development tools.

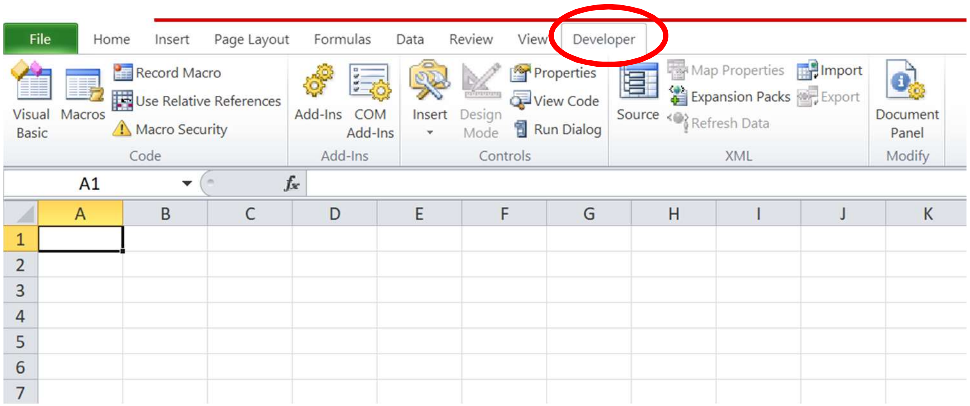


Figure 2.2. Elements of the Developer tab

The **Workspace**, which is the main part of the sheet, is divided into a grid of columns and rows that form separate “**cells**” at their intersections. A cell is referred to by a letter and a number, e.g., A1, B3, H2, etc. The letter represents the cell’s column, while the number represents its row. The **Name box** shows the location of the current cell. As Figure 2.1 shows, a cell can simply contain a character data, such as “Saeed” and “Salim”, or a numerical data, such as 62.5 and 70. A cell can also contain a formula for data manipulation using the numerous built-in functions provided by Excel. The formula bar in Figure 2.1 reveals the formula typed in cell H2 that uses the built-in function “**AVERAGE**” to determine the average score of the first student in the list (Saeed) in the five subjects as 64.0. Note that, unlike simple numerical or character cells, a cell that includes a formula must include the equal sign “=”. The role of the **Formula bar** will be explained in more details in the following section.

2.2. Excel’s formulae

In general, Excel’s formulae include mathematical or logical operators, built-in functions, and cell references. Excel provides two ways to refer a particular cell in a formula; either by its location in the sheet, e.g., A2, C10, etc., or by giving it a relevant name, e.g., efficiency, diameter, etc. The two methods will be illustrated below.

2.2.1. Cell reference by location

To illustrate this method, let us write a formula that calculates the area of a circle that has a radius of 5 m. Open a new Excel sheet and type the number 5, which is the radius of the circle, in cell A1 as shown in Figure 2.3. Now, go to cell A2 and type the following formula:

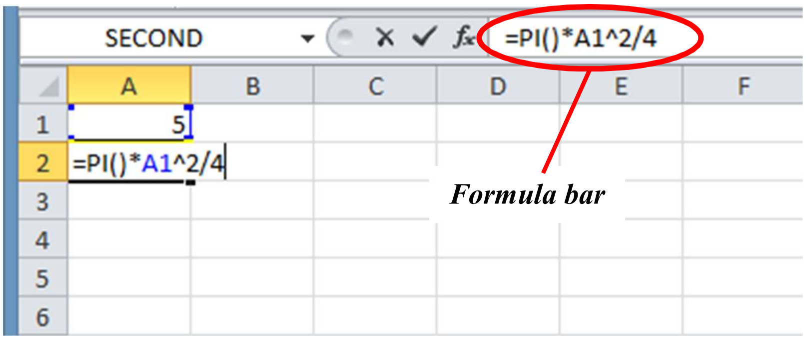


Figure 2.3. Writing an Excel formula to determine the area of a circle

=PI()*A1^2/4

The function "**PI()**" is a built-in function that returns the value of Archimedes' constant π. The formula also contains a reference to cell A1 that stores the value of the circle's radius, the multiplication operator *, the division operator /, the power operator ^, and the constants 2 and 4. Note that the formula is shown in the formula bar which can also be used to edit the formula. Pressing the **Enter** key after typing the formula, the result will be as shown in Figure 2.4; which is 19.63495 square metres. The following example shows how Excel's formulae and built-in functions can be used in a simple thermodynamic analysis.

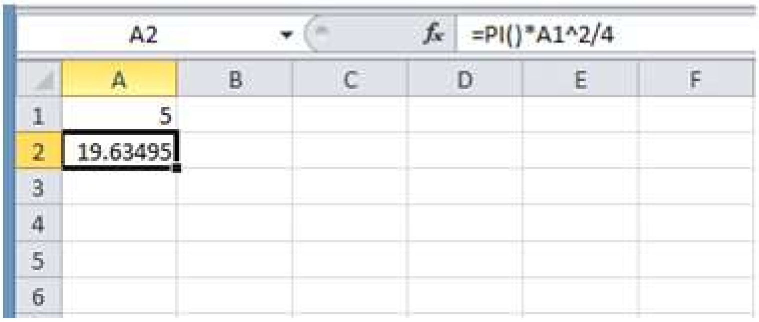


Figure 2.4. The Excel sheet with a formula that determines the area of a circle

Example 2-1. Determining the error in the specific volume of R134a calculated by using the ideal-gas law

Develop an Excel sheet that calculates the error in the specific volume (*v*) of superheated refrigerant R134a that results from applying the ideal-gas law at a pressure of 200 kPa (T_{sat} = -10.09°C) and temperatures in the range 0°C to 100°C (273 to 373 K).

Solution

Figure 2.5 shows the Excel sheet prepared for this example. The pressure (*P*), the gas constant (*R*), and the temperature (*T*) are stored in columns A, B, and C, respectively.

E2 f_x =B2*C2/A2

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal		
2	200	0.08149	273	0.10438	0.1112339		
3	200	0.08149	283				
4	200	0.08149	293				
5	200	0.08149	303				
6	200	0.08149	313				
7	200	0.08149	323				
8	200	0.08149	333				
9	200	0.08149	343				
10	200	0.08149	353				
11	200	0.08149	363				
12	200	0.08149	373				
13							

Figure 2.5.The sheet developed for Example 2-1

Column D stores the values of v obtained from relevant property tables and column E stores the corresponding values obtained from the ideal-gas law:

$$v = RT / P \tag{2.1}$$

Where, P and T are the absolute pressure and temperature, respectively, and R is the gas constant for R134a ($R = 0.08149$ kJ/kg.K). Note that the formula bar in Figure 2.5 reveals the formula in cell E2 that applies Equation (2.1). Since the pressure (P) and the gas constant (R) do not change in this example, Excel allows for these parameters to be stored in single cells instead of being repeated as shown in Figure 2.5 (see Section 2.4).

Figure 2.5 shows the tabulated value of the specific volume (v_table) and that determined by Equation (2.1) (v_ideal) at 273K. The percentage error of the ideal-gas law in estimating the specific volume is given by:

$$Error = \frac{v_{Ideal} - v_{Table}}{v_{Table}} \times 100 \tag{2.2}$$

To determine the percentage error at 273K, go to cell F2 as shown in Figure 2.6 and type the following formula, which is equivalent to Equation (2.2):

=(E2 – D2)/D2*100

Note that the formula bar in Figure 2.6 reveals the above formula for 273K. When you press the **Enter** key, the number **6.566** will appear in cell F2 as shown in the figure.

F2 | f_x =(E2-D2)/D2*100

	A	B	C	D	E	F	G
1	P	R	T	v_table	v_ideal	error_v_ideal	
2	200	0.08149	273	0.10438	0.1112339	6.5662483	
3	200	0.08149	283	0.10922	0.1153084	5.5743911	
4	200	0.08149	293	0.11394	0.1193829	4.776944	
5	200	0.08149	303	0.11856	0.1234574	4.1306933	
6	200	0.08149	313	0.12311	0.1275319	3.5917878	
7	200	0.08149	323	0.12758	0.1316064	3.1559414	
8	200	0.08149	333	0.13201	0.1356809	2.7807363	
9	200	0.08149	343	0.13639	0.1397554	2.4674463	
10	200	0.08149	353	0.14073	0.1438299	2.2026931	
11	200	0.08149	363	0.14504	0.1479044	1.974869	
12	200	0.08149	373	0.14932	0.1519789	1.7806389	
13							

Figure 2.6.The completed sheet developed for Example 2-1

To find the percentage errors at other temperatures you can simply copy the formula in cell F2 and paste it on cells F3 to F12. Figure 2.6 shows the completed Excel sheet for the required temperature range. The calculated values of the errors show that the maximum error occurs at the lowest temperature, which is 273K. Note that the error decreases gradually as the temperature increases.

2.2.2. Use of cell labels

The usual reference to cells by their columns and rows suits perfectly statistical analyses in which the same formula is applied to a large body of data that is stored column-wise or row-wise. For example, we may want to determine the average value, maximum value, or minimum value of the tabulated data. However, thermofluid analyses usually involve numerous formulae but a small set of data, e.g. the diameter of a pipe, the density or viscosity of a fluid, the effectiveness of a heat exchanger, etc. For such analyses, it is more convenient to give the cell a meaningful name or “label” that matches its content. The cell can then be referred to by its label instead of its relative location. This method makes it easier to recognise the quantities involved in the Excel formulae.

For the purpose of illustration, let us develop an Excel sheet to compare the density of air before and after an isentropic compression process from an initial condition of P_1 = 100 kPa and T_1 =300K to a final pressure of P_2 = 800 kPa. The two air densities involved can be calculated from the ideal-gas law as follows:

$$\rho_1 = P_1 / RT_1 \tag{2.3}$$

$$\rho_2 = P_2 / RT_2 \tag{2.4}$$

Where R is the gas constant (for air, R = 0.287 kJ/kg.K). For an isentropic process, T_2 is related to T_1 according to the following approximate relationship:

$$T_2 = T_1 \times \left(P_2 / P_1\right)^{\frac{k-1}{k}} \tag{2.5}$$

Where k is the ratio of specific heat at constant pressure (c_p) and at constant volume (c_v). At the given temperature range, k for air can be taken as 1.4. Note that k is also used for the thermal conductivity in Appendix A, Table A.1.

Figure 2.7 shows the sheet prepared for this analysis in which the respective cell labels are typed in the column to the left of the different pressures and temperatures, while the corresponding units are written in the column to the right of each quantity. This is also done to the other quantities in the calculations. The sheet also shows the units of the different properties involved for more clarification.

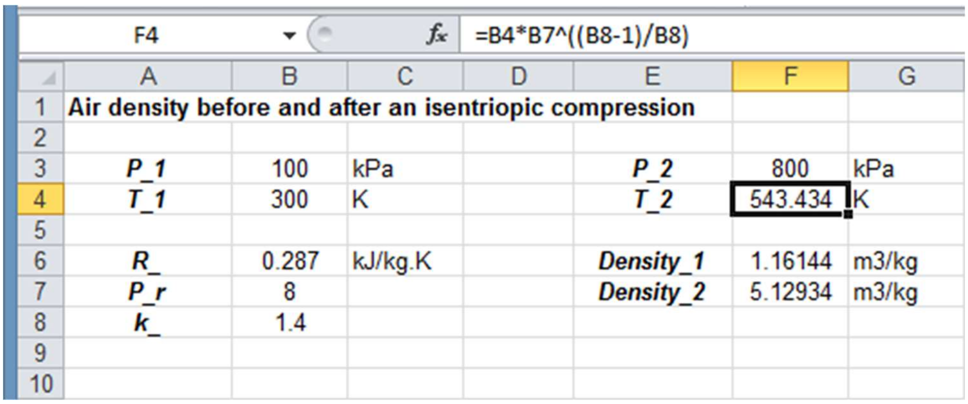


Figure 2.7. Excel sheet for calculating the air densities before and after compression

Placing the cursor on cell F4 makes the formula bar reveal the formula used in the calculation of the temperature T_2 according to Equation (2.5) which is:

=B4*B7^((B8-1)/B8)

The above formula can be made more understandable by using familiar labels to refer to the different cells involved. To do that, select the cells in columns A and B as shown in Figure 2.8, then go to **Formulas** and, at the **Name Manager**, select **Create** from **Selection**. When the form shown in Figure 2.8 appears to you, tick the “Left column” option. Pressing the “**OK**” button will make Excel create names for the different values in the selection box according to the labels written on the left column. The cell F3 that stores the value of P_2 can also be associated with its corresponding label in cell E3.

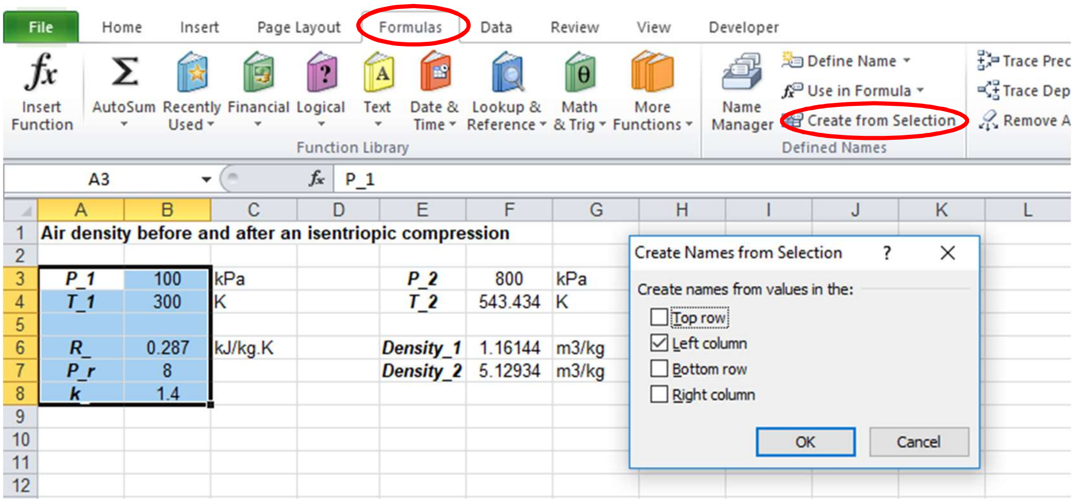


Figure 2.8. Creating names for a selected group of cells

Now, retype the formula in cell F4 that determines T_2 as follows:

=T_1*P_r^((k_-1)/k_)

The formula bar in the sheet shown in Figure 2.9 reveals the formula with the corresponding labels instead of location references.

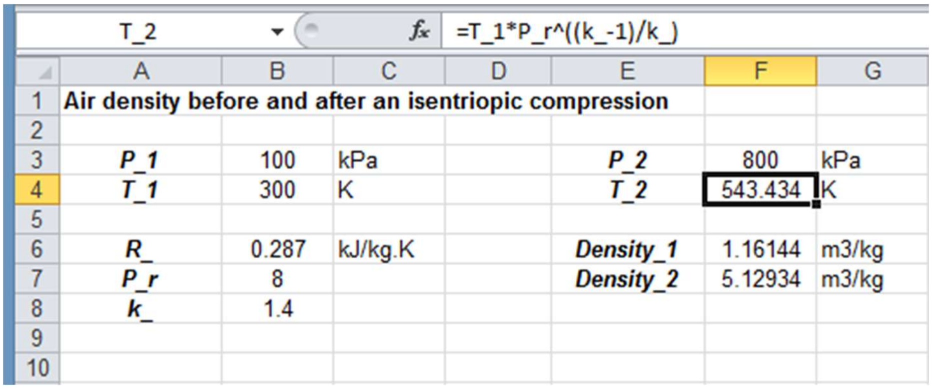


Figure 2.9. Formulae using cells labels instead of locations

Labelled formulae are easier to edit than those using location referencing particularly when intricate formulas are involved. Another advantage of cell-labelling is that if you copy a labelled formula and paste it in any other cell you will get the same answer, but if you copy a formula that uses the usual referencing by location in another cell you will get a different answer. To reveal or hide all the formulae in the sheet, press the control key (ctrl) with the tilde key (~). When naming your cells, choose suitable representative names for the variables involved, e.g. P_1 and T_1 for P_1 and T_1. Note that Excel does not accept “P1” or “T1” as labels since these can be confused with usual cell references by locations. Therefore, Excel automatically changes these labels to “P1_” and “T1_”. More information about Excel’s formulae can be found in Walkenbach [1].

2.3. Excel’s built-in functions

Excel provides a large library of built-in functions for data manipulation, like the **AVERAGE** function, and other functions commonly used in engineering analyses like the **PI**, **SIN**, and **COS** functions. To view the full range of Excel’s functions, type “=” in any Excel cell as shown in Figure 2.10 and then place the cursor on the **Insert Function** “*fx*” button in the formula bar and click it. The dialog box shown in Figure 2.11 will appear to you. List all the categories via the “**Select a category**” slot.

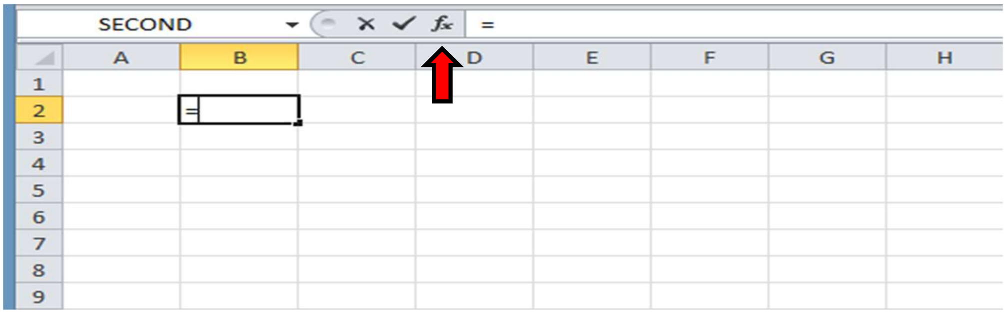


Figure 2.10. Exploring Excel’s built-in functions

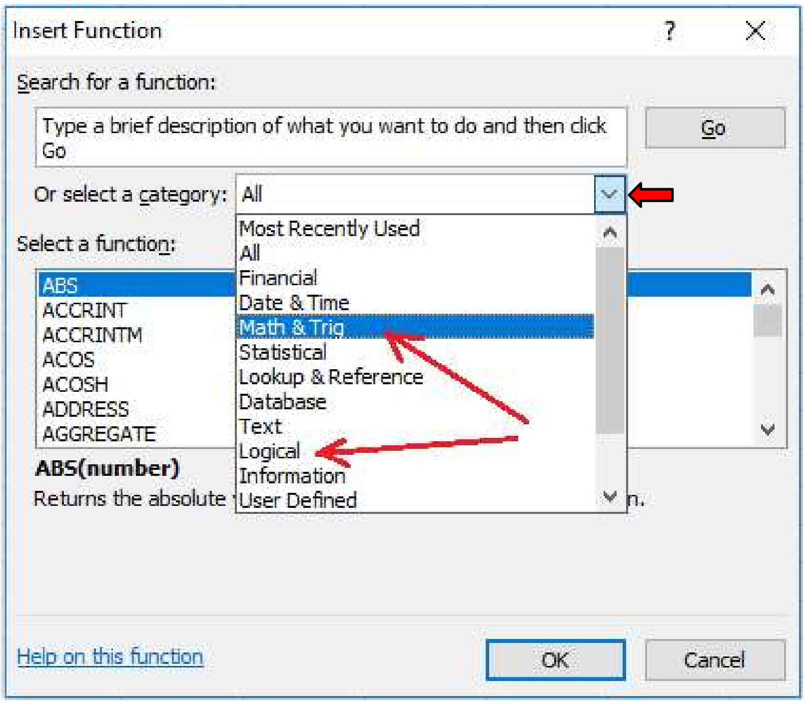


Figure 2.11. The various categories of Excel’s built-in functions

The **Math & Trig** group includes the mathematical and trigonometric functions used in different types of engineering analyses. Figure 2.12 shows some of the numerous functions in this group. Note that the dialog box gives a brief explanation of each function. For example, the explanation given to the **ABS** function is that it returns the absolute value of a number. The functions **ACOS**, **ASIN**, and **ATAN** apply the familiar inverse trigonometric functions: $\cos^{-1}$, $\sin^{-1}$, and $\tan^{-1}$, respectively. By scrolling down the list, you can find many other familiar functions. The following discussion focuses on two types of functions that are needed for the development of analytical models in subsequent chapters of the book, which are (a) the logical functions and (b) the functions for matrix operations.

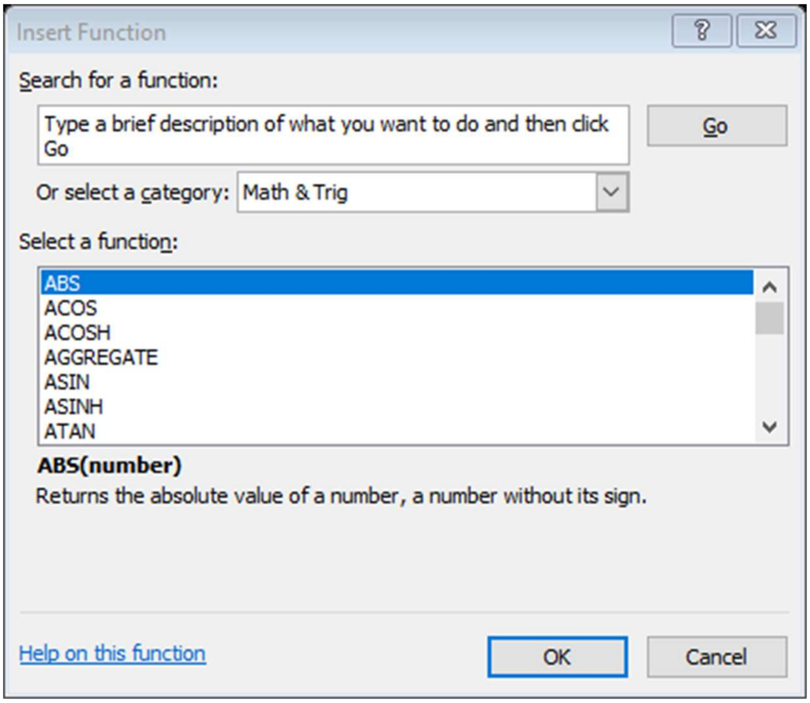


Figure 2.12. Common mathematical functions supported by Excel

2.3.1. Logical functions

To determine the major friction loss (h_f) in a pipe by using the Darcy-Weisbach equation we have to establish whether the flow is laminar or turbulent so as to select the relevant formula for the friction factor (f). The flow remains laminar before the Reynolds number (Re) reaches a certain value, which can be taken as 2,000, but the flow can only be considered fully turbulent beyond Re = 3000. There is a transitional region between laminar and turbulent flows when 2000 < Re < 3,000. Suppose that we want to write an Excel formula that determines the type of flow from the given value of the Reynolds number. Using a simple **IF** function, we can write the following formula:

=IF(Re<=2000, “Laminar”, “Turbulent or transitional”)

Note that the above **IF**-formula does not differentiate between a turbulent flow and a transitional flow. Therefore, we need to use a second logical test inside the first logical test. This can be done by using the following nested IF function:

=IF(Re<=2000, "Laminar", IF(Re>=3000, "Turbulent", "Transitional"))

Figure 2.13 shows an Excel sheet containing the above formula (shown in the formula bar) and the response of the formula when Re = 500, which is "Laminar". Depending on the value of Re stored in cell C2, the result of the If-formula can be "laminar", "Turbulent", or "Mixed". Excel supports six other logical functions; **AND**, **FALSE**, **IFERROR**, **NOT**, **OR** and **TRUE** that can be combined in the same formula so as to handle more intricate choices.

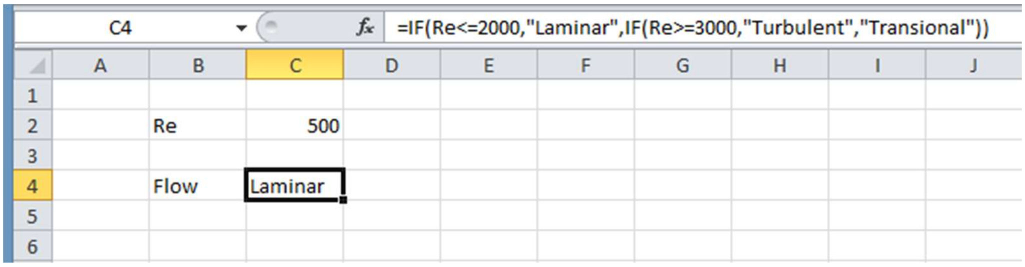


Figure 2.13. A formula using the nested IF function to determine the type of flow

2.3.2. Functions for matrix operations

Adjacent cells can be treated as a matrix or a vector and a group of Excel's formulae allow for the addition, subtraction, and multiplication of these matrices and vectors according to the established rules of matrix operations. Figure 2.14 shows a 3x3 matrix (A) stored in the cells B3:D5 and a vector (b) stored in cells F3:F5.

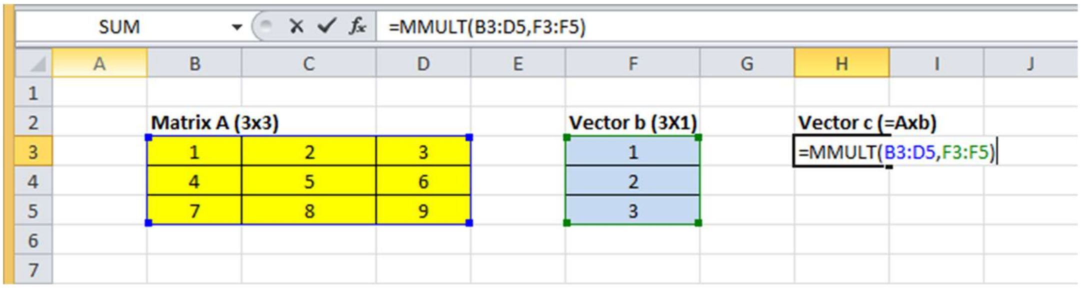


Figure 2.14. Step 1 of using the matrix multiplication function

Matrix (A) and vector (b) can be multiplied and the result stored in a third vector (c) by using the matrix function **MMULT**. The procedure is as follows:

1. After keying in the data of matrix (A) and vector (b) as shown in Figure 2.14, position the cursor at cell H3 and type the formula: =MMULT(B3:D5;F3:F5).

2. Now press ENTER key and cell H3 will take the value 14, which the result of multiplying the first row of the matrix with the vector (b) as Figure 2.15 shows.

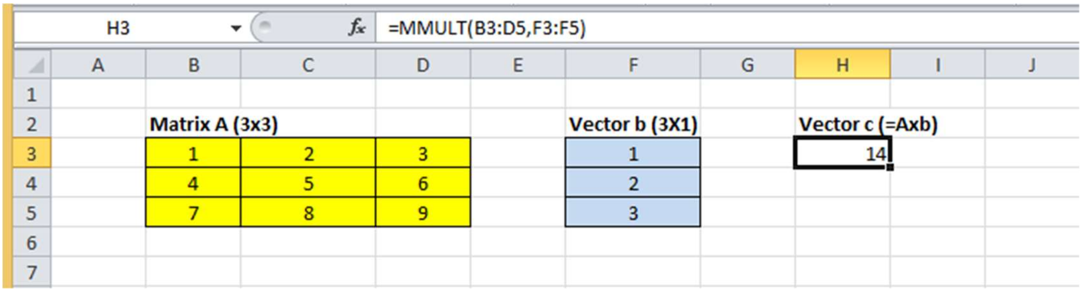


Figure 2.15. Step 2 of using the matrix multiplication function

The other two elements of the result vector will not appear automatically. To view the complete solution vector, do as follows:

1. Select the cells H3:H5 as shown in Figure 2.16,
2. Press the function key F2 once and then simultaneously hold the (**SHIFT** + **CONTROL**) keys together and press ENTER. The complete solution vector (c) will now appear as shown in Figure 2.17.

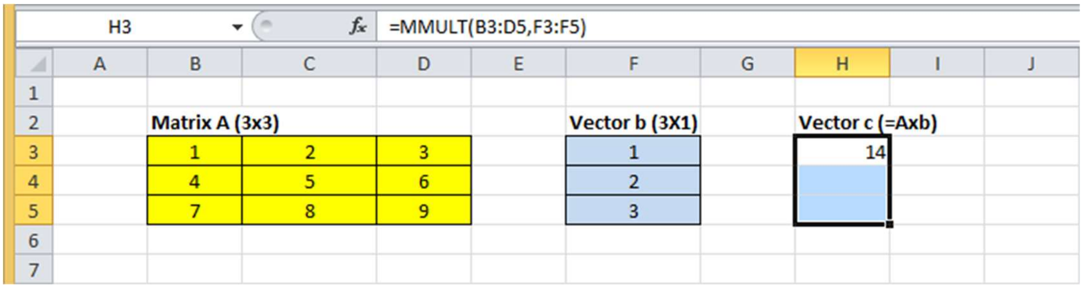


Figure 2.16. Step 3 of using the matrix multiplication function

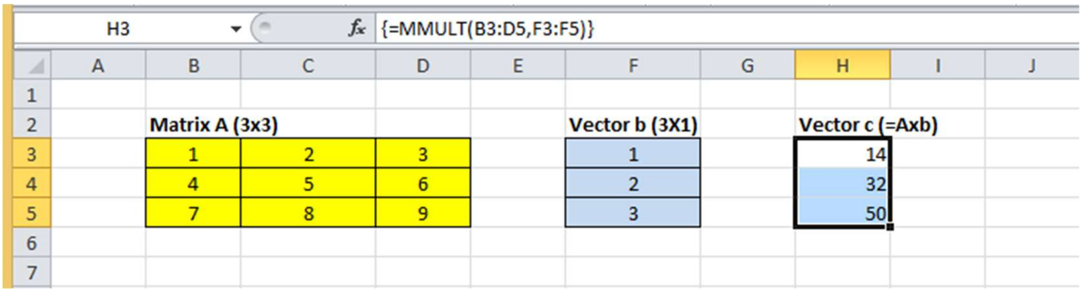


Figure 2.17. Step 4 of using the matrix multiplication function

An important matrix-operation function provided by Excel is the matrix-inversion function **MINVERSE** which is useful for the solution of linear systems of equations. The following example illustrates the use of this function.

Example 2-2. Using the matrix inversion function
By using the **MINVERSE** function, find the inverse of matrix [A] given by:

$$[A] = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 6 \\ 7 & 0 & 5 \end{bmatrix}$$

Solution
The first step is to enter the elements of the matrix as shown in Figure 2.18. After entering the data, go to cell F2 and type the formula "=MINVERSE(B2:D4)". When you press **ENTER**, this cell will have the value -0.3125, which is the first element of the inverse matrix $[A]^{-1}$ shown in Figure 2.19.

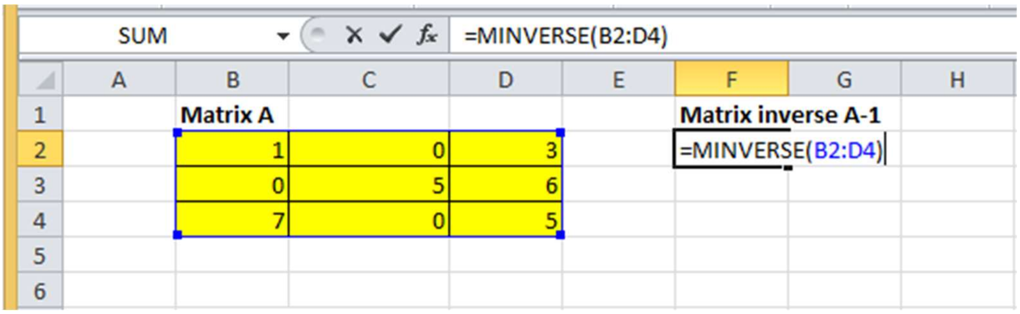


Figure 2.18. Step 1 of using the MINVERSE function

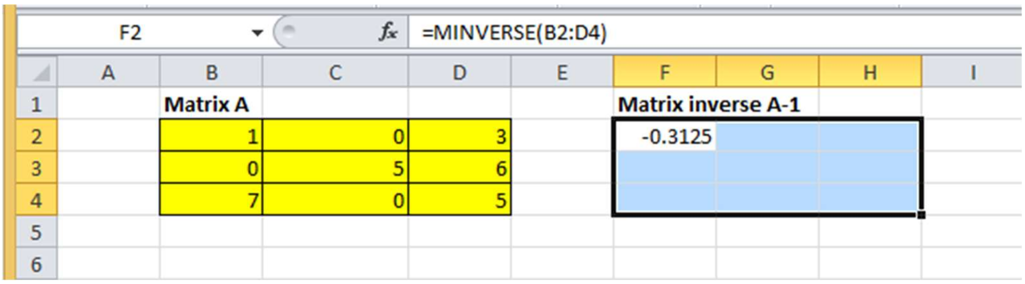


Figure 2.19. Step 2 of using the MINVERSE function

Starting with the formula in cell F2, select the range F2 to H4 as shown in Figure 2.19. Press and release the function key F2 and then simultaneously hold the **CTRL+SHIFT** keys and press **ENTER**. Other elements of the inverse matrix $[A]^{-1}$ will then appear as shown in Figure 2.20. You can check the solution by multiplying matrix [A] with its inverse by using the **MMULT** functions. The procedure is illustrated by Figures 2.21 to 2.23. As should be expected, Figure 2.23 shows that the resultant matrix is the identity matrix.

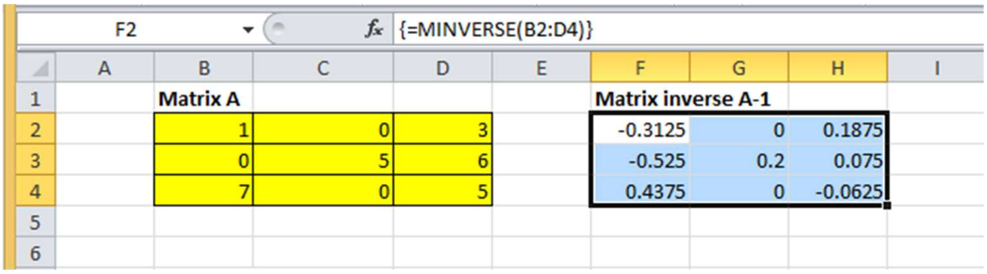


Figure 2.20. The complete inverse matrix $[A]^{-1}$

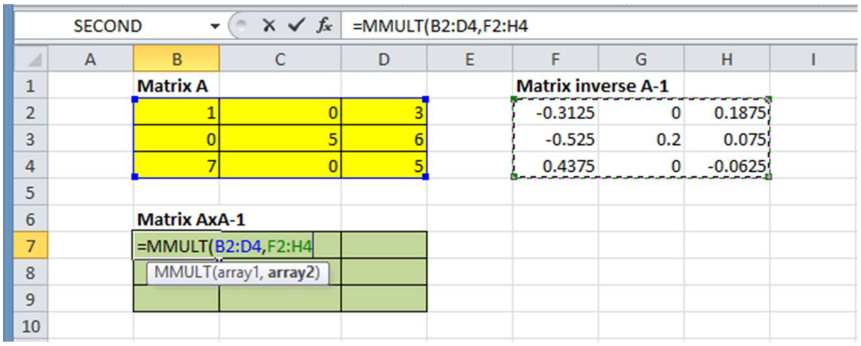


Figure 2.21. Multiplying matrix [A] by its inverse $[A]^{-1}$

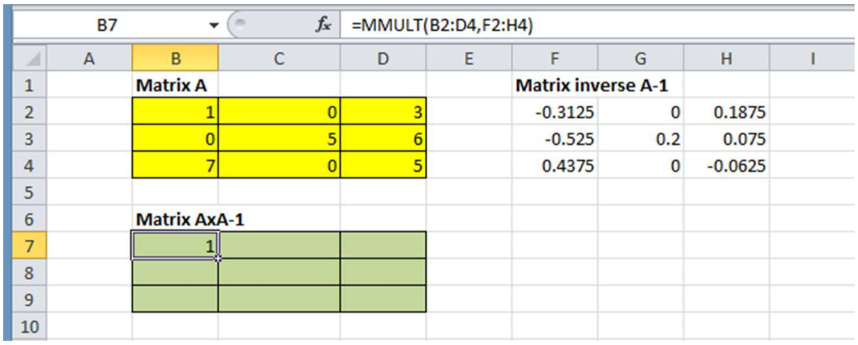


Figure 2.22. The first element of the product (identity) matrix

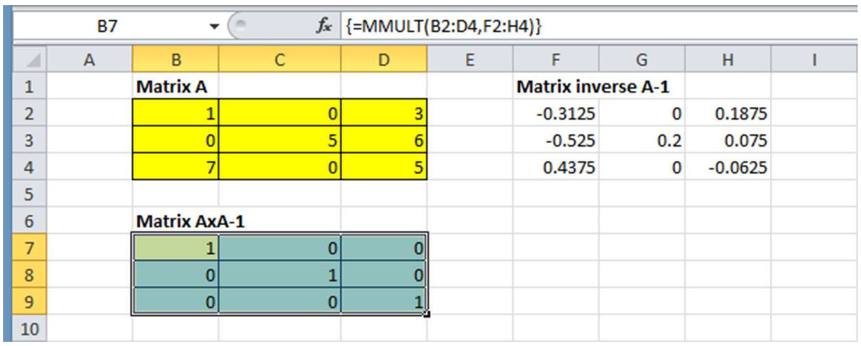


Figure 2.23. The complete solution which is the identity matrix

2.3.3. Solution of linear system of equations

An example of thermofluid analyses that involve systems of linear equations is the solution of the heat conduction equation by the finite-difference method. Linear systems of equations can be solved with Excel by applying the matrix-inversion method. For illustration, consider the following linear system written in matrix notation:

$$[A]\{x\}=\{y\} \tag{2.6}$$

Where [A] is the coefficient matrix, $\{x\}$ the vector of unknowns, and $\{y\}$ the right-side or "load" vector. By applying the matrix-inversion method, the solution vector $\{x\}$ can be obtained as follows:

$$\{x\}=[A]^{-1}\{y\} \tag{2.7}$$

Where $[A]^{-1}$ is the inverse of matrix [A]. The following example illustrates the procedure of applying the method by using Excel's matrix functions.

Example 2-3. Solution of a system of linear equations

Find the values of x_i in the following system of linear equations:

$$\begin{bmatrix} 14 & 14 & -9 & 3 & -5 \\ 14 & 52 & -15 & 2 & -32 \\ -9 & -15 & 36 & -5 & 16 \\ 3 & 2 & -5 & 47 & 49 \\ -5 & -32 & 16 & 49 & 79 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} -15 \\ -100 \\ 106 \\ 329 \\ 463 \end{Bmatrix} \tag{2.8}$$

Solution

Note that the system is symmetric; which is typically the case with linear systems that arise in the solution of heat-conduction problems by the finite-difference method. For larger systems of equations, the symmetry of the system can be utilised for reducing the required computer memory by storing only one half of the coefficient matrix. However, this requires a complicated computer programming. For small systems like the one considered here, it is more convenient to use Excel's matrix inversion and multiplication functions. Figure 2.24 shows the Excel sheet that stores both the coefficient matrix [A] and the load vector $\{y\}$. The inverse matrix $[A]^{-1}$, which is obtained by following the procedure described in the previous section, is stored below the coefficient matrix as shown in the figure. The inverse matrix $[A]^{-1}$ is then multiplied with the load vector {y} and the result stored below the load vector as shown in Figure 2.25. The complete solution is shown in Figure 2.26. The first element is practically zero and, therefore, the solution vector is {x} = (0, 1, 2, 3, 4).

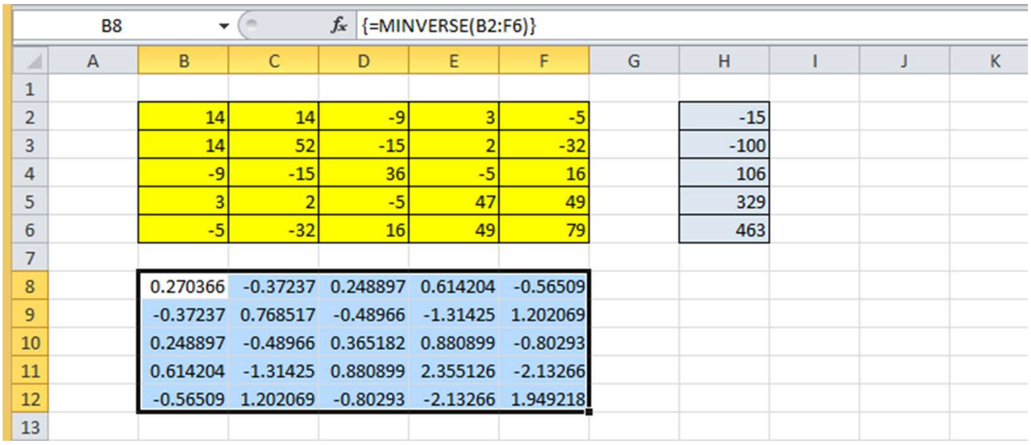


Figure 2.24.The coefficient matrix [A], the load vector {y}, and the inverse matrix $[A]^{-1}$

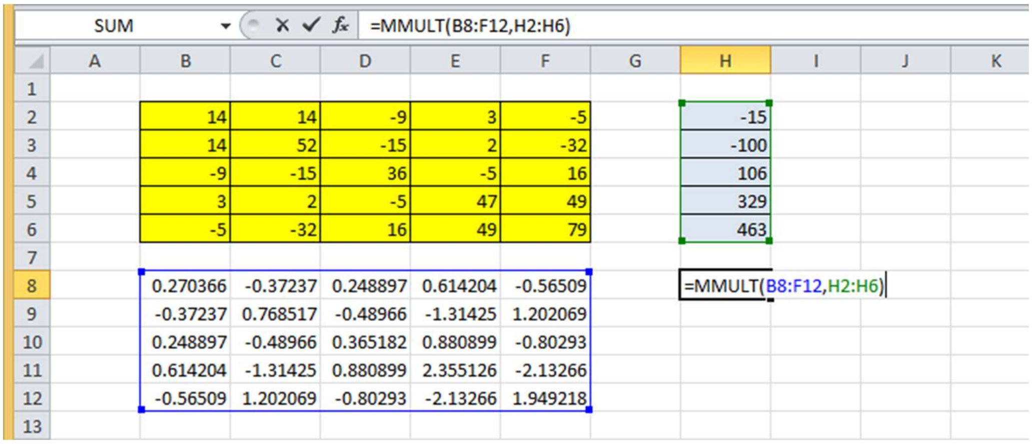


Figure 2.25. Multiplying the inverse matrix $[A]^{-1}$ with the load vector {y}

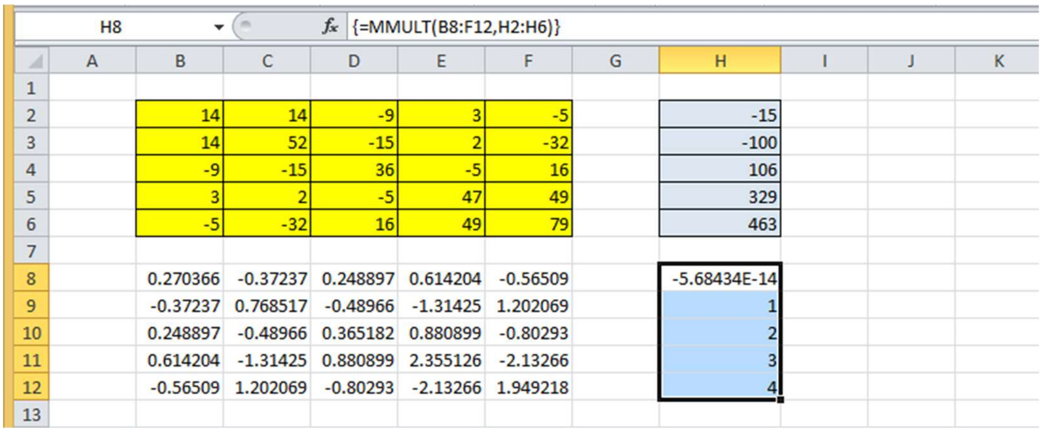


Figure 2.26. The complete solution vector {x}

It should be mentioned that the procedure described above by using Excel's functions suits best one-dimensional fluid-flow and heat-transfer analyses. This is because the linear systems generated in multi-dimensional analyses are usually too large to be solved efficiently by using the matrix-inversion method. Another method for solving small systems of linear equations using Solver is described in Chapter 3.

2.4. Itearative solutions with Excel

Excel offers its user two methods to perform iterative solutions: (i) by using the **Goal Seek** command and (ii) by using **circular calculations**. In what follows the two methods will be illustrated with the help of simple examples.

2.4.1. Itearative solutions with Goal Seek

The **Goal Seek** command is used for finding the value of an independent variable (x) that yields a specified value of a dependent variable (y). It is a simple, yet very useful tool for "What-if" analyses. The following example illustrates how this command can be used to solve a nonlinear equation.

Example 2-4. Solution of a nonlinear equation by Goal Seek

A centrifugal pump is used for lifting water from the utility network at the ground level to a tank at the top of a building that is 30-m high as shown in Figure 2.27. The pump's characteristic curve can be represented by the following formula:

$$h_p = h_0 - aQ - bQ^2 - cQ^3 \qquad (2.9)$$

Where h_p and Q are the pump's head (m) and discharge (m^3/s), respectively, and h_0, a, b, and c are constants the values of which are 47.22, 2.985×10^3, 1.549×10^5, and 2.348×10^8, respectively.

Neglecting friction losses in the pipe, determine the water flow rate (m^3/s) that can be delivered by the pump.

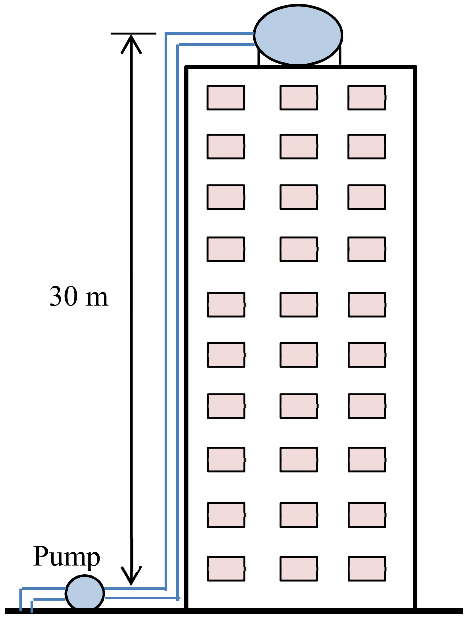


Figure 2.27. Schematic for Example 2-4

Solution

Figure 2.28 shows the Excel sheet prepared for this example in which the values of the four constants in Equation (2.9) are stored at the top of the sheet. The pump's head is calculated at various values of the discharge and plotted as shown in the figure. We can see from the plot that the value of Q that yield h_p = 30 m is approximately 0.003 m^3/s.

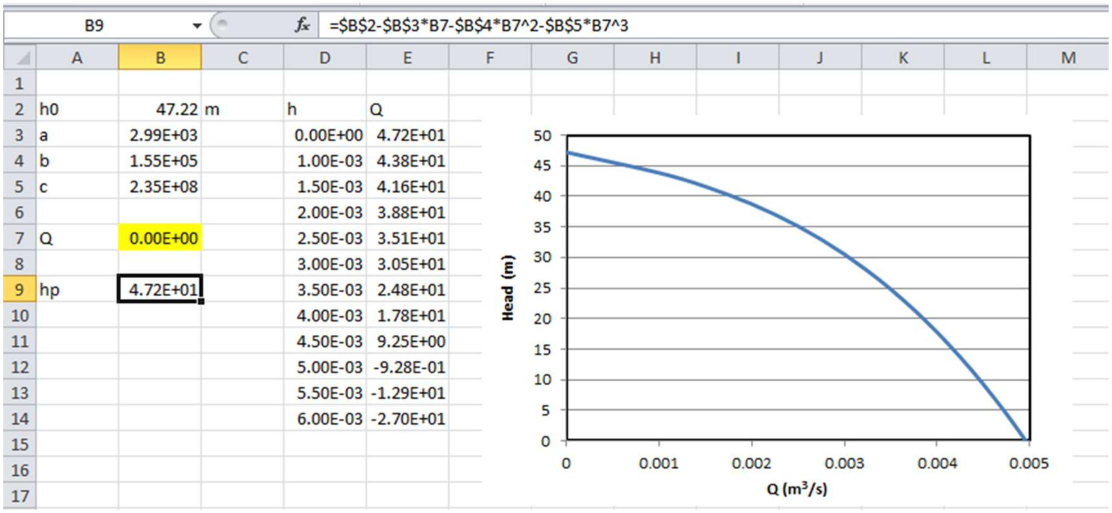


Figure 2.28. Excel sheet for Example 2-4

To solve the problem by using Goal Seek, enter an initial guess for Q in cell B7, say 0, and then enter the following formula that uses Equation (2.9) to calculate h_p in cell B9:

=B2-B3*B7-B4*B7^2-B5*B7^3

Note the dollar sign ($) that has been added to the references of the four constants, e.g. B2 has become B2. The formula bar in Figure 2.28 reveals the above formula by placing the cursor at cell B9.

To activate the Goal Seek command, go to the **Data** tab, select the **What-If-Analysis** option in the **Data Tools** group and then select **Goal Seek,** as shown in Figure 2.29. The Goal Seek dialog box shown in Figure 2.30.a will then appear to you. The dialog box asks you to select the "**Set cell**", i.e. the cell that contains the dependent variable, which is B9 in this case. You also have to specify the value sought for this cell and the adjustable cell that stores the parameter to be changed. In this case, we seek the value in the cell B9 to be 30 by changing the value of the cell B7. The completed form is shown in Figure 2.30.b.

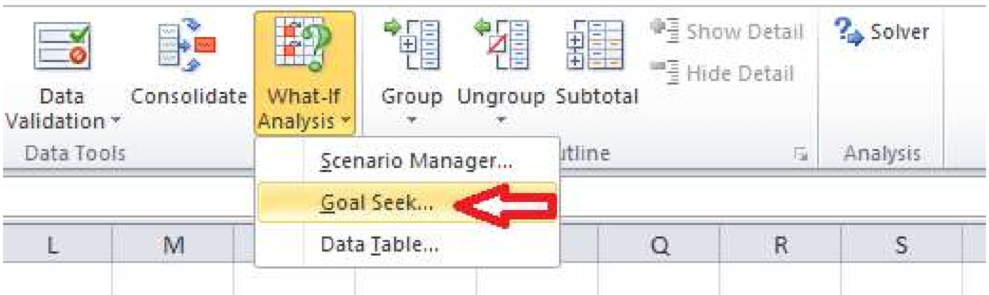
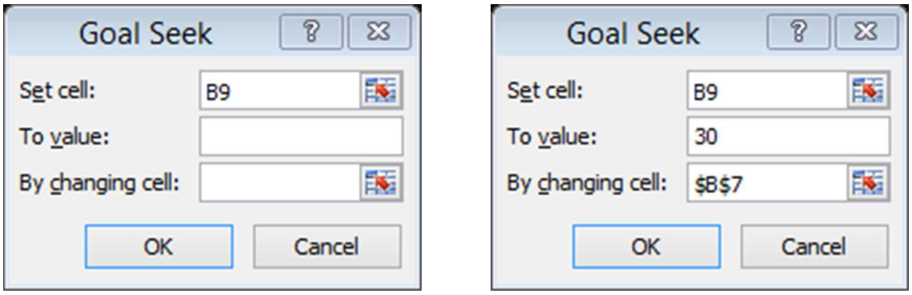


Figure 2.29. Activation of the Goal Seek command



(a) (b)

Figure 2.30. Goal Seek Set-up for Example 2-4: (a) before completion (b) the completed box

By pressing the “**OK**” button after completing the Goal Seek form, Excel will change the value in the adjustable cell (B7) until the Set cell (B9) acquires the required value. As shown in Figure 2.31, the answer obtained is Q = 0.003 m^3/s which agrees with the estimated value from the plot in Figure 2.28.

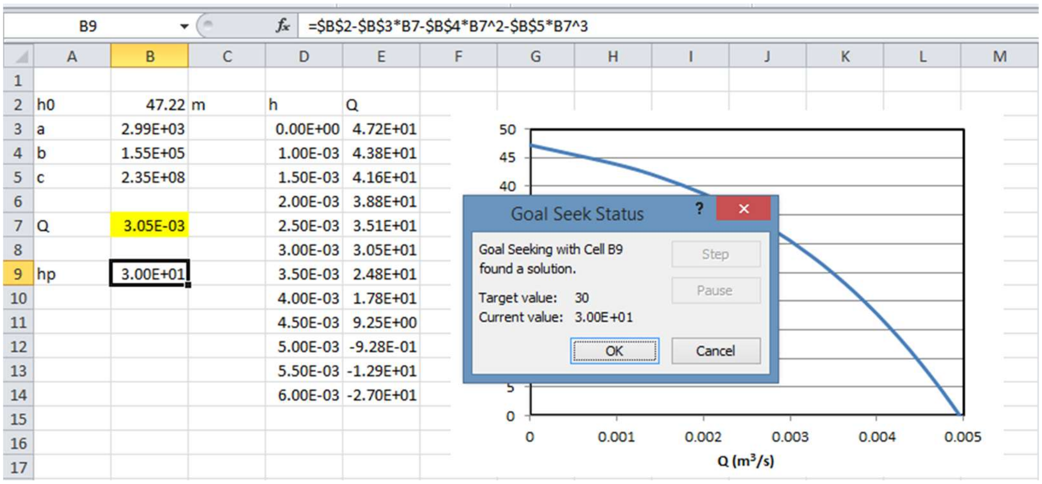


Figure 2.31. Goal Seek solution for Example 2-4

2.4.2. Iterative solution with circular calculations

A circular reference occurs when an Excel formula refers to its own cell in a direct or indirect manner. In standard programming this is not allowed, but Excel gives its user the option to use “**circular calculation**” if intended. In this case, Excel will itearate until all the formulae involved are satisfied. The following example illustrates this special feature which is useful for thermofluid analyses.

Example 2-5. Determining the final temperature of heated air

Heat is added to a piston-cylinder device that contains one kg of air initially at 300K. If 100 kJ of heat is added to the air at constant pressure, determine the final temperature of air taking into consideration that its molar specific-heat ($\tilde{c}_p$) varies with temperature according to the following formula:

$$\tilde{c}_p = a + bT + cT^2 + dT^3 \quad \text{[kJ/kmol]} \qquad (2.10)$$

Where a = 28.11, b =1.97x10^{-03}, c = 4.80x10^{-06}, and d = -1.97x10^{-09}.

Solution

From the defenition of specific heat, the final temperature (T_2) is given by:

$$T_2 = T_1 + Q/\left(\tilde{c}_p / M\right) \qquad (2.11)$$

Where T_1 is the initial temperature, Q is the amount of heat added, and M is the molar mass for air (M=29). If the variation of $\tilde{c}_p$ with temperature is ignored and its value at T_1 alone is used, Equation (2.11) determines T_2 as 399.73K. However, we can be more accurate by using Equation (2.10) to determine $\tilde{c}_p$ at the average temperature, T_{avr} = (T_1+T_2)/2. Figure 2.32 shows the Excel sheet developed for this method which reveals the formulae inserted in cells F2, F4, and F6.

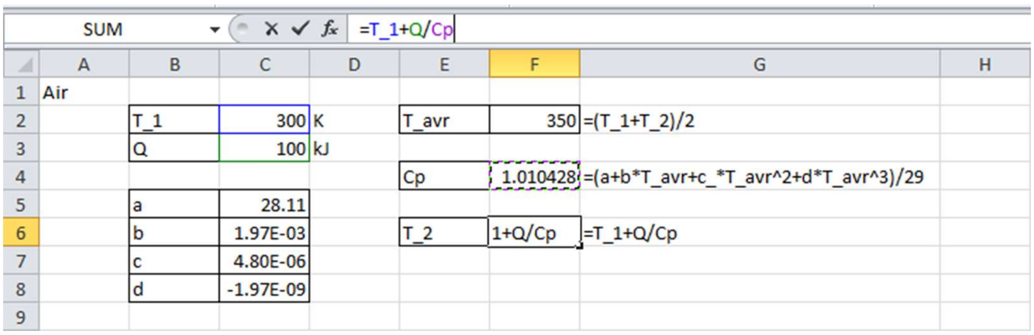


Figure 2.32. Excel sheet developed for Example 2-5

As soon as we type Equation (2.11) in cell F6, Excel will make the warning message that there is a circulare refernce as shown in Figure 2.33.

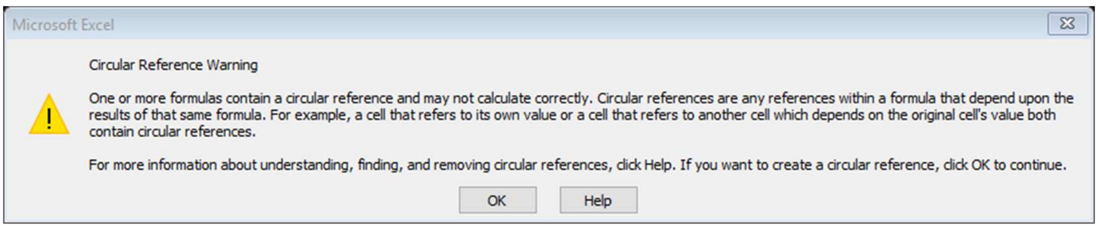


Figure 2.33. The circular-reference prompt

The circular reference occurs because T_2 depends on $\tilde{c}_p$ according to Equation (2.11) while $\tilde{c}_p$ itself depends on T_2 according to Equation (2.10). If we press the "OK" button shown in Figure 2.33, the cells involved in the circular reference whill be identified as shown in Figure 2.34. In this case, three cells are involved in the circular reference, which are F2, F4, and F6.

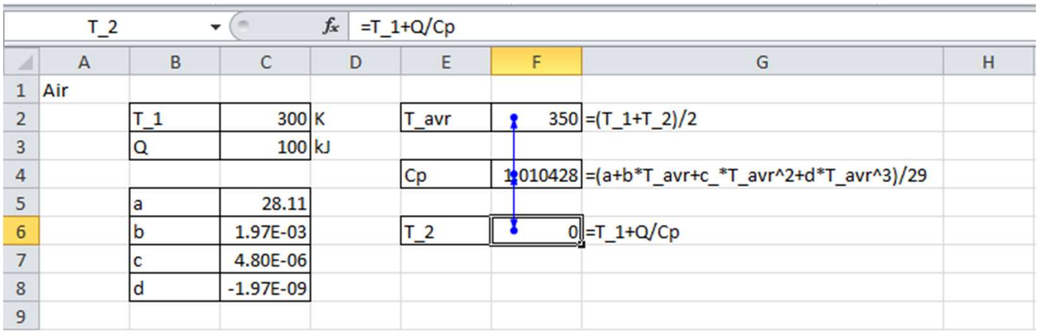


Figure 2.34. The cells involved in the circular reference

Excel can iterate to determine the values of both T_2 and $\tilde{c}_p$ that satisfy the relevant equations but the iterative-calculation option is not allowed by default. To allow it, go to **File** and select **Options**. The **Backstage View** form shown in Figure 2.35 will appear to you. Select **Formulas**, then the form will appear as shown in Figure 2.36. Enable iterative calculations by ticking (√) the box indicated in the figure and press the "OK" button. Excel can now iterate to find the values of T_2 and c_p that simultaneously satisfy Equations (2.10) and (2.11). Figure 2.37 shows the solution found by this method, which is T_2 =398.976K.

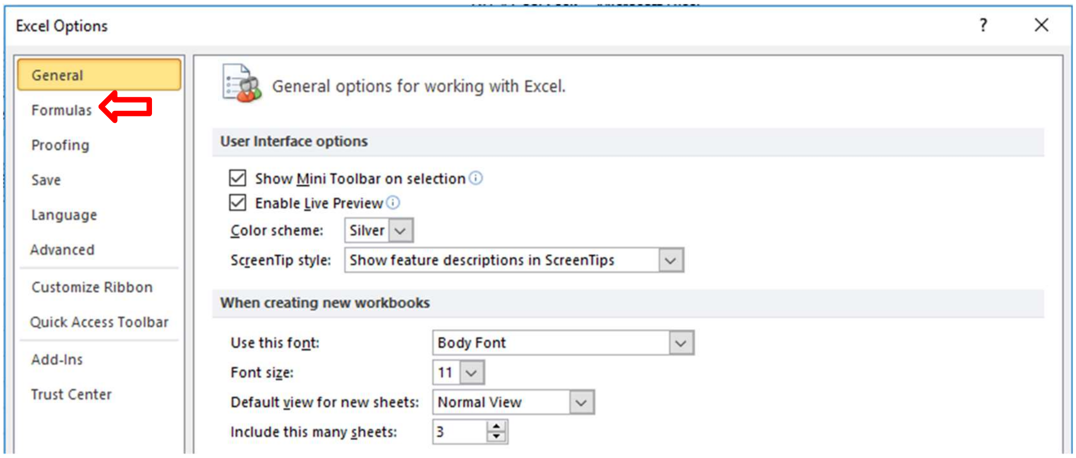


Figure 2.35. Selecting Excel's option (Formulas)

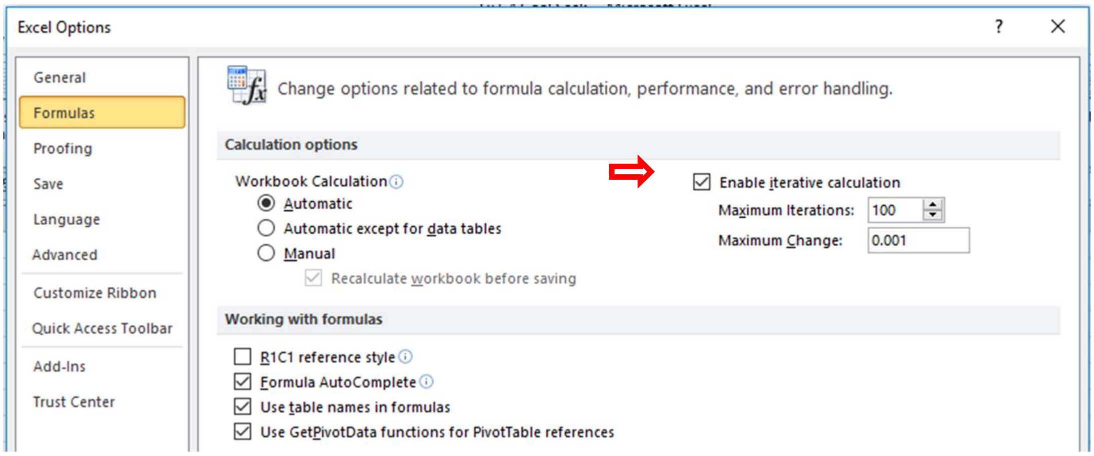
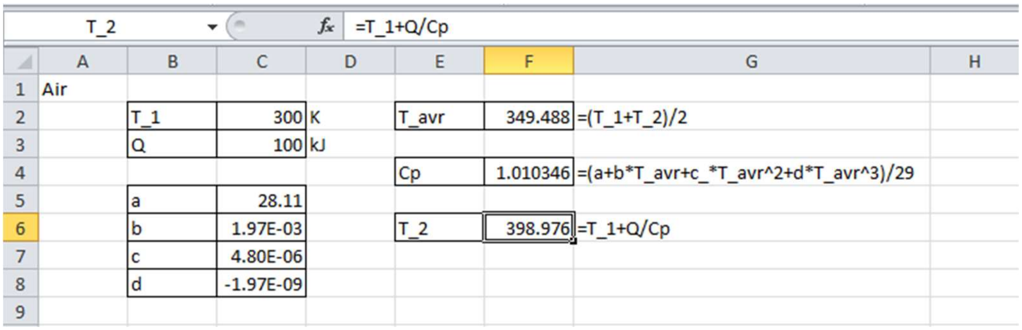


Figure 2.36. Enabling iterative calculations from Excel's Formulas option


T_2 | =T_1+Q/Cp

	A	B	C	D	E	F	G	H
1	Air							
2		T_1	300	K	T_avr	349.488	=(T_1+T_2)/2	
3		Q	100	kJ				
4					Cp	1.010346	=(a+b*T_avr+c_*T_avr^2+d*T_avr^3)/29	
5		a	28.11					
6		b	1.97E-03		T_2	398.976	=T_1+Q/Cp	
7		c	4.80E-06					
8		d	-1.97E-09					
9								

Figure 2.37. Solution of Example 2-5 by circular calculations

This example can also be solved by using the Goal Seek command. In this case, we have to start the iterative solution by providing Excel with a guessed value for T_2, call it T_{2o}, based on which a new value for T_2 is calculated, called it T_{2c}. Since the guessed value T_{2c} is unlikely to be correct, it will be different from T_{2o}. Goal Seek is then used to adjust the value of T_{2o} until the difference (Diff = T_{2c} - T_{2o}) vanishes.

Most iterative solutions in this book are obtained by using the Goal Seek command. However, the circular-calculation option is more useful in certain situations as demonstrated in Chapter 6 that deals with the numerical solution of the heat- conduction equation with the finite-difference method and Chapter 7 that deals with the hydraulic analyses of pump-pipe systems.

2.5. Excel's graphical tools for data presentation and analysis

Excel has numerous graphical tools that can be used to present the stored data in a variety of charts. Figure 2.38 shows one type of Excel charts that displays the annual variation of temperature and relative humidity at one location in a certain day. The figure shows a line chart in which the temperature is scaled on the primary *y*-axis (on the left) while the humidity is scaled on the secondary *y*-axis (on the right). This arrangement is useful for displaying two or more types of data that differ significantly in magnitude such as the net specific work and thermal efficiency of a power cycle.

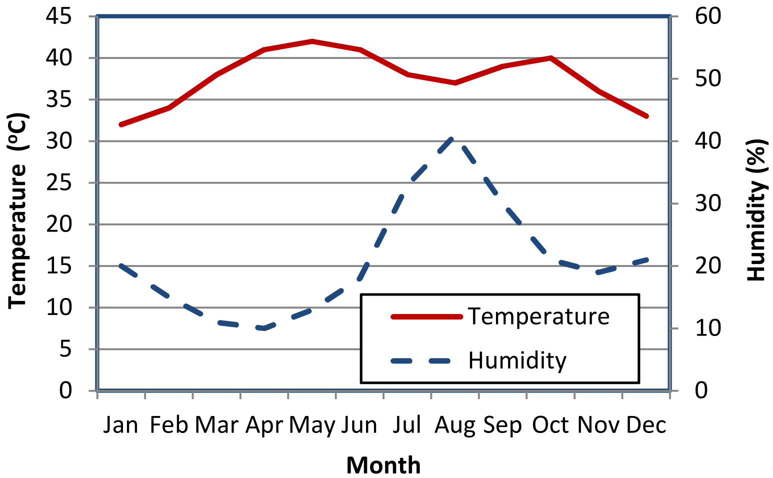


Figure 2.38. An example of line charts

Excel supports other types of charts that allow the user to select the most appropriate way to display his/her data in the form of bar, area, or scatter charts. For more information about the different types of Excel's charts, the reader can refer to specialised references such as Walkenbach [2]. A number of tutorials and videos that show how to create different types of charts can also be found in the internet.

Excel's charts provide a curve-fitting capability of numerical data by using the **Trendline** feature. This capability is particularly useful for computer-aided thermofluid analyses because it can be used to convert tabulated fluid-properties and other data into analytical equations that make the data more suitable for iterative solutions and optimisation analyses. To illustrate the use of this feature, consider Table 2.1 that shows properties of saturated water in the range 0.001ºC – 60ºC. These values of the saturation pressure (P_{sat}) and saturated liquid enthalpy (h_f) are used in psychrometric analyses of air-conditioning applications. For computer-aided analyses, it is useful to convert these data into relevant equations.

The trendline feature provides a number of options, which include exponential, linear, logarithmic, polynomial, and power equations as shown in Figure 2.39. To fit a trendline to the data, we have to create line charts for the two properties as shown in Figures 2.40.a and 2.40.b. Trendlines can then be added on the line charts. Figures 2.40.a and 2.40.b also show the corresponding trendline equations of the tabulated data as determined by using polynomial equations. As Figure 2.40.b shows, a linear equation is adequate for the h_f data since its variation over the given temperature range is mild. However, a third-order polynomial is required to represent the variation of P_{sat} with temperature as shown in Figure 2.40.a.

Table 2.1. Properties of saturated water at temperatures in the range 0ºC- 60ºC taken from Cengel and Boles [3]

*T*ºC	P_{sat} [kPa]	v_f [m³/kg]	v_g [m³/kg]	u_f [kJ/kg]	u_g [kJ/kg]	h_f [kJ/kg]	h_g [kJ/kg]	s_f [kJ/kg.K]	s_g [kJ/kg.K]
0.01	0.6117	0.001000	206.00	0.000	2374.9	0.001	2500.9	0.0000	9.1556
5	0.8725	0.001000	147.03	21.019	2381.8	21.020	2510.1	0.0763	9.0249
10	1.2281	0.001000	106.32	42.020	2388.7	42.022	2519.2	0.1511	8.8999
15	1.7057	0.001001	77.885	62.980	2395.5	62.982	2528.3	0.2245	8.7803
20	2.3392	0.001002	57.762	83.913	2402.3	83.915	2537.4	0.2965	8.6661
25	3.1698	0.001003	43.340	104.83	2409.1	104.83	2546.5	0.3672	8.5567
30	4.2469	0.001004	32.879	125.73	2415.9	125.74	2555.6	0.4368	8.4520
35	5.6291	0.001006	25.205	146.63	2422.7	146.64	2564.6	0.5051	8.3517
40	7.3851	0.001008	19.515	167.53	2429.4	167.53	2573.5	0.5724	8.2556
45	9.5953	0.001010	15.251	188.43	2436.1	188.44	2582.4	0.6386	8.1633
50	12.352	0.001012	12.026	209.33	2442.7	209.34	2591.3	0.7038	8.0748
55	15.763	0.001015	9.5639	230.24	2449.3	230.26	2600.1	0.7680	7.9898
60	19.947	0.001017	7.6670	251.16	2455.9	251.18	2608.8	0.8313	7.9082

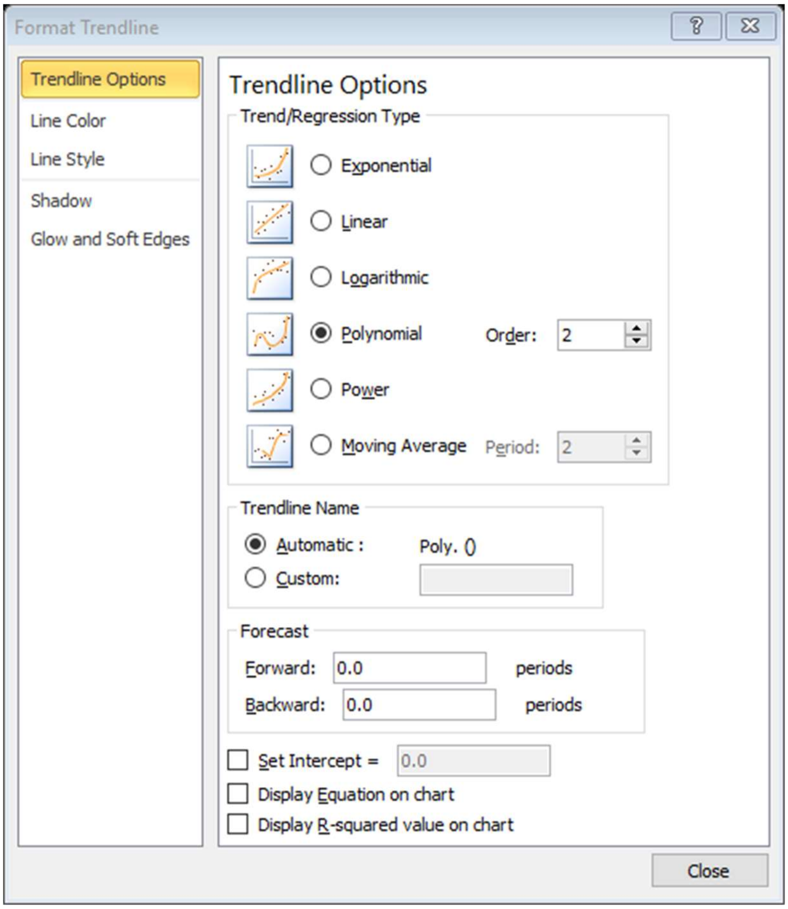


Figure 2.39. The Format Trendline window

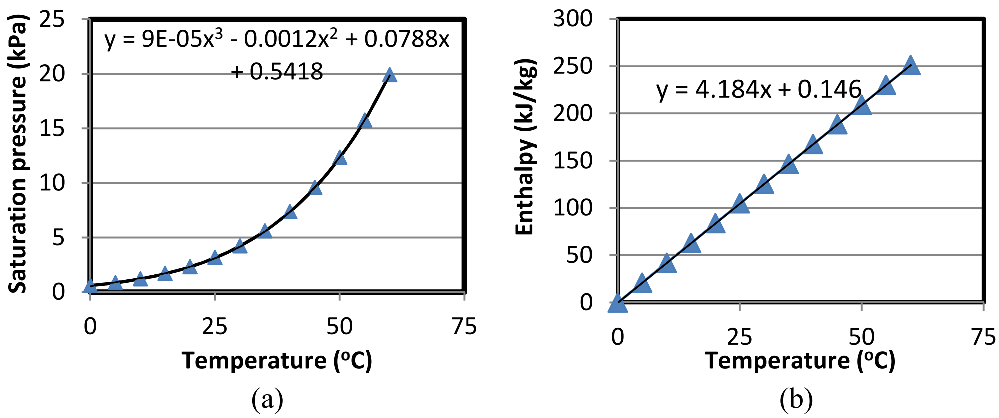


Figure 2.40. Fitting trendlines on tabulated data for water of (a) saturation pressure and (b) saturated liquid enthalpy

2.6. Closure

This chapter focussed on the basic features of Excel that are needed for thermofluid analyses. The chapter highlighted the importance of using cell labelling with Excel's formulae and illustrated the use of Excel's general mathematical functions, logical functions, and the functions for matrix operations. The chapter also demonstrated the use of Excel's two iterative tools: Goal Seek and circular calculations. In spite of its simplicity, the Goal Seek command is very useful for iterative solutions as shown in later chapters of this book. Finally, the chapter illustrated the usefulness of Excel's charting tools for the presentation of tabulated data particularly the trendline feature.

It should be mentioned that the **Developer** tab of Excel's user-interface provides a number of useful features that enhance the effectiveness of Excel as modelling platform for thermofluid analyses, but are not discussed in the chapter. One of these features is the ability to develop user-defined functions with VBA; which will be discussed in Chapter 3. Another useful feature of Excel is the ability to record **macros** for conducting repetitive calculations and parametric analyses.

References

[1] J. Walkenbach, *Excel 2010 Formulas*, Wiley Publishing Inc., 2010

[2] J. Walkenbach, *Excel 2007 Charts*, Wiley Publishing Inc., 2007

[3] Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007

[4] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*, 6th Edition, McGraw Hill, 2010

Exercises

1. The following table shows measured values of the temperature by two different methods compared to the correct corresponding values. Find the average error for each method.

***Correct T* (ºC)**	***Method* 1**	***Method* 2**
0	0.1044	0.1112
10	10.1092	10.1153
20	20.1139	20.1194
30	30.1186	30.1235
40	40.1231	40.1275
50	50.1276	50.1316
60	60.1320	60.1357
70	70.1364	70.1397
80	80.1407	80.1438
90	90.1450	90.1479
100	100.1493	100.1520

2. The following table shows the data for the saturation pressure of a certain fluid. Use a nested IF statement to develop an interpolation formula that determines the saturation pressure of the fluid at any temperature in the range 5ºC ≤ T ≤ 30ºC.

T(ºC)	P_{sat} (kPa)
5	0.872
10	1.228
15	1.705
20	2.339
25	3.169
30	4.246

3. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

Solve the system of equations to determine the values of the three unknowns a, b, and c. This exercise is based on Example 9.11 in Chapra and Canale [4]. The answer is: a = 8.5941, b=34.4118, and c = 36.7647.

4. Figure 2.P4 shows a triangular fin attached to the surface of a wall. Solution of the conduction heat transfer equation with the finite-difference method resulted in the

following system of linear equations the solution of which gives the temperatures in °C at different distances from the fin base as shown in the figure:

$-8.008\ T_1 + 3.5\ T_2 = -900.209$

$3.5\ T_1 - 6.008\ T_2 + 2.5\ T_3 = -0.209$

$2.5\ T_2 - 4.008\ T_3 + 1.5\ T_4 = -0.209$

$1.5\ T_3 - 2.008\ T_4 + 0.5\ T_5 = -0.209$

$T_4 - 1.008\ T_5 = -0.209$

Use Excel functions to solve the above system of linear equations.

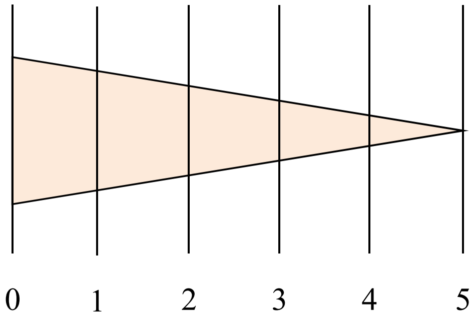


Figure 2.P4. Triangular fin

5. Adopting suitable names in your formulae, prepare an Excel sheet for calculating the frictional loss (h_f) in a circular pipe of diameter D, length L, and roughness k_s. Use your sheet to determine h_f in the following cases:

   (a) D = 25 cm, L = 150 m, V = 2 m/s, k_s = 0.045 mm, carrying water at 20°C.
   (b) D = 25 cm, L = 150 m, V = 0.2 m/s, k_s = 0.045 mm, carrying oil at 20°C.
   (c) D = 25 cm, L = 150 m, V = 7 m/s, k_s = 0.045 mm, carrying air at 20°C.

   Use the Darcy-Weisbach equation and determine the values of the kinematic viscosity from relevant property tables.
6. Using a line chart, plot the variation of sine θ for $-180 \leq \theta \leq 180$ in steps of 10°then add cosine θ on the same chart.
7. Using the data shown in Table 2.1, make a line chart for v_f and v_g. Add polynomial trendlines for both properties and comment on the trendlines equations.
8. The table below shows some of the thermo-physical properties of air at atmospheric pressure and different temperatures. Use Excel charts to show the variation of the

properties ρ, β, c_p, k, α, μ, v, and Pr with temperature and use trendline to obtain suitable equations for these properties.

T (K)	ρ (kg/m^3)	$\beta\times10^3$ (1/K)	c_p (J/kg.K)	k (W/m.K)	α (m^2/s)	$\mu\times10^6$ (N S/m^2)	$v\times10^6$ (m^2/s)	Pr
273	1.252	3.66	1011	0.0237	19.2	17.456	13.9	0.71
293	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71
313	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71
333	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71
353	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71
373	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71
473	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71
573	0.596	1.75	1047	0.0429	68.9	29.322	49.2	0.71
673	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72
773	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72

9. The volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid by:

   $V = \pi h^2(3r - h)/3$

   Using the Goal Seek command, determine the value of h for the tank with r=1 m and V = 0.5 m^3.

   This exercise is based on Problem 8.9 in Chapra and Canale [4]. Answer: h = 0.431 m.

3

Solver, VBA, and Thermax

As the main component of the modelling platform used in this book for thermofluid analyses, Excel provides the user-interface, numerous built-in functions, two iterative tools, and the graphic tools. However, what makes Excel particularly effective for thermofluid analyses are the three auxiliary components; Solver, VBA, and Thermax. This chapter focuses on these three components and illustrates their use by means of relevant examples. With respect to Solver, the chapter shows how its three solution methods can be used for performing optimisation analyses and solving systems of linear and nonlinear equations. The chapter also shows how VBA can be used for developing custom functions and how Thermax functions can be used in Excel formulae.

3.1. Solver

Solver is a multi-purpose iterative tool developed by Frontline Systems [1] as an Excel add-in for "What-if" analyses. Compared to Excel's own iterative tool, which is the Goal Seek command described in Chapter 2, Solver offers the following advantages:

1. While Goal-Seek can be used for simple problems that involve only one decision variable, Solver can deal with more difficult problems in which the objective cell is affected by numerous cells and decision variables.
2. Goal Seek allows only a required value of the objective cell to be achieved, but beside that Solver enables Excel to perform an optimisation analysis by finding the maximum or minimum value for the formula in the objective cell.
3. Solver allows constraints to be applied on the iterative solution. This ability, which is particularly important for optimisation analyses, is not possible with Goal Seek.
4. Solver offers three solution methods that suit different types of problems and gives the user a number of numerical options for applying these methods.

3.1.1. Activation of Solver

Like the Goal Seek command, Solver is found in the **Data** tab as shown in Figure 3.1. If it doesn't appear in your **Data** tab, then go to **File**, click **Options**, and select **Add-Ins**. From the **Manage** option at the bottom of the menu select **Excel Add-ins** and then click "**Go**". The **Add-Ins** dialog box shown in Figure 3.2 will be shown. To add **Solver** to the add-ins menu, tick (√) on the "**Solver"** option and return to the **Data** tab. When you click the **Solver** button from the **Data** tab, the **Solver Parameters** dialog box shown in Figure 3.3 will be shown.

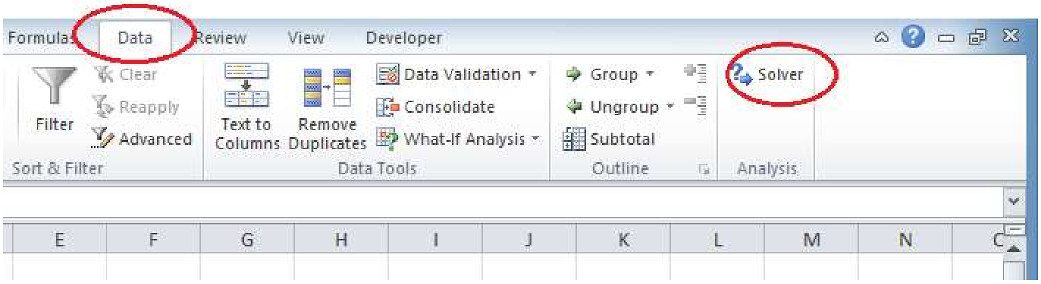


Figure 3.1. The Solver add-in in the Data tab

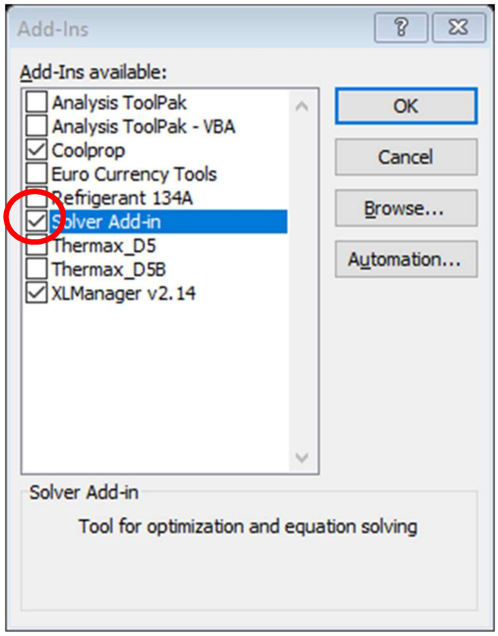


Figure 3.2. Activating Solver from the menu of Excel add-ins

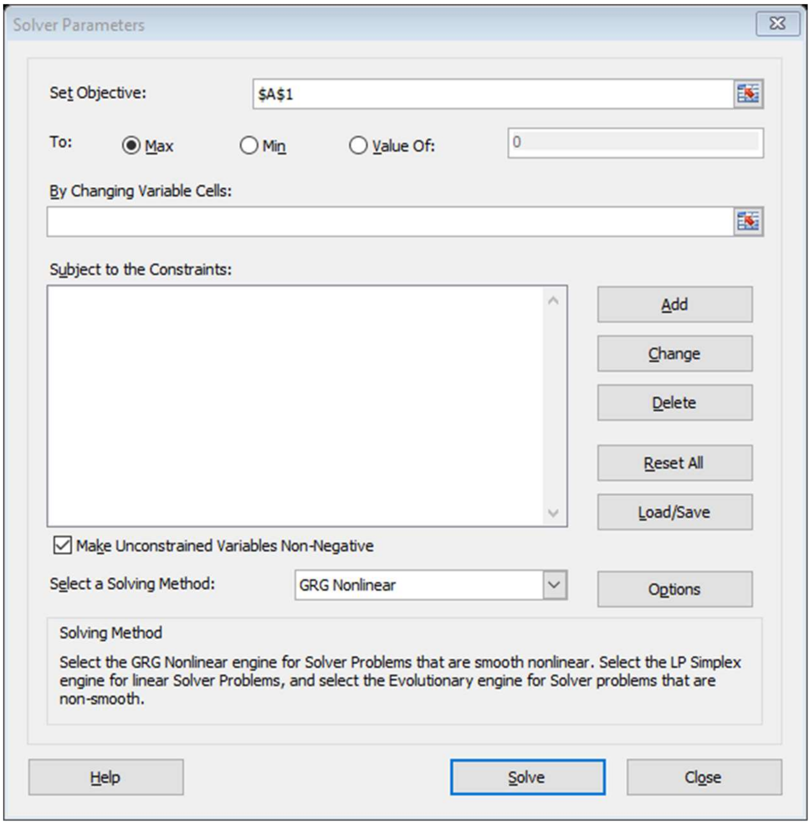


Figure 3.3. Solver Parameters dialog box

Solver Parameters dialog box helps the user to select a formula in one cell, called the **objective cell**, and a group of other cells, called **decision variables** or variable cells, that are directly or indirectly related to the formula in the objective cell. By adjusting the **decision variables**, the **objective cell** can be maximised, minimised, or made to acquire a certain value. As shown in Figure 3.3, **constraints** can be applied on the sought values of the decision variables. To suit different types of problems, Solver offers three solution methods which are:

1. The **GRG Nonlinear** method,
2. The **Evolutionary** method
3. The **Simplex LP** method.

The **GRG Nonlinear** method involves the determination of the function's gradient like the Steepest Descent method [2, 3]. The **Evolutionary** method adopts a variety of genetic algorithms and local search methods [4]. Both the **GRG Nonlinear** method and the **Evolutionary** method are suitable for non-linear problems. The **Simplex LP** method, which is a linear-programming method, is suitable for linear problems. As shown in Figure 3.3, Solver uses the **GRG Nonlinear** method by default. The following sections give examples of using the three solution methods.

3.1.2. The GRG Nonlinear method

In an optimisation analysis we may require the objective function to be maximised or minimised depending on the situation at hand. For example, the optimisation of pipe insulation requires its total cost to be minimised, while the optimisation of a central thermal power plant requires its thermal efficiency to be maximised. The following example illustrates the use of the **GRG Nonlinear** method in optimisation analyses.

Example 3-1. Finding the minimum value of a quadratic function

Find the minimum value of the following quadratic function in the specified range.

$$f(x) = x^2 - 2x - 1; \qquad -2 \le x \le 3 \tag{3.1}$$

Solution

Figure 3.4 shows the Excel sheet developed for this example. The line chart inserted in the figure shows the variation of f with x from which we can estimate that the minimum value of f is -2 and occurs at x =1. An initial value for x is entered in cell **B3** based on which the function $f(x)$ is calculated in cell **B6** according to Equation (3.1). Note the curser is placed on cell **B6** to reveal the formula typed in the cell, which is:

= B3^2-2*B3−1

We can now use Solver to determine the minimum value of the function. To do so, select **Solver** from the **Data** tab and fill its parameters dialog-box as shown in Figure 3.5 that shows the top part of the completed box.

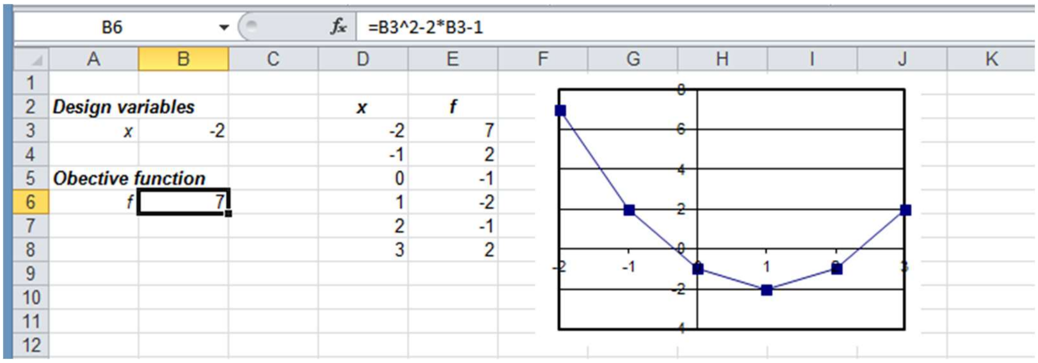


Figure 3.4. Excel sheet for determining the local minimum of the quadratic function

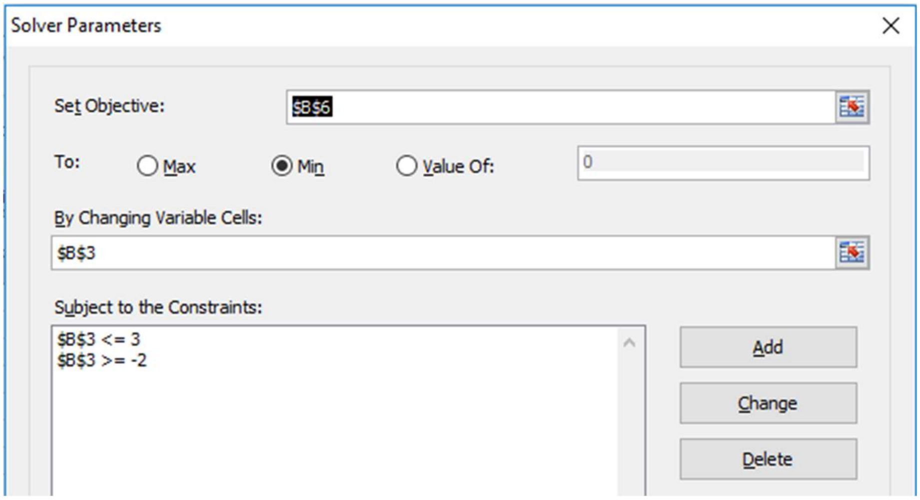


Figure 3.5. The completed Solver dialog box for Example 3-1

The dialog box in Figure 3.5 has been filled as follows:

Set Objective:	**B6** and **Min** have been selected for this option since want the value of the function in cell **B6** to be minimised.
By Changing Variable Cells:	**B3** which is the cell that stores the value of the independent variable *x*.
Subject to the Constraints:	Two constraints have been added that specify the minimum and maximum values of *x* as $x \geq -2$ and $x \leq 3$, respectively.
Select a Solving Method:	The **GRG Nonlinear** method (the default option).

Pressing the "**Solve**" button of the completed parameters box sparks Solver to find the required solution. As Figure 3.6, shows, the answer found by Solver, which is $x = 1$, $f = -2$, agrees with the graphical solution shown in Figure 3.4.

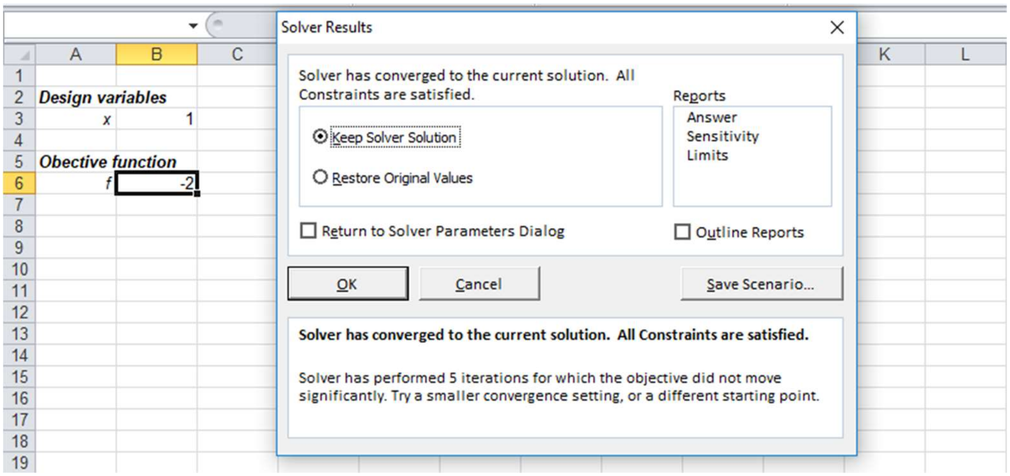


Figure 3.6. Solver solution for Example 3-1

3.1.3. The Evolutionary method

When the function to be optimised has more than one point of inflection, the solution found by the **GRG Nonlinear** method may only be a local minimum or maximum. The following example illustrates the capability of the **Evolutionary** method to find the global optimum solution in this situation for a simple function.

Example 3-2. Finding the global minimum of a function

Determine the global minimum value for the following function:

$$f(x) = x\cos(x) \qquad 3 \leq x \leq 14 \qquad (3.2)$$

Solution

Figure 3.7 shows the Excel sheet developed for solving this example. The insert shows that the function has two minima in the specified range of x; at $x \approx 5$ and $x \approx 11$.

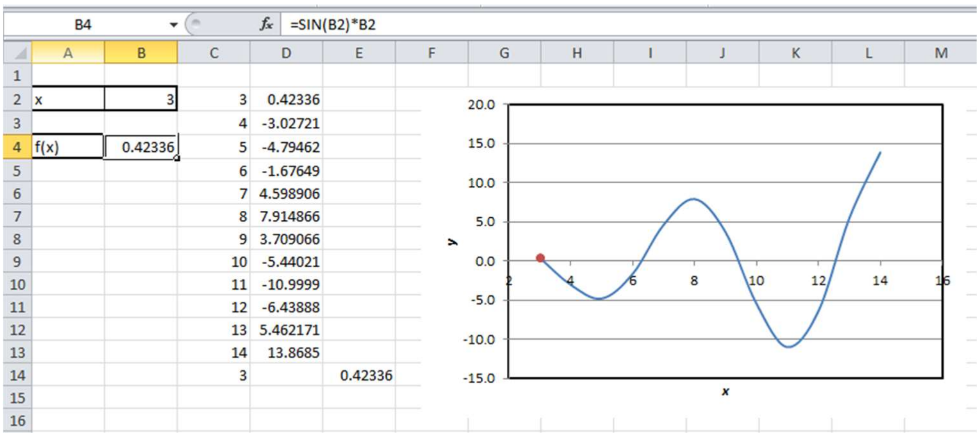


Figure 3.7. The Excel sheet for Example 3-2

Let us first try to solve the problem with the **GRG Nonlinear** method. An initial value for *x* has been entered in cell **B2** based on which the function *f* is calculated in cell **B4**. The formula bar in Figure 3.7 reveals the formula entered in the cell **B4**. Figure 3.8 shows the completed Solver parameters dialog-box with two constraints that specify the upper and lower limits for *x*. From Figure 3.9 that shows the solution found by Solver it is clear that it found the local minimum ($y = -4.81$) which is nearer to the initially specified value and not the global minimum.

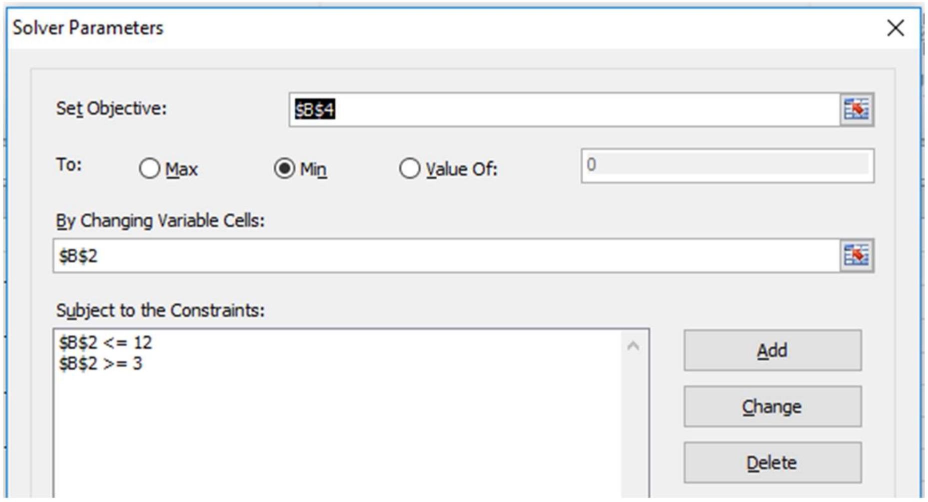


Figure 3.8. Solver set-up for Example 3-2 with GRG Nonlinear method

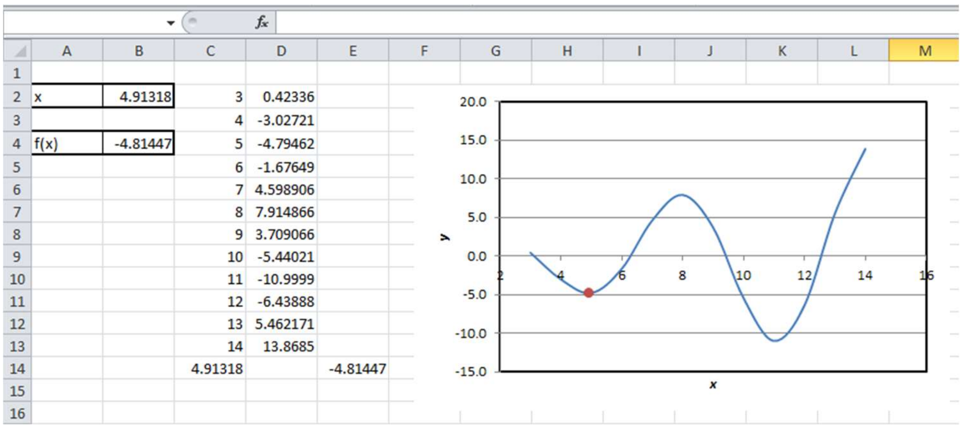
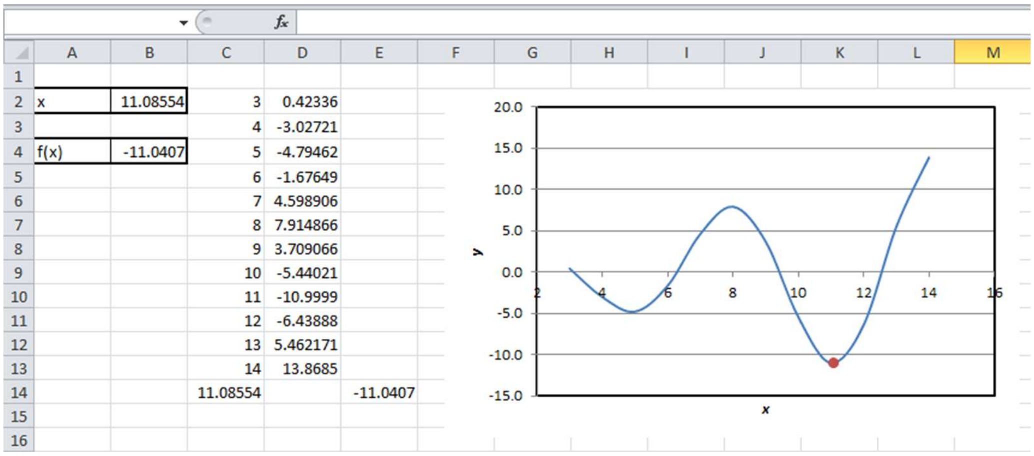


Figure 3.9. Solver solution for Example 3-2 with the GRG Nonlinear method

In order to locate the global minimum by the **GRG Nonlinear** method, the solution has to be started with an initial guess that is closer to the global minimum, e.g., $x = 9$. The advantage of the **Evolutionary** method is that such an arrangement is not required. To use this method, the set-up shown in Figure 3.8 only needs Solver's solution method to be changed to "**Evolutionary**". Figure 3.10 that shows the solution obtained by this method confirms that the method produced the global minimum ($y = -11.041$).



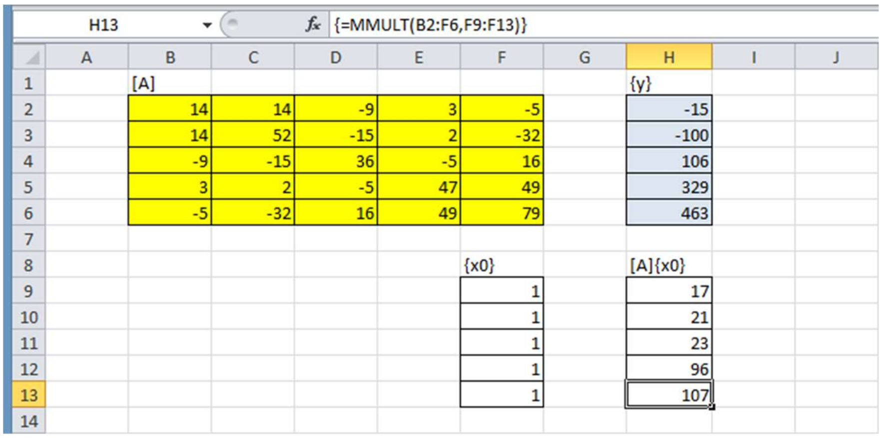
	A	B	C	D	E
1					
2	x	11.08554	3	0.42336	
3			4	-3.02721	
4	f(x)	-11.0407	5	-4.79462	
5			6	-1.67649	
6			7	4.598906	
7			8	7.914866	
8			9	3.709066	
9			10	-5.44021	
10			11	-10.9999	
11			12	-6.43888	
12			13	5.462171	
13			14	13.8685	
14			11.08554		-11.0407

Figure 3.10. Solver solution for Example 3-2 with the Evolutionary method

The **Evolutionary** method is useful for optimisation analyses that involve non-smooth and discontinuous functions, which are difficult to solve with the **GRG Nonlinear** method. However, its disadvantage is that it takes long computer times. While the **GRG Nonlinear** method took less than a second to solve the above problem, the **Evolutionary** method took more than a minute with the same computer.

3.1.4. The Simplex LP method

This option provides a method for solving small systems of linear equations that can be used instead of the method described in the Chapter 2 by using Excel's matrix functions. The method will be illustrated by reconsidering the problem of Example 2-3. Figure 3.11 shows a new Excel sheet for solving the problem with the present method.



H13 {=MMULT(B2:F6,F9:F13)}

	A	B	C	D	E	F	G	H
1		[A]						{y}
2		14	14	-9	3	-5		-15
3		14	52	-15	2	-32		-100
4		-9	-15	36	-5	16		106
5		3	2	-5	47	49		329
6		-5	-32	16	49	79		463
7								
8						{x0}		[A]{x0}
9						1		17
10						1		21
11						1		23
12						1		96
13						1		107
14								

Figure 3.11. Excel sheet for solving Example 2-3 with Solver

The top part of the sheet stores the coefficient matrix [A] and the right-hand vector {y} of the system of linear equations to be solved. The procedure starts with a guessed solution which is stored as vector {x0} in cells **F9:F13**. All the elements of this vector are given a value of 1 as shown in Figure 3.11. The coefficient matric [A] is then multiplied by the guessed vector {x0} using Excel's "**MMULT**" function and the result stored in cells **H9:H13**. If this initial guess were the correct answer, the multiplication [A]{x0} would have been the same as the true right-hand side vector, i.e,:

$$[A]\{x0\} = \{y\} \tag{3.3}$$

However, Figure 3.11 shows that the vector [A]{x0} is different form the true right-hand side vector {y} stored in cells **H2:H6**. Solver can now be used to adjust the variable cells **D9:D13** so that all elements of the vector [A]{x0} become equal to their counterparts in vector {y}, i.e:

H9 = H2
H10 = H3
H11 = H4
H12 = H5
H13 = H6

Solver set-up for this task is shown in Figure 3.12.

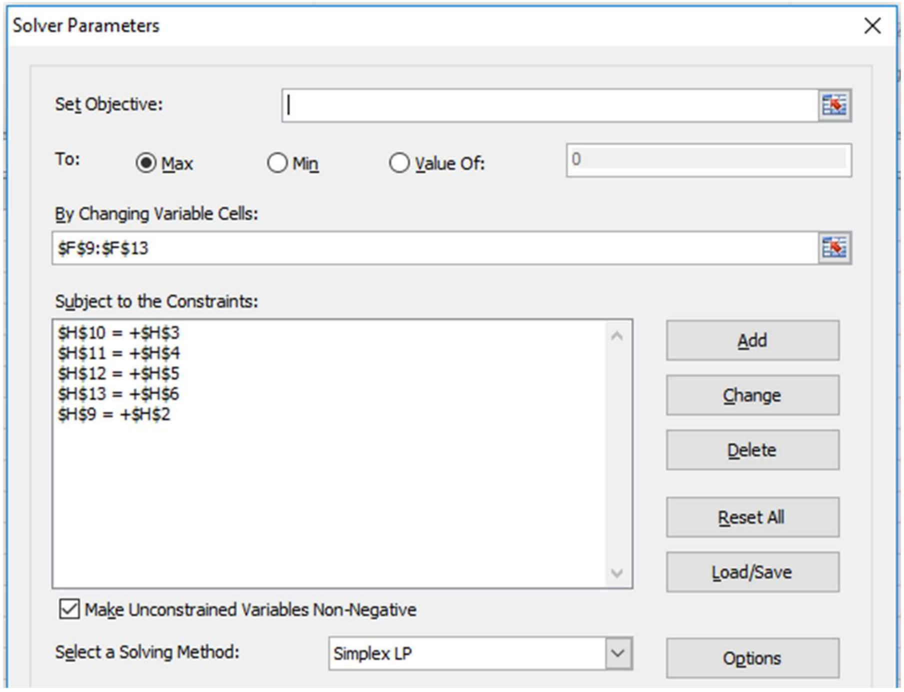
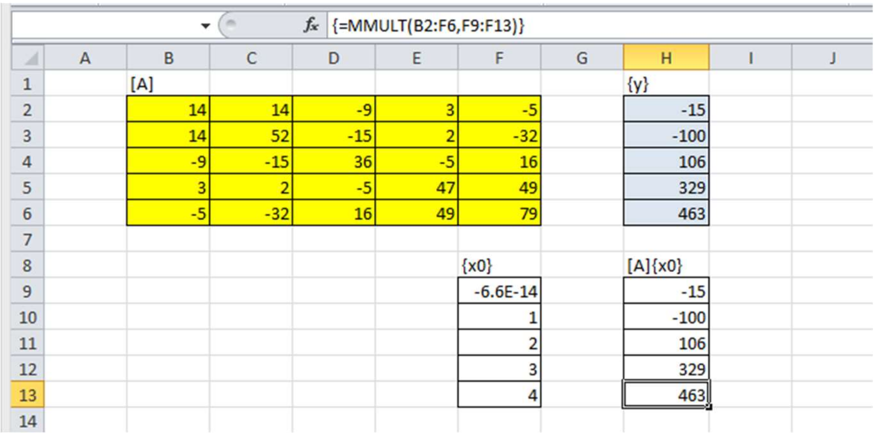


Figure 3.12. Solver set-up for Example 2-3 with the Simplex LP method

Note that the objective cell is left blank and the **Simplex LP** method is selected as the solution option. In this case, Solver will iterate to find the values of the decision variables that satisfy all the imposed constraints. The solution found by Solver using the above set-up is shown in Figure 3.13.



	A	B	C	D	E	F	G	H	I	J
1		[A]						{y}		
2		14	14	-9	3	-5		-15		
3		14	52	-15	2	-32		-100		
4		-9	-15	36	-5	16		106		
5		3	2	-5	47	49		329		
6		-5	-32	16	49	79		463		
7										
8						{x0}		[A]{x0}		
9						-6.6E-14		-15		
10						1		-100		
11						2		106		
12						3		329		
13						4		463		
14										

Figure 3.13. Solution of Example 2-3 with the Simplex LP method

All the elements of the [A]{x0} are now equal to their corresponding elements in the vector {y}. The first element of the solution vector, which is -6.6×10^{-16}, is practically zero. Therefore, the solution is [0,1,2,3,4], which is the same as that obtained in Example 2-3 by using the matrix-inversion method. The advantage of Solver compared to the matrix-inversion method is that it can be used for solving systems of nonlinear equations by following a similar procedure (Refer to Problem 3.5 in the Exercises).

3.1.5. The default settings of Solver options

Solver gives it user additional flexibility by allowing alternative options for monitoring and adjusting the precision and computer time of its three solution methods. While some options are common to all three solution methods, others are particular to the **GRG Nonlinear** or the **Evolutionary** method. By clicking the "**Options**" button in Solver's parameters dialog box shown in Figure 3.14, the dialog box shown in Figure 3.15 will be shown.

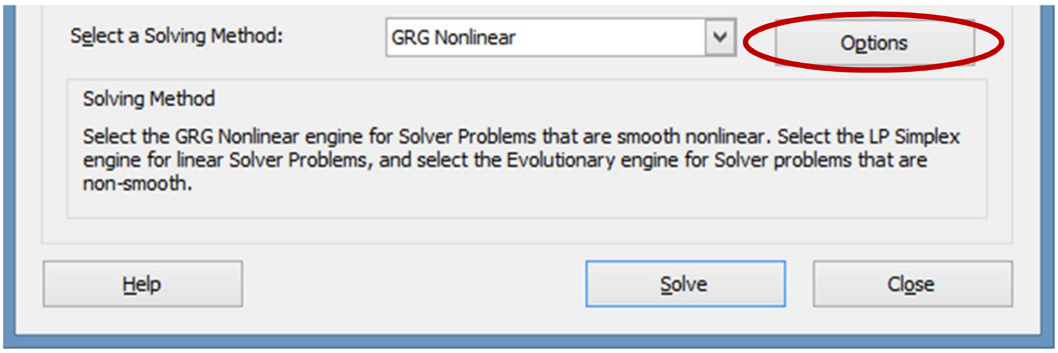


Figure 3.14. Solver options in the Properties dialog box

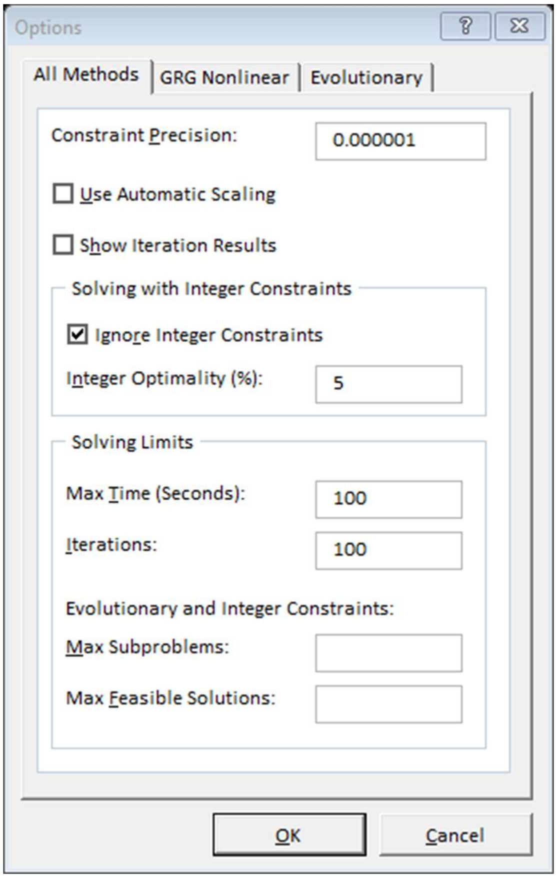
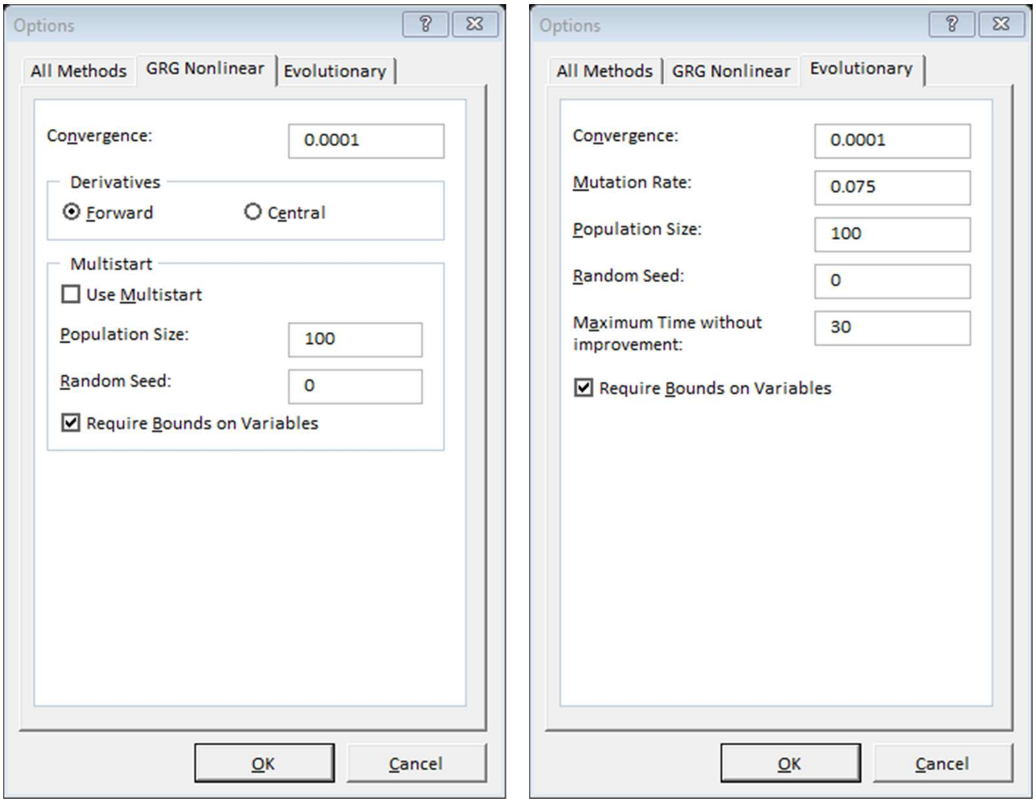


Figure 3.15. Default Solver options adopted in the analyses for all solution methods

Figure 3.15 shows the default settings of the options that are common to all three solution methods. In certain situations, some of these default settings may have to be changed in order to reduce the computation time or increase the precision of the solution. Sometimes, Solver fails altogether to find the solution if the default options are used. For example, the use of automatic-scaling (AS) is favourable in certain situations but not always. AS enables Solver to handle a poorly-scaled model, i.e., a model in which the values of the objective and constraint functions differ by several orders of magnitude. By using AS, the values of the objective and constraint functions are scaled internally in order to minimise the differences between them.

Figures 3.16.a and 3.16.b show the default settings which are particular to the **GRG Nonlinear** method and the **Evolutionary** method, respectively. The **GRG Nonlinear** method uses by default the forward difference (FD) approximation of derivatives. Most of the analyses presented in later chapters of the book used the **GRG Nonlinear** method with the default FD approximation. More information about Solver options for the **GRG Nonlinear** method can be obtained from the developer's website [5].



(a) (b)

Figure 3.16. The default Solver options specific to: (a) the GRG Nonlinear method and (b) the Evolutionary method

Figure 3.16.b shows the default settings used by the **Evolutionary** method. Note that this method has more adjustable parameters than the **GRG Nonlinear** method. According to the default set-up, the population size is 100 and the maximum allowable time without improvement is 30 seconds. The few cases solved in this book with the **Evolutionary** method show that, with this set-up, the method needs long computer times. Although the time required by the method can be reduced by reducing the population size, number of iterations, or the maximum allowable time, the optimsation analyses presented in later chapters to do not show a clear advantage to this method over the **GRG Nonlinear** method which is easier to apply for optimisation analyses with a single objective.

3.2. VBA and the development of user-defined functions

Although Excel's user-interface provides numerous built-in functions for data analyses in general, there are situations where the analytical model requires the development of a customised user-defined function (UDF) that is not provided by Excel. This arises, for example, in thermodynamic analyses that require functions that determine the properties of fluids at various pressures and/or temperatures. This section illustrates the process of activating VBA and using it to develop simple UDFs.

3.2.1. Activation of VBA

As shown in Figure 3.17, VBA is found on the left side of the **Developer** tab. If the **Developer** tab is not shown in the ribbon of your Excel sheet, then go to **File**, select **Options**, and then select **Customise Ribbon** from the **Backstage View** shown in Figure 3.18. From the **Main Tabs**, select the **Developer** check box and then click "**OK**". The **Developer tab** will now be shown in the ribbon of your Excel sheet.

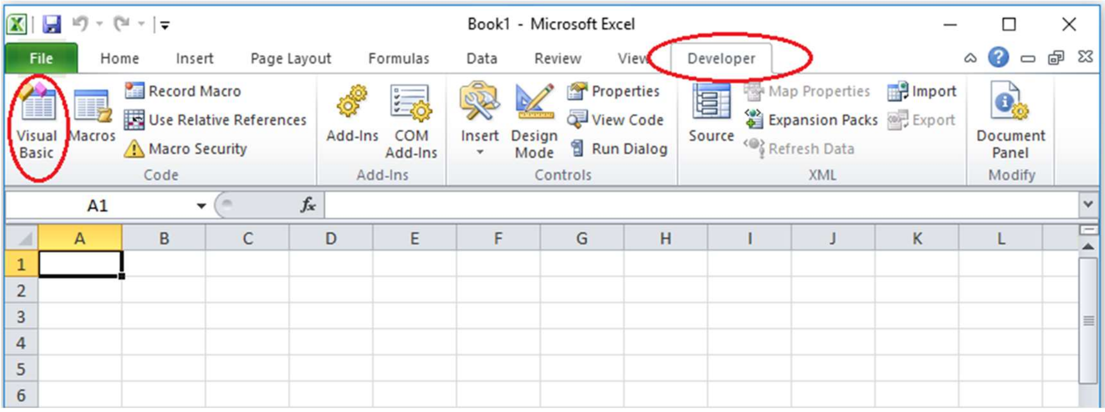


Figure 3.17. Selection of VBA from the Developer tab

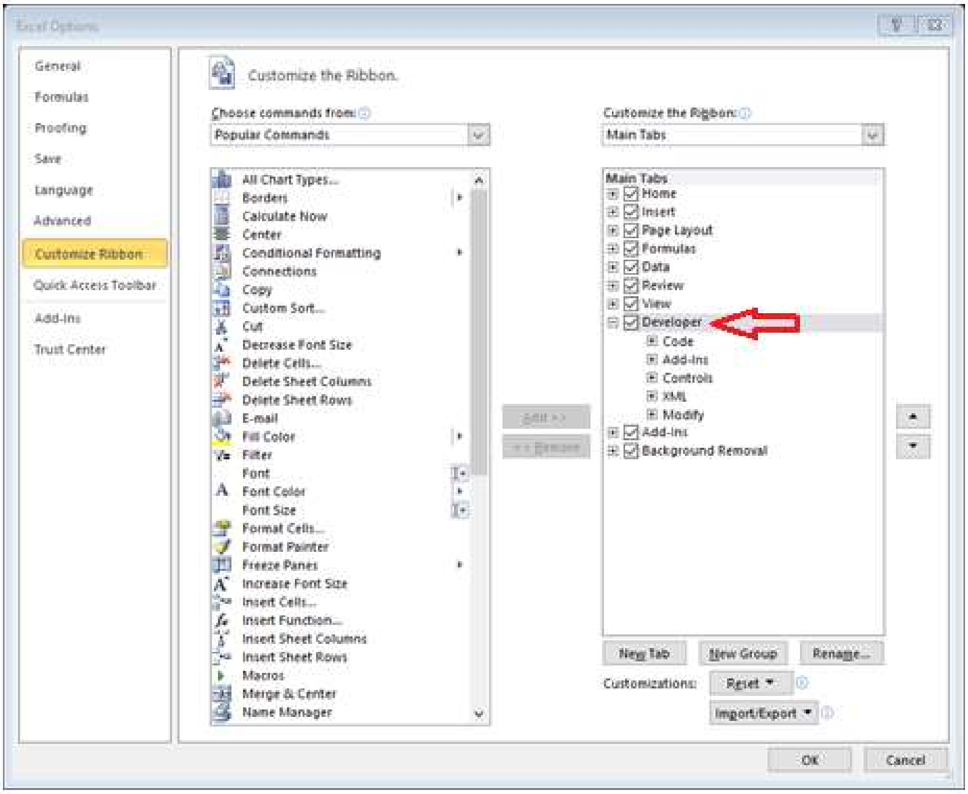


Figure 3.18. Adding VBA to the Developer tab

3.2.2. Development of UDFs

To start writing the UDF, go to **Developer** tab menu and select **Visual Basic**. The Visual Basic editor will appear to you as shown in Figure 3.19. Select **Insert** → **Module** and the blank page shown in Figure 3.20 will be open for you to type the VBA code.

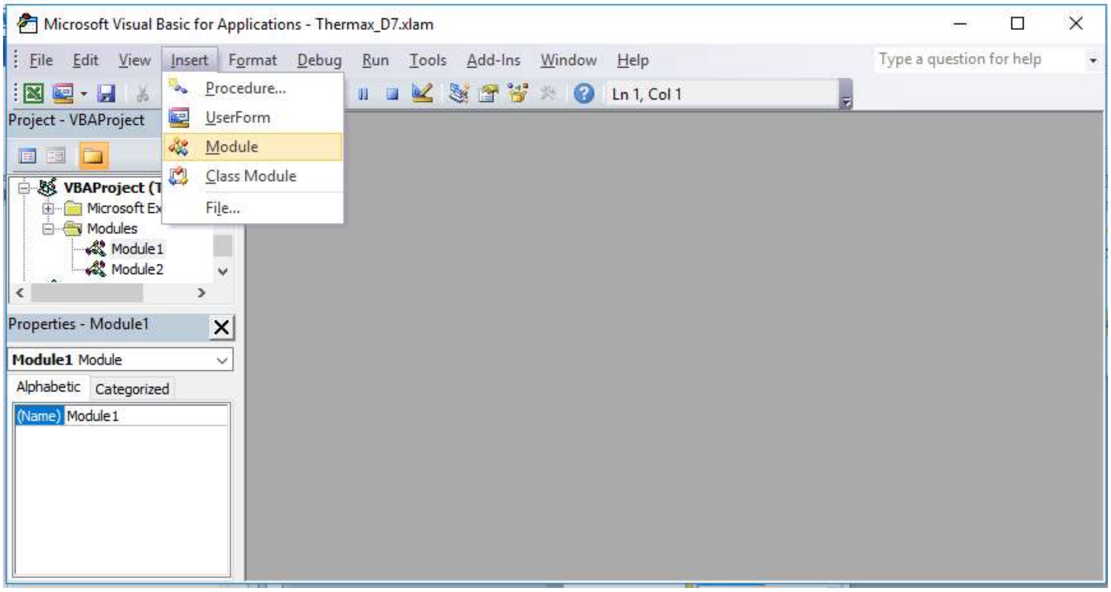


Figure 3.19. Inserting a new module

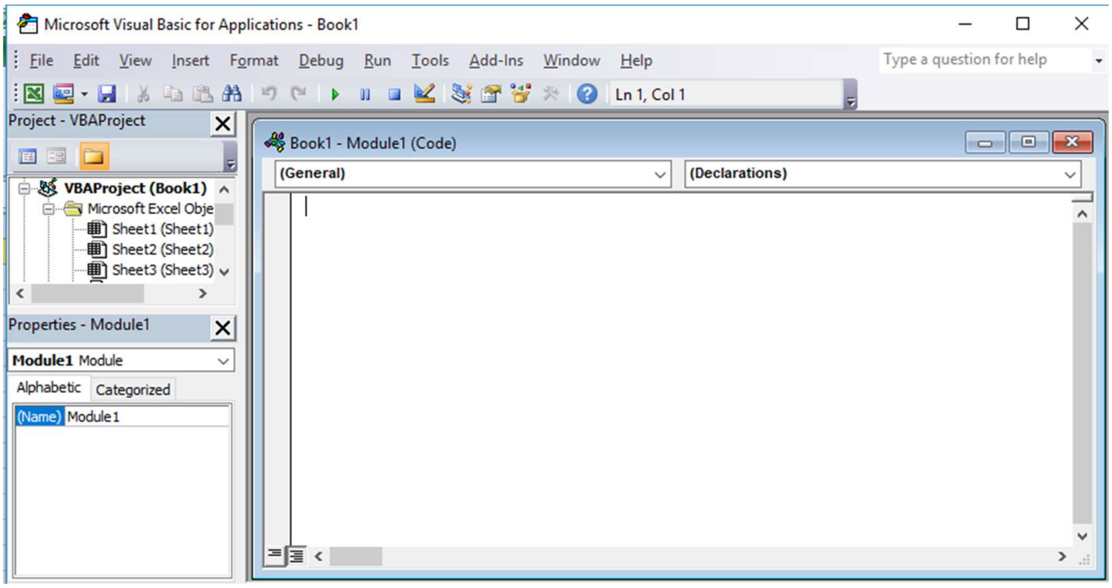


Figure 3.20. A new VBA module

As a first example let us write a VBA function for determining the area (A) of a circle given its diameter (D) using the following mathematical equation:

$$A = \pi D^2 / 4 \tag{3.4}$$

The following UDF determines the circle's area according to Equation (3.4):

```
Function Circ_area(Dia)
Pi = 3.141593
Circ_area = Pi * Dia ^2 / 4
End Function
```

Note that the first line in the code starts with the word "**Function**" followed by the name given to the function, which is "**Circ_area**". The required input parameters are specified between two brackets after the function name. The present function has only one input parameter, which is the diameter (Dia). As soon as you type the first line of the code and press the "**Enter"** key, the editor will automatically add the **End** line of the function. Now, type the rest of the code as shown in Figure 3.21.

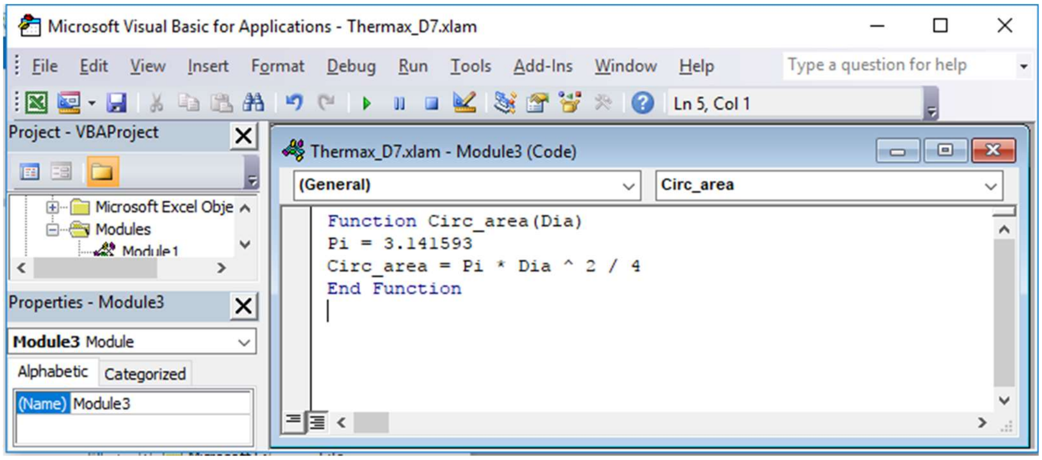


Figure 3.21. A UDF for caclcuating the area of a circle with a given diameter

After typing the code correctly, the function can be used via Excel UI just like any built-in function as shown in Figure 3.22. Note that the formula bar in Figure 3.22 reveals the formula in cell B2 as:

= Circ_area(10)

Where the number 10 refers to the diameter of the circle, which is the only input to the UDF. You can now check the answer of your user-defined function by calculating the circle's area with a normal Excel formula by typing in any cell "**=pi()*10^2/4**".

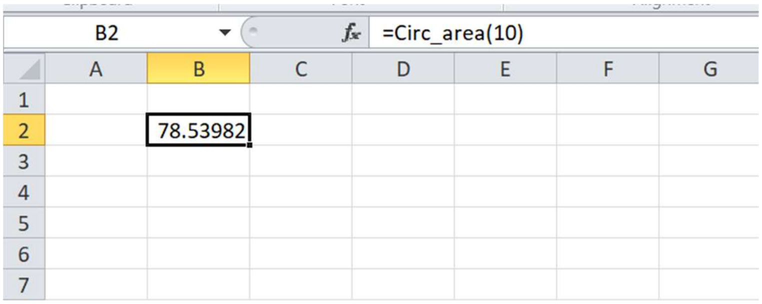


Figure 3.22. Using the "Circ_area" function in Excel

In writing the UDF shown in Figure 3.21 we assigned a value for the constant π because VBA does not provide a built-in function for it like Excel. However, it is possible to call the built-in functions **PI** provided by Excel within the VBA function as follows:

```
Function Circ_area(Dia)
Circ_area = Application.WorksheetFunction.Pi() * Dia ^ 2 / 4
End Function
```

This is very useful when built-in functions, like MIN and MAX, can be used to minimise the programming effort needed for developing the required UDF. More information about this and other features of the VBA language can be found in specialised references [7-9].

For thermodynamic analyses, VBA is useful for developing UDFs for fluid properties. As an example, let us develop a UDF that determines the molar specific-heat at constant pressure ($\tilde{c}_p$) for air. For an ideal gas, $\tilde{c}_p$ is given by the following polynomial [6]:

$$\tilde{c}_p = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad \text{[kJ/kmol.K]} \qquad (3.5)$$

Where T is the absolute temperature and a_0, a_1, a_2, and a_3 are constants that have different values for different gases. For air, the constants' values are 28.11, 0.1967×10^{-2}, 0.4802×10^{-5}, and -1.966×10^{-9} in this order. The following VBA function, called "cp_air", determines $\tilde{c}_p$ for air based on Equation (3.5):

```
Function cp_air(TempK)
a0 = 28.11
a1 = 0.00196
a2 = 0.000004802
a3 = -0.000000001966
M = 28.97
cpbar = a0 + a1 * TempK + a2 * TempK ^ 2 + a3 * TempK ^ 3
cp_air = cpbar / M
End Function
```

The only input for the function is the absolute temperature (TempK). Figure 3.23 shows the VBA function and the formula bar in Figure 3.24 shows how the function can be used in an Excel formula to determine $\tilde{c}_p$ for air at 300K. The value returned by the function is 29.0771 kJ/kmol.K. By making suitable extensions, the function can easily be used to determine the values of $\tilde{c}_p$ for other ideal gases in addition to air (Refer to Exercise 3.8). The various functions provided by Thermax for the thermo-physical properties of water, refrigerants, etc., have been developed by writing similar functions with VBA.

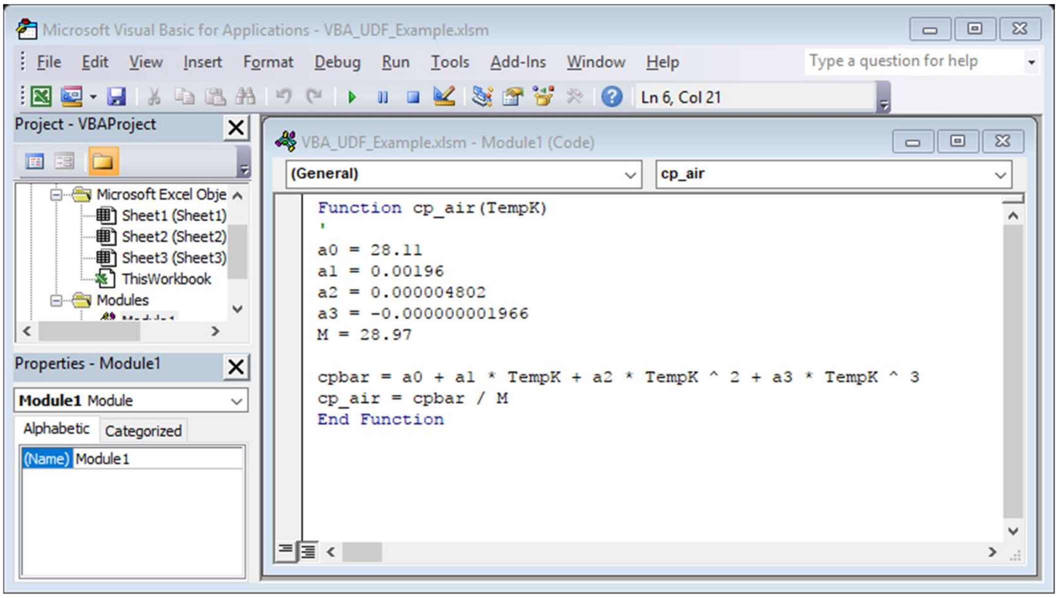


Figure 3.23. A UDF for caclcuating the molar specific-heat for air

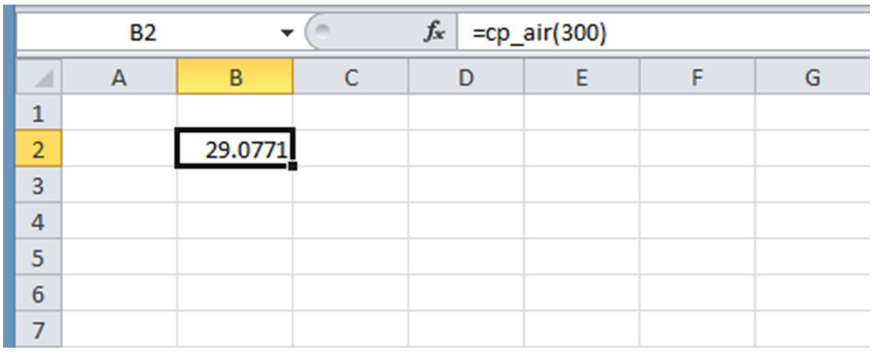


Figure 3.24. Using the cp_air function in Excel

3.3. Thermax installation and use

Thermax enables Excel to be used as an educational platform for a wide range of thermofluid analyses [10]. Before the add-in can be recognised by Excel you have to install it in your computer. To do that, open the **Thermax.xla** file and then save it as an

“**Excel Add-in**”. Recent Excel versions locate all add-ins in a certain folder in the computer and automatically direct you to that location when you want to save a new add-in. Save the Thermax add-in in the specified location and **restart** Excel in order to activate it. Open a new Excel sheet and then do the following:

1. Go to **File** and then click **Options**.
2. Select **Add-Ins**. From the **Manage** ribbon at the bottom of the menu select **Excel Add-ins** and then press **Go**. The pull-down menu shown in Figure 3.25 will appear to you.
3. To add **Thermax** to the add-ins menu, tick (√) the corresponding box.
4. If for any reason you saved the add-in in a location that is different from the default folder, then click on **Browse** and search for it in the destination folder and select it.

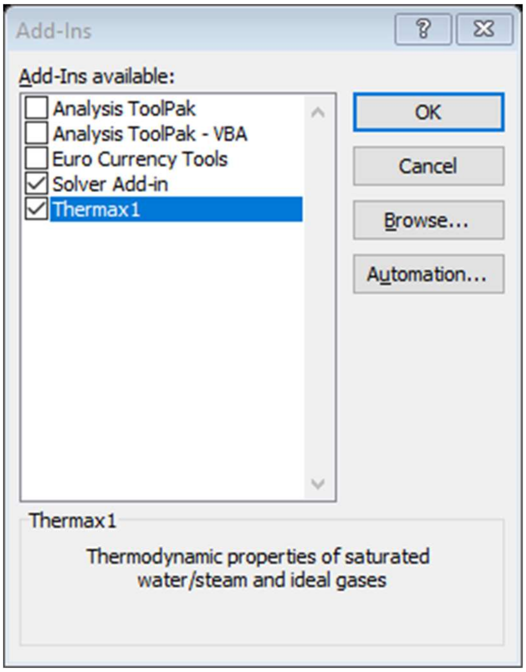


Figure 3.25. Adding Thermax to the menu of Excel add-ins

Once installed, Thermax functions can be used in Excel's formulae just like the built-in functions. For illustration, let us start a formula by entering the equal sign (=) in any cell (say cell B2). If you now press the *fx* button in the formula ribbon, the **Function Wizard** shown in Figure 3.26 will be shown. The Function Wizard first lists the various categories of built-in functions provided by Excel. Scroll down the list of function categories and select the **User-defined** functions. Then, all the functions provided by Thermax will be listed alphabetically as shown in Figure 3.27.

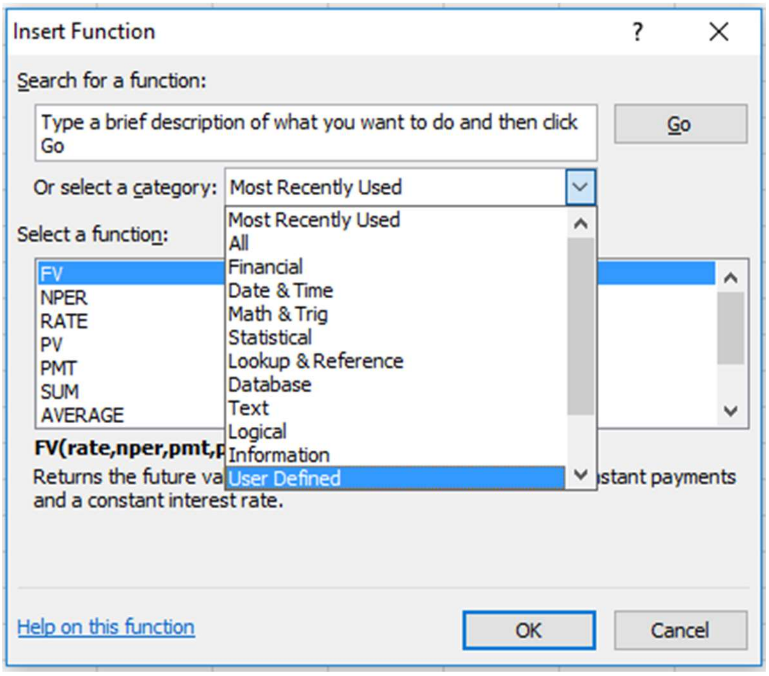


Figure 3.26. Finding the add-in user-defined functions in the Function Wizard

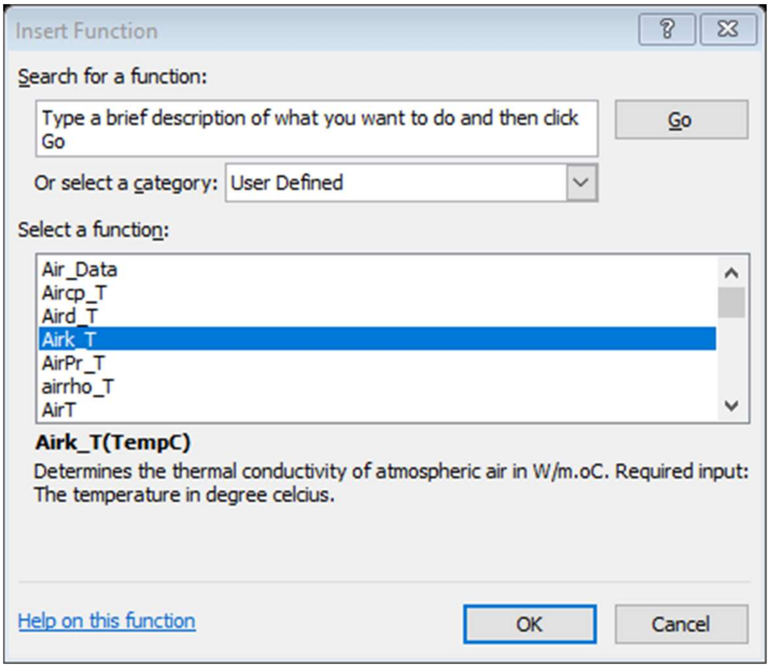


Figure 3.27. Thermax functions listed alphabetically in the User Defined category

The first function in the list, **Air_Data**, is the auxiliary function that stores the data for the thermo-physical properties of air at standard atmospheric pressure. This function is called by other functions in the same category to obtain the values of these properties at the required temperature. To start using the add-in functions, scroll down the list and select the function **Airk_T** that determines the thermal conductivity (*k*) of air at a given temperature. Upon pressing the **OK** button, the **Function Arguments** box shown in Figure 3.28 will be shown.

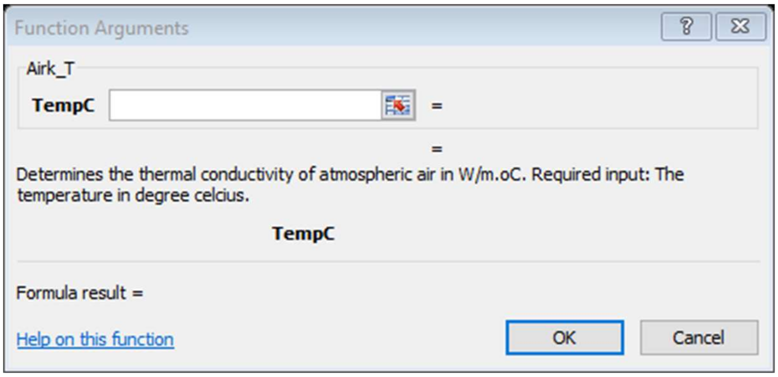


Figure 3.28. The Function Arguments box for the "Airk_T" function

Figure 3.28 shows that this function has one input parameter, which is the temperature in °C "TempC", and gives a brief description of its intended use. Let us use the function to determine the thermal conductivity for air at 25°C. Fill the form by entering the value of the temperature, 25, as shown in Figure 3.29.

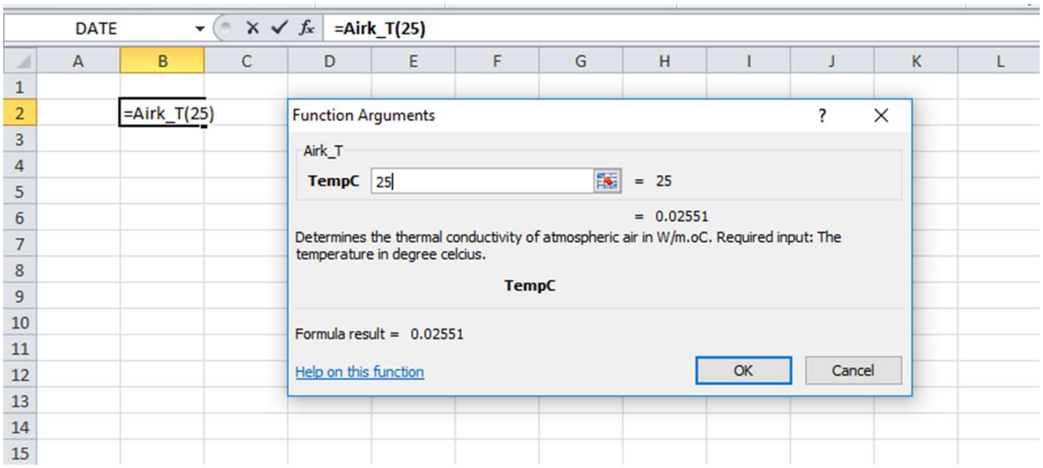


Figure 3.29. Using the function "Airk_T" to determine the thermal conductivity of atmospheric air at 25°C

Note that the formula ribbon now shows the formula in cell **B2**, which is **"=Airk_T(25)"**, and the function form shows the calculated value of *k*, which is 0.02551 W/m.°C. When you press the "OK" button, this value will appear in the cell **B2**. You can check this value with that given in Table A.1 of Appendix A.

In certain situations the confinement of Excel's formula to one cell becomes too restrictive for developing the analytical model. This situation arises, for example, when an iterative process involves a non-linear equation such as the Colebrook-White equation or the Soave-Redlich-Kwong equation of state. In this case, a UDF is needed solve the nonlinear equation and return the result to Excel's formula like a built-in function. For such a case, Thermax provides a Newton-Raphson solver for nonlinear equations in addition to its property functions. Appendix B shows how to use this tool. Appendix B also describes two interpolation functions provided by Thermax for tabulated data which are useful for including additional fluid properties or other tabulated data needed in a thermofluid analysis. Another useful function provided by Thermax is a custom function for determining the standard schedule-40 pipe diameter.

3.4. Closure

This chapter introduces the three auxiliary components of the Excel-based modelling platform used in this book for thermofluid analyses; Solver, VBA, and Thermax. The chapter gives Examples of using the three solution methods provided by Solver to deal with different types of problems. It also shows how VBA can be used for developing user-defined functions not provided by Excel and explained the procedure for installing Thermax and using its functions in Excel formulae. Appendix B demonstrates the use of the two interpolation functions for tabulated data provided by Thermax and its Newton-Raphson solver for solving nonlinear equations.

References

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[5] FrontlineSolvers, internet: https://www.solver.com/excel-solver-change-options-all-solving-methods

[6] Y. A. Cengel and M. A. Boles. *Thermodynamics an Engineering Approach*, McGraw-Hill, 7th Edition, 2007

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[8] J. L. Latham, *Programming in Microsoft Excel VBA, An Introduction*, 2008. Internet: http://ies.fsv.cuni.cz/default/file/download/id/21101 (Last accessed November 29, 2015).

[9] http://www.fontstuff.com/vba/vbatut01.htm (Last accessed November 26, 2017).

[10] M.M. El-Awad, *Thermax: Thermodynamic Analyses of Energy-Utilisation Systems using Excel* -

https://www.academia.edu/24381392/Thermodynamic_Analyses_of_Energy_Utilisation_Systems_Using_Excel
[11] S.C. Chapra and R.P. Canale, *Numerical Methods for Engineers*, 6th Edition, McGraw Hill, 2010.

Exercises

1. A system of algebraic equations can be expressed in matrix form as follows:

$$\begin{bmatrix} 70 & 1 & 0 \\ 60 & -1 & 1 \\ 40 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 636 \\ 518 \\ 307 \end{Bmatrix}$$

   Solve the system of equations by using Solver to determine the values of the three unknowns *a*, *b*, and *c*. This exercise is based on Example 9.11 in Chapra and Canale [11]. The answer is: $a = 8.5941$, b=34.4118, and $c = 36.7647$.

2. Draw a line chart with Excel to show the variation of the following function in the range $0 \le x \le 4$:

$$f(x) = 2 \sin x - x^2/10$$

   Use Solver to find the maximum of the function in the same range. Based on Example 13.1 in Chapra and Canale [11]. The answer is: $f(x) = 1.7757$ at $x = 1.4276$.

3. The curve shown in Figure 3.P3 is a plot of the function:

$$f = e^{\theta} \sin(2\theta)$$

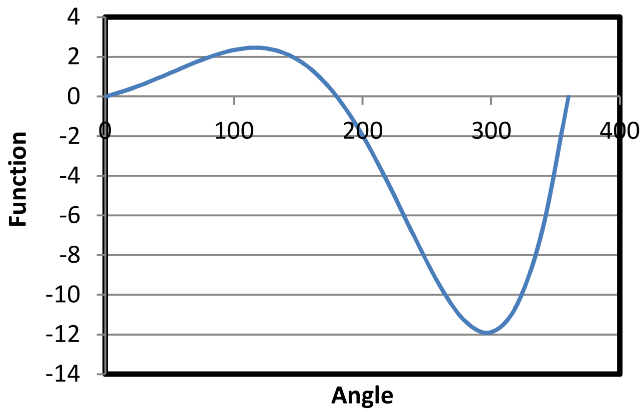


Figure 3.P3 A composite function

   Use Solver to find:

a) The minimum value of the function and the corresponding angle
b) The maximum value of the function and the corresponding angle
c) The angle at which value of the function equals 4.0

4. Using the Excel sheet developed to solve Example 3-1 by the **GRG Nonlinear** method, study the effect of using central-difference approximation of derivatives instead of the default forward-difference approximation on the solution.

5. Consider the following set of simultaneous nonlinear equations:

$$x^2 + xy = 10 \qquad \text{(A)}$$

$$y + 3xy^2 = 57 \qquad \text{(B)}$$

To solve the system with Solver, rearrange the equations as follows:

$$u(x, y) = x^2 + xy - 10 = 0 \qquad \text{(C)}$$

$$v(x, y) = y + 3xy^2 - 57 = 0 \qquad \text{(D)}$$

Create two cells (**B1** and **B2**) to hold initial guesses for x and y. Enter the function values themselves, $u(x, y)$ and $v(x, y)$ into two other cells (**B3** and **B4**). The initial guesses may result in function values of u and v that are far from zero. Determine the sum of the function squares, i.e. $u^2 + v^2$, and store it in cell **B5**.

Use Solver to find the values of x and y in cells **B1** and **B2** (the Changing cells) that make the value in cell **B5** (the objective cell) equal to zero. Using this procedure, find the roots of the above system starting with initial guesses of x =1 and y = 3.5.

This exercise is based on Example 7.5 in Chapra and Canale [11]. The correct pair of roots are x=2 and y=3.

6. The volume V of liquid in a spherical tank of radius r is related to the depth h of the liquid by:

$$V = \pi h^2(3r - h)/3$$

Using VBA, develop a user-defined function that determines h at any given values of r [m] and V [m^3]. Check your function with r=1 m and V = 0.5 m^3. Answer: h = 0.431 m.

7. Extend the UDF developed in Section 3.2 for determining the specific heat for air so that it can be used for other ideal gases as well. Note that in this case, the function will have two input parameter: the name of the gas and the temperature.

8. Using suitable formulae for the thermodynamic properties of superheated steam, develop user-defined functions with VBA for determining the specific enthalpy and entropy of superheated steam from its pressure and temperature.
9. Using suitable formulae for the thermodynamic properties of superheated refrigerant R134a, develop user-defined functions with VBA for determining properties, e.g., enthalpy and entropy, of superheated R134a from its temperature and pressure.
10. Using the data for properties of air at atmospheric pressure given in Appendix A, Table A.1, develop an Excel sheet that can be used to determine the kinematic viscosity of air at any given temperature in the range 200 – 1000K by using

    a. The trendline feature of Excel
    b. The linear-interpolation function (**Interpl**) provided by Thermax.

11. Develop a VBA function to determine the friction factor from the Colebrook-White equation and use it with the NRM solver provided by Thermax (refer to Appendix B) to determine the frictional losses (h_f) in a circular pipe that carries air at 20°C with the following data:

    D = 25 cm, L = 150 m, V = 7 m/s, k_s = 0.045 mm.

4

Iterative solutions

The need for iterative solutions in thermofluid analyses arises for a number of reasons. This chapter gives examples of such analyses and shows how they can be handled by using the two iterative tools provided by Excel; the Goal Seek command and Solver. While the Goal Seek command can be used for the simplest type of iterative solutions that involve a single parameter, Solver is needed for those involving multiple variables and requiring certain constraints to be satisfied by the iterative solution. When the analytical model involves a nonlinear equation, such as the Colebrook-White equation, it becomes difficult to use only Goal Seek and Solver. For such problems, the chapter shows how the Newton-Raphson solver provided by Thermax can be used to deal with the nonlinear equation, leaving the main iteration to Goal Seek or Solver.

4.1. Iterative solutions by using Goal Seek

Most thermofluid problems that require iterative solutions can be solved by using the Goal Seek command. This section presents three examples that demonstrate its use for typical analyses that require iterative solutions in fluid-dynamics, thermodynamics, and heat-transfer.

4.1.1. Type-2 and type-3 pipe-flow analyses

The frictional head loss (h_f) in a pipe depends on a number of factors that characterise the pipe itself as well as the fluid being transported. For a straight pipe with no fittings carrying a viscous Newtonian and incompressible fluid, the frictional head loss is determined by the following Darcy-Weisbach equation:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{4.1}$$

Where f is the Darcy friction factor, L the length of the pipe, D its diameter, V the fluid velocity, and g the gravity acceleration constant. The friction factor f can be obtained from Equation (1.22) if the flow is laminar and from Equation (1.24) or (1.25) if it is turbulent. Practical pipe-flow problems that involve Equation (4.1) can be divided into three types [1]:

1. Type-1 problems – require the determination of h_f when both the pipe's diameter and fluid velocity (or flow rate) are known.
2. Type-2 problems – require the flow rate to be determined for specified values of h_f and the pipe diameter.
3. Type-3 problems – require the pipe diameter to be determined for given values of h_f and the flow rate.

Type-1 problems can be solved in a straight-forward manner by using Equation (4.1) to determine the friction head loss. However, both type-2 and type-3 problems require iterative solutions because the Reynolds number and, therefore, the friction factor, f, cannot be determined without knowing D or V. For type-2 problems (i.e. unknown flow rate), the iterative procedure can be avoided by using extended Moody diagrams that require the determination of the following dimensionless parameter [2]:

$$\mathrm{Re}\, f^{0.5} = \frac{D^{1.5}}{\nu}\left(\frac{2gh_f}{L}\right)^{0.5}. \qquad (4.2)$$

Apart from the inaccuracy of visual chart interpolation, the procedure is difficult to adopt in optimisation or parametric analyses. By using the Goal Seek command, both type-2 and type-3 problems can be solved more accurately. The following example, which is based on Example 8.4 in Cengel and Cimbala [1], shows how Goal Seek can be used to solve a type-3 problem.

Example 4-1. Solution of type-3 pipe flow problems

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct (ε=0.045 mm) as shown in Figure 4.1 at a rate, Q, of 0.35 m^3/s. If the head loss in the duct is not to exceed 20 m, determine the smallest required diameter for the duct.

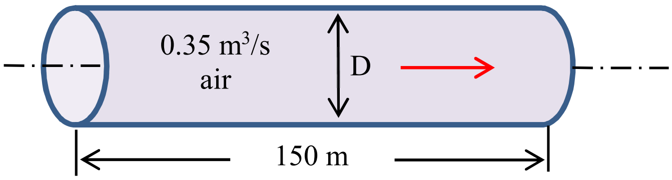


Figure 4.1. Schematic for Example 4-1 (adapted from Cengel and Cimbala [1])

Solution with Goal Seek

The problem can be solved by calculating the friction head loss at different diameters of the duct and then selecting the diameter that gives the required head loss which is 20 m. The iterative solution proceeds as follows:

1. Select a diameter for the inner pipe (D)
2. Calculate the velocity of the hot air (V), $V=Q/A$, $A=\pi D^2/4$
3. Calculate the Reynolds number in the pipe (Re), Re=VD/ν
4. Calculate the friction factor (f) using Equation (1.22) or (1.24)
5. Calculate the friction head loss (h_f) from Equation (4.1)
6. If $h_f \neq 20$ m, repeat steps 1 to 5

Figure 4.2 shows the Excel sheet developed for this example which is divided into three parts: (i) problem data (ii) calculations, and (iii) results. The data part shows the information given in the question. The value of the kinematic viscosity of air at 35°C (ν = 1.655x10^{-5} m^2/s) was obtained from Cengel and Cimbala [1] and fixed throughout the calculations. Cell-labelling is applied in the formulae and Figure 4.2 reveals the formulae used in each cell of the calculations part. Note the If-function in cell **F10**.

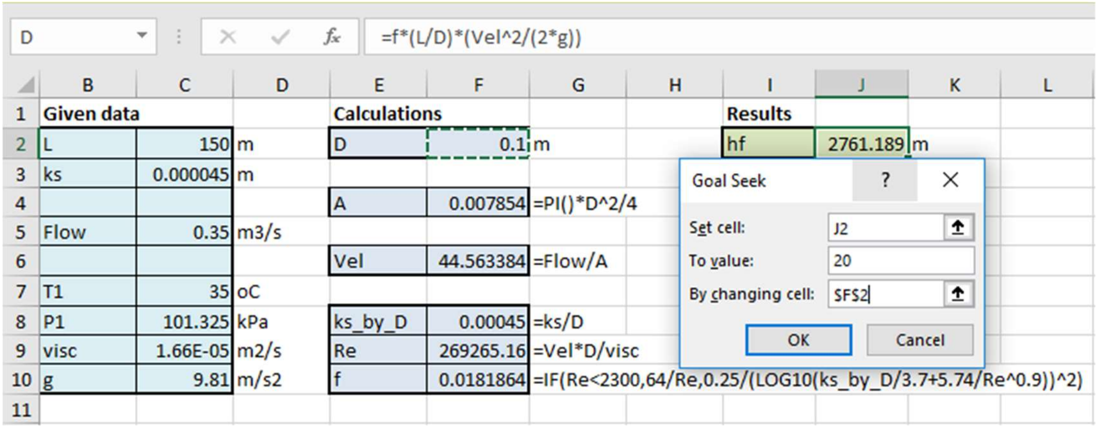


Figure 4.2. Excel sheet and Goal Seek set-up for Example 4-1

As Figure 4.2 shows, for an assumed duct diameter of 0.1 m the friction head loss exceeds 2761 m. Figure 4.2 also shows the completed Goal Seek dialog box that requires Goal Seek to change the diameter in cell **F2** and iterate until the friction head loss in cell **J2** attains the required value of 20 m. Figure 4.3 shows the answer found by Goal Seek, which is $D \geq 0.27$ m. This answer agrees with that given by Cengel and Cimbala [1]. A similar procedure can be used to solve type-2 flow problems by iterating over the flow rate instead of the diameter.

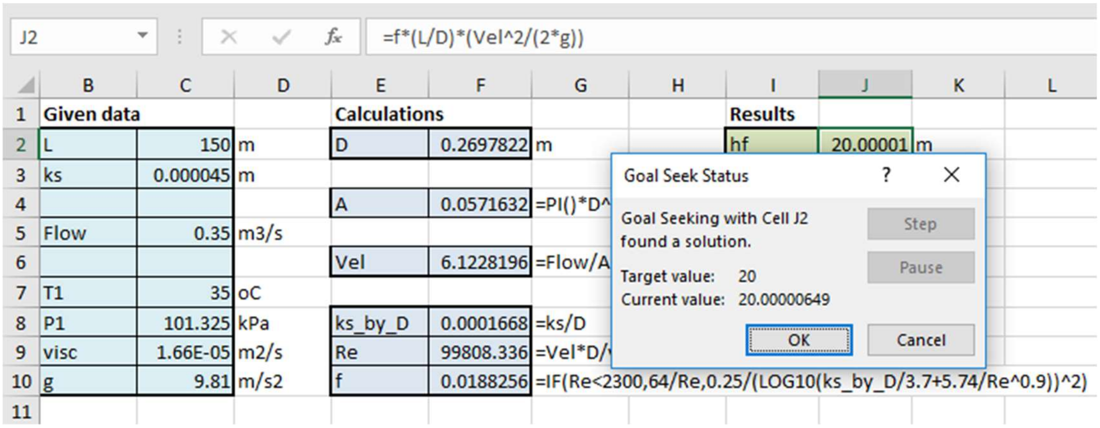


Figure 4.3. Goal Seek solution for Example 4-1

4.1.2. Thermodynamic analyses involving ideal-gas mixtures

Without the usual idealisations and simplifications applied in thermodynamic analyses, most of these analyses, if not all, would require iterative solutions. A commonly used thermodynamic approximation is treating air as a pure gas even though it is known to be a mixture of nitrogen, oxygen, and water vapour with small traces of other gases.

Computer-aided analyses with fluid property functions such as those provided by Thermax enable more realistic models to be used by treating air as a mixture of gases instead of a single gas. However, if the temperature of the gas mixture is not known and has to be determined, an iterative solution will be required by this model. The following example shows how the problem can be solved by using Goal Seek.

Example 4-2. Constant-pressure expansion of air

Figure 4.4 shows a piston-cylinder device that initially contains a mixture of 21% oxygen and 79% nitrogen by volume. Initially at 100 kPa, 330K, the gas mixture occupies 0.1 m^3. Fifty kJ of heat is then transferred to the gas causing it to expand at constant pressure. Treating oxygen and nitrogen as ideal gases, determine the final temperature of the gas inside the cylinder.

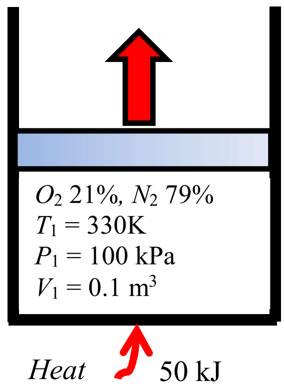


Figure 4.4. Schematic diagram for Example 4-2

The analytical model

The solution procedure applies the first-law of thermodynamics to the expansion process. For a mixture of O_2 and N_2, the first law reads:

$$Q = m_{O2}\left(h_{2\_O2} - h_{1\_O2}\right) + m_{N2}\left(h_{2\_N2} - h_{1\_N2}\right) \tag{4.3}$$

Where Q is the amount of heat added, m_{O2} and m_{N2} are the masses of oxygen and nitrogen in the device, h_{1_O2} and h_{2_O2} are enthalpies of oxygen at the initial and final temperatures, respectively, and h_{1_N2} and h_{2_N2} are the corresponding enthalpies for nitrogen. The correct value of the final temperature is that at which the amount of heat added as obtained from Equation (4.3) is equal to the given value, which is 50 kJ.

The values of enthalpy for O_2 and N_2 in Equation (4.3) can be determined by using the relevant Thermax function for ideal gases, **Gash_TK**, and the masses m_{O2} and m_{N2} can be obtained from the ideal-gas law using the corresponding partial pressures as follows:

$$m_{O2} = \frac{(0.21P_1)V_1}{R_{O2}T_1} \tag{4.4}$$

$$m_{N2} = \frac{(0.79P_1)V_1}{R_{N2}T_1} \tag{4.5}$$

Where R_{O2} and R_{N2} are the gas constants for oxygen and nitrogen, which are 0.2598 kJ/kg.K and 0.2968 kJ/kg.K, respectively.

Solution with Goal Seek

Figure 4.5 shows the Excel sheet developed for this example. The data part includes the initial pressure, temperature, and volume of the gas mixture together with the mole fractions and gas constants of oxygen and nitrogen. The initial partial pressures of oxygen and nitrogen, P1_O2 and P1_N2, are calculated from the total initial pressure (P_1) and the respective volume fractions, y_O2 and y_N2, as shown in cells **E2** and **E3**, respectively. The masses of the two gases in the mixture (m_O2 and m_N2) are calculated in cells **E5** and **E6**, respectively, and the total mass (m_total) in cell **E8**.

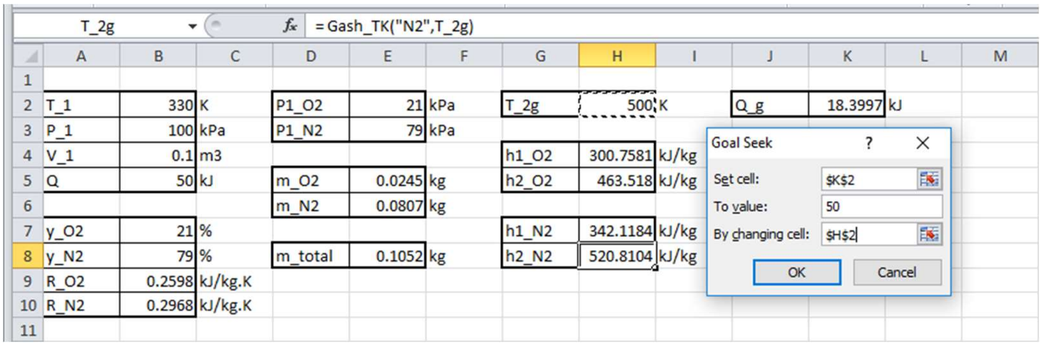


Figure 4.5. The Excel sheet developed for Example 4-2 by using Thermax functions

Starting with a guessed value for the final temperature, T_2g, which is 500K, the initial and final enthalpies of oxygen and nitrogen are determined by using Thermax function **Gash_TK** at the corresponding temperatures. Equation (4.3) is then used to determine the total amount heat added in the process (Q_g). With the guessed final temperature, Equation (4.3) determined the total amout of heat as 18.4 kJ, which is less than the actual values of 50 kJ. To find the appropriate final temperature, the guessed temperature T_2g has to be adjusted by Goal Seek so that the value of Q_2g equals 50 kJ. Figure 4.5 shows the required Goal Seek set-up and Figure 4.6 shows the solution obtained by Goal Seek, which is 780.444K. The value determined for T_2 by treating air as a single pure gas and using the approximate constant specific-heat method (c_p = 1.005 kJ/kg) is 801.2K while the value determined by using the exact method is 781.6K. Although these results confirm the accuracy of treating air as a single pure gas with the exact method of analysis, the deviation from the present model is expected to increase as T_2 increases.

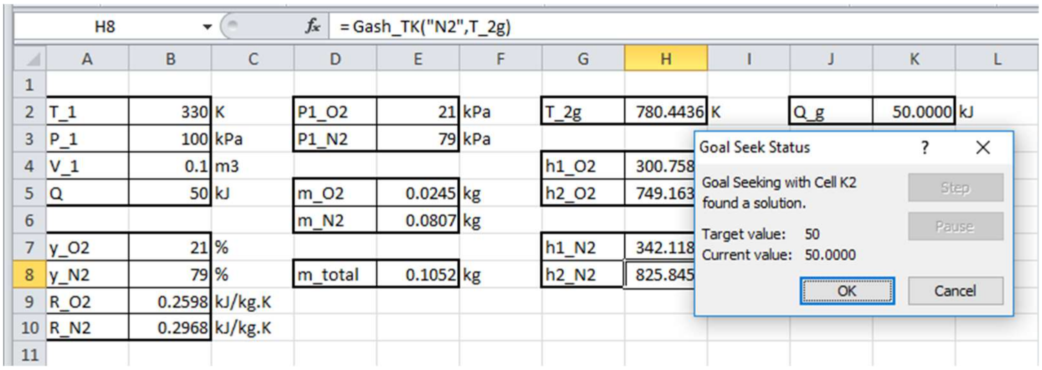


Figure 4.6. Goal Seek solution for Exampe 4.2 by using Thermax functions

4.1.3. Convection heat-transfer analyses

Like the friction factor (*f*) in pipe-flow analyses, the convection heat-transfer coefficient (*h*) is not constant but depends on the flow itself. Therefore, convection heat-transfer analyses frequently involve iterative solutions. The following example shows how Excel's Goal Seek command can be used for such analyses. The example is based on Example 10.1 in Holman [3].

Example 4-3. Overall heat-transfer coefficient for pipe in air

Hot water at 98ºC flows through a 2-in schedule 40 horizontal steel pipe (k =54 W/m·◦C) and is exposed to atmospheric air at 20ºC as shown in Figure 4.7. The water velocity is 25 cm/s.

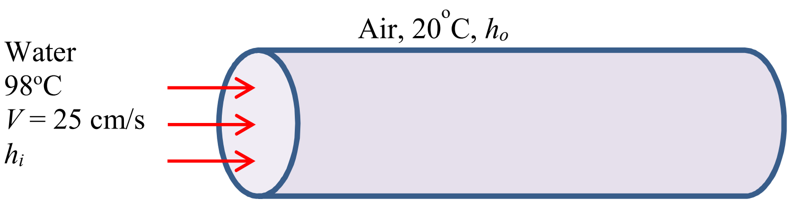


Figure 4.7. Schematic for Example 4-3 (adapted from Holman [3])

Calculate:

(a) the rate of heat-transfer through the pipe,
(b) the temperatures at the inside and outside surfaces of the pipe, and
(c) the overall heat-transfer coefficient based on the outer area of the pipe.

Properties of water at 98ºC are: ρ = 960 kg/m^3, μ = 2.82 x 10^{-4} kg/m.s, k = 0.68 W/m.ºC, Pr = 1.76. For a 2-in schedule 40 pipe, D_i = 5.25 cm and D_o = 6.033 cm.

The analytical model

Using the thermal-resistance concept, the rate of heat-transfer through the pipe, Q, is given by:

$$Q = (T_w - T_\alpha)/R_{th} \tag{4.6}$$

Where T_w and T_∞ are the water temperature and air-temperature, respectively, and R_{th} is the total thermal resistance that consists of the thermal resistances due to heat-transfer by convection inside the pipe (R_i), by conduction through the steel pipe (R_p), and by convection outside the pipe (R_o). The three resistances are given by:

$$R_i = \frac{1}{A_I h_i} \tag{4.7}$$

$$R_p = \frac{\ln(D_i / D_o)}{2\pi k} \tag{4.8}$$

$$R_o = \frac{1}{A_o h_o} \tag{4.9}$$

Where A_i and A_o are the inside and outside areas of the pipe and h_i and h_o are the corresponding heat-transfer coefficiens. The internal heat-transfer coefficien h_i is determined from the corresponding Nusselt number (Nu) by:

$$h_i = Nu \frac{k_w}{D_i} \tag{4.10}$$

Where, k_w is the thermal conductivity of water. The Nusselt number itself is determined from emprirical equations depending on the type of the flow, i.e., natural or forced, laminar or turbulent. For the turbulent forced internal flow (to be confirmed later), Nu is obtained from the Dittus-Boelter equation, Equation (1.31) [3]:

$$Nu = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4} \tag{4.11}$$

Where, Re and Pr are the Reynolds number and Prandtl number, respectively. For the external flow, Holman [3] used the following simplified equation for free laminar convection from a horizontal pipe to air at atmospheric pressure:

$$h_o = 1.32 \left(\frac{T_o - T_\infty}{D_o} \right)^{1/4} \tag{4.12}$$

Both R_i and R_p can be determined directly from the given data, but R_o depends on h_o which cannot be determined directly since T_o is not known. Therefore, the problem has to be solved by adopting an iterative approach by assuming a value for T_o based on which h_o is determined and, consequently, Q. The value of Q thus obtained can be used to calculate corresponding values for T_i and T_o from:

$$T_i = T_w - Q.R_i \tag{4.13}$$

$$T_o = T_i + Q.R_p \tag{4.14}$$

If the guessed value for T_o is correct, then it will be the same as that obtained from Equation (4.14). Otherwise, a new guess for T_o has to be made repeatedly until this condition is met. Once this is achieved, the overall heat-transfer coefficient (U_o) based on the outside area (A_o) can be obtained from:

$$U_o = \frac{1}{A_o\left(R_i + R_o + R_p\right)} \tag{4.15}$$

Solution with Goal Seek

The Excel sheet developed for this example is shown in Figure 4.8. The given information about the pipe, water, and air properties are entered in the data part on the left side of the sheet. The cells are labelled and the figure shows the formulae used in the calculations. The calculations part at the central part of the sheet starts with a guessesd value for the pipe's outside temperature (T_og) of 50°C. Based on this value, the sheet determines the outside heat-transfer coefficient (h_o) from Equation (4.12) and the thermal resistance associated with it (R_o) from Equation (4.9). Following the analytical model described above, the sheet determines the three thermal resistances (R_i, R_p, and R_o), and then calculates the rate of heat-transfer (Q), inside temperature (T_i), outside temperature (T_o), and overall-heat transfer coefficient (U).

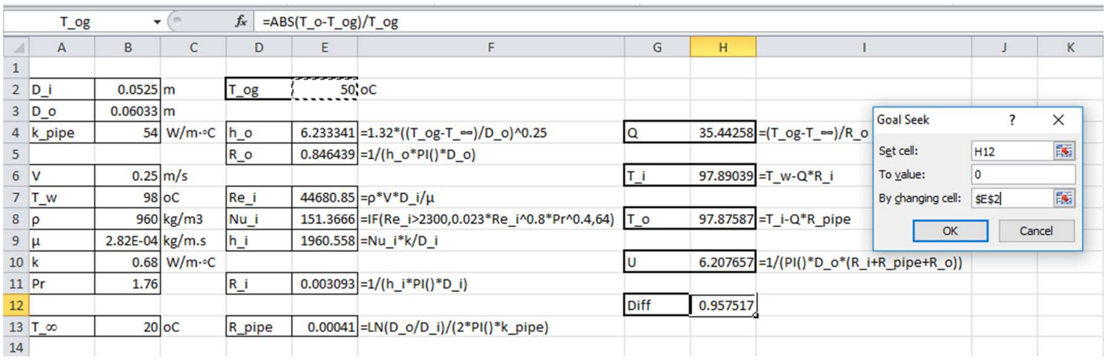


Figure 4.8. Excel sheet developed for Example 4-3

As Figure 4.8 shows, the value of T_o calculated from Equation (4.14) is 97.876°C, which is different from the initially guessed value (T_og = 50°C). The formula bar reveals the formula entered in cell **H12** that calculates the difference between the calculated exit temperature (T_o) and the guessd value (T_og) as a fraction of T_og. The exit temperature that makes the difference vanishes can be found by using the Goal Seek command and Figure 4.8 shows the required set-up. The solution found by Goal Seek is shown in Figure 4.9. Table 4.1 comapres the presents results with those given by Holman [3] to confirm the accuracy of the iterative solution with Goal Seek.

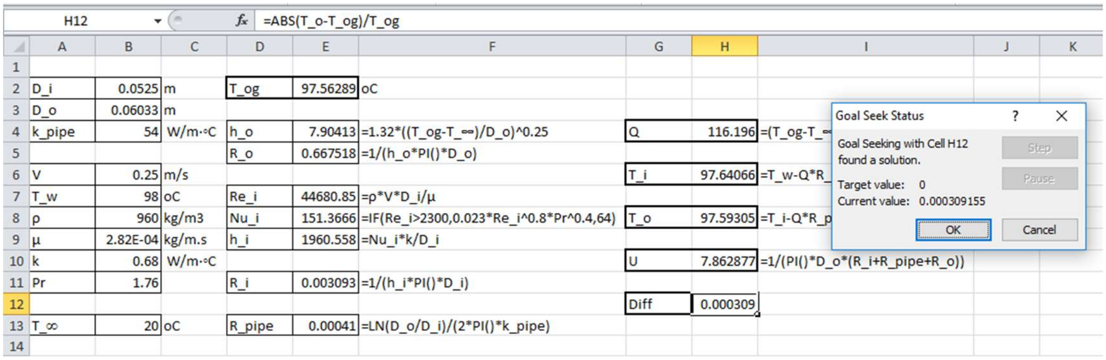


Figure 4.9. Solution obtained by Goal Seek for Example 4-3

Table 4.1. Comparison of the presnt Goal Seek solution with that given by Holman [3]

	Holman [3]	Goal Seek solution
T_i	97.65	97.64
T_o	97.6	97.59
h_i	1961.0	1960.56
h_o	7.91	7.90
U_o	7.87	7.86

4.2. Constrained iterative solutions with Solver

Compared to Goal Seek, Solver offers greater flexibility for dealing with iterative solutions because it allows for multiple changeable cells and for constraints to be imposed on the iterative solution. This section illustrates the need for these additional features in thermofluid analyses by two examples from the areas of fluid-dynamics and thermodynamics.

Example 4-4. Determining the maximum water flow rate to avoid cavitation

Water at 20°C (γ = 9810 N/m^3 and ν =1.006x10^{-6} m^2/s) is to be pumped from a large reservoir via a pump-pipe system as shown in Figure 4.10. The pump is positioned vertically at a level which is 9 m above the surface of the reservoir and horizontally at 1 m from the vertical section of the pipe. The pipe is made of commercial steel pipe (ε = 0.046 mm) and has a 2″ nominal diameter.

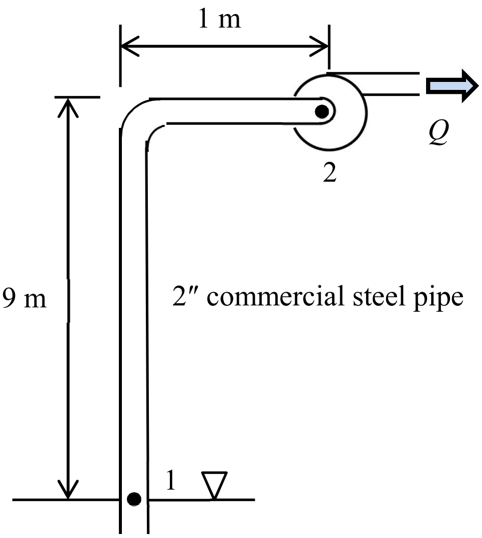


Figure 4.10. Schematic for the pump-pipe system in Example 4-4

Determine the maximum allowable water flow rate (Q) such that:

1. The water velocity (V) is to be in the range 1.4-2.8 m/s for economic considerations.
2. The pressure at the pump inlet must be greater than the saturation pressure of water at 20°C, which is 2.338 kPa, to avoid cavitation.

This example, which is basically a type-2 pipe flow problem, is based on a similar example given by Schumack [4]. The example highlights two important design considerations for pump-pipe systems which are the two conditions mentioned above.

The analytical model

The energy equation between the pipe inlet (point 1) and the pump inlet (point 2) is:

$$\frac{p_1}{\gamma}+Z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+Z_2+\frac{V_2^2}{2g}+h_f \tag{4.16}$$

Where γ stands for the specific weight of water, z for the elevation, V for the water velocity, g for the gravitational acceleration, and h_f for the friction loss in the pipe. For a large reservoir $V_1 \approx 0$. Taking point 1 as a reference, i.e. $Z_1 = 0$, and noting that the water velocity in the pipe is uniform, i.e. $V_2 = V$, the energy equation reduces to:

$$p_2=\gamma\left(\frac{p_1}{\gamma}-Z_2-\frac{V^2}{2g}-h_f\right) \tag{4.17}$$

The velocity V is related to the pipe diameter (D) and water flow rate (Q) as follows:

$$V = 4Q / \pi D^2 \tag{4.18}$$

Neglecting minor losses, the friction loss can be calculated from the Darcy-Weisbach equation, Equation (4.1), which needs an auxiliary formula to determine the friction factor (f) depending on whether the flow is laminar or turbulent.

Solution with Solver

Figure 4.11 shows the Excel sheet developed for this example. The data part on the left side stores the problem data such as the diameter, roughness, and length of the pipe, etc. The central part stores a guessed value for the water velocity (V=1.0 m/s) in cell **E2**. Based on the guessed water velocity, the sheet performs the necessary calculations according to the analytical model given above. Figure 4.11 reveals the formulae used in these calculations. Note that an **IF-**statement is used to calculate the friction factor (f) depending on the value of the Reynolds number (Re). Cell **E6** calculates the friction loss (hf). Based on the calculated value of friction loss, the pressure at point 2 (P_2) is calculated from Equation (4.17) and stored in cell **E7**. The right side of the sheet contains the single cell **I2** that determines the flow rate (Q) and the formula in this cell is shown in the formula bar.

Q | fx =0.25*PI()*D^2*V

	A	B	C	D	E	F	G	H	I	J	K
1											
2	D	0.05252	m	V	1.00	m/s		Q	0.0021664	m3/s	
3	ε	0.000046	m								
4	L	10	m	Re	52520	=V*D/v					
5	Z_2	9	m	f	0.023651	=IF(Re<2000,64/Re,0.25/(LOG10(ε/3.7/D+5.74/Re^0.9))^2)					
6	P_1	100	kPa	hf	0.229523	=f*L/D*V^2/(2*gc)					
7	v	1.00E-06	Pa.s	P_2	8.958382	=(P_1*1000/γ-Z_2-V^2/(2*gc)-hf)*γ/1000					
8	γ	9810	N/m3								
9	gc	9.81	m/s2								
10											

Figure 4.11. Excel sheet developed for Example 4-4

Based on the assumed water velocity of 1.0 m/s, the calculated values of hf and P_2 are 0.2295 m and 8.958 kPa, respectively. Since the pressure at point 2 is higher than the minimum desired level of 2.338 kPa, while the water velocity (V) is less than the minimum economic value of 1.4 m, there is room to increase the flow rate. The task can be left to Solver and Figure 4.12 shows the set-up that requires Solver to maximise the value of the flow rate Q while satisfying the three constraints shown in the figure. The first constraint on the iterative solution requires the value of P_2 in cell **E7** to be higher than or equal to 2.338 kPa. The two other constraints are to satisfy the limits on the water velocity imposed by economic limits, i.e., 1.4 m $\leq V \leq$ 2.8 m.

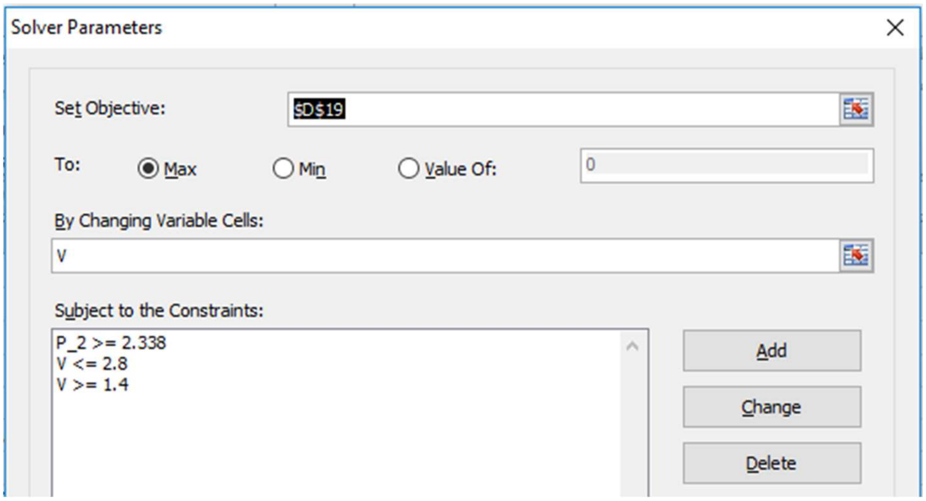


Figure 4.12. Solver parameters dialog box for Example 4-4

Pressing the “Solve” button will trigger Solver to search for the solution. The solution found by the **GRG Nonlinear** method is shown in Figure 4.13. The value determined for the water velocity is 1.90 m/s. Note that this velocity lies within the limits imposed by the economic constraint. The pressure at point 2 is equal to 2.338 kPa which is the minimum pressure level required to prevent cavitation. Therefore, the corresponding flow rate, which is 0.00413 m^3/s, is the maximum flow rate to be recommended.

Q fx =0.25*PI()*D^2*V

	A	B	C	D	E	F	G	H	I	J	K
1											
2	D	0.05252	m	V	1.90	m/s		Q	0.004125	m3/s	
3	ε	0.000046	m								
4	L	10	m	Re	100001.5	=V*D/v					
5	Z_2	9	m	f	0.021901	=IF(Re<2000,64/Re,0.25/(LOG10(ε/3.7/D+5.74/Re^0.9))^2)					
6	P_1	100	kPa	hf	0.770567	=f*L/D*V^2/(2*gc)					
7	v	1.00E-06	Pa.s	P_2	2.338005	=(P_1*1000/γ-Z_2-V^2/(2*gc)-hf)*γ/1000					
8	γ	9810	N/m3								
9	gc	9.81	m/s2								
10											

Figure 4.13. Solver solution for Example 4-4

Example 4-5. Restrained expansion of air inside a piston-cylinder device

Figure 4.14 shows a piston-cylinder device that initially contains 0.05 m^3 of air at 200 kPa and 317K. At this state, a linear spring is touching the piston but exerting no force on it before 72.7 kJ of heat is transferred to the air, causing the piston to rise and compress the spring. The cross-sectional area of the piston is 0.25 m^2 and the spring’s constant (k) is 150 kN/m, Air can be treated as an ideal gas with a specific heat at constant volume (c_v) that varies linearly with the temperature according to the formula:

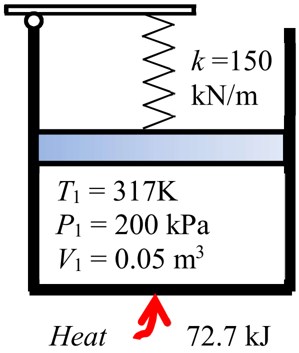


Figure 4.14. Schematic and pressure-volume diagrams for Example 4-5 (adapted from Cengel and Boles [5])

$$c_v = 0.645+0.0002T \tag{4.19}$$

Where T is the temperature in K and c_v is in kJ/kg.K. Determine the final volume, pressure, and temperature of the air inside the cylinder after the heat-addition process.

Comment

This example is based on Example 4-4 given by Cengel and Boles [5]. However, unlike the present example, that given by Cengel and Boles [5] specified the final volume to be 0.1 m^3 instead of specifying the amount of heat added. When the final volume (or final pressure) is given, the problem can be solved in a straightforward manner without iteration. However, in the present example T_2, V_2, and P_2 at the final state depend on the amount of heat added. Another factor that makes the present example more difficult than the one given by Cengel and Boles [5] is the use of Equation (4.19) to determine the specific heat c_v for air. The specific value of 72.7 kJ given in this example has been chosen such that the final volume will be 0.1 m^3 as specified by Cengel and Boles [5] so that the present final pressure on the piston and total work will be comparable to their corresponding values even though the data of the two examples are different.

The analytical model

Like Example 4-2, the problem can be solved by using the first-law of thermodynamics together with the ideal-gas law, but the variation of the specific heat with temperature makes it necessary to adopt an iterative solution. Moreover, the addition of the linear spring in this example introduces a new factor, which is the variation of pressure with air expansion. Since the present iteration process involves both the temperature and the volume (or pressure), the Goal Seek command cannot be used. Therefore, this example requires Solver to start the iterative procedure with assumed values for both the final temperature (T_2^*) and the final volume (V_2^*).

The final pressure P_2 is given by:

$$P_2 = P_1 - \frac{k\Delta x}{A} \quad (4.20)$$

Where A is the base area of the piston and Δx is the reduction in the spring's length given by:

$$\Delta x = \frac{V_2^* - V_1}{A} \quad (4.21)$$

The total work (W), i.e., the summation of the air expansion work and the work done against the spring, can now be obtained from:

$$W = \frac{(P_1 + P_2)}{2}\left(V_2^* - V_1\right) \quad (4.22)$$

The final temperature can be determined by applying the first-law of thermodynamics to the piston-cylinder device as a closed system:

$$Q - W = m(u_2 - u_1) = m\bar{c}_v(T_2 - T_1) \quad (4.23)$$

Where Q is the amount of heat added, u is the internal energy, m is the mass of air inside the cylinder, and $\bar{c}_v$ is the average specific heat of air at constant volume. The mass and specific heat of air can be obtained from:

$$m = P_1 V_1 / R T_1 \quad (4.24)$$

$$\bar{c}_v = 0.645 + 0.0002\left(T_1 + T_2^*\right)/2 \quad (4.25)$$

Rearranging Equation (4.23), the final temperature T_2 is given by:

$$T_2 = T_1 + \frac{Q - W}{m\bar{c}_v} \quad (4.26)$$

Using the values obtained for T_2 and P_2, the final volume V_2 can be determined from the ideal-gas law:

$$V_2 = mRT_2 / P_2 \quad (4.27)$$

If the initially guessed volumes of T_2^* and V_2^* are correct, then they will be the same as T_2 and V_2 obtained from Equation (4.24) and Equation (4.26), respectively. Otherwise, new values for T_2^* and V_2^* have to be used until the differences between the calculated and guessed values become negligibly small. This multi-variable iterative process can be performed with Solver as shown below.

Solution with Solver

Figure 4.15 shows the Excel sheet developed for this example and reveals the formulae used in it. The left side of the sheet accommodates the problem data. The calculations part start by an assumed values for the final temperature (T_2g = 500K) and final volume V_2g = 0.15 m^3. Based on the assumed final volume, the sheet determines the compression of the spring (Δx), spring force (Fspring), final pressure (P_2), and total work involved (Work). The final temperature (T_2) is then calculated from the first-law according to Equation (4.26), and the final volume (V_2) from the ideal-gas law, Equation (4.27).

T_2 | fx | =T_1+(Q-Work)/m/Cv

	A	B	C	D	E	F	G	H	I	J
1										
2		P_1	200	kPa	T_2g	500		T_2	826.5425	K
3		T_1	317	kPa	V_2g	0.15		V_2	0.059259	m3
4		V_1	0.05	m3						
5					m	0.109915	=P_1*V_1/Rgas/T_1			
6		Q	72.7	kJ						
7		Area	0.25	m	Δx	0.4	=(V_2g-V_1)/Area			
8					Fspring	60	=kspring*Δx			
9		kspring	150	kN/m	P_2	440	=P_1+Fspring/Area			
10					Work	32	=0.5*(P_1+P_2)*(V_2g-V_1)			
11		Rgas	0.287	kJ/kg.K						
12					Cv	0.7267	=0.645+0.0002*(T_1+T_2g)/2			
13										

Figure 4.15. Excel sheet developed for Example 4-5

As Figure, 4.15 shows, the calculated values T_2 and V_2 are different from the initial values T_2g and V_2g. Solver can now be used to adjust the guessed value of T_2g and V_2g until they become the same as the calculated values. Figure 4.16 shows the set-up that requires Solver to change the values of T_2g and V_2g in cells F2 and F3, respectively, until the two specified constraints are satisfied: (i) T_2 = T_2g and (ii) V_2=V_2g. Note that the "**Set Objective**" option has been left blank. The "**Changing Variable Cells**" are **F2** and **F3**.

Figure 4.17 shows the solution obtained by the **GRG Nonlinear** method of Solver, which is T_2g = 1014.864K and V_2g = 0.1 m^3. At this state, the final pressure on the piston is 320.0 kPa and the total work is 13.0 kJ. These values agree with their corresponding values given by Cengel and Boles [5] whose analysis also gave P_2 = 320 kPa and W = 13 kJ.

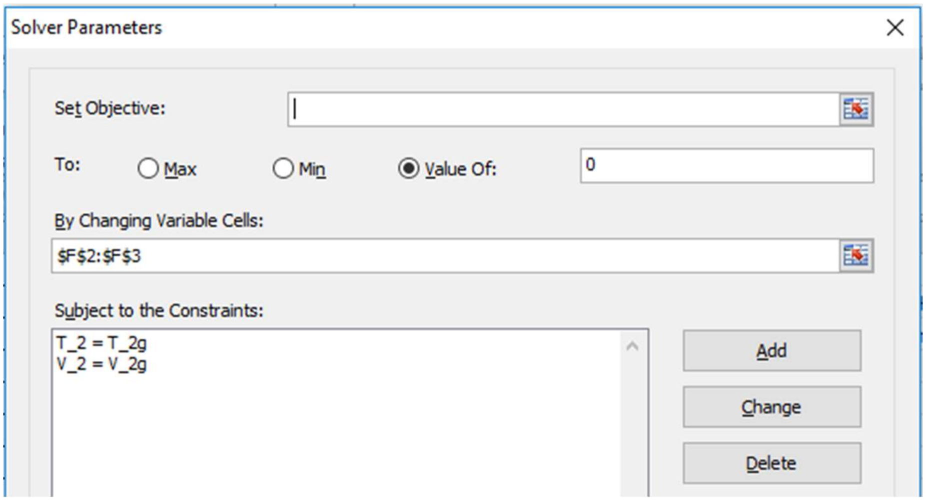
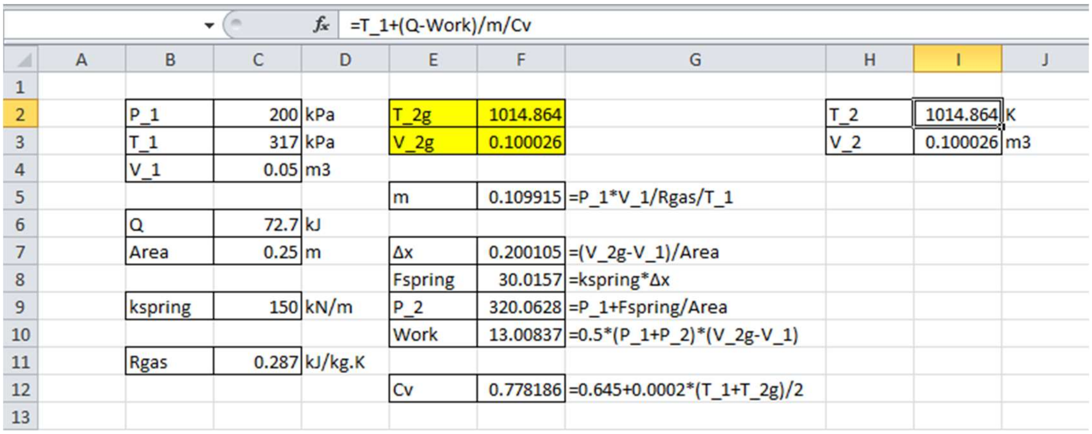


Figure 4.16. Solver set-up for Example 4-5



=T_1+(Q-Work)/m/Cv

	A	B	C	D	E	F	G	H	I	J
1										
2		P_1	200	kPa	T_2g	1014.864		T_2	1014.864	K
3		T_1	317	kPa	V_2g	0.100026		V_2	0.100026	m3
4		V_1	0.05	m3						
5					m	0.109915	=P_1*V_1/Rgas/T_1			
6		Q	72.7	kJ						
7		Area	0.25	m	Δx	0.200105	=(V_2g-V_1)/Area			
8					Fspring	30.0157	=kspring*Δx			
9		kspring	150	kN/m	P_2	320.0628	=P_1+Fspring/Area			
10					Work	13.00837	=0.5*(P_1+P_2)*(V_2g-V_1)			
11		Rgas	0.287	kJ/kg.K						
12					Cv	0.778186	=0.645+0.0002*(T_1+T_2g)/2			
13										

Figure 4.17. Solver solution for Example 4-5

4.3. Iterative solutions involving nonlinear equations

To determine the head loss due to friction (h_f) in Example 4-1, the friction factor (f) for the turbulent pipe flow was obtained from Equation (1.24) which is an explicit equation. However, for a turbulent pipe flow f can be determined more accurately by using the following Colebrook-White equation [1]:

$$\sqrt{\frac{1}{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{4.28}$$

Since the equation involves the friction factor f in both sides, it needs to be solved iteratively in order to determine f. Therefore, for type-2 and type-3 flow problems using this equation involves two nested iterations; an inside iteration to determine f and an outside iteration to determine the pipe's diameter or flow rate. To allow the Colebrook-

White equation to be used in iterative solutions and optimisation analyses, Thermax provides a Newton-Raphson solver (**NRM**) for this equation which can also be used for other nonlinear equations. Appendix B illustrates the use of the **NRM** solver by considering another nonlinear equation which is the Benedict-Webb-Rubin equation. In the present situation, the **NRM** solver will be used to solve the Colebrook-White equation leaving the main iteration for Solver or Goal Seek.

For illustration, let us reconsider Example 4-1 and solve it by the using Equation (4.28) to determine f instead of Equation (1.24). The **NRM** solver requires the intended nonlinear equation to be provided as a separate user-defined function. The one needed for the Colebrook-White equation is listed below:

```
Function colebrook(x, e, Re)
' Colebrook equation for the friction factor
        colebrook = 1/Sqr(x) + (2/log(10))*Log(e /3.7 + 2.51/Re/Sqr(x))
End Function
```

Note that in VBA syntax the term "log" is used for the natural logarithm "ln", which is different from Excel. Figure 4.18 shows the Excel sheet developed for solving Example 4-1 with the Colebrook-White equation.

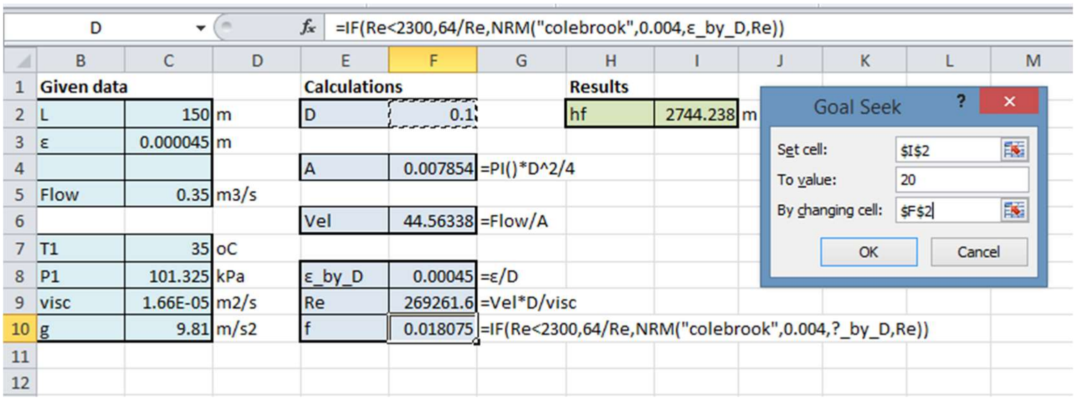


Figure 4.18. Excel sheet for Example 4-1 using the Colebrook-White equation

The only difference from the sheet shown in Figure 4.2 is the content of the cell **F10** that calculates friction factor. Figure 4.18 shows the formula typed in this cell as:

=IF(Re<2300,64/Re,NRM("colebrook",0.004,ε_by_D,Re))

The first input to the **NRM** solver, "**colebrook**", refers to the function that contains the Colebrook-White equation while the second input, **0.004**, is an initial guess for f. The last two arguments, **ε_by_D** and **Re**, respectively, are values of the two cells **F8** and **F9** that store the roughness-diameter ratio (ε/D) and the Reynolds number (Re) at which f is to be determined. Relevant input parameters are used for the Benedict-Webb-Rubin equation in Appendix B that considers this equation.

Figure 4.18, which shows the calculations for a selected diameter of 0.1 m, shows that the value of the friction factor obtained by the Colebrook-White equation is 0.018075 and the corresponding friction loss is 2744.2 m. These values are slightly different from those obtained with Equation (1.24) as shown in Figure 4.2. The diameter that keeps the loss below 20 m can be determined by using Goal Seek and Figure 4.18 also shows Goal Seek set-up for finding the value of D that makes the friction head loss equal to 20 m. As Figure 4.19 shows, the answer found by Goal Seek with the Colebrook-White equation is $D \geq 0.27$ m, which is the same answer obtained earlier in Example 4-1.

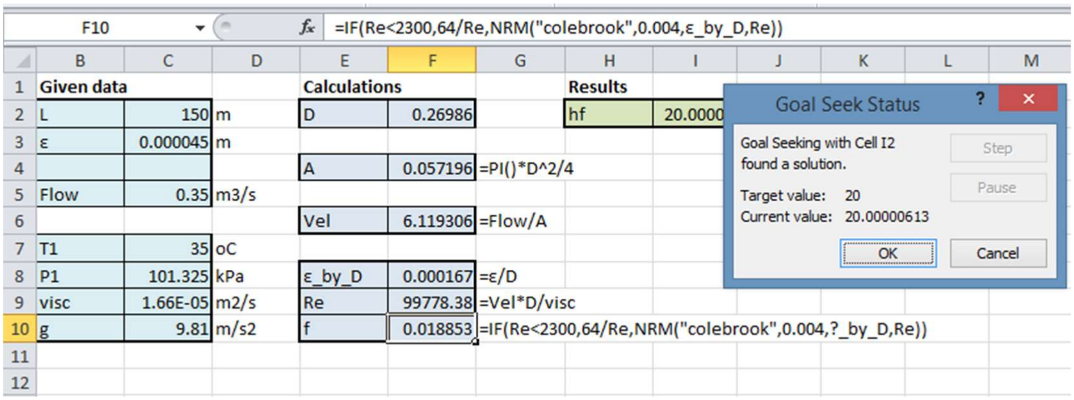


Figure 4.19. Goal Seek solution for Example 4-1 using the Colebrook-White equation

4.4. Closure

This chapter deals with thermofluid analyses that require iterative solutions and shows how Excel's Goal Seek command and Solver can be used for solving typical problems from the areas of fluid dynamics, thermodynamics, and heat-transfer. While the Goal Seek command can easily perform the simple type of iterative solutions that involve a single parameter, Solver can perform the more difficult iterative solutions that involve multiple changeable cells and require constraints to be applied to the iterative solution. The chapter also shows how the Newton-Raphson solver provided by Thermax can be used to deal with the solutions that involve nonlinear equations such as the Colebrook-White equation and the Benedict-Webb-Rubin equation.

References

[1] Y. A. Cengel and J. M. Cimbala, *Fluid Mechanics Fundamentals and Applications*, 3rd edition, McGraw-Hill, 2006.

[2] C. T. Crowe, D. F. Elger, B. C. Williams, and J. A. Roberson, *Engineering Fluid Mechanics*, 9th edition, John Wiley, 2009.

[3] J. P. Holman, *Heat Transfer*, 10th edition, McGraw-Hill. 2010.

[4] M. Schumack, Solution of complex pipe flow problems using spreadsheets in an introductory fluid mechanics course, Proceedings of the 2004 American Society for

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[5] Y. A. Cengel, and M. A. Boles, *Thermodynamics. An Engineering Approach*. 5th edition, McGraw-Hill, New York, 2006.
[6] M.J. Moran and H.N. Shapiro, *Fundamentals of Engineering Thermodynamics*, 5th edition, John Wiley, & Sons. Inc. 2006.
[7] M.M. El-Awad, Use of Excel's 'Goal Seek' feature for Thermal-Fluid Calculations, the Electronic Journal of Spreadsheets in Education (eJSiE), Vol. 9, Iss. 1, Article 3, 2016.

Exercises

1. Consider the case in Example 4-1. Suppose that the only available pipe diameter is 20 cm and we want to maintain the same maximum limit on the friction head loss of 20 m by reducing the water flow rate. Using the Goal Seek command, determine the water flow rate that gives the required result. Answer: 0.157 m^3.
2. Using the Excel sheet developed for Example 4-2, determine the final temperature for air when the amount of heat added is 50, 100, 150, and 200 kJ. Also calculate the final temperature from Equation (4.3) by using a constant specific heat (c_p) of 1.043 kJ/kg.K. Plot the values obtained for the final temperature (T_2) with the amount of heat added by the two methods and comment on the result.
3. A gas mixture consisting of O_2 and CO_2 with mole fractions 0.2 and 0.8, respectively, expands isentropically and at steady state through a nozzle from 700 K, 500 kPa to an exit pressure of 100 kPa as shown in Figure 4.P3. Determine the temperature at the nozzle exit, in K.

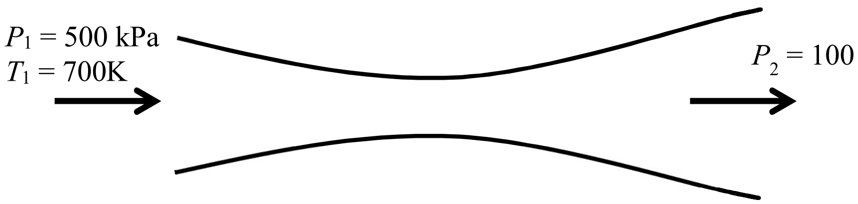

Figure 4.P3. Isentropic expansion in a nozzle

Using the approximate constant-specific heat method, the exit temperature (T_2) can be determined from:

$$T_2 = T_1 \times \left(P_2 / P_1\right)^{\frac{k-1}{k}} \qquad \text{(A)}$$

Where k is the ratio of the specific heats for the mixture. Using k = 1.304, the resulting exit temperature is 480.9K. Using the exact variable specific heat method, T_2 is determined by requiring the total entropy change to be zero, i.e.:

$$y_{O2}\left[s^0_{O2}(T_2)-s^0_{O2}(T_1)-R_{O2}\ln\frac{P_2}{P_1}\right]$$
$$+\,y_{CO2}\left[s^0_{CO2}(T_2)-s^0_{CO2}(T_1)-R_{CO2}\ln\frac{P_2}{P_1}\right]=0 \qquad \text{(B)}$$

Where y_{O2} and y_{CO2} are the volume fractions of O_2 and Co_2, respectively, and R_{O2} and R_{CO2} are the molar masses for O_2 and Co_2, respectively. The values of s^0_{O2} and s^0_{CO2} can be determined by using the relevant function provided by Thermax. Equation (B), that requires an iterative solution, can be solved by using the Goal Seek command. This exercise is based on Example 12.4 in Moran and Shapiro [6]. Answer: T_2 = 514.05K.

4. Steam is be condensed at 30°C on the shell side of the multi-pass shell-and-tube heat exchanger shown in Figure 4.P4. The condenser has 8-tube-passes with 50 tubes in each pass. Its overall heat transfer coefficient is 1000 W/m^2·°C. Cooling water (C_p = 4180 J/kg·°C) enters the tubes at 15°C at a rate of 55,000 kg/h. The tubes are thin-walled, and have a diameter of 1.5 cm and length of 2 m per pass. Develop an Excel sheet to determine the outlet temperature of the cooling water by using Goal Seek and the LMTD method instead of the ε-NTU method [7].

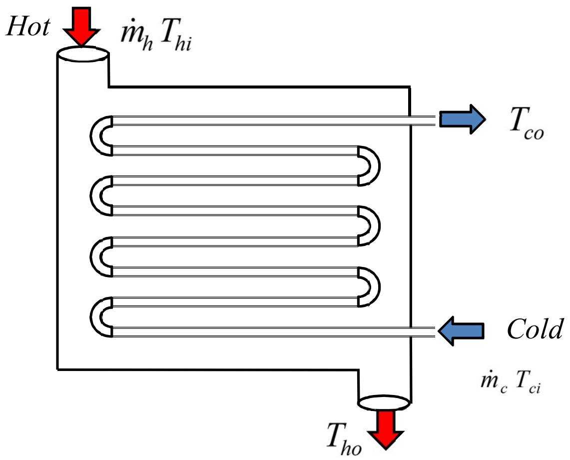

Figure 4.P4. A multi-pass shell-and-tube heat exchanger

5. Reconsider the case in Example 4-5. An alternative solution of this problem that also takes into consideration the variation of specific heat for air with temperature can be obtained by using the ideal-gas property functions provided by Thermax instead of Equation (4.19). Show that this solution can be obtained by using the Goal Seek command instead of Solver and compare your solution with that given in Example 4-5.

6. Consider the semi-infinite slab shown in Figure 4.P6 that is suddenly exposed to convection environment at T_∞. The temperature (T) at a depth x from the surface at any time is given by [3]:

$$\frac{T - T_i}{T_\infty - T_i} = 1 - erf(X) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha \tau}{k^2} \right) \right] \times \left[1 - erf\left(X + \frac{h\sqrt{\alpha\tau}}{k} \right) \right] \quad \text{(A)}$$

Where α and k are the diffusivity and thermal conductivity of the slap material, respectively, T_i is the initial temperature of the solid, T_∞ is the environmental temperature, τ is the elapsed time in seconds, and $X = \left(2\sqrt{\alpha\tau}\right)$.

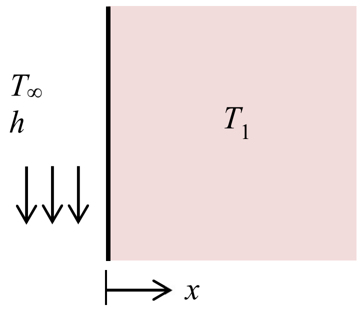

Figure 4.P6. Semi-infinite slab with convection heat-transfer

Equation (A) requires an iterative procedure because the time (τ) appears in both terms on the right-hand side of the equation.

A large slab of aluminium (k = 215 W/m.ºC, $\alpha = 8.4 \times 10^{-5}$ m²/s) at a uniform temperature of 200ºC is suddenly exposed to a convection-surface environment of 70ºC with a heat-transfer coefficient of 525 W/m²·ºC. Calculate the time required for the temperature to reach 120ºC at the depth of 4.0 cm for this circumstance.

This problem is based on Example 4-5 in Holman [3] where the answer is approximately 3000 seconds.

7. Water at 60ºC enters a tube of 3-cm diameter at a mean flow velocity of 1.2 cm/s. If the tube is 3.0 m long and the wall temperature is constant at 80ºC, what will be the exit water temperature?

Use Goal Seek to perform the iterative solution of this problem. To determine the viscosity of water at any temperature, develop a user-defined function based on the data shown in Table A.2 in Appendix A. This exercise is based on Example 4-2 in Holman [3]. Answer: 73.0ºC.

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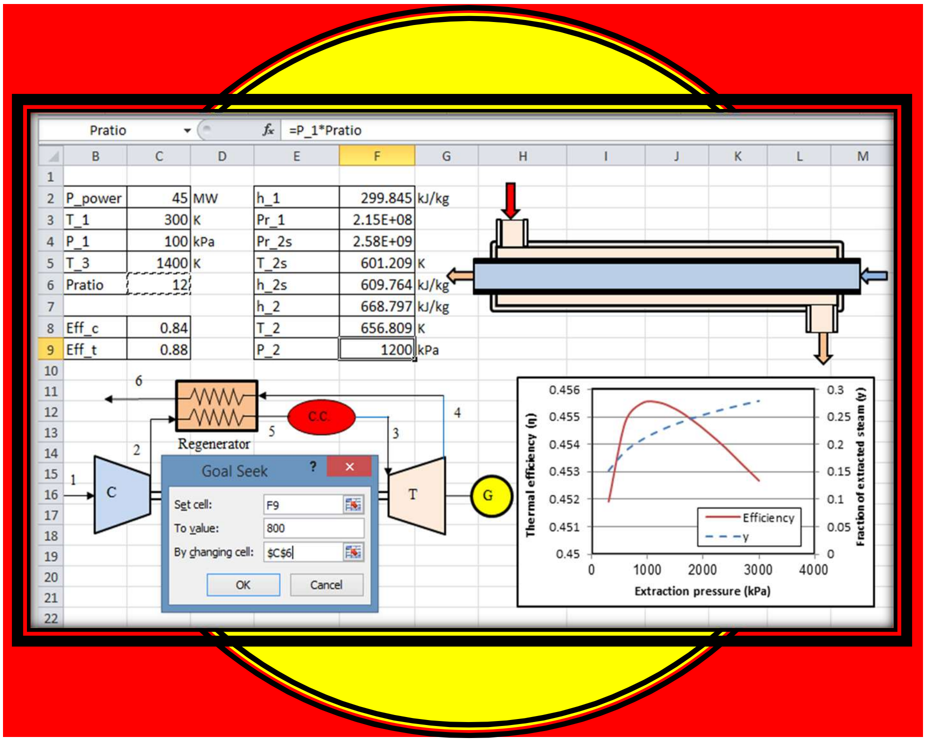
1 Fundamentals 2 Design 3 Research Excel

1

Computer-Aided Analyses and Optimisation of Fluid-Thermal Systems using Excel



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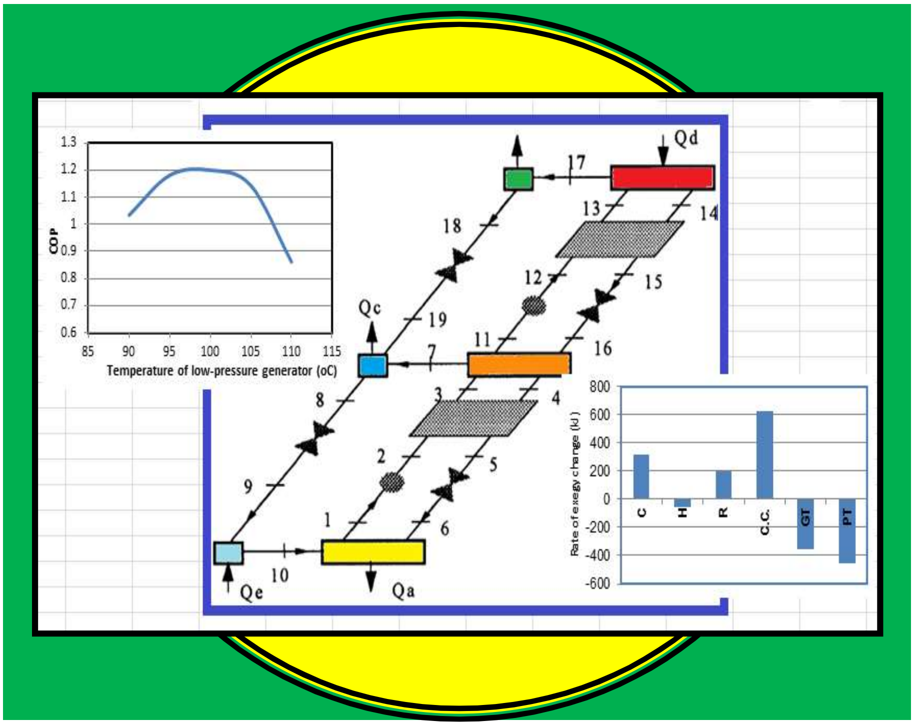
2

Thermodynamic Analyses and Optimisation of Energy-Conversion Systems using Excel



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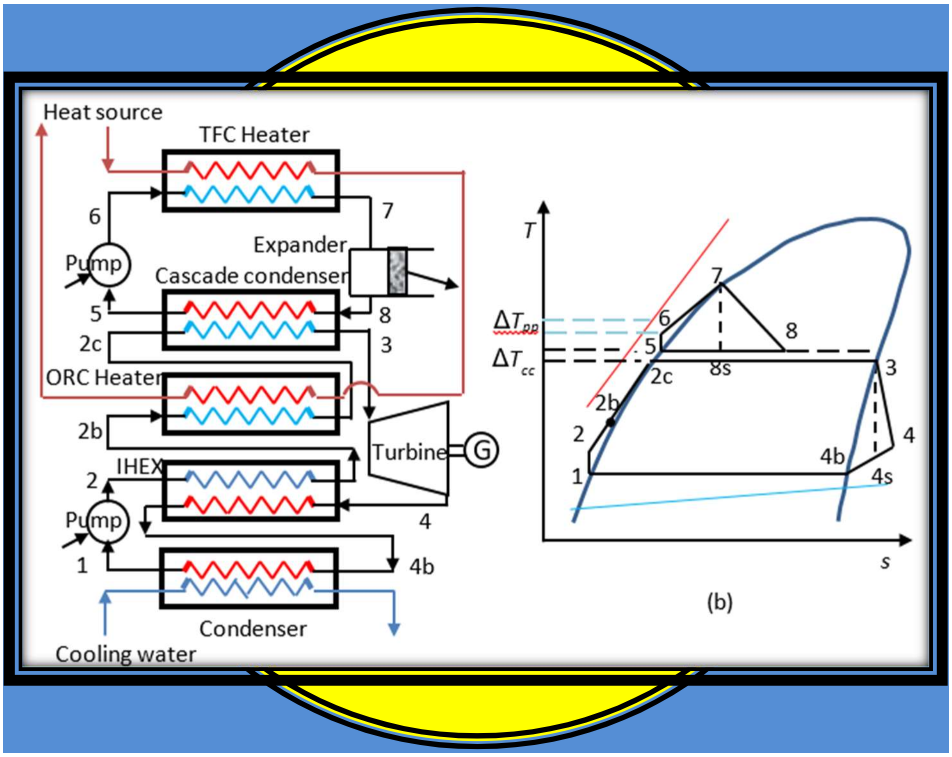
3

Introduction to Exergoeconomic Analyses and Multi-Objective Optimisation of Energy-Conversion Systems using Excel



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