Foundations of (m, n)-SuperHyperFuzzy, SuperHyperNeutrosophic, and SuperHyperPlithogenic Sets

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Abstract

Uncertainty modeling is fundamental to decision-making across diverse domains, and numerous frameworks—such as Fuzzy Sets [1], Rough Sets [2, 3], Vague Sets [4, 5], Intuitionistic Fuzzy Sets [6, 7], Hesitant Fuzzy Sets [8, 9], Soft Sets [10, 11], Neutrosophic Sets [12, 13], and Plithogenic Sets [14, 15]—have been developed to capture different facets of imprecision. Among these extensions are Hyperfuzzy Sets [16] and their recursive generalizations, SuperHyperfuzzy Sets [17], which assign set-valued membership degrees at multiple hierarchical levels. Similarly, corresponding hyper and superhyper extensions have been proposed for Neutrosophic and Plithogenic frameworks. These constructions enable clear, intuitive modeling of inherently hierarchical and complex uncertainties. In this paper, we review the notions of (m, n)-SuperHyperfuzzy Sets, (m, n)-SuperHyperneutrosophic Sets, and (m, n)-SuperHyperPlithogenic Sets, and illustrate their use through several concrete examples.

Keywords: Fuzzy set, HyperFuzzy Set, Neutrosophic Set, Plithogenic Set, HyperNeutrosophic Set, HyperPlithogenic Set

Structure of this paper

The format of this paper is described below.

1 Preliminaries

This section provides an introduction to the foundational concepts and definitions required for the discussions in this paper. In addition, all concepts addressed herein are assumed to be finite rather than infinite.

1.1 Fuzzy and Hyperfuzzy Sets

A *fuzzy set* assigns to each element a membership degree in the interval $[0, 1]$, thereby capturing uncertainty through graded, rather than binary, membership [1, 18, 19]. Fuzzy sets have been applied across many domains, and a variety of extensions—such as Bipolar Fuzzy Sets [20, 21] and Picture Fuzzy Sets [22, 23]—have been proposed to handle richer kinds of ambiguity. A *hyperfuzzy set* further generalizes this framework by assigning each element a nonempty subset of $[0, 1]$ [24, 25], thus representing multiple plausible membership values to model imprecision, variability, and evaluator disagreement. This concept is extended even further by *superhyperfuzzy sets* [17], which employ iterated nonempty power sets to encode hierarchical, multi-level fuzzy uncertainty.

Definition 1.1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 1.2 (Power set)**.** (cf. [26,27]) The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 1.3 (n-th Powerset)**.** (cf. [28–30])

The n-th powerset of a set H, denoted $\mathcal{P}_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$\mathcal{P}_1(H) = \mathcal{P}(H), \quad \mathcal{P}_{n+1}(H) = \mathcal{P}(\mathcal{P}_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $\mathcal{P}_n^*(H)$, is defined recursively as:

$$\mathcal{P}_1^*(H) = \mathcal{P}^*(H), \quad \mathcal{P}_{n+1}^*(H) = \mathcal{P}^*(\mathcal{P}_n^*(H)).$$

Here, $\mathcal{P}^*(H)$ represents the powerset of H with the empty set removed.

Example 1.4 (Multi-UAV Fleet Configuration via the 3rd Powerset)**.** Let

$$H = \{\text{Navigation, Perception, Actuation}\}$$

denote the core control subsystems of an unmanned aerial vehicle (UAV). Then

$$\mathcal{P}_1(H) = \mathcal{P}(H)$$

is the set of all possible subsystem combinations, for example {Navigation, Perception} or {Perception, Actuation}, each representing a specific control mode.

The second powerset

$$\mathcal{P}_2(H) = \mathcal{P}\big(\mathcal{P}_1(H)\big)$$

collects all *vehicle configurations* as sets of subsystem-sets. For instance,

$$A = \big\{\{\text{Navigation, Perception}\},\ \{\text{Perception, Actuation}\}\big\} \in \mathcal{P}_2(H)$$

models a UAV capable of switching between navigation-perception and perception-actuation modes.

Finally, the third powerset

$$\mathcal{P}_3(H) = \mathcal{P}\big(\mathcal{P}_2(H)\big)$$

describes *fleet-level strategies* as collections of vehicle configurations. For example,

$$B = \big\{A,\ \{\{\text{Actuation}\}\}\big\} \in \mathcal{P}_3(H)$$

represents a two-UAV fleet with one dual-mode vehicle A and one actuator-only vehicle. This hierarchical construction via the n-th powerset enables systematic design and analysis of multi-level control architectures in engineering applications.

Definition 1.5 (Fuzzy Set)**.** [1,31] A *Fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \to [0,1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is called a *fuzzy relation on* τ if

$$\delta(y,z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Definition 1.6 (Hyperfuzzy Set)**.** [16,25,32–34] Let X be a non-empty universe. A *hyperfuzzy set* $\tilde{A}$ on X is defined by a mapping

$$\tilde{\mu} : X \longrightarrow \tilde{P}([0,1]),$$

where $\tilde{P}([0,1])$ denotes the collection of all non-empty subsets of the interval $[0,1]$.

For each element $x \in X$, $\tilde{\mu}(x) \subseteq [0,1]$ represents the *set of possible membership degrees* of x in the set $\tilde{A}$. This formulation allows for representing uncertainty or variability in the degree of membership, extending the classical fuzzy set (which assigns a single real number in $[0,1]$) to a set-valued interpretation.

Thus, a hyperfuzzy set captures both fuzziness and imprecision by associating each element not with a fixed degree, but with a range or subset of plausible membership values.

Example 1.7 (Urban Air Quality Assessment via a Hyperfuzzy Set)**.** Let

$$X = \{\text{Downtown, Suburb, IndustrialArea}\}$$

be three monitoring locations in a city. We evaluate "Good Air Quality" at each site using three different sensor types—$PM_{2.5}$, NO_2, and O_3 monitors—each calibrated to produce a fuzzy score in $[0, 1]$. We model their variability as a hyperfuzzy set $\tilde{Q}$ on X, defined by

$$\tilde{\mu}(x) = \{\, \text{PM}_{2.5}\text{-score, NO}_2\text{-score, O}_3\text{-score}\} \subseteq [0, 1].$$

Concretely,

$$\begin{aligned} \tilde{\mu}(\text{Downtown}) &= \{0.45,\ 0.50,\ 0.55\}, \\ \tilde{\mu}(\text{Suburb}) &= \{0.70,\ 0.75,\ 0.80\}, \\ \tilde{\mu}(\text{IndustrialArea}) &= \{0.30,\ 0.35,\ 0.40\}. \end{aligned}$$

Here each set of three values captures the range of sensor judgments on whether the air quality at x is "good," allowing us to represent both measurement imprecision and inter-sensor variability.

1.2 Neutrosophic Set

Neutrosophic Sets generalize Fuzzy Sets by introducing an additional component: indeterminacy, alongside truth and falsity [13, 35–37]. A hyperneutrosophic set assigns each element a nonempty set of neutrosophic triplets, modeling multiple truth, indeterminacy, falsity evaluations across methods [38, 39].

Definition 1.8 (Neutrosophic Set)**.** [40, 41] Let X be a non-empty set. A *Neutrosophic Set (NS) A* on X is characterized by three membership functions:

$$T_A : X \to [0, 1], \quad I_A : X \to [0, 1], \quad F_A : X \to [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Definition 1.9 (HyperNeutrosophic Set)**.** (cf. [17, 38, 42]) Let X be a non-empty set. A *HyperNeutrosophic Set (HNS) $\tilde{A}$* on X is a mapping:

$$\tilde{\mu} : X \to \mathcal{P}([0, 1]^3),$$

where $\mathcal{P}([0, 1]^3)$ is the family of all non-empty subsets of the unit cube $[0, 1]^3$. For each $x \in X$, $\tilde{\mu}(x) \subseteq [0, 1]^3$ is a set of neutrosophic membership triplets (T, I, F) that satisfy:

$$0 \le T + I + F \le 3.$$

Example 1.10 (Attitude Sensor Reliability via a HyperNeutrosophic Set)**.** Let

$$X = \{\text{Gyroscope, Accelerometer, Magnetometer}\}$$

be the set of three sensors in an unmanned aerial vehicle (UAV) attitude-control system. We evaluate each sensor's "high reliability" using three assessment methods—simulation, bench testing, and flight trials—and represent their neutrosophic judgments as a hyperneutrosophic set $\tilde{R}$ on X:

$$\tilde{\mu}(x) = \{\, (T, I, F) \mid (T, I, F) \text{ from each method}\} \ \subseteq\ [0, 1]^3.$$

Concretely,

$$\begin{aligned} \tilde{\mu}(\text{Gyroscope}) &= \{\, (0.85, 0.10, 0.05),\ (0.80, 0.15, 0.05),\ (0.88, 0.08, 0.04)\}, \\ \tilde{\mu}(\text{Accelerometer}) &= \{\, (0.75, 0.20, 0.05),\ (0.70, 0.25, 0.05),\ (0.78, 0.18, 0.04)\}, \\ \tilde{\mu}(\text{Magnetometer}) &= \{\, (0.65, 0.25, 0.10),\ (0.60, 0.30, 0.10),\ (0.68, 0.22, 0.10)\}. \end{aligned}$$

Here each triplet (T, I, F) gives the degrees of truth (reliability), indeterminacy (measurement ambiguity), and falsity (failure likelihood) from one evaluation method. This hyperneutrosophic set thus captures both the inherent imprecision and method-dependent uncertainty in sensor reliability for UAV attitude control.

1.3 Plithogenic Set

A Plithogenic Set is a mathematical framework that incorporates multi-valued degrees of appurtenance and contradictions, making it suitable for complex decision-making processes. Various studies have been conducted on Plithogenic Sets [15,43–46]. A hyperplithogenic set assigns each element hyper-valued attribute-value combinations to set-valued membership degrees, capturing inter-attribute contradictions and hierarchical uncertainty structures. The definition is presented below.

Definition 1.11 (Plithogenic Set)**.** [47,48] Let S be a universal set, and $P \subseteq S$. A *Plithogenic Set* PS is defined as:

$$PS = (P, v, Pv, pdf, pCF)$$

where:

- v is an attribute.
- Pv is the range of possible values for the attribute v.
- $pdf : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)* [1]
- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

These functions satisfy the following axioms for all $a, b \in Pv$:

1. *Reflexivity of Contradiction Function*:
$$pCF(a, a) = 0$$
2. *Symmetry of Contradiction Function*:
$$pCF(a, b) = pCF(b, a)$$

Example 1.12 (Control Algorithm Selection via a Plithogenic Set)**.** Let

$$S = \{\text{PID\_Basic, PID\_Adaptive, FuzzyLogic}\}, \quad P = S,$$

and consider the attribute

$$v = \text{SettlingTime}.$$

Its value domain is

$$Pv = \{\text{Short, Medium, Long}\}.$$

Define the Degree of Appurtenance Function $pdf : P \times Pv \to [0,1]$ by:

$$\begin{aligned} pdf(\text{PID\_Basic, Short}) &= 0.30, & pdf(\text{PID\_Basic, Medium}) &= 0.70, & pdf(\text{PID\_Basic, Long}) &= 0.40,\\ pdf(\text{PID\_Adaptive, Short}) &= 0.80, & pdf(\text{PID\_Adaptive, Medium}) &= 0.60, & pdf(\text{PID\_Adaptive, Long}) &= 0.20,\\ pdf(\text{FuzzyLogic, Short}) &= 0.65, & pdf(\text{FuzzyLogic, Medium}) &= 0.85, & pdf(\text{FuzzyLogic, Long}) &= 0.50. \end{aligned}$$

Next, define the Degree of Contradiction Function $pCF : Pv \times Pv \to [0,1]$ by:

$$pCF(a, b) = \begin{cases} 0, & a = b,\\ 0.6, & \{a, b\} = \{\text{Short, Medium}\},\\ 0.8, & \{a, b\} = \{\text{Medium, Long}\},\\ 1.0, & \{a, b\} = \{\text{Short, Long}\}. \end{cases}$$

One checks immediately the Plithogenic axioms:

[1] It is important to note that the definition of the Degree of Appurtenance Function varies across different papers. Some studies define this concept using the power set, while others simplify it by avoiding the use of the power set [49]. The author has consistently defined the Classical Plithogenic Set without employing the power set.

- Reflexivity: $pCF(a,a) = 0$ for all $a \in Pv$.
- Symmetry: $pCF(a,b) = pCF(b,a)$ by construction.

Thus $PS = (P, v, Pv, pdf, pCF)$ models how each control algorithm belongs to different settling-time categories, while quantifying contradictions between those categories in control-system design.

Definition 1.13 (HyperPlithogenic Set)**.** (cf. [17, 50, 51]) Let X be a non-empty set, and let A be a set of attributes. For each attribute $v \in A$, let Pv be the set of possible values of v. A *HyperPlithogenic Set* HPS over X is defined as:

$$HPS = (P, \{v_i\}_{i=1}^n, \{Pv_i\}_{i=1}^n, \{\tilde{pdf}_i\}_{i=1}^n, pCF)$$

where:

- $P \subseteq X$ is a subset of the universe.
- For each attribute v_i, Pv_i is the set of possible values.
- For each attribute v_i, $\tilde{pdf}_i : P \times Pv_i \to \tilde{P}([0,1]^s)$ is the *Hyper Degree of Appurtenance Function (HDAF)*, assigning to each element $x \in P$ and attribute value $a_i \in Pv_i$ a set of membership degrees.
- $pCF : \left(\bigcup_{i=1}^n Pv_i\right) \times \left(\bigcup_{i=1}^n Pv_i\right) \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

Example 1.14 (Control Strategy Selection via a HyperPlithogenic Set)**.** Let

$$X = \{\text{PID, LQR, MPC}\}, \quad P = \{\text{PID, MPC}\},$$

and consider two attributes:

$$v_1 = \text{SpeedResponse}, \quad Pv_1 = \{\text{Fast}, \text{Moderate}, \text{Slow}\},$$

$$v_2 = \text{Robustness}, \quad Pv_2 = \{\text{Low}, \text{Medium}, \text{High}\}.$$

Define the Hyper Degree of Appurtenance Functions $\tilde{pdf}_i : P \times Pv_i \to \tilde{P}([0,1])$ by

$$\tilde{pdf}_1(\text{PID}, \text{Fast}) = \{\{0.60, 0.70\}, \{0.75\}\}, \quad \tilde{pdf}_1(\text{PID}, \text{Moderate}) = \{\{0.50\}\},$$

$$\tilde{pdf}_1(\text{MPC}, \text{Fast}) = \{\{0.80, 0.85\}\}, \quad \tilde{pdf}_1(\text{MPC}, \text{Moderate}) = \{\{0.65, 0.70\}, \{0.75\}\},$$

$$\tilde{pdf}_2(\text{PID}, \text{Medium}) = \{\{0.55, 0.60\}, \{0.65\}\}, \quad \tilde{pdf}_2(\text{MPC}, \text{High}) = \{\{0.90\}, \{0.95\}\}.$$

The Degree of Contradiction Function $pCF : (Pv_1 \cup Pv_2) \times (Pv_1 \cup Pv_2) \to [0,1]$ is defined by:

$$pCF(a,a) = 0, \quad pCF(\text{Fast}, \text{Slow}) = 0.8, \quad pCF(\text{Low}, \text{High}) = 0.9,$$

with symmetry $pCF(a,b) = pCF(b,a)$. All other pairs receive moderate contradiction values (e.g. 0.5).

Thus

$$HPS = \left(P, \{v_1, v_2\}, \{Pv_1, Pv_2\}, \{\tilde{pdf}_1, \tilde{pdf}_2\}, pCF\right)$$

models how each control strategy belongs hyper-fuzzily to speed-response and robustness categories, while quantifying contradictions among attribute values in control-system design.

2 Review and Examples: (m,n)-superhyperfuzzy set

An (m,n)-superhyperfuzzy set maps each m-level subset to a family of n-level membership sets, modeling hierarchical recursive uncertainty and complexity.

Definition 2.1 (m, n-SuperHyperFuzzy Set)**.** (cf. [17,42]) Let X be a nonempty set and let $m, n \in \mathbb{N}_0$. Define the nonempty k-th powerset of a set Y by

$$\mathcal{P}_0^*(Y) = Y, \quad \mathcal{P}_k^*(Y) = \mathcal{P}\big(\mathcal{P}_{k-1}^*(Y)\big) \setminus \{\emptyset\}, \quad k \geq 1.$$

In particular, $\mathcal{P}_m^*(X)$ is the family of all nonempty elements of the m-th iterated powerset of X, and $\mathcal{P}_n^*([0,1])$ is defined analogously. Then an (m, n)-SuperHyperFuzzy Set on X is a function

$$\tilde{\mu}_{m,n} : \mathcal{P}_m^*(X) \longrightarrow \tilde{\mathcal{P}}_n^*\big([0,1]\big), \quad A \mapsto \tilde{\mu}_{m,n}(A),$$

where $\tilde{\mathcal{P}}_n^*([0,1])$ denotes the collection of all nonempty subsets of $\mathcal{P}_n([0,1])$. Thus each $A \in \mathcal{P}_m^*(X)$ is assigned a nonempty family of membership-degree sets $\tilde{\mu}_{m,n}(A) \subseteq \mathcal{P}_n([0,1])$, capturing hierarchical uncertainty across both the m- and n-levels.

Example 2.2 (Engine Temperature Monitoring via a $(1,2)$-SuperHyperFuzzy Set)**.** Let

$$X = \{\text{Sensor}_1,\ \text{Sensor}_2,\ \text{Sensor}_3\}$$

be the set of three coolant-temperature sensors in an engine control system. Fix $m = 1$, $n = 2$. Then $\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$, and consider the sensor pair

$$A = \{\text{Sensor}_1, \text{Sensor}_2\} \in \mathcal{P}_1^*(X).$$

We define the $(1,2)$-superhyperfuzzy membership mapping

$$\tilde{\mu}_{1,2} \colon \mathcal{P}_1^*(X) \longrightarrow \tilde{\mathcal{P}}_2^*([0,1]),$$

by assigning to A the family

$$\tilde{\mu}_{1,2}(A) = \Big\{L_1 = \{\{0.80, 0.85\},\ \{0.90\}\}, \quad L_2 = \{\{0.75\},\ \{0.88, 0.92\}\}\Big\} \subseteq \tilde{\mathcal{P}}_2^*([0,1]).$$

Here:

- Each inner set—for example $\{0.80, 0.85\}$—collects membership degrees from two calibration runs (bench vs. simulation).
- The singleton $\{0.90\}$ is from a high-accuracy laboratory test.
- L_1 thus captures "controlled-environment" evaluations, while
- $L_2 = \{\{0.75\}, \{0.88, 0.92\}\}$ groups "in-field" measurements (road test vs. heat-soak test).

This $(1,2)$-SuperHyperFuzzy Set models one level of sensor grouping ($m = 1$) and two nested levels of membership-uncertainty ($n = 2$) in engine temperature monitoring.

Example 2.3 (ECU Fault Diagnosis via a $(3,2)$-SuperHyperFuzzy Set)**.** Let

$$X = \{\text{S1},\ \text{S2},\ \text{S3},\ \text{S4}\}, \quad m = 3, \quad n = 2.$$

Then $\mathcal{P}_1^*(X)$ contains all nonempty sensor subsets, $\mathcal{P}_2^*(X)$ all nonempty subsystem groupings, and $\mathcal{P}_3^*(X)$ all nonempty collections of those groupings. Choose two level-2 groupings:

$$P_{2a} = \{\{\text{S1}, \text{S2}\},\ \{\text{S3}\}\}, \quad P_{2b} = \{\{\text{S2}, \text{S4}\}\},$$

and form the level-3 element

$$A = \{P_{2a},\ P_{2b}\} \in \mathcal{P}_3^*(X).$$

Define the $(3,2)$-superhyperfuzzy membership mapping

$$\tilde{\mu}_{3,2} \colon \mathcal{P}_3^*(X) \longrightarrow \tilde{\mathcal{P}}_2^*([0,1])$$

by

$$\tilde{\mu}_{3,2}(A) = \big\{L_1 = \{\{0.70, 0.75\}, \{0.80\}\},\ L_2 = \{\{0.65\}, \{0.85, 0.90\}\}\big\}.$$

Here:

- Each inner set, e.g. $\{0.70, 0.75\}$, aggregates fuzzy fault-likelihood scores from simulation vs. bench tests.
- The singleton $\{0.80\}$ is from high-precision lab calibration.
- L_1 thus captures the "design-phase" evaluation scenario.
- L_2 groups $\{0.65\}$ field-trial scores and $\{0.85, 0.90\}$ expert-review judgments.

This $(3, 2)$-SuperHyperFuzzy Set models a three-level sensor-to-subsystem-to-system hierarchy ($m = 3$) with two nested levels of fuzzy uncertainty ($n = 2$) in automotive ECU fault diagnosis.

3 Review and Examples: (m, n)-superhyperneutrosophic set

A (m, n)-superhyperneutrosophic set maps each m-level subset to hierarchical nested families of truth, indeterminacy, and falsity triplets across n levels.

Definition 3.1 ((m, n)-SuperHyperNeutrosophic Set)**.** Let X be a nonempty set and $m, n \in \mathbb{N}_0$. Define recursively the nonempty k-th powerset of a set Y by

$$\mathcal{P}_0^*(Y) = Y, \qquad \mathcal{P}_k^*(Y) = \mathcal{P}\big(\mathcal{P}_{k-1}^*(Y)\big) \setminus \{\emptyset\}, \quad k \geq 1.$$

Let $\mathcal{P}_m^*(X)$ be the family of all nonempty elements of the m-th iterated powerset of X, and let $\mathcal{P}_n^*([0, 1]^3)$ be defined analogously on the unit cube. An (m, n)–*superhyperneutrosophic set* on X is a mapping

$$\tilde{\nu}_{m,n} : \mathcal{P}_m^*(X) \longrightarrow \tilde{\mathcal{P}}_n^*\big([0, 1]^3\big), \quad A \mapsto \tilde{\nu}_{m,n}(A),$$

where $\tilde{\mathcal{P}}_n^*([0, 1]^3)$ denotes the collection of all nonempty subsets of $\mathcal{P}_n([0, 1]^3)$. Thus each $A \in \mathcal{P}_m^*(X)$ is assigned a nonempty family of n-level neutrosophic-membership sets $\tilde{\nu}_{m,n}(A) \subseteq \mathcal{P}_n([0, 1]^3)$.

Example 3.2 (Medical Diagnosis Confidence via a $(1, 2)$-SuperHyperNeutrosophic Set)**.** Let

$$X = \{\text{BloodTest, MRI, Questionnaire}\}, \quad m = 1, \quad n = 2.$$

Then $\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$, and consider the symptom-group

$$A = \{\text{BloodTest, MRI}\} \in \mathcal{P}_1^*(X).$$

We assign to A a family of second-level neutrosophic membership sets:

$$\tilde{\nu}_{1,2}(A) = \big\{L_1 = \{\, (T, I, F) \in [0, 1]^3 \mid (0.80, 0.10, 0.10),\ (0.85, 0.10, 0.05)\},$$

$$L_2 = \{\, (T, I, F) \in [0, 1]^3 \mid (0.75, 0.15, 0.10),\ (0.90, 0.05, 0.05)\}\big\} \subseteq \tilde{\mathcal{P}}_2^*([0, 1]^3).$$

Here each (T, I, F) triple represents one evaluation scenario:

- In L_1, $(0.80, 0.10, 0.10)$ comes from lab-based predictive modeling, and $(0.85, 0.10, 0.05)$ from expert panel consensus.
- In L_2, $(0.75, 0.15, 0.10)$ is derived from population-level statistical analysis, and $(0.90, 0.05, 0.05)$ from high-precision imaging interpretation.

Thus this $(1, 2)$-superhyperneutrosophic set captures one level of symptom grouping ($m = 1$) and two nested levels of neutrosophic uncertainty ($n = 2$) in medical diagnosis confidence.

Example 3.3 (Supply Chain Reliability via a $(2, 2)$-SuperHyperNeutrosophic Set)**.** Let

$$X = \{\text{FactoryA, FactoryB, FactoryC}\}, \quad m = 2, \quad n = 2.$$

Then $\mathcal{P}_2^*(X) = \mathcal{P}(\mathcal{P}^*(X)) \setminus \{\emptyset\}$, and one representative element is

$$A = \big\{\{\text{FactoryA, FactoryB}\},\ \{\text{FactoryB, FactoryC}\}\big\},$$

modeling overlapping production clusters. We assign

$$\tilde{\nu}_{2,2}(A) = \Big\{L_1,\ L_2\Big\} \ \subseteq\ \tilde{\mathcal{P}}_2^*([0,1]^3),$$

where

$$L_1 = \big\{\{(0.80, 0.10, 0.10),\ (0.85, 0.05, 0.10)\},\ \{(0.75, 0.15, 0.10)\}\big\},$$
$$L_2 = \big\{\{(0.90, 0.05, 0.05)\},\ \{(0.70, 0.20, 0.10),\ (0.65, 0.25, 0.10)\}\big\}.$$

Here each innermost set contains two neutrosophic triples (T, I, F):

- In L_1, the first set's triples come from *simulation-based reliability* scenarios, and the second set from *historical-failure-rate* analyses.
- In L_2, the first singleton triple is from *real-time sensor fusion*, while the second set's triples derive from *third-party audit* and *customer feedback*.

Thus this $(2, 2)$-superhyperneutrosophic set captures two-level supplier groupings ($m = 2$) and two nested neutrosophic uncertainty levels ($n = 2$) in supply chain reliability assessment.

Example 3.4 (Investment Portfolio Robustness via a $(1, 3)$-SuperHyperNeutrosophic Set)**.** Let

$$X = \{\text{Stocks, Bonds, Commodities}\}, \quad m = 1, \quad n = 3.$$

Then $\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$, and consider the asset-bundle

$$A = \{\text{Stocks, Bonds}\} \ \in\ \mathcal{P}_1^*(X).$$

We assign to A a family of third-level neutrosophic membership sets:

$$\tilde{\nu}_{1,3}(A) = \big\{\, M_1,\ M_2\big\} \ \subseteq\ \tilde{\mathcal{P}}_3^*([0,1]^3),$$

where

$$M_1 = \big\{\{\,(0.70, 0.20, 0.10),\ (0.75, 0.15, 0.10)\},\ \{\,(0.65, 0.25, 0.10)\}\big\},$$
$$M_2 = \big\{\{\,(0.80, 0.10, 0.10)\},\ \{\,(0.85, 0.05, 0.05),\ (0.90, 0.03, 0.07)\}\big\}.$$

Here each triple (T, I, F) quantifies:

- T: degree of expected return confidence,
- I: indeterminacy due to market volatility,
- F: degree of downside risk.

Specifically,

- In M_1, the first inner set's triples come from *macroeconomic scenario analysis*, and the second from *interest-rate shock stress test*.
- In M_2, the first singleton triple is derived from *quantitative momentum strategy*, while the second set's triples reflect *liquidity-crunch simulations* and *counterparty default stress tests*.

Thus this $(1, 3)$-superhyperneutrosophic set models one level of asset grouping ($m = 1$) and three nested levels of neutrosophic uncertainty ($n = 3$) in portfolio robustness assessment.

Theorem 3.5 (Recovery of HyperNeutrosophic Sets)**.** *If $m = 0$ and $n = 1$, then any (m, n)-superhyperneutrosophic set $\tilde{\nu}_{m,n}$ reduces exactly to a hyperneutrosophic set*

$$\tilde{\nu}_{0,1} : X \to \tilde{P}([0,1]^3),$$

assigning to each $x \in X$ a nonempty subset of neutrosophic triples (T, I, F).

Proof. Setting $m = 0$ gives $\mathcal{P}_0^*(X) = X$. Likewise, $n = 1$ yields $\mathcal{P}_1^*([0,1]^3) = \mathcal{P}([0,1]^3) \setminus \{\emptyset\}$. Hence

$$\tilde{\nu}_{0,1} : X \to \tilde{\mathcal{P}}_1^*([0,1]^3) = \{ S \subseteq [0,1]^3 \mid S \neq \emptyset \},$$

which is precisely the definition of a hyperneutrosophic set (mapping each element to a set of neutrosophic membership triplets). □

Theorem 3.6 (Reduction to (m,n)-SuperHyperfuzzy Sets)**.** *Suppose in an (m,n)-superhyperneutrosophic set $\tilde{\nu}_{m,n}$ each neutrosophic membership set $\tilde{\nu}_{m,n}(A)$ contains only triples of the form $(t,0,0)$. Then $\tilde{\nu}_{m,n}$ induces an (m,n)-superhyperfuzzy set*

$$\tilde{\mu}_{m,n} : \mathcal{P}_m^*(X) \to \tilde{\mathcal{P}}_n^*([0,1]), \quad A \mapsto \{ t \mid (t,0,0) \in S \text{ for some } S \in \tilde{\nu}_{m,n}(A) \}.$$

Proof. If every element of each $S \in \tilde{\nu}_{m,n}(A) \subseteq \mathcal{P}_n([0,1]^3)$ is a set of triples $(t,0,0)$, then projecting each triple onto its truth-component t defines a mapping

$$\pi : \mathcal{P}_n([0,1]^3) \to \mathcal{P}_n([0,1]), \quad S \mapsto \{ t \mid (t,0,0) \in S \}.$$

Since π preserves nonemptiness and compactness, composing $\tilde{\nu}_{m,n}$ with π yields $\tilde{\mu}_{m,n} = \pi \circ \tilde{\nu}_{m,n} : \mathcal{P}_m^*(X) \to \tilde{\mathcal{P}}_n^*([0,1])$, which is exactly an (m,n)-superhyperfuzzy set. □

Theorem 3.7 (Classical Neutrosophic Set as Special Case)**.** *If $m = 0$, $n = 0$, and each $\tilde{\nu}_{0,0}(x) = \{(T,I,F)\}$ is a singleton for all $x \in X$, then the (m,n)-superhyperneutrosophic set reduces to a classical neutrosophic set*

$$A : X \to [0,1]^3, \quad x \mapsto (T(x), I(x), F(x)).$$

Proof. With $m = n = 0$, we have $\mathcal{P}_0^*(X) = X$ and $\mathcal{P}_0^*([0,1]^3) = [0,1]^3$. If each $\tilde{\nu}_{0,0}(x)$ is a singleton $\{(T,I,F)\}$, then defining

$$A(x) = (T,I,F) \quad \text{whenever } \tilde{\nu}_{0,0}(x) = \{(T,I,F)\}$$

yields a classical neutrosophic set on X. □

4 Review and Examples: (m,n)-superhyperplithogenic Set

An (m,n)-SuperHyperPlithogenic Set assigns hierarchical attribute-value subsets and multi-level membership plus contradiction degrees for complex decision modeling.

Definition 4.1 ((m,n)-SuperHyperPlithogenic Set)**.** Let X be a nonempty universe, $V = \{v_1, \ldots, v_\ell\}$ a finite attribute set, and for each v_i let P_{v_i} be its value domain. Fix $m,n \in \mathbb{N}_0$ and positive integers s,t. Define the nonempty k-th powerset $\mathcal{P}_k^*(Y)$ recursively by

$$\mathcal{P}_0^*(Y) = Y, \quad \mathcal{P}_k^*(Y) = \mathcal{P}(\mathcal{P}_{k-1}^*(Y)) \setminus \{\emptyset\}.$$

Then an (m,n)–*SuperHyperPlithogenic Set* is a structure

$$SHPS_{m,n} = \big(P_m,\ V,\ \{P_{v_i}\}_{i=1}^{\ell},\ \{\tilde{pdf}_i^{(m,n)}\}_{i=1}^{\ell},\ pCF^{(m,n)}\big),$$

where:

- $P_1 \subseteq X$, and for $k \geq 2$, $P_k = \mathcal{P}_1^*(P_{k-1})$. In particular $P_m \subseteq \mathcal{P}_m^*(X)$.
- Each $\tilde{pdf}_i^{(m,n)} : P_m \times P_{v_i} \to \tilde{\mathcal{P}}_n^*([0,1]^s)$ is the *superhyper-degree of appurtenance*, assigning to each (A,a) a nonempty family of n-level membership-vectors in $[0,1]^s$.
- The *superhyper-degree of contradiction*

$$pCF^{(m,n)} : \Big(\bigcup_i P_{v_i}\Big) \times \Big(\bigcup_i P_{v_i}\Big) \longrightarrow \tilde{\mathcal{P}}_n^*([0,1]^t)$$

  satisfies for all a,b:

$$pCF^{(m,n)}(a,a) = \{0\}, \qquad pCF^{(m,n)}(a,b) = pCF^{(m,n)}(b,a).$$

Example 4.2 (Smartphone Selection via a $(1, 2)$-SuperHyperPlithogenic Set)**.** Let

$$X = \{\mathrm{A},\ \mathrm{B},\ \mathrm{C}\}, \quad m = 1, \quad n = 2, \quad V = \{\mathrm{Price},\ \mathrm{Battery},\ \mathrm{Camera}\}.$$

Then $\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$, and choose the candidate pair

$$A = \{\mathrm{A}, \mathrm{B}\} \ \in \ \mathcal{P}_1^*(X).$$

Define the attribute domains:

$$P_{\mathrm{Price}} = \{\mathrm{Low}, \mathrm{Medium}, \mathrm{High}\}, \quad P_{\mathrm{Battery}} = \{\mathrm{Short}, \mathrm{Medium}, \mathrm{Long}\}, \quad P_{\mathrm{Camera}} = \{\mathrm{Std}, \mathrm{Good}, \mathrm{Excellent}\}.$$

The superhyper-degree of appurtenance functions $\tilde{pdf}_i^{(1,2)} : P_1 \times P_{v_i} \to \tilde{\mathcal{P}}_2^*([0, 1])$ are specified by:

$$\tilde{pdf}^{(1,2)}_{\mathrm{Price}}(A, \mathrm{Low}) = \{\{0.85,\ 0.90\},\ \{0.80\}\}, \quad \tilde{pdf}^{(1,2)}_{\mathrm{Price}}(A, \mathrm{Medium}) = \{\{0.60\},\ \{0.65,\ 0.70\}\},$$

$$\tilde{pdf}^{(1,2)}_{\mathrm{Battery}}(A, \mathrm{Long}) = \{\{0.75,\ 0.80\},\ \{0.85\}\}, \quad \tilde{pdf}^{(1,2)}_{\mathrm{Camera}}(A, \mathrm{Excellent}) = \{\{0.70\},\ \{0.75,\ 0.80\}\}.$$

The superhyper-degree of contradiction

$$pCF^{(1,2)} : \Big(\bigcup_i P_{v_i}\Big) \times \Big(\bigcup_i P_{v_i}\Big) \ \longrightarrow \ \tilde{\mathcal{P}}_2^*([0, 1])$$

satisfies reflexivity $pCF^{(1,2)}(a, a) = \{0\}$ and symmetry, for example:

$$pCF^{(1,2)}(\mathrm{Low}, \mathrm{High}) = \{\{0.40\},\ \{0.45,\ 0.50\}\}.$$

Thus this $(1, 2)$-SuperHyperPlithogenic Set models

- one-level grouping of smartphone models ($m = 1$),
- two nested levels of appurtenance uncertainty,
- two nested levels of attribute contradiction.

Example 4.3 (Catering Vendor Selection via a $(2, 3)$-SuperHyperPlithogenic Set)**.** Let

$$X = \{\mathrm{V1},\ \mathrm{V2},\ \mathrm{V3}\}, \quad m = 2, \quad n = 3, \quad V = \{\mathrm{Cost},\ \mathrm{Quality},\ \mathrm{Reliability}\}.$$

Then $\mathcal{P}_2^*(X) = \mathcal{P}(\mathcal{P}^*(X)) \setminus \{\emptyset\}$, and choose the vendor cluster

$$A = \big\{\{\mathrm{V1}, \mathrm{V2}\}, \{\mathrm{V2}, \mathrm{V3}\}\big\} \ \in \ \mathcal{P}_2^*(X).$$

Define attribute domains:

$$P_{\mathrm{Cost}} = \{\mathrm{Low}, \mathrm{Medium}, \mathrm{High}\},$$

$$P_{\mathrm{Quality}} = \{\mathrm{Acceptable}, \mathrm{Good}, \mathrm{Superior}\},$$

$$P_{\mathrm{Reliability}} = \{\mathrm{Uncertain}, \mathrm{Reliable}, \mathrm{HighlyReliable}\}.$$

The superhyper-degree of appurtenance functions $\tilde{pdf}_i^{(2,3)} : P_2 \times P_{v_i} \to \tilde{\mathcal{P}}_3^*([0, 1])$ are specified by, for example:

$$\tilde{pdf}^{(2,3)}_{\mathrm{Cost}}(A, \mathrm{Low}) = \big\{L_1 = \{\{\{0.70, 0.75\}, \{0.80\}\}, \{\{0.85\}\}\}, L_2 = \{\{\{0.65\}, \{0.78, 0.82\}\}, \{\{0.90\}\}\}\big\},$$

$$\tilde{pdf}^{(2,3)}_{\mathrm{Quality}}(A, \mathrm{Superior}) = \big\{Q_1 = \{\{\{0.80, 0.85\}, \{0.90\}\}, \{\{0.95\}\}\}, Q_2 = \{\{\{0.88\}, \{0.92, 0.94\}\}, \{\{0.98\}\}\}\big\},$$

$$\tilde{pdf}^{(2,3)}_{\mathrm{Reliability}}(A, \mathrm{Reliable}) = \big\{R_1 = \{\{\{0.75\}, \{0.80, 0.82\}\}, \{\{0.85\}\}\}, R_2 = \{\{\{0.88\}, \{0.90, 0.93\}\}, \{\{0.96\}\}\}\big\}.$$

The superhyper-degree of contradiction

$$pCF^{(2,3)} : \Big(\bigcup_i P_{v_i}\Big) \times \Big(\bigcup_i P_{v_i}\Big) \ \longrightarrow \ \tilde{\mathcal{P}}_3^*([0, 1])$$

satisfies reflexivity and symmetry, for instance:

$$pCF^{(2,3)}(\mathrm{Low}, \mathrm{High}) = \big\{\{\{0.40\}, \{0.45, 0.50\}\}, \{\{0.55\}\}\big\},$$

$$pCF^{(2,3)}(\mathrm{Good}, \mathrm{Acceptable}) = \big\{\{\{0.30\}, \{0.35, 0.38\}\}, \{\{0.40\}\}\big\}.$$

Thus this $(2, 3)$-SuperHyperPlithogenic Set models:

- Two-level grouping of vendors ($m = 2$),
- Three nested levels of membership uncertainty ($n = 3$) for cost, quality, and reliability,
- Three nested levels of attribute contradiction to capture conflicting criteria.

Example 4.4 (Job Candidate Assessment via a $(1, 2)$-SuperHyperPlithogenic Set)**.** Let

$$X = \{\text{Alice, Bob, Carol}\}, \quad m = 1, \ n = 2, \quad V = \{\text{Experience, Education, SkillFit}\}.$$

Then $\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$, and consider the candidate pair

$$A = \{\text{Alice, Carol}\} \ \in \ \mathcal{P}_1^*(X).$$

Define attribute value domains:

$$P_{\text{Experience}} = \{\text{Junior, Mid, Senior}\}, \quad P_{\text{Education}} = \{\text{BSc, MSc, PhD}\}, \quad P_{\text{SkillFit}} = \{\text{Low, Medium, High}\}.$$

The superhyper-degree of appurtenance functions $\tilde{pdf}_i^{(1,2)} : P_1 \times P_{v_i} \to \tilde{\mathcal{P}}_2^*([0, 1])$ are given by:

$$\tilde{pdf}_{\text{Experience}}^{(1,2)}(A, \text{Senior}) = \big\{L_1 = \{\{0.80, 0.85\}, \{0.90\}\}, \ L_2 = \{\{0.75\}, \{0.88, 0.92\}\}\big\},$$

$$\tilde{pdf}_{\text{Education}}^{(1,2)}(A, \text{PhD}) = \big\{E_1 = \{\{0.70, 0.75\}, \{0.80\}\}, \ E_2 = \{\{0.78\}, \{0.85, 0.88\}\}\big\},$$

$$\tilde{pdf}_{\text{SkillFit}}^{(1,2)}(A, \text{High}) = \big\{S_1 = \{\{0.85, 0.88\}, \{0.90\}\}, \ S_2 = \{\{0.82\}, \{0.93, 0.95\}\}\big\}.$$

The superhyper-degree of contradiction

$$pCF^{(1,2)} : \Big(\bigcup_i P_{v_i}\Big) \times \Big(\bigcup_i P_{v_i}\Big) \ \longrightarrow \ \tilde{\mathcal{P}}_2^*([0, 1])$$

satisfies reflexivity and symmetry, for example:

$$pCF^{(1,2)}(\text{Junior}, \text{Senior}) = \{\{0.30\}, \{0.35, 0.40\}\}, \quad pCF^{(1,2)}(\text{BSc}, \text{PhD}) = \{\{0.20\}, \{0.25, 0.30\}\}.$$

Thus this $(1, 2)$-SuperHyperPlithogenic Set models:

- One-level grouping of candidates ($m = 1$),
- Two nested levels of membership uncertainty in experience, education, and skill-fit,
- Two levels of attribute contradiction capturing conflicts between values.

Theorem 4.5 (Unification of HyperPlithogenic, SuperHyperfuzzy, SuperHyperneutrosophic)**.** *The (m, n)-SuperHyperPlithogenic Set $SHPS_{m,n}$ specializes to:*

(i) ***HyperPlithogenic Set*** *when $m = 0, n = 1$ (superhyper reduces to hyper) and $s = 1, t = 1$.*

(ii) *(m, n)-**SuperHyperfuzzy Set** when $\ell = 1$, $P_{v_1} = [0, 1]$, $s = 1$, and $pCF^{(m,n)}$ is trivial.*

(iii) *(m, n)-**SuperHyperNeutrosophic Set** when $\ell = 1$, $P_{v_1} = [0, 1]^3$, $s = 3$, $t = 1$, and $pCF^{(m,n)}$ trivial.*

Proof. *(i) HyperPlithogenic:* If $m = 0$, then $P_0 = X$. If $n = 1$, $\tilde{\mathcal{P}}_1^*([0, 1]^s) = \tilde{P}([0, 1]^s)$, recovering the HDAF of a HyperPlithogenic Set. Taking $s = t = 1$ yields scalar degrees.

(ii) SuperHyperfuzzy: With a single attribute v_1, $P_{v_1} = [0, 1]$, and requiring each vector in $[0, 1]^s$ to be scalar ($s = 1$), the mapping $\tilde{pdf}_1^{(m,n)} : P_m \times [0, 1] \to \tilde{\mathcal{P}}_n^*([0, 1])$ coincides with the (m, n)-superhyperfuzzy membership $\tilde{\mu}_{m,n}$. A trivial contradiction function (always $\{0\}$) imposes no further structure.

(iii) SuperHyperneutrosophic: Similarly, with one attribute whose values are neutrosophic triples $P_{v_1} = [0, 1]^3$ and $s = 3$, the same $\tilde{pdf}_1^{(m,n)}$ becomes $\tilde{\nu}_{m,n}$. Setting $t = 1$ and trivial pCF yields the (m, n)-superhyperneutrosophic definition. □

Theorem 4.6 (Reduction to Classical Plithogenic Sets)**.** *Let* $SHPS_{m,n} = (P_m, V, \{P_{v_i}\}, \{\tilde{pdf}_i^{(m,n)}\}, pCF^{(m,n)})$ *be an* (m, n)*-SuperHyperPlithogenic Set. If* $m = 0$ *and* $n = 1$*, then*

$$P_0 = X, \quad \tilde{\mathcal{P}}_1^*([0,1]^s) = [0,1]^s,$$

and each $\tilde{pdf}_i^{(0,1)}$ *becomes a classical Degree of Appurtenance Function* $pdf_i \colon X \times P_{v_i} \to [0,1]^s$*. Moreover,* $pCF^{(0,1)}$ *is a classical Degree of Contradiction Function. Hence*

$$SHPS_{0,1} \cong (X, V, \{P_{v_i}\}, \{pdf_i\}, pCF),$$

i.e. an ordinary Plithogenic Set.

Proof. By definition $P_1 \subseteq X$ and for $m = 0$ we have $P_0 = X$. Also $\mathcal{P}_0^*(X) = X$ and $\tilde{\mathcal{P}}_1^*([0,1]^s)$ is precisely the family of nonempty subsets of $\mathcal{P}_1([0,1]^s) = \mathcal{P}([0,1]^s)$, which collapses to singletons when $n = 1$. Thus each superhyper-degree $\tilde{pdf}_i^{(0,1)}$ assigns to $(x, a) \in X \times P_{v_i}$ a singleton $\{v\} \subseteq [0,1]^s$, identified with the scalar v. Likewise $pCF^{(0,1)}$ assigns single-valued contradictions. These coincide exactly with the classical Plithogenic DAF and DCF. □

Theorem 4.7 (Monotonicity in m and n)**.** *Let* $m_1 \leq m_2$ *and* $n_1 \leq n_2$*. Then any* (m_1, n_1)*-SuperHyperPlithogenic Set* $SHPS_{m_1,n_1}$ *naturally embeds into an* (m_2, n_2)*-SuperHyperPlithogenic Set* $SHPS_{m_2,n_2}$ *by restricting the domain and inflating membership families:*

$$P_{m_1} \subseteq P_{m_2}, \quad \tilde{\mathcal{P}}_{n_1}^*([0,1]^s) \subseteq \tilde{\mathcal{P}}_{n_2}^*([0,1]^s).$$

Proof. Since $m_1 \leq m_2$, the iterated powersets satisfy $\mathcal{P}_{m_1}^*(X) \subseteq \mathcal{P}_{m_2}^*(X)$, so $P_{m_1} \subseteq P_{m_2}$. Likewise $\mathcal{P}_{n_1}([0,1]^s) \subseteq \mathcal{P}_{n_2}([0,1]^s)$ implies $\tilde{\mathcal{P}}_{n_1}^*([0,1]^s) \subseteq \tilde{\mathcal{P}}_{n_2}^*([0,1]^s)$. Therefore we can define an embedding

$$\iota \colon SHPS_{m_1,n_1} \hookrightarrow SHPS_{m_2,n_2}$$

by sending each $(A, a) \mapsto (\iota(A), a)$ and each family of n_1-level membership sets to the same family viewed in the larger n_2-level codomain. Contradiction degrees extend similarly. This map preserves all Plithogenic axioms, establishing the embedding. □

5 Conclusion

In this paper, we reviewed the concepts of (m, n)-SuperHyperfuzzy Sets, (m, n)-SuperHyperneutrosophic Sets, and (m, n)-SuperHyperPlithogenic Sets, and illustrated their utility through several concrete examples. Future work will develop efficient algorithms for these structures and explore their applications across various engineering and decision-making domains.

Funding

This study did not receive any financial or external support from organizations or individuals.

Acknowledgments

We extend our sincere gratitude to everyone who provided insights, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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