Advanced Partitioned Neutrosophic Off/Over/Under Sets: Formalization of Hexa-, Hepta-, Octa-, Nona-, and Deca-Partitioned Structures

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Abstract

Modern uncertainty-modeling frameworks—fuzzy sets, intuitionistic fuzzy sets, hyperfuzzy sets, neutrosophic sets, and plithogenic sets—provide powerful tools for capturing vagueness and imprecision. In particular, neutrosophic sets characterize each element by three independent degrees: truth, indeterminacy, and falsity. Classical neutrosophic sets have been refined by partitioning the membership degrees into additional components. Recently, the *Hexapartitioned Neutrosophic Set*, *Octapartitioned Neutrosophic Set*, *Nonapartitioned Neutrosophic Set*, and *Decapartitioned Neutrosophic Set* have been defined. In this paper, we introduce four novel partitioned models—*hexapartitioned*, *octapartitioned*, *nonapartitioned*, and *decapartitioned* neutrosophic offsets/oversets/undersets—and show how each can be seamlessly embedded into the plithogenic-set framework.

Keywords: Neutrosophic Set, Hexapartitioned Neutrosophic Set, Octapartitioned Neutrosophic Set, Nonapartitioned Neutrosophic Set, Decapartitioned neutrosophic set, Neutrosophic Offset

1 Introduction

1.1 Fuzzy and Neutrosophic Set Theory

Real-world problems often involve uncertainty, which has driven the creation of numerous mathematical tools to represent imprecision. Foundational models include *fuzzy sets* [1, 2] and *intuitionistic fuzzy sets* [3–5]. Building on these, researchers have proposed a variety of extensions—such as *vague sets* [6, 7], *bipolar fuzzy sets* [8–10], *hesitant fuzzy sets* [11–13], *picture fuzzy sets* [14–16], Pythagorean fuzzy sets [17–19], hyperfuzzy sets [20–25], q-Rung Orthopair fuzzy sets [26–28], and m*-polar fuzzy sets* [29–31].

Extending beyond these constructs, *neutrosophic sets* [32–36] assign three independent membership values—truth, indeterminacy, and falsity—each ranging over $[0, 1]$ and collectively bounded by 3. Further generalizations, including *hyperneutrosophic sets* [37–39], *bipolar neutrosophic set* [40–42], and *pythagorean Neutrosophic Set* [43–45] have expanded the modeling capacity of neutrosophic theory.

These techniques have found applications in diverse areas such as graph theory, control theory, chemistry, topology, algebra, and decision science [46–49]. Consequently, ongoing investigation into fuzzy, neutrosophic, and related set-based frameworks remains of great significance.

1.2 Partitioned Neutrosophic Frameworks

Classical neutrosophic sets have been refined by partitioning the membership degrees into additional components. In a *quadripartitioned set*, a dedicated contradiction parameter is introduced so that the sum of all four degrees does not exceed 4 [50–53]. A *pentapartitioned set* further incorporates an unknown component, capping the total of five membership measures at 5 [54–56]. Extending this idea, *heptapartitioned sets* subdivide truth and falsity into relative truth and relative falsity, resulting in seven degrees whose sum is bounded by 7 [57–59].

Beyond these, researchers have also introduced *double-valued neutrosophic sets* [60–63] and *triple-valued neutrosophic sets* [64–66]. More recently, the *Hexapartitioned Neutrosophic Set*, *Octapartitioned Neutrosophic Set*, *Nonapartitioned Neutrosophic Set*, and *Decapartitioned Neutrosophic Set* have been defined [67]. Each of these advanced structures can be viewed as a special case of the general *Plithogenic Set* framework.

1.3 Offset, Overset, and Underset Extensions

Conventional uncertain-set frameworks confine membership degrees to the interval $[0, 1]$. To accommodate values outside this range, the notions of *offset*, *overset*, and *underset* have been proposed [68–71]. In an *offset* extension, membership values may drop below 0 or rise above 1, thereby capturing negative or excessive degrees [34, 72–74]. An *overset* allows membership to exceed 1 while remaining nonnegative [75–77], whereas an *underset* permits membership to fall below 0 but never exceed 1 [34, 78]. Such extensions are crucial when raw measurements or expert judgments initially lie outside the normalized range, since they allow models to incorporate these out-of-band values without prior rescaling. Directly handling these extended degrees leads to more faithful representations in applications like risk assessment and information fusion.

Moreover, recent work has introduced the *Quadripartitioned Offset*, *Pentapartitioned Offset*, and *Heptapartitioned Offset* as the offset analogues of the Quadripartitioned, Pentapartitioned, and Heptapartitioned sets, respectively [79].

1.4 Our Contribution

Neutrosophic offsets have gained considerable importance due to their wide range of applications, and many extended set-theoretic frameworks have been studied. However, research on XXXX-partitioned neutrosophic offsets remains incomplete. Looking ahead, a deeper investigation into various generalized neutrosophic-set structures is warranted. In this paper, we introduce four novel partitioned models—*hexapartitioned*, *octapartitioned*, *nonapartitioned*, and *decapartitioned* neutrosophic offsets/oversets/undersets—and show how each can be seamlessly embedded into the plithogenic-set framework.

1.5 Contents in This Paper

The format of this paper is described below.

2 Preliminaries

Throughout this paper, all sets are assumed to be finite. For the basic operations associated with each concept, the reader is referred to the respective references.

2.1 Neutrosophic Sets

Neutrosophic Sets extend classical fuzzy sets by introducing an explicit *indeterminacy* degree, thereby accommodating propositions that are not wholly true nor wholly false. Each element in a neutrosophic set is characterized by three independent membership values: truth, indeterminacy, and falsity [32, 33, 36, 80]. Building on this framework, several generalized variants have been proposed, including *bipolar neutrosophic sets* [81–83], *interval-valued neutrosophic sets* [84–86], and *complex neutrosophic sets* [87–89].

Definition 2.1 (Neutrosophic Set)**.** [32, 90] Let X be a non-empty set. A *Neutrosophic Set (NS)* A on X is characterized by three membership functions:

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

2.2 Single-Valued Neutrosophic Offset

A *Single-Valued Neutrosophic Offset* relaxes the usual neutrosophic-set constraints by permitting the truth, indeterminacy, or falsity degrees of some elements to lie outside the unit interval (cf. [91–94]). This models situations of extreme uncertainty where membership can be “underset” (below 0) or “overset” (above 1) without prior normalization.

Definition 2.2 (Single-Valued Neutrosophic Offset)**.** [34,95,96] Let U_{off} be a universe of discourse and choose real bounds $\Psi < 0 < 1 < \Omega$. A *Single-Valued Neutrosophic Offset* is a collection

$$A_{\text{off}} = \Big\{ (x,\ \langle T(x),\ I(x),\ F(x) \rangle) \ \Big|\ x \in U_{\text{off}},\ T(x), I(x), F(x) \in [\Psi, \Omega],\ \exists \mu \in \{T, I, F\} : \mu(x) \notin [0,1] \Big\}.$$

Here $T(x)$, $I(x)$, and $F(x)$ denote the truth-membership, indeterminacy-membership, and falsity-membership degrees, respectively, each allowed to range over $[\Psi, \Omega]$ so that some degrees may exceed 1 or fall below 0.

Example 2.3 (Weather Forecast Confidence as a Single-Valued Neutrosophic Offset)**.** Let $U_{\text{off}} = \{\text{Day}_1, \text{Day}_2\}$ be two consecutive days for which a meteorologist assigns probabilistic forecasts with an explicit indeterminacy component. Fix under- and over-limits $\Psi = -0.1$ and $\Omega = 1.1$. For each day $x \in U_{\text{off}}$, define:

$$\begin{aligned} T(x) &: \text{degree of confidence that it will rain,} \\ I(x) &: \text{degree of indeterminacy due to model ambiguity,} \\ F(x) &: \text{degree of confidence that it will not rain.} \end{aligned}$$

Each of $T(x), I(x), F(x)$ lies in $[\Psi, \Omega]$, and at least one lies outside $[0, 1]$.

Day	T	I	F
Day_1	1.05	0.10	−0.05
Day_2	0.80	−0.10	0.40

On Day_1, the rain–confidence $T = 1.05$ exceeds 1 (overset) and the no-rain confidence $F = -0.05$ is below 0 (underset), capturing exceptionally strong but conflicting model signals. On Day_2, the indeterminacy $I = -0.10$ falls below 0, indicating overconfidence in the prediction. In both cases, the triple $\langle T, I, F \rangle \in [\Psi, \Omega]^3$ with at least one component outside $[0, 1]$ constitutes a valid single-valued neutrosophic offset.

2.3 Hexapartitioned Neutrosophic Set

A *Hexapartitioned Neutrosophic Set* refines the classical neutrosophic set by introducing six independent membership degrees—truth, contradiction, ignorance, unknown, hesitation, and falsity—whose sum is bounded by 6 [67].

Definition 2.4 (Hexapartitioned Neutrosophic Set)**.** [67,97] Let U be a nonempty universe. A *hexapartitioned neutrosophic set* on U is given by

$$N = \big\{ \langle x,\ T(x),\ C(x),\ G(x),\ U(x),\ H(x),\ F(x) \rangle \ \big|\ x \in U \big\},$$

where each function

$$T,\ C,\ G,\ U,\ H,\ F : U \longrightarrow [0,1]$$

assigns the degrees of truth, contradiction, ignorance, unknown, hesitation, and falsity, respectively, and satisfies

$$0 \ \le\ T(x) + C(x) + G(x) + U(x) + H(x) + F(x) \ \le\ 6, \quad \forall x \in U.$$

In this formulation, the "ambiguous" component of earlier six-valued schemes is replaced by the *ignorance* degree $G(x)$.

Example 2.5 (Restaurant Quality Assessment as a Hexapartitioned Neutrosophic Set)**.** Let $U = \{\mathrm{R}_1, \mathrm{R}_2\}$ be two candidate restaurants. We evaluate each on six criteria, yielding a hexapartitioned neutrosophic set:

$$N = \big\{ \langle x,\ T(x),\ C(x),\ G(x),\ U(x),\ H(x),\ F(x) \rangle \ \big|\ x \in U \big\},$$

where

$T(x)$: degree of genuine culinary excellence,

$C(x)$: degree of contradictory reviews,

$G(x)$: degree of missing information,

$U(x)$: degree of novel or unexpected features,

$H(x)$: degree of reviewer hesitation,

$F(x)$: degree of genuine shortcomings.

Each value lies in $[0, 1]$ and their sum is bounded by 6. A possible assessment is:

Restaurant	T	C	G	U	H	F
R_1	0.80	0.10	0.30	0.20	0.15	0.05
R_2	0.60	0.05	0.40	0.10	0.25	0.20

For R_1, the sum is $0.80 + 0.10 + 0.30 + 0.20 + 0.15 + 0.05 = 1.60 \le 6$. For R_2, the sum is $0.60 + 0.05 + 0.40 + 0.10 + 0.25 + 0.20 = 1.60 \le 6$. Thus both tuples satisfy the hexapartitioned neutrosophic-set constraints.

2.4 Octapartitioned Neutrosophic Set

An *octapartitioned neutrosophic set* refines the classical neutrosophic set by assigning eight independent membership degrees—truth, relative truth, contradiction, unknown, ignorance, hesitancy, relative falsity, and falsity—to each element.

Definition 2.6 (Octapartitioned Neutrosophic Set)**.** [67] Let U be a nonempty universe. An *octapartitioned neutrosophic set* on U is given by

$$O = \big\{ \langle x,\ T_O(x),\ M_O(x),\ C_O(x),\ U_O(x),\ I_O(x),\ H_O(x),\ K_O(x),\ F_O(x) \rangle \ \big|\ x \in U \big\},$$

where

$$T_O,\ M_O,\ C_O,\ U_O,\ I_O,\ H_O,\ K_O,\ F_O : U \ \longrightarrow\ [0,1]$$

are the membership functions for truth, relative truth, contradiction, unknown, ignorance, hesitancy, relative falsity, and falsity, respectively. These satisfy, for every $x \in U$,

$$0 \ \le\ T_O(x) + M_O(x) + C_O(x) + U_O(x) + I_O(x) + H_O(x) + K_O(x) + F_O(x) \ \le\ 8.$$

Example 2.7 (Smartphone Review Assessment as an Octapartitioned Neutrosophic Set)**.** Let $U = \{\text{Phone}_A, \text{Phone}_B\}$ be two smartphone models under consideration. We evaluate each on eight neutrosophic membership degrees:

$$\begin{aligned} T_O(x) &: \text{genuine performance quality,} \\ M_O(x) &: \text{relative performance (compared to peers),} \\ C_O(x) &: \text{conflicting user feedback,} \\ U_O(x) &: \text{unknown or emergent features,} \\ I_O(x) &: \text{missing data (ignorance),} \\ H_O(x) &: \text{reviewer hesitation,} \\ K_O(x) &: \text{relative falsity (overhyped claims),} \\ F_O(x) &: \text{clear shortcomings (falsity).} \end{aligned}$$

Each degree lies in $[0, 1]$ and their sum does not exceed 8. A sample assessment is:

Model	T_O	M_O	C_O	U_O	I_O	H_O	K_O	F_O
Phone$_A$	0.80	0.60	0.10	0.20	0.30	0.15	0.05	0.10
Phone$_B$	0.70	0.50	0.20	0.25	0.10	0.20	0.15	0.05

For each model,

$$0 \le T_O + M_O + C_O + U_O + I_O + H_O + K_O + F_O \le 8,$$

so this table provides a valid octapartitioned neutrosophic set representation of the smartphone evaluations.

2.5 Nonapartitioned Neutrosophic Set

A *nonapartitioned neutrosophic set* further generalizes this idea by introducing nine membership degrees—truth, strongly relative truth, weakly relative truth, contradiction, unknown, ignorance, strongly relative falsity, weakly relative falsity, and falsity.

Definition 2.8 (Nonapartitioned Neutrosophic Set)**.** [67] Let U be a nonempty universe. A *nonapartitioned neutrosophic set* on U is defined as

$$N = \left\{ \langle x,\ T_N(x),\ ST_N(x),\ WT_N(x),\ C_N(x),\ U_N(x),\ I_N(x),\ SF_N(x),\ WF_N(x),\ F_N(x) \rangle \;\middle|\; x \in U \right\},$$

where

$$T_N,\ ST_N,\ WT_N,\ C_N,\ U_N,\ I_N,\ SF_N,\ WF_N,\ F_N : U \longrightarrow [0, 1]$$

are the membership functions corresponding to truth, strongly relative truth, weakly relative truth, contradiction, unknown, ignorance, strongly relative falsity, weakly relative falsity, and falsity. They satisfy, for every $x \in U$,

$$0 \le T_N(x) + ST_N(x) + WT_N(x) + C_N(x) + U_N(x) + I_N(x) + SF_N(x) + WF_N(x) + F_N(x) \le 9.$$

Example 2.9 (University Applicant Evaluation as a Nonapartitioned Neutrosophic Set)**.** Let $U = \{\text{Alice, Bob}\}$ be two applicants for a graduate program. We assess each on nine neutrosophic membership degrees, all valued in $[0, 1]$ and summing to at most 9:

$$\begin{aligned} T_N(x) &: \text{baseline suitability (truth),} \\ ST_N(x) &: \text{strong relative suitability,} \\ WT_N(x) &: \text{weak relative suitability,} \\ C_N(x) &: \text{conflicting indicators,} \\ U_N(x) &: \text{unknown or novel qualifications,} \\ I_N(x) &: \text{data gaps (ignorance),} \\ SF_N(x) &: \text{strong relative unsuitability,} \\ WF_N(x) &: \text{weak relative unsuitability,} \\ F_N(x) &: \text{baseline unsuitability (falsity).} \end{aligned}$$

A possible evaluation is:

Applicant	T_N	ST_N	WT_N	C_N	U_N	I_N	SF_N	WF_N	F_N
Alice	0.80	0.50	0.10	0.05	0.20	0.10	0.05	0.08	0.12
Bob	0.70	0.30	0.20	0.10	0.15	0.05	0.10	0.12	0.10

For Alice,

$$T_N + ST_N + WT_N + C_N + U_N + I_N + SF_N + WF_N + F_N = 2.00 \le 9,$$

and similarly for Bob. Thus this table constitutes a valid nonapartitioned neutrosophic set representation of the applicants' evaluations.

2.6 Decapartitioned Neutrosophic Set

A *Decapartitioned Neutrosophic Set* refines the neutrosophic framework by assigning ten independent membership degrees—two levels of truth, contradiction, unknown, ignorance, hesitation, two levels of falsity, and standard truth and falsity—to each element.

Definition 2.10 (Decapartitioned Neutrosophic Set)**.** [67] Let U be a nonempty universe. A *decapartitioned neutrosophic set* on U is a collection

$$D = \left\{ \langle x,\ T(x),\ \mathrm{SRT}(x),\ \mathrm{WRT}(x),\ C(x),\ U(x),\ I(x),\ H(x),\ \mathrm{SRF}(x),\ \mathrm{WRF}(x),\ F(x)\rangle \;\middle|\; x \in U \right\},$$

where the ten functions

$$T,\ \mathrm{SRT},\ \mathrm{WRT},\ C,\ U,\ I,\ H,\ \mathrm{SRF},\ \mathrm{WRF},\ F \;:\; U \longrightarrow [0,1]$$

denote, respectively:

- $T(x)$: truth,
- $\mathrm{SRT}(x)$: strongly relative truth,
- $\mathrm{WRT}(x)$: weakly relative truth,
- $C(x)$: contradiction,
- $U(x)$: unknown,
- $I(x)$: ignorance,
- $H(x)$: hesitation,
- $\mathrm{SRF}(x)$: strongly relative falsity,
- $\mathrm{WRF}(x)$: weakly relative falsity,
- $F(x)$: falsity.

These satisfy, for every $x \in U$,

$$0 \le T(x) + \mathrm{SRT}(x) + \mathrm{WRT}(x) + C(x) + U(x) + I(x) + H(x) + \mathrm{SRF}(x) + \mathrm{WRF}(x) + F(x) \le 10.$$

Example 2.11 (Investment Opportunity Evaluation as a Decapartitioned Neutrosophic Set)**.** Let $U = \{\mathrm{Inv}_A, \mathrm{Inv}_B\}$ be two investment projects under consideration. We assign ten membership degrees—truth (T), strongly relative truth (SRT), weakly relative truth (WRT), contradiction (C), unknown (U), ignorance (I), hesitation (H), strongly relative falsity (SRF), weakly relative falsity (WRF), and falsity (F)—to each project, all valued in $[0,1]$. These degrees satisfy

$$0 \le T(x) + \mathrm{SRT}(x) + \mathrm{WRT}(x) + C(x) + U(x) + I(x) + H(x) + \mathrm{SRF}(x) + \mathrm{WRF}(x) + F(x) \le 10, \quad x \in U.$$

Project	T	SRT	WRT	C	U	I	H	SRF	WRF	F
Inv_A	0.80	0.70	0.50	0.20	0.30	0.10	0.20	0.15	0.10	0.05
Inv_B	0.60	0.50	0.40	0.30	0.20	0.20	0.10	0.20	0.10	0.40

Here, for example, Inv_A has strong baseline confidence ($T = 0.80$), significant positive signals (SRT = 0.70), moderate uncertainty ($U = 0.30$), and low outright rejection ($F = 0.05$). The total for each project is well below 10, satisfying the decapartitioned neutrosophic set constraints.

3 Results: Partitioned Neutrosophic Offset

As the main outcome of this work, we introduce and formally define the concept of a *Partitioned Neutrosophic Offset*.

3.1 Hexapartitioned Neutrosophic Offset

We begin by defining a six–component offset model that subsumes both the single-valued neutrosophic offset and the classical hexapartitioned neutrosophic set.

Definition 3.1 (Hexapartitioned Neutrosophic OffSet (H-NOS))**.** Let U be a nonempty set and fix real constants

$$\Psi < 0 < 1 < \Omega \quad (\text{the } \textit{UnderLimit} \text{ and } \textit{OverLimit}).$$

A *hexapartitioned neutrosophic offset* is a collection

$$N_{\text{off}} = \left\{ \langle x,\, T(x),\, C(x),\, G(x),\, U(x),\, H(x),\, F(x)\rangle \;\middle|\; x \in U \right\},$$

where the six functions

$$T,\, C,\, G,\, U,\, H,\, F : U \longrightarrow [\Psi, \Omega]$$

are called, respectively, the *truth*, *contradiction*, *ignorance*, *unknown*, *hesitation*, and *falsity* degrees. They satisfy, for every $x \in U$,

$$\Psi \;\le\; T(x) + C(x) + G(x) + U(x) + H(x) + F(x) \;\le\; \Omega + 5.$$

Moreover, at least one of the six values must lie outside the unit interval $[0, 1]$ (i.e. exhibit *overset* or *underset* behavior); otherwise N_{off} reduces to the ordinary hexapartitioned neutrosophic set.

Example 3.2 (Medical Diagnosis as a Hexapartitioned Neutrosophic Offset)**.** Consider a clinical setting where two patients, $U = \{\mathrm{P}_1, \mathrm{P}_2\}$, are evaluated for a complex syndrome. We fix under- and overlimits $\Psi = -0.2$ and $\Omega = 1.2$. For each patient x, clinicians assign six "degrees":

$$\begin{aligned} T(x) &: \text{evidence supporting the diagnosis,} \\ C(x) &: \text{contradictory findings,} \\ G(x) &: \text{missing information (ignorance),} \\ U(x) &: \text{novel or atypical features (unknown),} \\ H(x) &: \text{clinical hesitation,} \\ F(x) &: \text{evidence against the diagnosis.} \end{aligned}$$

These values lie in $[\Psi, \Omega]$ and must satisfy $\Psi \le T + C + G + U + H + F \le \Omega + 5$, with at least one component outside $[0, 1]$.

Patient	T	C	G	U	H	F
P_1	1.10	0.05	0.15	0.10	0.08	0.02
P_2	0.80	0.10	0.05	0.20	0.15	−0.05

For P_1, the truth-evidence degree $T = 1.10$ slightly exceeds 1 (overset), reflecting exceptionally strong supporting data, while all other values remain within $[0, 1]$. For P_2, the falsity-evidence degree $F = -0.05$ falls below 0 (underset), capturing negative confidence in the diagnosis. In both cases,

$$-0.2 \;\le\; T + C + G + U + H + F \;\le\; 1.2 + 5,$$

so the table constitutes a valid hexapartitioned neutrosophic offset.

The allowance of $[\Psi, \Omega]$ enables each degree to take *offset* values: strictly negative (underset) or exceeding 1 (overset).

Theorem 3.3. *The hexapartitioned neutrosophic offset (H-NOS) strictly generalises both*

1. *the single-valued neutrosophic offset (SV-NOS), and*
2. *the classical hexapartitioned neutrosophic set (H-NS).*

Proof.

1. **(i) Reduction to SV-NOS.** Given $\langle x, T, C, G, U, H, F\rangle \in N_{\text{off}}$, set
$$I(x) = G(x) + U(x) + H(x), \qquad F'(x) = F(x) + C(x).$$
Since $T, C, G, U, H, F \in [\Psi, \Omega]$, we have $T, I, F' \in [\Psi, \Omega]$ and
$$T + I + F' = T + G + U + H + F + C \leq \Omega + 5.$$
Moreover, $G + U + H \leq 3$ and $C + F \leq \Omega + 2$, so $T + I + F' \leq \Omega + 2$. Hence $\langle x, T, I, F'\rangle$ satisfies the SV-NOS axioms. Injectivity follows from the fact that (C, G, U, H) can be recovered uniquely.
2. **(ii) Reduction to H-NS.** If $\Psi = 0$, $\Omega = 1$, and $T, C, G, U, H, F \in [0, 1]$ for all x, then the defining inequality becomes
$$0 \leq T + C + G + U + H + F \leq 6,$$
which is exactly the condition for a classical hexapartitioned neutrosophic set.

Since neither inclusion can be reversed without collapsing degrees or disallowing offset values, H-NOS strictly generalises both SV-NOS and H-NS. □

3.2 Octapartitioned Neutrosophic Offset

We generalize the offset concept to an eight–component framework, unifying several earlier models.

Definition 3.4 (Octapartitioned Neutrosophic OffSet (O-NOS))**.** Let U be a nonempty universe and fix real bounds
$$\Psi < 0 < 1 < \Omega \quad \text{(UnderLimit and OverLimit)}.$$
An *octapartitioned neutrosophic offset* is a function
$$\mathbf{n}\colon U \longrightarrow [\Psi, \Omega]^8, \quad x \mapsto \big(T(x),\, M(x),\, C(x),\, U(x),\, I(x),\, K(x),\, H(x),\, F(x)\big),$$
where the eight components represent:

T : truth, M : relative truth,
C : contradiction, U : unknown,
I : ignorance, K : relative falsity,
H : hesitation, F : falsity,

and for each $x \in U$ the normalization condition
$$\Psi \leq T(x) + M(x) + C(x) + U(x) + I(x) + K(x) + H(x) + F(x) \leq \Omega + 7$$
holds. Moreover, there must exist at least one $x \in U$ for which one of these eight values lies outside the unit interval $[0, 1]$, otherwise the structure reduces to the classical octapartitioned neutrosophic set.

Example 3.5 (Corporate Credit Assessment as an Octapartitioned Neutrosophic Offset)**.** Consider two firms, $U = \{\mathrm{Firm}_A, \mathrm{Firm}_B\}$, whose bond default risk is evaluated using eight neutrosophic offset degrees. We set the under- and overlimits to $\Psi = -0.1$ and $\Omega = 1.2$. For each firm x, analysts assign:

$$\begin{aligned} T(x) &: \text{baseline creditworthiness}, & M(x) &: \text{relative credit strength},\\ C(x) &: \text{contradictory indicators}, & U(x) &: \text{unknown market factors},\\ I(x) &: \text{data gaps (ignorance)}, & K(x) &: \text{relative falsity (overstated strength)},\\ H(x) &: \text{analyst hesitation}, & F(x) &: \text{negative signals (falsity)}. \end{aligned}$$

All eight values lie in $[\Psi, \Omega]$ and satisfy $\Psi \le T + M + C + U + I + K + H + F \le \Omega + 7$, with at least one entry outside $[0, 1]$.

Firm	T	M	C	U	I	K	H	F
Firm_A	1.15	0.10	0.05	0.08	0.07	0.00	0.03	0.02
Firm_B	0.85	0.20	0.10	0.05	0.02	−0.05	0.04	0.03

For Firm_A, the baseline credit degree $T = 1.15$ exceeds 1 (overset), indicating exceptionally strong fundamentals. For Firm_B, the relative falsity degree $K = -0.05$ falls below 0 (underset), reflecting slight negative bias in reported strengths. Both rows satisfy

$$-0.1 \ \le\ T + M + C + U + I + K + H + F \ \le\ 1.2 + 7,$$

validating this as an octapartitioned neutrosophic offset.

When $\Psi = 0$ and $\Omega = 1$, Definition 3.4 specializes exactly to the ordinary octapartitioned neutrosophic set.

Theorem 3.6. *The octapartitioned neutrosophic offset (O-NOS) strictly generalizes:*

1. *the single-valued neutrosophic offset (SV-NOS),*
2. *the hexapartitioned neutrosophic offset (H-NOS), and*
3. *the classical octapartitioned neutrosophic set (O-NS).*

Proof. We prove each embedding in turn.

(i) Embedding SV-NOS into O-NOS. Given $\mathbf{n}(x) = (T, M, C, U, I, K, H, F)$, define

$$T' = T + M, \quad I' = C + U + I + H, \quad F' = K + F.$$

Since each original component lies in $[\Psi, \Omega]$, we have $T', I', F' \in [\Psi, \Omega]$ and

$$T' + I' + F' = T + M + C + U + I + H + K + F \le \Omega + 7.$$

Moreover, at most two of M, C, U, H, K sum to 5, so $T' + I' + F' \le \Omega + 2$, satisfying the SV-NOS normalization. Thus the projection $(T, M, C, U, I, K, H, F) \mapsto (T', I', F')$ yields a surjective homomorphism from O-NOS onto SV-NOS.

(ii) Embedding H-NOS into O-NOS. Setting $M(x) = K(x) = 0$ for all $x \in U$ reduces the eight-tuple to the six-tuple $(T, C, G := I, U := U, H, F)$, which by Definition 3.1 satisfies the H-NOS axioms. Hence H-NOS arises as the subclass $\{M = K = 0\} \subset \mathrm{O{-}NOS}$.

(iii) Recovering O-NS. Restricting $\Psi = 0, \Omega = 1$, and requiring all components to lie in $[0, 1]$ enforces

$$0 \ \le\ T + M + C + U + I + K + H + F \ \le\ 8,$$

exactly the constraint defining the classical O-NS.

Since each reduction either merges components (SV-NOS), fixes some to zero (H-NOS), or disallows offset values (O-NS), all containments are strict, proving the theorem. □

3.3 Nonapartitioned Neutrosophic Offset

We conclude the hierarchy of neutrosophic offsets with a nine–component model that simultaneously subsumes all previously defined offset and classical partitioned structures.

Definition 3.7 (Nonapartitioned Neutrosophic OffSet (N-NOS))**.** Let U be a nonempty universe and fix real bounds

$$\Psi < 0 < 1 < \Omega \quad (\text{underlimit and overlimit}).$$

A *nonapartitioned neutrosophic offset* is a mapping

$$\mathbf{n}\colon U \to [\Psi, \Omega]^9, \qquad x \mapsto \big(T(x),\ ST(x),\ WT(x),\ C(x),\ U(x),\ I(x),\ SF(x),\ WF(x),\ F(x)\big),$$

where the nine components are interpreted as

$$\begin{aligned} T &: \text{truth}, & ST &: \text{strong relative truth}, \\ WT &: \text{weak relative truth}, & C &: \text{contradiction}, \\ U &: \text{unknown}, & I &: \text{ignorance}, \\ SF &: \text{strong relative falsity}, & WF &: \text{weak relative falsity}, \\ F &: \text{falsity}, \end{aligned}$$

and satisfy the normalization condition

$$\Psi \ \le\ T(x) + ST(x) + WT(x) + C(x) + U(x) + I(x) + SF(x) + WF(x) + F(x) \ \le\ \Omega + 8 \quad \text{for all } x \in U.$$

Moreover, for at least one $x \in U$, at least one of these nine values must lie outside the unit interval $[0, 1]$, else the model reduces to the ordinary (non-offset) nonapartitioned neutrosophic set.

Example 3.8 (Fraud Detection Scores as a Nonapartitioned Neutrosophic Offset)**.** Let $U = \{\mathrm{Txn}_1, \mathrm{Txn}_2\}$ be two financial transactions under review for potential fraud. Fix underlimit $\Psi = -0.1$ and overlimit $\Omega = 1.1$. For each transaction $x \in U$, an automated system assigns nine "offset" degrees:

$$\begin{aligned} T(x) &: \text{baseline fraud likelihood}, \\ ST(x) &: \text{strong relative likelihood}, \\ WT(x) &: \text{weak relative likelihood}, \\ C(x) &: \text{contradictory signals}, \\ U(x) &: \text{unknown/unseen patterns}, \\ I(x) &: \text{data gaps (ignorance)}, \\ SF(x) &: \text{strong relative non-fraud}, \\ WF(x) &: \text{weak relative non-fraud}, \\ F(x) &: \text{baseline non-fraud likelihood}. \end{aligned}$$

These values lie in $[\Psi, \Omega]$ and satisfy $\Psi \le \sum T + ST + WT + C + U + I + SF + WF + F \le \Omega + 8$, with at least one component outside $[0, 1]$.

Transaction	T	ST	WT	C	U	I	SF	WF	F
Txn_1	1.05	0.10	0.05	0.07	0.03	0.04	0.06	0.02	0.08
Txn_2	0.90	0.25	0.05	0.03	0.02	0.01	0.04	−0.03	0.10

For Txn_1, the baseline fraud score $T = 1.05$ exceeds 1 (overset), reflecting an unusually strong risk signal. For Txn_2, the weak non-fraud score $WF = -0.03$ falls below 0 (underset), indicating slight negative confidence in non-fraud. In both cases

$$-0.1 \ \le \sum_{d \in \{T, ST, \ldots, F\}} d(x) \ \le\ 1.1 + 8,$$

so these assignments form a valid nonapartitioned neutrosophic offset.

Theorem 3.9. *The nonapartitioned neutrosophic offset* (N-NOS) *strictly generalises:*

1. *the* single-valued neutrosophic offset *(SV-NOS),*
2. *the* octapartitioned neutrosophic offset *(O-NOS),*
3. *the classical* nonapartitioned neutrosophic set *(N-NS).*

Proof. **(i) Embedding SV-NOS.** Given any SV-NOS element $\langle x, T^*, I^*, F^* \rangle$, define

$$T(x) = T^*, \quad ST(x) = WT(x) = 0, \quad C(x) = I^*, \quad U(x) = I(x) = 0, \quad SF(x) = F^*, \quad WF(x) = F(x) = 0.$$

Then each component lies in $[\Psi, \Omega]$ and

$$T + ST + \cdots + F = T^* + I^* + F^* \in [\Psi, \Omega + 2] \subseteq [\Psi, \Omega + 8],$$

so the resulting nine-tuple satisfies Definition 3.7. This injection is strict since SV-NOS has only three degrees.

(ii) Embedding O-NOS. Starting from an O-NOS profile (T, M, C, U, I, K, H, F), set

$$WT(x) = WF(x) = 0, \quad ST(x) = M(x), \quad SF(x) = K(x).$$

The resulting nine-tuple obeys the same bounds $(\Psi, \Omega + 8)$, and the constraint of Definition 3.4 is recovered. Thus O-NOS embeds as the subclass $\{WT = WF = 0\}$.

(iii) Recovering N-NS. Specializing $\Psi = 0, \Omega = 1$ and requiring all components in $[0, 1]$ transforms the offset bound into

$$0 \le \sum_{\alpha \in \{T, ST, \ldots, F\}} \alpha(x) \le 9,$$

which exactly matches the classical nonapartitioned neutrosophic set constraint. Hence N-NS is obtained by $(\Psi, \Omega) \to (0, 1)$.

In each case, collapsing components or disallowing offset values would violate the nine-component offset structure, so all containments are strict. □

3.4 Decapartitioned Neutrosophic Offset

We now introduce the most expressive offset model to date, featuring ten independent membership degrees.

Definition 3.10 (Decapartitioned Neutrosophic OffSet (D-NOS))**.** Let U be a nonempty universe and fix real constants

$$\Psi < 0 < 1 < \Omega$$

(called the *UnderLimit* and *OverLimit*). A *decapartitioned neutrosophic offset* is a map

$$\mathbf{n} : U \longrightarrow [\Psi, \Omega]^{10}, \quad x \longmapsto \big(T(x), \text{SRT}(x), \text{WRT}(x), C(x), U(x), I(x), H(x), \text{SRF}(x), \text{WRF}(x), F(x)\big),$$

where the ten functions

$$T, \text{ SRT}, \text{ WRT}, C, U, I, H, \text{ SRF}, \text{ WRF}, F : U \longrightarrow [\Psi, \Omega]$$

are called respectively: truth, strongly relative truth, weakly relative truth, contradiction, unknown, ignorance, hesitation, strongly relative falsity, weakly relative falsity, and falsity. They satisfy the normalization

$$\Psi \le T(x) + \text{SRT}(x) + \text{WRT}(x) + C(x) + U(x) + I(x) + H(x) + \text{SRF}(x) + \text{WRF}(x) + F(x) \le \Omega + 9, \quad \forall x \in U,$$

and there must exist at least one $x \in U$ and one component μ among these ten such that $\mu(x) \notin [0, 1]$, ensuring genuine offset behavior. If all values lie in $[0, 1]$, the model reduces to the classical decapartitioned neutrosophic set.

Example 3.11 (Bridge Health Monitoring as a Decapartitioned Neutrosophic Offset)**.** Consider two bridges, $U = \{\text{Bridge}_A, \text{Bridge}_B\}$, whose structural integrity is assessed using ten neutrosophic offset degrees. Fix underlimit $\Psi = -0.2$ and overlimit $\Omega = 1.2$. For each bridge x, engineers assign:

$$\begin{aligned}
T(x) &: \text{baseline integrity (truth)},\\
\text{SRT}(x) &: \text{strongly relative integrity},\\
\text{WRT}(x) &: \text{weakly relative integrity},\\
C(x) &: \text{conflicting sensor readings},\\
U(x) &: \text{unknown environmental factors},\\
I(x) &: \text{data gaps (ignorance)},\\
H(x) &: \text{inspection hesitation},\\
\text{SRF}(x) &: \text{strongly relative failure},\\
\text{WRF}(x) &: \text{weakly relative failure},\\
F(x) &: \text{baseline failure (falsity)}.
\end{aligned}$$

Each degree lies in $[\Psi, \Omega]$, and for every x, $\Psi \le T + \text{SRT} + \text{WRT} + C + U + I + H + \text{SRF} + \text{WRF} + F \le \Omega + 9$, with at least one component outside $[0, 1]$. A possible evaluation is:

Bridge	T	SRT	WRT	C	U	I	H	SRF	WRF	F
Bridge_A	1.10	0.08	0.05	0.06	0.04	0.03	0.02	0.07	0.01	0.04
Bridge_B	0.85	0.20	0.05	0.10	0.03	0.02	0.01	0.09	−0.05	0.15

For Bridge_A, the truth degree $T = 1.10$ exceeds 1 (overset), indicating exceptionally high confidence in stability. For Bridge_B, the weak failure degree $\text{WRF} = -0.05$ is below 0 (underset), reflecting slight negative confidence in failure predictions. Both satisfy

$$-0.2 \;\le\; T + \text{SRT} + \text{WRT} + C + U + I + H + \text{SRF} + \text{WRF} + F \;\le\; 1.2 + 9,$$

so this table provides a valid decapartitioned neutrosophic offset.

Theorem 3.12 (Unifying Power of D-NOS)**.** *The decapartitioned neutrosophic offset* D−NOS *strictly generalizes:*

1. *the single-valued neutrosophic offset (*SV−NOS*),*
2. *the nonapartitioned neutrosophic offset (*N−NOS*),*
3. *the classical decapartitioned neutrosophic set (*D−NS*).*

Proof. **(i) Embedding** SV−NOS**.** Given $\langle x, T^*, I^*, F^* \rangle$ in SV-NOS, define a D-NOS profile by

$$T(x) = T^*, \quad \text{SRT}(x) = \text{WRT}(x) = 0, \quad C(x) + U(x) + I(x) + H(x) = I^*, \quad \text{SRF}(x) + \text{WRF}(x) + F(x) = F^*,$$

with any remaining degrees chosen in $[\Psi, \Omega]$. Since $T^*, I^*, F^* \in [\Psi, \Omega]$ and $T^* + I^* + F^* \le \Omega + 2$, the D-NOS normalization holds. This injection is strict because D-NOS has ten independent degrees versus three.

(ii) Embedding N−NOS**.** Starting with an N-NOS element $\langle x, T, ST, WT, C, U, I, SF, WF, F \rangle$, set $\text{SRT}(x) = ST(x)$, $\text{SRF}(x) = SF(x)$, and $\text{WRT}(x) = \text{WRF}(x) = 0$. The resulting ten-tuple satisfies the D-NOS axioms with the same (Ψ, Ω), so N−NOS ⊂ D−NOS strictly.

(iii) Recovering D−NS**.** If $\Psi = 0$ and $\Omega = 1$ and all degrees lie in $[0, 1]$, the normalization becomes

$$0 \;\le\; \sum\nolimits_{\mu \in \{T, \dots, F\}} \mu(x) \;\le\; 10,$$

which matches the classical D-NS definition. Thus $\text{D}-\text{NS} = \text{D}-\text{NOS}\big|_{\Psi=0,\Omega=1}$.

Strictness. Each embedding either collapses or fixes components, or removes offsets, so no reverse inclusion can hold. □

4 Additional Result: Representing Partitioned Neutrosophic OffSets as Plithogenic OffSets

A *plithogenic set* enriches each element with multiple attribute-based membership degrees alongside a measure of contradiction, offering a highly flexible framework for uncertainty modeling [49, 98–102].

Definition 4.1 (Plithogenic Set)**.** [103, 104] Let S be a universe and $P \subseteq S$ a subset. A *plithogenic set* is the quintuple

$$PS = \big(P,\ v,\ P_v,\ \text{pdf},\ \text{pCF}\big),$$

where:

- v is an attribute;
- P_v is the set of all possible values of v;
- $\text{pdf} : P \times P_v \to [0,1]^s$ is the *degree of appurtenance function*;
- $\text{pCF} : P_v \times P_v \to [0,1]^t$ is the *degree of contradiction function*.

These satisfy, for all $a, b \in P_v$:

$$\begin{aligned} \text{pCF}(a,a) &= 0, && \text{(Reflexivity)} \\ \text{pCF}(a,b) &= \text{pCF}(b,a). && \text{(Symmetry)} \end{aligned}$$

Example 4.2 (House-Hunting with a Plithogenic Set)**.** Suppose S is the set of all houses in a city and $P = \{H_1, H_2\} \subseteq S$ are two finalists. We take the attribute

$$v = \text{“amenity category”}, \quad P_v = \{\,\text{Safety, Walkability, Affordability, SchoolQuality}\}.$$

Define the *degree of appurtenance function* $\text{pdf} : P \times P_v \to [0,1]$ by the membership table:

House	Safety	Walkability	Affordability	SchoolQuality
H_1	0.90	0.70	0.60	0.80
H_2	0.40	0.90	0.80	0.50

Next, the *degree of contradiction function* $\text{pCF} : P_v \times P_v \to [0,1]$ encodes pairwise conflict between amenities:

	Safety	Walk.	Afford.	SchoolQ.
Safety	0	0.20	0.30	0.15
Walkability	0.20	0	0.50	0.25
Affordability	0.30	0.50	0	0.40
SchoolQuality	0.15	0.25	0.40	0

Note that $\text{pCF}(a,a) = 0$ and $\text{pCF}(a,b) = \text{pCF}(b,a)$.

Hence

$$PS_{\text{off}} = \big(P,\ v,\ P_v,\ \text{pdf},\ \text{pCF}\big)$$

is a valid plithogenic set: each house in P has a vector of membership degrees across four amenity categories, and the symmetric contradiction matrix quantifies how strongly two categories conflict.

Remarkably, every partitioned neutrosophic offset (hexapartitioned, octapartitioned, nonapartitioned, decapartitioned) can be cast as a plithogenic offset. Below we extend this construction to oversets, undersets, and offsets [94, 105–109].

Definition 4.3 (Plithogenic Offset)**.** [94, 110] Let S be a universal set and $P \subseteq S$. A *plithogenic offset* is a quintuple

$$PS_{\text{off}} = \big(P,\ v,\ P_v,\ \text{pdf},\ \text{pCF}\big),$$

where

- v is an attribute and P_v its set of possible values;
- pdf: $P \times P_v \longrightarrow [\Psi_v,\ \Omega_v]^s$ is the *degree of appurtenance function*, with real bounds $\Psi_v < 0 < 1 < \Omega_v$, so that membership degrees may fall below 0 (*under-membership*) or exceed 1 (*over-membership*);
- pCF: $P_v \times P_v \longrightarrow [\Psi_v,\ \Omega_v]^t$ is the *degree of contradiction function*, satisfying $\text{pCF}(a, a) = 0$ and $\text{pCF}(a, b) = \text{pCF}(b, a)$.

When $\Psi_v = 0$ the model reduces to a *plithogenic overset*; when $\Omega_v = 1$ it becomes a *plithogenic underset*; and when $[\Psi_v, \Omega_v] = [0, 1]$ one recovers the ordinary plithogenic set.

Example 4.4 (Software Module Evaluation as a Plithogenic Offset)**.** Let S be the universe of all software modules and

$$P = \{\text{M}_1,\ \text{M}_2\} \subseteq S$$

the two candidate modules. We choose the attribute

$$v = \text{“quality criterion”}, \quad P_v = \{\text{Performance},\ \text{Security},\ \text{Maintainability},\ \text{Usability},\ \text{Scalability}\}.$$

We fix under- and over-limits $\Psi_v = -0.1$ and $\Omega_v = 1.2$.

Degree of Appurtenance Function pdf**.** Each module $x \in P$ is assigned a vector of five membership degrees in $[\Psi_v, \Omega_v]$:

Module	Performance	Security	Maintainability	Usability	Scalability
M_1	1.20	0.80	0.50	−0.05	0.90
M_2	0.90	1.15	0.70	0.60	0.00

Here $1.20 > 1$ for *Performance* of M_1 (overset) and $-0.05 < 0$ for *Usability* of M_1 (underset).

Degree of Contradiction Function pCF**.** We encode pairwise conflicts between criteria by a symmetric matrix in $[\Psi_v, \Omega_v]$:

	Perf.	Sec.	Maint.	Usab.	Scal.
Perf.	0	0.30	0.20	0.10	0.15
Sec.	0.30	0	0.25	0.05	0.10
Maint.	0.20	0.25	0	0.15	0.05
Usab.	0.10	0.05	0.15	0	0.20
Scal.	0.15	0.10	0.05	0.20	0

Note that $\text{pCF}(a, a) = 0$ and $\text{pCF}(a, b) = \text{pCF}(b, a)$.

Thus the quintuple

$$PS_{\text{off}} = \big(P,\ v,\ P_v,\ \text{pdf},\ \text{pCF}\big)$$

is a valid *plithogenic offset*, integrating five-dimensional overset/underset appurtenance with a symmetric conflict measure.

We now show that the plithogenic offset subsumes all of the partitioned neutrosophic offset models introduced above.

Theorem 4.5. *Let* $PS_{\text{off}} = (P, v, P_v, \text{pdf}, \text{pCF})$ *be any plithogenic offset with bounds* $\Psi_v < 0 < 1 < \Omega_v$*. Then by choosing the attribute-value set* P_v *appropriately and defining* pdf *to enumerate the component degrees, one recovers exactly:*

1. *the* hexapartitioned neutrosophic offset *(H-NOS) when* $|P_v| = 6$ *and*

$$\text{pdf}(x, a) = \begin{cases} T(x), & a = T, \\ C(x), & a = C, \\ G(x), & a = G, \\ U(x), & a = U, \\ H(x), & a = H, \\ F(x), & a = F, \end{cases}$$

2. *the* octapartitioned neutrosophic offset *(O-NOS) when* $|P_v| = 8$ *and*

$$\text{pdf}(x, a) = \begin{cases} T(x), & a = T, \\ M(x), & a = M, \\ C(x), & a = C, \\ U(x), & a = U, \\ I(x), & a = I, \\ K(x), & a = K, \\ H(x), & a = H, \\ F(x), & a = F, \end{cases}$$

3. *the* nonapartitioned neutrosophic offset *(N-NOS) when* $|P_v| = 9$ *and*

$$\text{pdf}(x, a) = \begin{cases} T(x), & a = T, \\ ST(x), & a = ST, \\ WT(x), & a = WT, \\ C(x), & a = C, \\ U(x), & a = U, \\ I(x), & a = I, \\ SF(x), & a = SF, \\ WF(x), & a = WF, \\ F(x), & a = F, \end{cases}$$

4. *the* decapartitioned neutrosophic offset *(D-NOS) when* $|P_v| = 10$ *and*

$$\text{pdf}(x, a) = \begin{cases} T(x), & a = T, \\ \text{SRT}(x), & a = \text{SRT}, \\ \text{WRT}(x), & a = \text{WRT}, \\ C(x), & a = C, \\ U(x), & a = U, \\ I(x), & a = I, \\ H(x), & a = H, \\ \text{SRF}(x), & a = \text{SRF}, \\ \text{WRF}(x), & a = \text{WRF}, \\ F(x), & a = F. \end{cases}$$

Moreover, in each case the contradiction function pCF *may be taken identically zero or defined to reflect any desired inter-component conflicts. Hence every partitioned neutrosophic offset is a special case of the plithogenic offset.*

Proof. Fix one of the four offset types and its carrier set U. Let the plithogenic attribute v range over exactly the named components of that offset (six, eight, nine, or ten values). Define

$$P_v = \{\text{component labels}\}, \quad \mathrm{pdf}(x, a) = \text{the membership degree of component } a \text{ at } x.$$

Since each component degree lies in $[\Psi_v, \Omega_v]$ and their sum satisfies the corresponding normalization bound (offset plus partition size minus one), the plithogenic degree function reproduces exactly the offset constraints. The symmetry and reflexivity axioms for pCF may be met by setting $\mathrm{pCF}(a, b) = 0$ for all a, b, or by importing any nontrivial contradiction structure without altering the underlying offset semantics.

Thus, by this straightforward identification of P_v and pdf, each hexapartitioned, octapartitioned, nonapartitioned, or decapartitioned neutrosophic offset is realized as an instance of a plithogenic offset. □

5 Conclusion and Future Work

This paper introduced four new partitioned neutrosophic offset families—*hexapartitioned*, *octapartitioned*, *nonapartitioned*, and *decapartitioned*—and demonstrated that each of them can be embedded naturally within the plithogenic–offset framework. Looking ahead, we hope that the ideas developed here will be explored further in a variety of settings, including *neutrosophic graph theory* [111–117], *neutrosophic algebra* [118–120], neutrosophic Probability [121–123], neutrosophic statistics [124–126], *neutrosophic topology* [127–129], *neutrosophic control theory* [130–133], and *neutrosophic decision science* [134–136]. Investigating concrete applications and computational techniques in these domains remains an open and promising direction for future research.

Acknowledgments

We extend our sincere gratitude to everyone who provided insights, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Author Contributions

The paper has been solely authored by the corresponding author at this stage.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Considerations

This work does not involve any experiments or studies involving human participants or animals, and therefore no ethical approvals were required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Code Availability

No code or software was developed for this study.

Clinical Trial

This study did not involve any clinical trials.

Consent to Participate

Not applicable.

Disclaimer on Scope and Accuracy

The theoretical models and concepts proposed in this manuscript have not yet undergone empirical testing or practical deployment. Future work may investigate their utility in applied or experimental contexts. While the authors have taken care to maintain accuracy and provide appropriate citations, inadvertent errors or omissions may remain. Readers are encouraged to consult original references for confirmation and further study.

The authors assert that all mathematical results and justifications included in this work have been carefully reviewed and are believed to be correct. Should any inaccuracies or ambiguities be discovered, the authors welcome constructive feedback and will provide clarification upon request.

The conclusions presented are valid only within the specific theoretical framework and assumptions described in the text. Generalizing these results to other mathematical contexts may require further investigation. All opinions and interpretations expressed herein are solely those of the authors and do not necessarily reflect the views of their respective institutions.

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