Experimental Report on Generating Gaussian White Noise Based on Generalized Mapping Theory

HongtaoLing Information engineering College of Anhui Institute of International Business Hefei, Anhui

Abstract

This experiment aims to generate Gaussian white noise using a custom algorithm based on the Generalized Mapping Theory (GMT) and verify the statistical properties of the generated signal. Through the Central Limit Theorem (CLT), uniform distributions are transformed into a Gaussian distribution, and the Kolmogorov-Smirnov (KS) test and visualization techniques are employed to evaluate the quality of the generated noise. The results demonstrate that the generated signal closely approximates the target Gaussian distribution with a mean of 0 and a standard deviation of 1.

1. Introduction

Gaussian white noise is widely used in various fields such as signal processing, communication systems, and simulation experiments. Generating high-quality Gaussian white noise is crucial for these applications. The Generalized Mapping Theory (GMT) provides a theoretical framework for transforming between different mathematical structures, and in this experiment, we leverage GMT combined with the Central Limit Theorem to generate Gaussian white noise, avoiding reliance on existing standard algorithms.

2. Methodology

2.1 Generalized Mapping Theory (GMT) Components

In the context of this experiment, the GMT consists of the following components:

- **Object Set (A)**: Twelve groups of uniformly distributed random numbers generated using `np.random.rand(12, N)`, serving as the initial seeds for the transformation.

Operation Set (F): A custom transformation function that sums the uniformly distributed numbers (exploiting the Central Limit Theorem) and then standardizes and scales them to obtain the Gaussian distribution.

Probability Set (P): Contains the target distribution parameters, specifically the mean (`mu`) and standard deviation (`sigma`).

Result Set (B): The final generated Gaussian white noise signal.

2.2 Generation Process

1. **Generating Initial Seeds**: Twelve groups of uniform random numbers in the range [0, 1) are generated.
2. **Applying CLT**: The sums of each group of uniform numbers are calculated. According to the Central Limit Theorem, the sum of a sufficient number of independent and identically distributed random variables approaches a normal distribution. Here, with 12 groups, the sum approximates a normal distribution with a mean of $12 \times 0.5 = 6$ and a variance of $12 \times \frac{1}{12} = 1$.
3. **Standardization and Scaling**: The summed values are standardized to a standard normal distribution (mean 0, standard deviation 1) and then scaled using the target mean and standard deviation from the probability set P.

2.3 Evaluation Methods

Statistical Metrics: Calculation of the actual mean and standard deviation of the generated signal to compare with the target values.

Kolmogorov-Smirnov (KS) Test: A non-parametric test to determine if the generated signal follows the specified normal distribution. A large p-value (typically > 0.05) indicates that we fail to reject the null hypothesis that the data is from the normal distribution.

Visualization: Time-domain analysis, frequency-domain analysis (power spectral density), and distribution characteristic analysis (histogram with theoretical Gaussian distribution overlay) to visually assess the signal properties.

3. Experimental Results

3.1 Statistical Characteristics

The generated Gaussian white noise signal has the following statistical properties:

Actual mean: -0.002984 (Target: 0)

Actual standard deviation: 0.985394 (Target: 1)

The actual mean and standard deviation are very close to the target values, indicating that the generated signal has the desired central tendency and spread.

3.2 KS Test Results

The results of the Kolmogorov-Smirnov test are:

KS statistic D $= 0.0067$

p-value $= 0.7569$

Since the p-value is much larger than the typical significance level of 0.05, we fail to reject the null hypothesis. This means there is insufficient evidence to suggest that the generated signal does not follow the normal distribution with the target mean and standard deviation.

3.3 Visualization

3.3.1 Time Domain Analysis

The time-domain plot (first 500 samples) shows that the signal has random fluctuations around zero, which is characteristic of white noise. There is no obvious periodicity or trend, indicating the random nature of the generated noise.

3.3.2 Frequency Domain Analysis

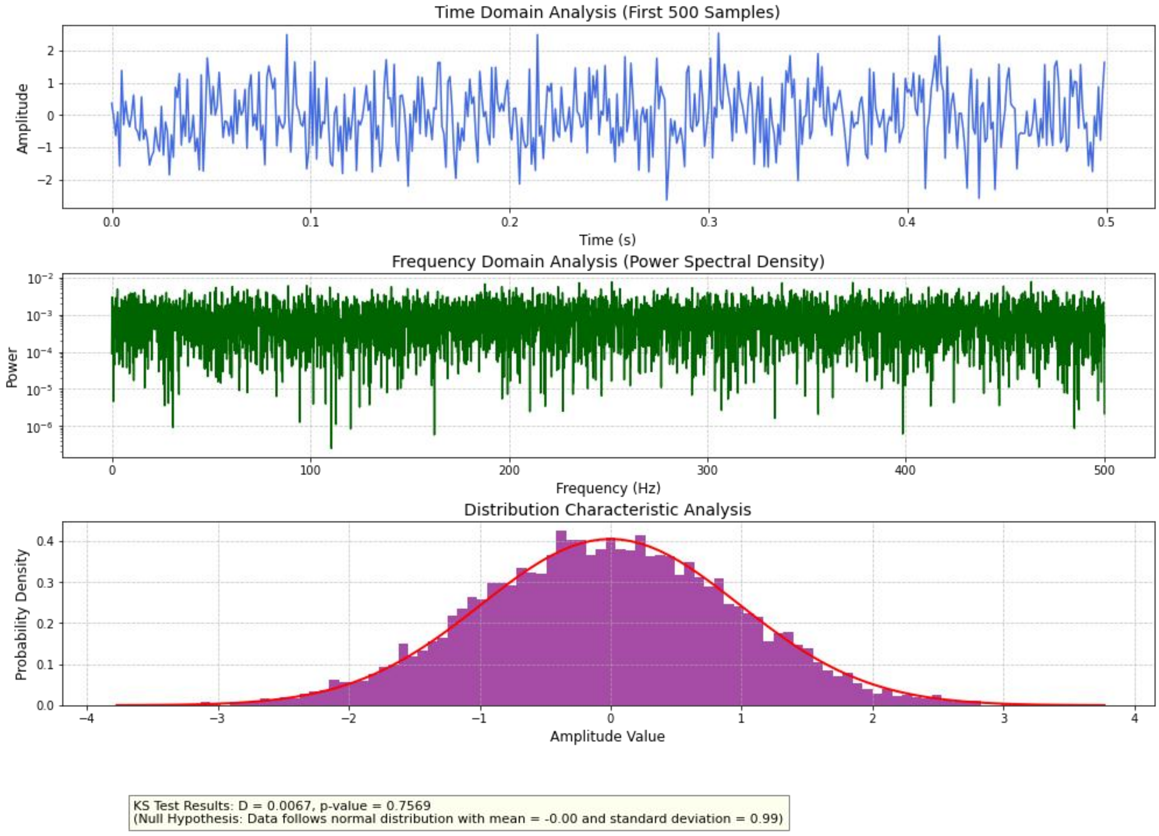
The power spectral density plot shows that the power is relatively evenly distributed across different frequencies, which is a key feature of white noise (constant power spectral density over frequency).

3.3.3 Distribution Characteristic Analysis

The histogram of the signal amplitudes, overlaid with the theoretical Gaussian distribution curve, shows a close match. The histogram follows the bell-shaped curve of the normal distribution, further confirming that the generated signal has a Gaussian distribution.

Link to the experimental code:https://gitee.com/ririririiriririiriir/First-Study-on-Generalized-Mapping-and-random-signal

The experimental result graph is as follows：



4. Discussion

The experimental results demonstrate that the custom algorithm based on GMT and the Central Limit Theorem successfully generates Gaussian white noise that closely matches the target distribution. The small deviation between the actual and target mean and standard deviation can be attributed to the inherent randomness of the generation process and the finite sample size. The large p-value from the KS test and the consistent visualizations all support the conclusion that the generated signal is a high-quality Gaussian white noise.

This method avoids using existing standard Gaussian generation algorithms, providing a novel approach based on GMT. It can be useful in scenarios where custom or theory-driven generation methods are preferred.

5. Conclusion

The experiment successfully generated Gaussian white noise using a GMT-based custom algorithm. The statistical metrics, KS test, and visualizations all confirm that the generated signal meets the requirements of a Gaussian distribution with the target mean and standard deviation. This method provides an effective alternative for generating Gaussian white noise, especially in contexts where GMT or custom generation approaches are of interest.

References

[1] Ling H. Generalized Mapping Theory — Used to Describe Phenomena That Cannot Be Characterized by Generalized Functions. Preprints 2025, 2025080640. https://doi.org/10.20944/preprints202508.0640.v1