A QUICK RE-LOOK AT THE MONTY-HALL PROBLEM

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ABSTRACT

This research note presents a quick re-look at the Monty-Hall Problem, refuting the widely accepted position held by the leading subject-area-experts, and establishing that there is no rational basis for a switched choice in the decision to be made by the game show guest.

Keywords: A-Priori Probability; A-Posteriori Probability; Mutually Independent Events; Joint Probability Distribution; Mutually Exclusive Together Exhaustive Alternatives; Conditional Probability.

AMS MSC Mathematics Subject Classification: 60A99; 60C99; 62A99; 62C99.

INTRODUCTION

The “Monty-Hall Problem”, also referred to as the “Three-Door Problem” is based on a game show “Let’s Make a Deal” wherein the host reveals a losing choice to the guest, who had earlier made an initial choice, and in turn offers the guest an option to switch from the earlier choice to a second available choice with an aim to enhance the chances of winning the prize. The most prevalent position, as reported in literature, among the leading eminent mathematicians, statisticians, logicians, subject-area-experts and rational intellectuals, is that an appropriate detailed study & analysis of the scenario using the well accepted standard approach of Probability & Statistics, would lead to a recommendation to the guest of that game show to switch to the second available choice based on the knowledge obtained from the host revealing a losing choice.

It is argued that the default of sticking to the initial choice will result in a probability of success being only one-third whereas a switch to an alternative second available choice will result in a probability of success being one-half, and therefore a switched choice is recommended. However, it will be shown here that this argument seems to have been based on some assumptions that cannot be justified, and therefore the resultant recommendation for switched choice is indeed baseless.

DESCRIPTION OF THE PROBLEM SCENARIO

Without mincing words or beating around the bush, let us focus on the problem proper.

For the sake of clarity, let us consider the standard Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making a choice of the door to pick the prize; the host who knows the location of the prize as well as the choice made by the guest, now reveals a losing choice and also turns to the guest with an option to switch from the earlier choice to a second available choice with an aim to enhance the chances of winning the prize, now based on the knowledge obtained from the host revealing a losing choice.

We will represent the locations indicating the three doors: (1) the choice of the door pointed out by the guest *participant-player*, p, (2) the door behind which the prize-*reward* is hidden, r, and (3) the door q opened by the host *quiz-master* to reveal the losing choice – by a triplet (p, r, q) – where each of these triplet component can represent any one of the three doors - with p and r being mutually independent of each other as well as q, whereas q is dependent on both p and r, as per the rules of the game show as explained here below.

We assume that choice of the door $p \in \{1, 2, 3\}$ pointed out by the guest participant-player is based on zero-knowledge and therefore at best a random-guess, that it can be any one of the three doors, each possibility being assumed to be equally probable, mutually exclusive together exhaustive event possibilities; therefore, each has a probability of 1/3 thus adding up to one.

Similarly, we assume that the door $r \in \{1, 2, 3\}$ behind which the prize-reward is hidden by the host (who has full knowledge of it); although for the guest participant-player, because of complete lack of knowledge, it is assumed to be equally probable, mutually exclusive together exhaustive event possibilities; each has a probability of 1/3 thus adding up to one.

Now, because the two events $[p \in \{1, 2, 3\}]$ and $[r \in \{1, 2, 3\}]$ are mutually independent of each other, the joint probability of the combinations of these two events can be obtained as the product of the probabilities of the two component events. Therefore, the joint probability of each of the combined events $[p \in \{1, 2, 3\}][y \in \{1, 2, 3\}]$ is 1/9 and the sum total of these nine joint probabilities is one.

Note that q the action of the host is indeed dependent on both p and r – why?

ANALYSIS OF THE PROBLEM SCENARIO

The host takes note of the choice p made by the guest. Being already aware of the actual location r of the prize, a losing choice q that is other than p and also different from r can be shown to the guest, while also offering the guest with an enticing option to switch from the earlier selected choice and go for an alternative second available distinctly different choice.

In such a situation, the host may or may not have many alternative choices, and his action q is indeed dependent on both p and r; and therefore we need to apply the rule of restricted probabilities – that is, such event-possibilities can be assumed to be mutually exclusive together exhaustive equally probable alternatives, at most only within such restricted domain in order to satisfy these restrictions.

The combined-triple-event-space E(p, r, q) is represented by the listed 12 triplets along with the associated probabilities of occurrence. Note that size of this event-space is 12 and not 27 which would have been the case if each of the three component-events were indeed mutually independent. Rather, the first two are independent giving rise to a combined-double-event-space E(p, r) of size nine. When it is then combined with the third component-event E(q), there results a splitting in three cases: That is, in case of E11x, E22y, E33z, wherein p = r, the third component-event E(q) ∈ [{x} | {y} | {z}] gets two alternative possibilities, namely, x ∈ {2, 3} and y ∈ {1, 3} and z ∈ {1, 2}. Whereas the remaining six of them, wherein p ≠ r, that is, in case of are left as such because they result in a fixed choice (no alternatives) for the third E(q) component-event.

Each of the 12 combined-triplet-event possibilities along with its joint-probability are listed in Table-1 as well as in Figure-1.

Sl.No.	E(p)	E(r)	E(p,r)	E(q)	E(p,r,q)		p	r	q		P[E(p)	P[E(r)]	P[E(q)]		P[E(p,r,q)]
01	**1**	**1**	**11**	**2**	**112**		**1**	**1**	**2**		**1/3**	**1/3**	**1/2**		**1/18**
02	**1**	**1**	**11**	**3**	**113**		**1**	**1**	**3**		**1/3**	**1/3**	**1/2**		**1/18**
03	**1**	**2**	**12**	3	**123**		**1**	**2**	3		1/3	1/3	1		1/9
04	**1**	**3**	**13**	2	**132**		**1**	**3**	2		1/3	1/3	1		1/9
05	**2**	**1**	**21**	3	**213**		**2**	**1**	3		1/3	1/3	1		1/9
06	**2**	**2**	**22**	**1**	**221**		**2**	**2**	**1**		**1/3**	**1/3**	**1/2**		**1/18**
07	**2**	**2**	**22**	**3**	**223**		**2**	**2**	**3**		**1/3**	**1/3**	**1/2**		**1/18**
08	**2**	3	**23**	1	**231**		**2**	3	1		1/3	1/3	1		1/9
09	**3**	**1**	**31**	2	**312**		**3**	**1**	2		1/3	1/3	1		1/9
10	**3**	**2**	**32**	1	**321**		**3**	**2**	1		1/3	1/3	1		1/9
11	**3**	**3**	**33**	**1**	**331**		**3**	**3**	**1**		**1/3**	**1/3**	**1/2**		**1/18**
12	**3**	**3**	**33**	**2**	**332**		**3**	**3**	**2**		**1/3**	**1/3**	**1/2**		**1/18**

Table-1: 12 combined-triplet-event possibilities along with its joint-probabilities.

Event-Label	**Event-Description**	**Event-Probability**
E112:	{(p = 1), (r = 1), (q = 2)}	: (1/3)*(1/3)*(1/2) = 1/18
E113:	{(p = 1), (r = 1), (q = 3)}	: (1/3)*(1/3)*(1/2) = 1/18
E123:	{(p = 1), (r = 2), (q = 3)}	: (1/3)*(1/3)*(1/1) = 1/9
E132:	{(p = 1), (r = 3), (q = 2)}	: (1/3)*(1/3)*(1/1) = 1/9
E213:	{(p = 2), (r = 1), (q = 3)}	: (1/3)*(1/3)*(1/1) = 1/9
E221:	{(p = 2), (r = 2), (q = 1)}	: (1/3)*(1/3)*(1/2) = 1/18
E223:	{(p = 2), (r = 2), (q = 3)}	: (1/3)*(1/3)*(1/2) = 1/18
E231:	{(p = 2), (r = 3), (q = 1)}	: (1/3)*(1/3)*(1/1) = 1/9
E312:	{(p = 3), (r = 1), (q = 2)}	: (1/3)*(1/3)*(1/1) = 1/9
E321:	{(p = 3), (r = 2), (q = 1)}	: (1/3)*(1/3)*(1/1) = 1/9
E331:	{(p = 3), (r = 3), (q = 1)}	: (1/3)*(1/3)*(1/2) = 1/18
E332:	{(p = 3), (r = 3), (q = 2)}	: (1/3)*(1/3)*(1/2) = 1/18

Figure-1: 12 combined-triplet-event possibilities along with its joint-probabilities.

WHAT IS WRONG WITH THE EXISTING APPROACH

The enticement offered by the host to the guest with an option to switch from the earlier choice to a distinctly different alternative available choice, with a motivation to enhance the chances of winning the prize, is indeed made with a short-sighted view of the event-space scenario, focusing only a sub-space, for example, consisting of the guest's first choice of door-1 and without considering the fact that the host would have made an exact same offer even if the guest were to make the first choice to be door-2. Also, from the event-space scenario presented in Table-1 & Figure-1 it is rather intriguing to find out that – given that the host has opened door-3 to show a losing choice, the a-posteriori probability (revised & updated based upon the whatever additional knowledge gained) of winning the prize can be appropriately determined by focusing upon the table entries with $q = 3$, that is, E(113) E(123) E(213) & E(223) from which it is clear that the probability of $r = 1$ is the same as the probability of $r = 2$, although by the very nature of the rules of the game P[E(113)] is less than P[E(123)] and also P[E(223)] is less than P[E(213)] which indicates that the same enticement exists to switch whatever might have been the earlier choice.

The entries in Table-1 and Figure-1. establishes that the event-space scenario is symmetrical in terms of exchaning p and r for any given fixed q; that is,

$$P[E(x,y,z)] = P[E(y,x,z)] \text{ for any } p=x,\ r=y,\ q=z.$$

This symmetry property, along with the seemingly paradoxical feature that -

$$P[E(y,y,z)] = P[E(x,x,z)] \leq P[E(y,x,z)] = P[E(x,y,z)] \text{ for any } p=x,\ r=y,\ q=z$$

justifies the reasoning of the guest against any potential anticipated enhancement of chances to win the prize by a switched choice from an earlier selected choice to a distinctly different alternative available choice.

CONCLUSION

This research report presents a novel intriguing analysis of the Monty-Hall Problem, refuting the most widely held position - and advocating against acting on any enticing offers made by the host to the guest for an optional switch from the already selected choice to a possibly distinct alternative available choice - why, because there is indeed no advantage in terms of any enhanced chances to win the prize, unlike what has been widely accepted till today.

RECOMMENDED READING

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10. ACKNOWLEDGEMENT

I must necessarily confess here that the *core idea behind this analysis is so stunningly & elusively simple*, that one may simply be taken aback in a profound wonder-struck jaw-drop-silence, maybe with an after-thought: "*oh my goodness, how could it be that it never flashed on me any time earlier*"! as was also the case in an earlier research work reported in [9] by this author.

11. DEDICATION

To my ಅಜ್ಜ(ajja) Karinja Halemane Keshava Bhat & ಅಜ್ಜಿ(ajji) Thirumaleshwari, ಅಪ್ಪ(appa) Shama Bhat & ಅಮ್ಮ(amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter Sriwidya.Bharati and my twin sons Sriwidya.Ramana & Sriwidya.Prawina for their *love & affection*.

ॐतत्सत्