REFUTATION OF THE LOGICAL FALLACY COMMITED BY THE SUBJECT MATTER EXPERTS ON THE MONTY-HALL PROBLEM

Let xr∈{1,2,3} be the door r behind which the prize x is hidden. Let yp∈{1,2,3} be the initial choice p of the guest y. Let z∈{1,2,3} be the door q opened by the host z to show a losing choice. Also, xr and yp are mutually independent; but zq is dependent on both yp and xr, that is, zq ≠ (yp, xr). The symbol ai denotes the event [E{(a=i)}] for any 'agent' a∈{x,y,z} and 'door' i∈{r,p,q}={1,2,3}. The Table lists the 12 *mutually-exclusive together-exhaustive* possibilities for the *combined-triple-event* [xr&yp&zq] :

Sl.No.	[xr]	[yp]	[xr&yp]	[zq]	[xr&yp&zq]		P[xr]	P[yp]	P[zq | (xr & yp)]	P[xr&yp&zq]
01	1	1	11	2	112		1/3	1/3	1/2	1/18
02	1	1	11	3	113		1/3	1/3	1/2	1/18
03	1	2	12	3	123		1/3	1/3	1	1/9
04	1	3	13	2	132		1/3	1/3	1	1/9
05	2	1	21	3	213		1/3	1/3	1	1/9
06	2	2	22	1	221		1/3	1/3	1/2	1/18
07	2	2	22	3	223		1/3	1/3	1/2	1/18
08	2	3	23	1	231		1/3	1/3	1	1/9
09	3	1	31	2	312		1/3	1/3	1	1/9
10	3	2	32	1	321		1/3	1/3	1	1/9
11	3	3	33	1	331		1/3	1/3	1/2	1/18
12	3	3	33	2	332		1/3	1/3	1/2	1/18

Table: Twelve *combined-triplet-event* possibilities along with its *joint-probabilities*.
[xr]: prize x behind door r; [yp]: guest y choses door p; [zq]: host z reveals door q
Mutually-Exclusive Together-Exhaustive Alternative-Possibilities

COMMENT ON THE APPROACH ADOPTED BY LEADING SUBJECT-MATTER-EXPERTS

One of the various approaches adopted by the Leading Subject-Matter-Experts, is to consider the two joint probabilities P[x1y1z3] = 1/18 and P[x2y2z3] = 1/18 and compare them with the two joint probabilities P[x1y2z3] = 1/9 and P[x2y1z3] = 1/9; and somehow (wrongly) derive the probability of winning with initial choice to be 1/3 and the probability of winning with switched choice to be 2/3; without even realizing that each of these four values is an a-priori probability corresponding to the four events leading to the host revealing a losing choice; and none of them correspond to any updated a-posteriori probability that incorporates the knowledge of the losing choice revealed by the host.
Also, there seems to be some Logical Fallacy in the underlying reasoning used therefor.
The only one *a-posteriori conditionality* is (zq=z3) whereas (yp=y1) can't be pinned down as a conditionality.
Logical Consistency requires that any *conditionality* used in the *evaluation process* cannot be lifted *after the evaluation process while implementing the decision* arrived at *based on that very conditionality*. Or-Else, an *erroneous problem formulation* & *erroneous model* naturally leads to *erroneous results*, that gets confirmed through some *erroneous computer simulation* studies, etc.

PROPOSED APPROACH

The guest has two options, to switch or not to switch. The *mathematical model must capture the central crux of the decision-making process*; wherein the guest goes through a *two-step procedure* to arrive at the decision: (Step-1) withdraw/cancel the initial choice of door-1; (Step-2) evaluate the required *a-posteriori probabilities* based on the knowledge gained from the host regarding a losing choice.
What is required is to compare the values of the two conditional (w.r.t. z3) marginal (w.r.t. yp) probabilities,
P[x1 | z3] = (P[x1 | y1z3] + P[x1 | y2z3]) / P[z3] = (1/18 + 1/9) / (1/3) = 1/2; and
P[x2 | z3] = (P[x2 | y1z3] + P[x2 | y2z3]) / P[z3] = (1/9 + 1/18) / (1/3) = 1/2;
thus, leading to the recommendation to the guest that it really doesn't matter either way.

REFUTATION OF THE LOGICAL FALLACY COMMITED BY THE SUBJECT MATTER EXPERTS ON THE MONTY-HALL PROBLEM

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ABSTRACT

This research report presents a deep re-look at the classical Monty-Hall Problem, refuting the widely accepted position held by the Leading Subject-Matter-Experts, and establishing that there is no rational basis for a switched choice in the decision to be made by the guest of the game show.

Logical consistency requires that any conditionality used in the evaluation process cannot be lifted after the evaluation process, implementing the decision arrived at based on that very conditionality.

Keywords: A-Priori Probability; A-Posteriori Probability; Mutually Independent Events; Mutually Exclusive Together Exhaustive Alternatives; Joint Probability; Restricted Probability, Marginal Probability, Conditional Probability.

AMS MSC Mathematics Subject Classification: 60A99; 60C99; 62A99; 62C99.

1. INTRODUCTION

The classical "Monty-Hall Problem", also referred to as the "Three-Door Problem" is based on a game show "Let's Make a Deal" wherein the host reveals a losing choice to the guest, who had earlier made an initial choice, and in turn offers the guest an enticing option to switch from the initial choice to a second available choice with an aim to enhance the chances of winning the prize. The most prevalent & widely accepted position, as reported in literature, among the leading eminent mathematicians, statisticians, logicians, Subject-Matter-Experts and rational intellectuals, is that an appropriate detailed study & analysis of the scenario using the well accepted standard approach of Probability & Statistics, would lead to a recommendation to the guest to switch to the second available choice based on the knowledge obtained from the host revealing a losing choice.

It is argued that the default of *sticking to the initial choice* will result in a probability of success being only *one-third* whereas a *switch to the alternative second available choice* will result in a probability of success being *two-third* and hence a *switched choice is recommended*. However, it will be shown here that this approach itself cannot be justified, and therefore the resultant recommendation for switched choice is indeed baseless.

2. DESCRIPTION OF THE PROBLEM - BACKGROUND SCENARIO

We shall focus only on the so-called *classical* Monty-Hall Problem, for the purpose of this report.

For the sake of clarity, let us consider the standard classical Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making a choice of the door to pick the prize; the host who knows the location of the prize as well as the choice made by the guest, now *reveals a distinctly different yet a losing choice*. The host also offers the guest, an option to switch from the initial choice to the now available second choice, anticipating an enhanced chance of winning the prize, based on the knowledge obtained about a losing choice.

Let us represent the events/actions associated with the three doors: (1) let $xr \in \{1,2,3\}$ be the door r behind which the prize x is hidden (2) let $yp \in \{1,2,3\}$ be the initial choice of the door p chosen by the guest y and (3) let $zq \in \{1,2,3\}$ be the door q opened by the host z to reveal a losing choice. The symbol 'ai' denotes the event $[E\{(a=i)\}]$ for any 'agent' $a \in \{x,y,z\}$ and 'door' $i \in \{r,p,q\}=\{1,2,3\}$.

It is essential to note here that xr and yp being mutually independent of each other as well as independent of zq, whereas zq itself is dependent on both xr and yp, as per the rules of the game. Also, note that the focus is on the decision-making process & the action to be taken by the guest. So, the *problem formulation* (*modelling*) must necessarily be from the view-point of the guest.

Since the guest y has absolutely zero knowledge about xr the *door r behind which the prize x is hidden*, no *assumptions* need to be made, even about its possible probability distribution. Similarly, the initial choice yp of the *door p chosen by the guest y* is based on zero-knowledge without any strategy as such, and therefore at best a random (blind) guess. However, *to facilitate a concrete analysis of the problem scenario and to provide a framework towards a rigorous mathematical model*, it may be useful to make an *assumption* that these two events/actions are *equally probable among the three available mutually-exclusive together-exhaustive possible alternatives, each having an a-priori probability of 1/3 thus adding up to one*.

Now, because the two events/actions $[xr \in \{1,2,3\}]$ and $[yp \in \{1,2,3\}]$ are *mutually independent* of each other, the *joint probability* of the combinations of these two events can be obtained as the *product of the probabilities of the two independent component events*. Therefore, the joint probability of each of the *combined-duplet-event*s $P[E\{(xr \in \{1,2,3\}) \;\&\; (yp \in \{1,2,3\})\}]$ is 1/9 and the sum total of these nine *joint probabilities* is one.

Note that the event/action of the host z opening door q, $zq \in \{1,2,3\}$ to show a losing choice, is dependent on both yp and xr, as per the rules of the game show; that is, $(zq \neq yp) \;\&\; (zq \neq xr)$. Although the host has full & complete knowledge of the problem scenario, this dependency of zq on yp and xr does indeed limit his options. It turns out that when $yp \neq xr$ the host doesn't have any option except to turn to the one and only one remaining door $zq \neq (yp \neq xr)$; whereas when $yp = xr$ the host has the option of choosing between the two doors, that is, $zq \neq (yp = xr)$. Because the host has this option, at least in a restricted sense, of choosing his door zq, it introduces an uncertainty for the guest to predict/expect/anticipate the host's decision/action in this regard. However, as earlier, for the very same reasons as stated above, it may be useful to make an *assumption* that the host's choice between the two options, whenever available, in a restricted sense, is *equi-probable between the two available mutually-exclusive together-exhaustive possible alternatives, each having a restricted probability of 1/2 thus adding up to one*.

3. PROBLEM FORMULATION

With the above understanding of the background scenario of the classical Monty-Hall Problem, one can derive that there are exactly 12 possibilities for the *combined-triplet-event* [xr&yp&zq] as represented in the Table, listing each of the 12 triplets along with the associated *joint probabilities*. Note that the event space is of size 12 and not 27 which would have been the case if each of the three component-events were indeed mutually independent. The first two are independent giving rise to a *combined-duplet-event* space E{[xr&yp]} of size nine. When it is then combined with the third component-event [._.zq], there results a splitting, in three cases. In the three cases where [xr]=[yp], that is, [x1y1.], [x2y2.], [x3y3.] the third component-event [._.zq] gets two alternative possibilities; [zq]∈{[z2]∨[z3]} and [zq]∈{[z1]∨[z3]} and [zq]∈{[z1]∨[z2]} respectively. Whereas in the other six cases [x1y2.], [x1y3.], [x2y1.], [x2y3.], [x3y1.], [x3y2.] where [xr]≠[yp], the third component-event [._.zq] has a fixed choice since there is *one and only one single* possibility satisfying the game requirement {[zq]≠([yp]≠[xr])}; no splitting into multiple alternative options.

Sl.No.	[xr]	[yp]	[xr&yp]	[zq]	[xr&yp&zq]		P[xr]	P[yp]	P[zq | (xr & yp)]	P[xr&yp&zq]
01	1	1	11	2	112		1/3	1/3	1/2	1/18
02	1	1	11	3	113		1/3	1/3	1/2	1/18
03	1	2	12	3	123		1/3	1/3	1	1/9
04	1	3	13	2	132		1/3	1/3	1	1/9
05	2	1	21	3	213		1/3	1/3	1	1/9
06	2	2	22	1	221		1/3	1/3	1/2	1/18
07	2	2	22	3	223		1/3	1/3	1/2	1/18
08	2	3	23	1	231		1/3	1/3	1	1/9
09	3	1	31	2	312		1/3	1/3	1	1/9
10	3	2	32	1	321		1/3	1/3	1	1/9
11	3	3	33	1	331		1/3	1/3	1/2	1/18
12	3	3	33	2	332		1/3	1/3	1/2	1/18

Table: Twelve *combined-triplet-event* possibilities along with its *joint-probabilities*.
[xr]: prize x behind door r; [yp]: guest y choses door p; [zq]: host z reveals door q
Mutually-Exclusive Together-Exhaustive Alternative-Possibilities

4. GENERAL ANALYSIS OF THE DECISION-MAKING SCENARIO

In this section, a general analysis of the *decision-making scenario* (modelling, from the guest's viewpoint) is presented first, without being constrained by the three assumptions mentioned earlier. The *combined-triple-event* space represented by E{[xr & yp & zq]} is partitioned into 12 mutually exclusive together exhaustive possible available alternatives – although with *no assumptions* about the probability distributions, just to accommodate for possible specific scenarios, especially if & when someone wishes to try out *computer simulation studies*, etc. Specific results pertaining to the data entries given in the Table, may always be easily worked out by plugging the corresponding data values to each of the concerned parameters as needed.

The decision/choice/action of the host, represented by zq∈{1,2,3} being dependent on xr∈{1,2,3} and yp∈{1,2,3}; implying, that the joint probability of the *combined-triplet-event* referred therein be determined by the corresponding *conditional probability*:

That is, in general, for any [zq] and [xr] and [yp] we have,

P[zq & xr & yp] = P[zq | xr & yp] * P[xr & yp]; (Eqn.1)

From the rules of the game, when [zq] ≠ {[yp] ≠ [xr]} we have the *conditional probability*,

P[zq | xr & yp] = 1; (Eqn.2)

and therefore, we get the *joint probability*,

P[zq & xr & yp] = P[xr & yp]; (Eqn.3)

whereas, when [zq] ≠ {[yp] = [xr]} we cannot make any stronger statement, except the general condition for the *restricted (conditional) probability*:

0 ≤ P[zq | xr & yp] ≤ 1; (Eqn.4)

and therefore, we get the *joint probability*, as per Eqn.1 above,

P[zq & xr & yp] = P[zq | xr & yp] * P[xr & yp]; (Eqn.5)

The entries in the Table have been filled based on the computations as in Eqn.1 to 5 above.

With this information, we determine the *a-posteriori* (conditional on zq) *marginal-probability* of the prize being hidden behind door (x=r), as follows;

P[xr | zq] * P[zq] = P[zq | xr & yp] * P[xr & yp] + P[zq | xr & yr] * P[xr & yr] (Eqn.6)

That is, P[xr | zq] * P[zq] = P[xr & yp & zq] + P[xr & yr & zq] (Eqn.7)

Note that Eqn.6 is the correct application of the *Generalized Bayes Theorem* or equivalently Eqn.7 the correct method of combining the *joint probabilities* to determine the *marginal probability*.

Using Eqn.7 in specific instances, for example, with (zq=z3) as:

P[x1 | z3] * P[z3] = P[z3 | x1 & y2] + P[z3 | x1 & y1]; (Eqn.8)

P[x2 | z3] * P[z3] = P[z3 | x2 & y1] + P[z3 | x2 & y2]; (Eqn.9)

Notice in Eqn.8 & Eqn.9 above, that the six terms P[x1y2z3], P[x2y1z3], P[x1y3z2], P[x3y1z2], P[x2y3z1], P[x3y2z1] are *not under the control of the host*, as can be confirmed from Eqn.2 & Eqn.3 above; whereas, the other six terms P[x1y1z3], P[x2y2z3], P[x1y1z2], P[x3y3z2], P[x2y2z1], P[x3y3z1] are indeed *under the direct control of the host* (of course, within certain limits as per Eqn.4 for the *restricted probability*) based on whatever strategy that the host decides and acts accordingly, while following the rules of the game.

The decision of the guest as to whether to avail the offer of the host to opt for a switched choice, say, from door-1 to door-2 after knowing the losing choice behind door-3 as revealed by the host, must be based on a comparison between the two *a-posteriori(conditional) marginal probabilities* P[x1 | z3] given by Eqn.8 and P[x2 | z3] given by Eqn.9 as shown above. However, any specific answer needs to be derived based on the relative magnitudes of the four joint probabilities involved therein, namely, P[x1y1z3], P[x1y2z3], P[x2y1z3] and P[x2y2z3]. That is where the need arises to pin down certain uncertainties (at least the ones that are *not under the control of the host*) by *assuming* certain probability distribution, as for example, P[xr] and also P[yp] to be *uniformly distributed* among the available (in this case, three) alternatives. If the decision & action of the host can also be *assumed* to adhere to certain *probability distribution* or certain *strategy*, it can be used to make specific comparisons that will lead to a firm recommendation to the guest as to whether it is worth at all to consider a switched choice. The Table entries *assume* that the host adheres and follows a *uniform distribution*, in the sense that whenever faced with two/multiple alternatives, the specific choice of any one of them is equally-probable and together-exhaustive.

5. WHAT IS WRONG WITH THE EXISTING APPROACH

What is required is a comparison between the two values of the conditional (w.r.t. zq) marginal (w.r.t. yp) probabilities as can be derived using Eqn.6 & Eqn.7, or Eqn.8 & Eqn.9 above, that is,

P[x1 | z3] =(1/18 + 1/9)/(1/3)=1/2; and P[x2 | z3] =(1/9 + 1/18)/(1/3)=1/2; (Eqn.10)

thus, leading to the recommendation that it really doesn't matter either way.

One of the various approaches adopted by the Leading Subject-Matter-Experts, is to consider the two joint probabilities P113 = P[x1y1z3] = 1/18 and P223 = P[x2y2z3] = 1/18 and compare them with the two joint probabilities P123 = P[x1y2z3] = 1/9 and P213 = P[x2y1z3] = 1/9; and somehow (*error of commission* in the wrong application of the generalized result from the Bayes Theorem) derive the probability of *winning with initial choice* to be 1/3 and the probability of *winning with switched choice* to be 2/3; without even realizing that each of these four values is an a-priori probability corresponding to the four events leading to the host revealing a losing choice; none of them correspond to any updated a-posteriori probability that incorporates the knowledge of the losing choice revealed by the host. Also, there seems to be some Logical Fallacy in the underlying reasoning used therefor. Note that the only one *a-posteriori conditionality* is (zq=z3) whereas (yp=y1) can't be pinned down as a conditionality. *Logical Consistency* requires that any *conditionality* used in the *evaluation process* cannot be lifted *after the evaluation process while implementing the decision* arrived at *based on that very conditionality*. Or-Else, an *erroneous problem formulation* & *erroneous model* naturally leads to *erroneous results*, that gets confirmed through some *erroneous computer simulation* studies, etc.

Another approach taken by the Leading Subject-Matter-Experts seems to be based on an erroneous (*error of omission*) comparison between the values of the two *joint probabilities* –

P[x1y1z3] = 1/18 and P[x2y1z3] = 1/9; (Eqn.11)

which they seem to somehow wrongly *combine together* to get the two *conditional probabilities*

P[x1 | z3] = 1/3 and P[x2 | z3] = 2/3; (Eqn.12)

and then lift the conditionality (yp=y1);

thus, leading to the recommendation to the guest for switching over to door-2 (that is, yp=y2). Again, note that the only one *a-posteriori conditionality* is (zq=z3) *whereas* (yp=y1) *cannot be pinned down as an a-posteriori conditionality, since this very initial choice of the guest is indeed under re-evaluation and hence subject to change.*

By the very nature of the rules of the game, and the symmetry in the data as listed in the Table,

P[x1y1z3] ≤ P[x1y2z3] and also P[x2y2z3] ≤ P[x2y1z3] (Eqn.13)

which indicates that the same *counter-intuitively paradoxical enticement* exists for a switched choice, *whatever might have been the initial choice* – justifying that MHP is indeed a paradox.

Also note that, P[x2y1z3] = P[x1y2z3] and also P[x1y1z3] = P[x2y2z3]; (Eqn.14)

It is essential to recognize that the guest has two options, to switch to door-2 [y2] or to keep the initial choice of door-1 [y1]. Note that a NULL option is also to be counted as an option. The *mathematical model must capture the central crux of the decision-making process*; wherein the guest goes through an *effectively two-step* procedure to arrive at the decision;

(Step-1) withdraw/cancel the initial choice of door-1; followed by

(Step-2) re-evaluate the required *a-posteriori probabilities* based on the knowledge gained from the host regarding a losing choice; and implement the decision by taking action accordingly.

The clearly partitioned *triple-event* space, with 12 *mutually-exclusive* and *together-exhaustive* possible alternatives, represented in the Table, is a fail-safe framework to study, analyze & solve the problem – no possibility of missing any relevant (and/or including any irrelevant) component terms while going through the required calculations in order to derive whatever desired results.

6. A CHALLENGE TO THE LEADING SUBJECT MATTER EXPERTS

Let us rephrase the Monty-Hall Problem, now adorned with a *jewel-on-the-crown* as below:

MONTY-HALL-PROBLEM (MHP) TO-SWITCH-OR-NOT-TO-SWITCH : THAT IS THE QUESTION
(1.1) The prize is hidden behind one of the three doors.
(1.2) I the guest make an initial choice of which door it could be, say door-1, to claim my prize.
(1.3) Then Monty the host opens a different door, say door-3, revealing a losing choice.
(2.1) I am given an option to withdraw/cancel the earlier choice of door-1 and switch to door-2.
(2.2) I appreciate the knowledge of a losing choice and also Monty's offer of the option to switch.
(3.1) I grab Monty's offer, withdraw/cancel my earlier choice of door-1.
(3.2) Then I re-evaluate the two choices available for me now, namely door-1 or door-2.
(3.3) I find that the chances of winning are exactly the same between the two available choices;
(4.1) Now, I turn towards YOU seeking YOUR recommendation. What is YOUr recommendation?
(4.2) TO SWITCH OR NOT TO SWITCH : THAT IS THE QUESTION!

Note that your answer must necessarily be independent of my initial-choice (door-1); although Monty's choice of door-3 revealing a losing choice was dependent on my initial choice (door-1) which he had to avoid as per the rules of the game. Hope your expert advice is not an exemplification of the proverb "*the grass is always greener on the other side*"!

7. COOL-HEADED BRAVE-HEARTS PLAY WITH STRATEGIST HOST

This is somewhat *far from the so-called classical version* of the Monty-Hall Problem, wherein we allow the host to exercise whatever 'strategic game-playing' that one wishes to play with the guest. The situation can be captured by Eqn.8 & Eqn.9 above; wherein the terms P[z3 | x1 & y1] in Eqn.8 and P[z3 | x2 & y2] in Eqn.9 are fully under the control of the host. An extreme situation is when the host adopts whatever strategy that pulls down one of them to zero and pushes the other one to its maximum value of the restricted probability, namely 1/9. Then it turns out that the values of the two a-posteriori(conditional) marginal probabilities P[x1 | z3] obtained from Eqn.8 and P[x2 | z3] obtained from Eqn.9 cannot be the same anymore; in the extreme case, one will be 1/3 and the other will be 2/3; which then may lead to the two possibilities – *a first specific strategy* wherein opting for a *switched choice has an clear disadvantage* and *a second specific strategy* wherein opting for a *switched choice has an clear advantage*. This topic is *beyond the scope of the present paper* and therefore I leave it as an exercise to those cool-headed brave-hearts who may be interested to figure out the possible specific strategy adopted by the host that would lead to such extreme situations.

8. CONCLUSION

This research report presents a novel intriguing analysis of the Monty-Hall Problem, *refuting* the most widely accepted position held by the Leading-Subject-Matter-Experts - and *advocating against acting on any enticing offers* made by the host to the guest for an optional switch from the already selected initial choice to a distinct alternative available second choice - *why, because there is no advantage* gained by opting for such switched choice, in terms of any enhanced chances to win the prize, unlike whatever has been widely accepted till today.

Logical consistency requires that any conditionality used in the evaluation process cannot be lifted after the evaluation process, implementing the decision arrived at based on that very conditionality, which seems to has been violated by the Leading Subject Matter Experts.

The approach taken by the Leading Subject-Matter-Experts seems to be based on an *erroneous mathematical formulation* of the problem, leading to an *erroneous model* which therefore yields *erroneous results*, possibly further confirmed (!?!) by *erroneous computer simulation studies* etc.

The error can also be considered as either an *error of commission*, that is, a wrong application of the *Generalized Bayes Theorem*, or an *error of omission*, that is, considering only the *aposteriori*(conditional|z3)*joint*(x1y1z3 as against x1y2z3)*probabilities* rather than the required *aposteriori*(conditional|z3)*marginal*(x1y1 xor x1y2 as against x2y1 xor x2y2)*probabilities*.

Note that *any additional knowledge gained*, revealing a losing (undesirable) possibility, although may lead to an *updated/smaller sample-space*, may not and/or *need not necessarily be specific enough for a refinement/update in the relative distinction between/among the a-posteriori probabilities of the updated/now-available alternatives* in the resultant updated sample-space.

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10. ACKNOWLEDGEMENT

I must necessarily confess here that the *core idea behind this analysis is so stunningly & elusively simple*, that one may simply be taken aback in a profound wonder-struck jaw-drop-silence, maybe with an after-thought: "*oh my goodness, how could it be that it never flashed on me any time earlier*"! as was also the case in an earlier research work reported in [12] by this author.

11. DEDICATION

To my ಅಜ್ಜ(ajja) Karinja Halemane Keshava Bhat & ಅಜ್ಜಿ(ajji) Thirumaleshwari, ಅಪ್ಪ(appa) Shama Bhat & ಅಮ್ಮ(amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter Sriwidya.Bharati and my twin sons Sriwidya.Ramana & Sriwidya.Prawina for their *love & affection*.

Whereas *this Original Author-Creator* holds the (PIPR:©:) Perpetual Intellectual Property Rights, it is but natural that his legal heirs (three children mentioned above) may avail the same for perpetuity.

To all the *cool-headed brave-hearts*, eagerly awaited but probably yet to be visible among the world professionals, especially the *Subject-Matter-Experts*, who would be attracted to and certainly capable of effectively understanding without any prejudice and appreciating the deeper insights enshrined in this short research report, who may opt for innate rational-&-intellectual common-sense and simple creativity over any sophisticated and/or complex theory in problem-solving to resolve any seemingly paradoxical scenario.

ॐतत्सत्