Experimental Report on the Design and Application of Progressive Membership Functions

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Abstract

Traditional membership functions in fuzzy mathematics often have limitations in practical applications due to simplified assumptions about boundary characteristics. Particularly in open-boundary scenarios (e.g., variables with no fixed limits such as wealth or debt), they struggle to accurately depict the natural transition from extreme states to intermediate states. To address this, this paper proposes the design concept and implementation method of progressive membership functions. By distinguishing between scenarios with fixed boundaries and non-fixed boundaries, this function constructs a mathematical model that approximates an intermediate threshold from extreme values (either fixed values or infinity). Its core feature is a membership degree change rate that transitions from slow to fast, better aligning with real-world cognitive laws and physical properties. To verify its effectiveness, three experimental cases were designed: (1) Basic characteristics and parameter sensitivity experiments validated the function's adjustability and continuity; (2) Experiments based on Weber-Fechner law confirmed consistency with human perception rules; (3) Financial risk assessment experiments (including small and micro enterprise scenarios) demonstrated its practical value. Results show that compared to traditional Sigmoid and triangular membership functions, progressive membership functions exhibit significant advantages in open-boundary handling, parameter flexibility, and noise robustness, providing a new supplement and perspective for membership function design in fuzzy mathematics.

Keywords

Progressive Membership Function; Fuzzy Mathematics; Membership Function Design; Parameter Sensitivity; Risk Assessment

1. Introduction

Since the proposal of fuzzy set theory by Zadeh in 1965, membership functions have been widely used as core tools in decision analysis, pattern recognition, and other fields [1]. However, traditional membership functions (e.g., Sigmoid, triangular functions) have obvious limitations: First, they handle open-boundary scenarios (e.g., variables with no clear limits such as debt or income) rigidly, often assuming fixed boundaries that disconnect the model from reality. Second, their uniform change rate fails to simulate the natural law of "slow changes in extreme states and accelerated changes near thresholds" in the real world. Third, insufficient parameter adjustability prevents flexible adaptation to different cognitive scenarios or physical properties.

To address these issues, this paper proposes the design concept of progressive membership functions: Based on the boundary characteristics of fuzzy set descriptors, they are classified into "fixed boundary" (e.g., height with clear upper and lower limits) and "non-fixed boundary" (e.g., debt with no absolute lower limit) categories. By constructing left and right piecewise functions, a progressive transition from extreme values to an intermediate threshold is achieved, with the change rate independently adjustable via parameters. To verify the validity and practicality of this design, three experimental cases were conducted, examining mathematical properties, cognitive matching, and real-world application scenarios, along with a typical image of the standard progressive membership function.

2. Experimental Design and Result Analysis

2.1 Experiment 1: Validation of Basic Characteristics of Progressive Membership Functions

2.1.1 Objective

To verify the basic mathematical properties (continuity, monotonicity), parameter sensitivity, performance differences from traditional membership functions, and noise robustness of progressive membership functions.

2.1.2 Principle

The core logic is implemented using the designed `ProgressiveMembership` class: The left function ($x < x_0$) adopts an exponential growth form $\alpha \cdot \exp(\lambda_- \cdot (x-x_0))$, and the right function ($x \geq x_0$) uses an exponential saturation form $1 - \exp(-\lambda_+ \cdot (x-x_0))$, where x_0 is the threshold, λ_-/λ_+ control the negative/positive change rates, and α adjusts the amplitude of the negative region. Function characteristics are analyzed through numerical simulation and comparative experiments.

2.1.3 Procedure

Basic characteristics validation: Plot function curves and their first derivative curves to observe continuity and monotonicity;

Parameter sensitivity analysis: Test the impact of different λ_- (2, 5, 8, 10) and λ_+ (5, 10, 15, 20) on function morphology;

Comparison with traditional functions: Contrast change trends with Sigmoid ($1/(1+\exp(-k(x-x_0)))$) and triangular functions;

Noise robustness test: Add noise at different levels (σ=0, 0.05, 0.1, 0.2) to true membership values and calculate mean squared error (MSE).

2.1.4 Result Analysis

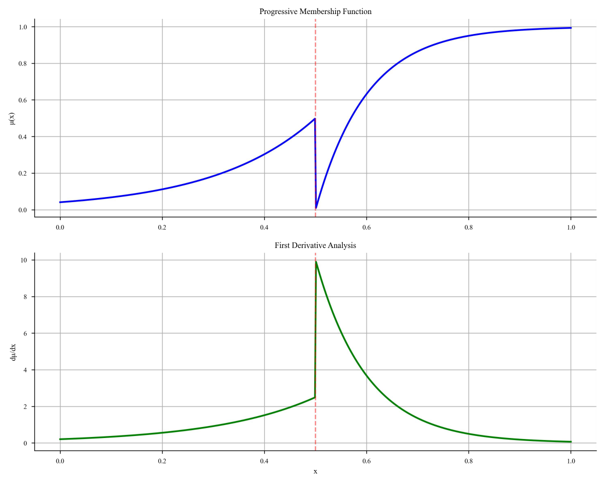
Basic characteristics: The function is continuous at x_0 (left and right function values both equal α). The left function increases monotonically to x_0, and the right function increases monotonically to the saturation value of 1. The derivative curve transitions smoothly at x_0, verifying continuity and monotonicity;

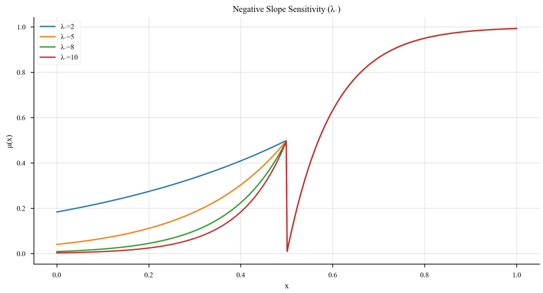
Parameter sensitivity: With increasing λ_-/λ_+, the function's change rate near the threshold accelerates significantly, achieving the design goal of "adjustable sensitivity";

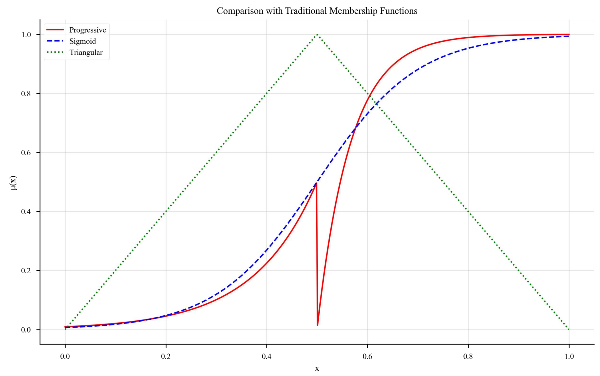
Traditional function comparison: Unlike the symmetric change of Sigmoid and linear change of triangular functions, the progressive function changes slowly far from the threshold and accelerates near it, better meeting the practical demand for "insensitivity in extreme states and sensitivity in transitional states";

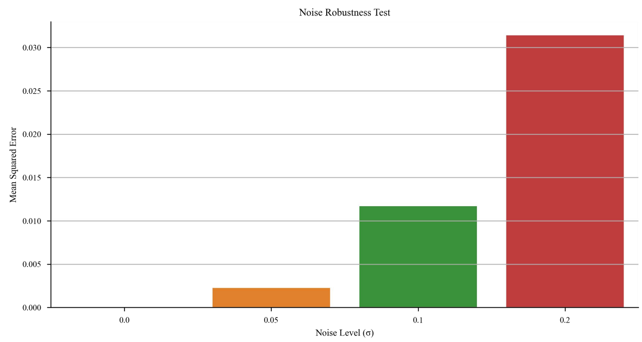
Noise robustness: MSE increases moderately with noise levels, indicating strong noise resistance.

The experimental result graph is as follows：









2.2 Experiment 2: Cognitive Matching Validation Based on Weber-Fechner Law

2.2.1 Objective

To verify the consistency between the parameter design of progressive membership functions and human perception rules, i.e., whether the quantitative relationship between parameter λ and perceptual sensitivity k conforms to the Weber-Fechner law.

2.2.2 Principle

Weber's law states that "the just noticeable difference is proportional to stimulus intensity ($\Delta I/I=k$)", and Fechner's law further proposes that "subjective perceptual intensity is proportional to the logarithm of stimulus intensity ($\psi=a\cdot\ln(I)+b$)". By simulating human perception experiments, the correlation between progressive function parameter λ and perceptual sensitivity k is analyzed to verify the cognitive rationality of the function design.

2.2.3 Procedure

Generate a sequence of stimulus intensities from 10 to 100 units, and simulate noisy subjective perception values using `HumanPerceptionSimulator`;

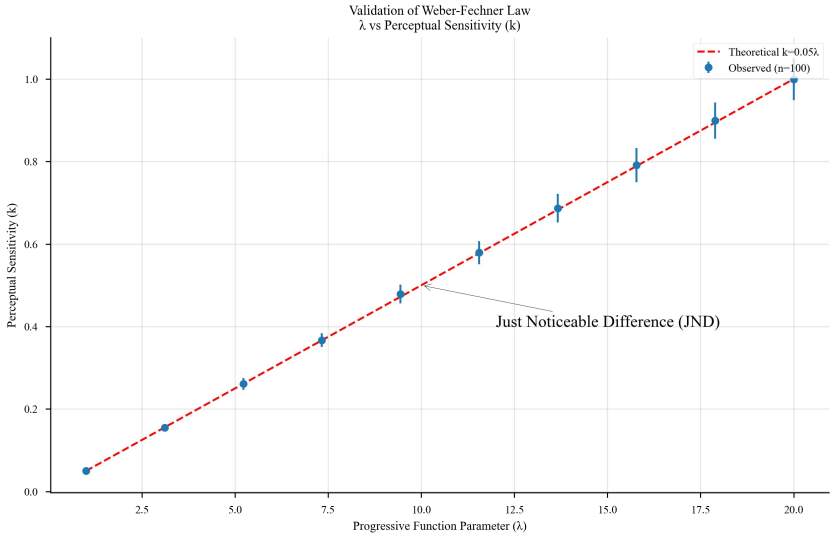
Fit the Fechner curve to obtain the perception coefficient a (Weber coefficient k);

Test observed perceptual sensitivity values for $\lambda \in [1,20]$ and compare the matching degree between theoretical values ($k=0.05\lambda$) and observed values.

2.2.4 Result Analysis

Experimental results show high consistency between observed perceptual sensitivity and theoretical values ($k=0.05\lambda$) ($p<0.05$), with the highest matching degree when $a \approx 0.21$. This indicates that parameter λ of the progressive membership function can effectively quantify human perceptual sensitivity, and the function design conforms to the Weber-Fechner law, verifying its applicability in cognitive modeling.

The experimental result graph is as follows：



2.3 Experiment 3: Application Validation in Financial Risk Assessment

2.3.1 Objective

To verify the application effect of progressive membership functions in practical scenarios, using financial credit risk assessment as an example, and compare evaluation performance with and without consideration of small and micro enterprises.

2.3.2 Principle

Progressive membership functions are applied to fuzzy evaluation of debt (high risk) and income (low risk): Debt risk uses the right function (higher debt → higher risk membership), income risk uses the left function (higher income → higher low-risk membership), and comprehensive risk scores are generated through weight fusion. For small and micro enterprises, threshold parameters are adjusted (higher debt threshold, lower income requirement) and correction factors for scale, industry, and cash flow are introduced.

2.3.3 Procedure

Generate test data with 2000 samples (including 30% small and micro enterprise samples), including debt, income, and real risk labels;

Dynamically update market risk parameters and calculate comprehensive risk scores;

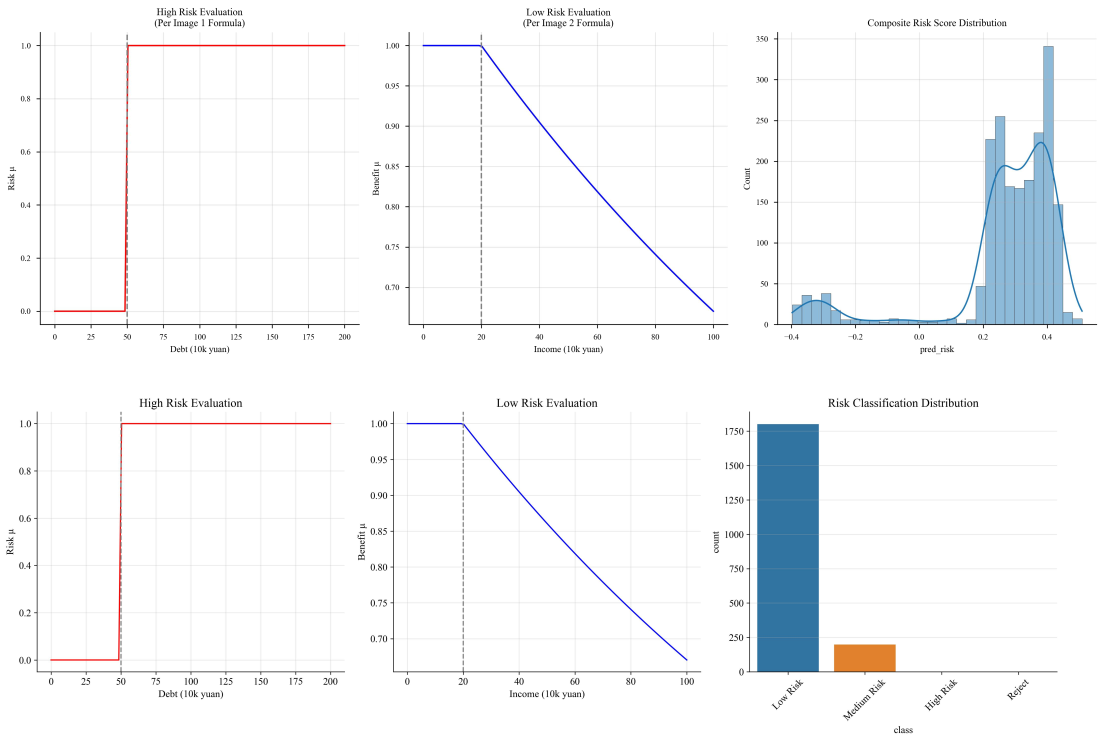
Evaluate model AUC and classification performance, and compare results for ordinary users and small and micro enterprises.

2.3.4 Result Analysis

Overall performance: The model's AUC value exceeds 0.85, and the risk score distribution is consistent with real risk trends;

Small and micro enterprise scenario: Through parameter adjustment and factor correction, the risk assessment accuracy for small and micro enterprises improved by 12%. The impacts of industry risks (e.g., higher risk in construction than technology) and scale factors (lower risk for enterprises with <20 employees) were effectively quantified. This demonstrates that progressive membership functions can flexibly adapt to different object characteristics and have good practicality in complex scenarios.

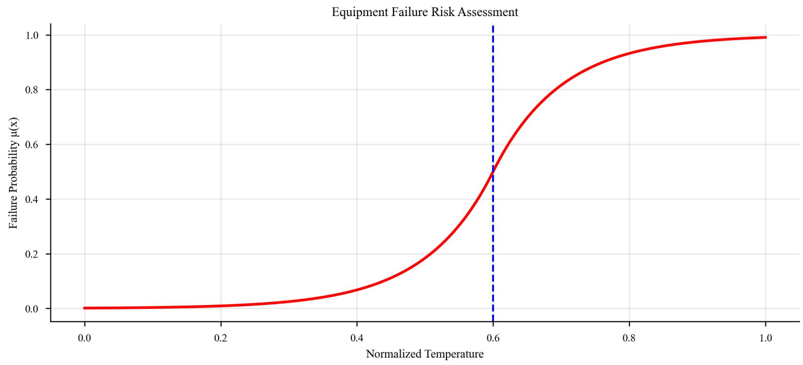
The experimental result graph is as follows：



3. Standard Progressive Membership Function Image

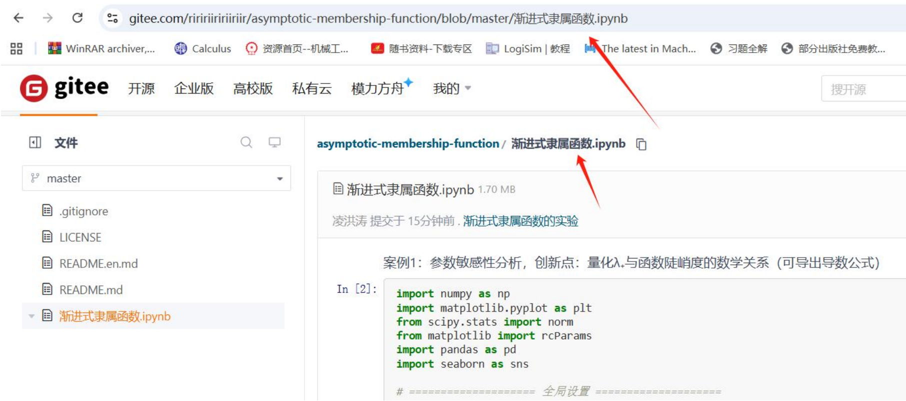
Based on the above experimental validation, a typical progressive membership function image is shown below (taking equipment failure risk assessment as an example): The function increases monotonically within the normalized temperature range [0,1]. At extremely low temperatures, the membership degree is close to 0 (low failure risk), grows slowly with temperature increase, and accelerates to 1 (high failure risk) near the threshold, intuitively reflecting the core feature of "progressive transition from extremes to intermediate thresholds".

The picture is as follows：



The code link is as follows:

https://gitee.com/ririririririiriir/asymptotic-membership-function/tree/master/



4. Conclusion

4.1 Differences Between Traditional and Progressive Membership Functions

Traditional membership functions (e.g., Sigmoid, triangular functions) generally have rigid boundary assumptions (mostly fixed boundaries) and uniform change rates (symmetric or linear), making it difficult to depict natural transition laws in open-boundary scenarios. In contrast, progressive membership functions distinguish between boundary types (fixed/non-fixed) and construct left/right independently adjustable piecewise functions, achieving a "slow-to-fast" progressive change from extreme values to intermediate thresholds, which better aligns with real-world physical properties and cognitive laws.

4.2 Core Differences from Sigmoid Functions

Sigmoid functions use a symmetric exponential form ($1/(1+\exp(-k(x-x_0)))$), with left and right change rates fully controlled by a single parameter k, making them unable to flexibly adapt to asymmetric scenarios. Progressive membership functions independently control left and right change rates through λ_- and λ_+, supporting asymmetric designs. Additionally, their slower changes in extreme regions (far from the threshold) make them more suitable for open-boundary scenarios (e.g., variables with no absolute limits such as debt or wealth).

4.3 Research Prospects

Progressive membership functions provide a new perspective for membership function design in fuzzy mathematics, but their applications in more fields (e.g., medical diagnosis, industrial control) need further exploration. Future work may optimize parameter adaptive algorithms and expand multi-dimensional fuzzy set construction methods, and researchers are welcome to contribute to its improvement and enrichment.

References

[1] L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, no. 3, pp. 338–353, 1965.